Distances & Angles

1. $|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
2. $M = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2})$
3. $d = \frac{|\overrightarrow{PS} \times \vec{\mathbf{v}}|}{|\vec{\mathbf{v}}|}$, $P \in \vec{\mathbf{L}}, \vec{\mathbf{L}} \parallel \vec{\mathbf{v}}$
4. $d = |\overrightarrow{PS} \cdot \frac{\vec{\mathbf{n}}}{|\vec{\mathbf{n}}|}|$, $\vec{\mathbf{n}}$ is normal to plane at point P
5. $\theta = cos^{-1}(\frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{u}}||\vec{\mathbf{v}}|})$

Vector Operations

5. $|\vec{\mathbf{v}}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
   $|\vec{\mathbf{v}}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
6. $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$
7. $k\vec{\mathbf{u}} = \langle ku_1, ku_2, ku_3 \rangle$
8. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = u_1v_1 + u_2v_2 + u_3v_3$
   $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}|cos\theta$
9. $proj_v\vec{\mathbf{u}} = (\frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|^2})\vec{\mathbf{v}} = (\frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|})\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$
10. $|\vec{\mathbf{u}}|cos\theta = \frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|} = \vec{\mathbf{u}} \cdot \frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$
11. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}}, \vec{\mathbf{D}} = \overrightarrow{PQ}$
12. $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = (|\vec{\mathbf{u}}||\vec{\mathbf{v}}|sin\theta)\vec{\mathbf{n}}$

Properties

Vector Algebra

1. $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}$
2. $(\vec{\mathbf{u}} + \vec{\mathbf{v}}) + \vec{\mathbf{w}} = \vec{\mathbf{u}} + (\vec{\mathbf{v}} + \vec{\mathbf{w}})$
3. $\vec{\mathbf{u}} + 0 = \vec{\mathbf{u}}$
4. $\vec{\mathbf{u}} + (-\vec{\mathbf{u}}) = 0$
5. $0\vec{\mathbf{u}} = 0$
6. $1\vec{\mathbf{u}} = \vec{\mathbf{u}}$
7. $a(b\vec{\mathbf{u}}) = (ab)\vec{\mathbf{u}}$
8. $a(\vec{\mathbf{u}} + \vec{\mathbf{v}}) = a\vec{\mathbf{u}} + a\vec{\mathbf{v}}$
9. $(a + b)\vec{\mathbf{u}} = a\vec{\mathbf{u}} + b\vec{\mathbf{u}}$

Dot Product

5. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{v}} \cdot \vec{\mathbf{u}}$
6. $(c\vec{\mathbf{u}}) \cdot \vec{\mathbf{v}} = \vec{\mathbf{u}} \cdot (c\vec{\mathbf{v}}) = c(\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})$
7. $\vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} + \vec{\mathbf{w}}) = \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{u}} \cdot \vec{\mathbf{w}}$
8. $\vec{\mathbf{u}} \cdot \vec{\mathbf{u}} = |\vec{\mathbf{u}}|^2$
9. $0 \cdot \vec{\mathbf{u}} = 0$

Cross Product

5. $(r\vec{\mathbf{u}}) \times (s\vec{\mathbf{v}}) = (rs)(\vec{\mathbf{u}} \times \vec{\mathbf{v}})$

Notes

- $\vec{\mathbf{u}} \perp \vec{\mathbf{v}} \iff \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 0$
- $\vec{\mathbf{u}} \parallel \vec{\mathbf{v}} \iff \vec{\mathbf{u}} \times \vec{\mathbf{v}} = 0$
- $proj_v\vec{\mathbf{u}}$ is the projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$