Distances & Angles

1. $|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
2. $M = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2})$
3. $d = \frac{|\overrightarrow{PS} \times \vec{\mathbf{v}}|}{|\vec{\mathbf{v}}|}$, $P \in \vec{\mathbf{L}}, \vec{\mathbf{L}} \parallel \vec{\mathbf{v}}$
4. $d = |\overrightarrow{PS} \cdot \frac{\vec{\mathbf{n}}}{|\vec{\mathbf{n}}|}|$, $\vec{\mathbf{n}}$ is normal to plane at point P
5. $\theta = cos^{-1}(\frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{u}}||\vec{\mathbf{v}}|})$

Vector Operations

5. $|\vec{\mathbf{v}}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
   $|\vec{\mathbf{v}}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
6. $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$
7. $k\vec{\mathbf{u}} = \langle ku_1, ku_2, ku_3 \rangle$
8. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = u_1v_1 + u_2v_2 + u_3v_3$
   $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}|cos\theta$
9. $proj_v\vec{\mathbf{u}} = (\frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|^2})\vec{\mathbf{v}} = (\frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|})\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$
10. $|\vec{\mathbf{u}}|cos\theta = \frac{\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|} = \vec{\mathbf{u}} \cdot \frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$
11. $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = (|\vec{\mathbf{u}}||\vec{\mathbf{v}}|sin\theta)\hat{\mathbf{n}}$
    $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = (u_2v_3 - u_3v_2)\hat{\mathbf{i}} - (u_1v_3 - u_3v_1)\hat{\mathbf{j}} + (u_1v_2 - u_2v_1)\hat{\mathbf{k}}$
    $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
12. $|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = |\vec{\mathbf{u}}||\vec{\mathbf{v}}|sin\theta = A$
    A is area of the parallelogram defined by $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$
    $\frac{A}{2}$ is area of triangle defined by $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$

Properties

Vector Algebra

1. $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}$
2. $(\vec{\mathbf{u}} + \vec{\mathbf{v}}) + \vec{\mathbf{w}} = \vec{\mathbf{u}} + (\vec{\mathbf{v}} + \vec{\mathbf{w}})$
3. $\vec{\mathbf{u}} + 0 = \vec{\mathbf{u}}$
4. $\vec{\mathbf{u}} + (-\vec{\mathbf{u}}) = 0$
5. $0\vec{\mathbf{u}} = 0$
6. $1\vec{\mathbf{u}} = \vec{\mathbf{u}}$
7. $a(b\vec{\mathbf{u}}) = (ab)\vec{\mathbf{u}}$
8. $a(\vec{\mathbf{u}} + \vec{\mathbf{v}}) = a\vec{\mathbf{u}} + a\vec{\mathbf{v}}$
9. $(a + b)\vec{\mathbf{u}} = a\vec{\mathbf{u}} + b\vec{\mathbf{u}}$

Unit Vectors

1. $|\hat{\mathbf{v}}| = 1$
2. $\hat{\mathbf{i}} = \langle 1, 0, 0 \rangle, \hat{\mathbf{j}} = \langle 0, 1, 0 \rangle, \hat{\mathbf{k}} = \langle 0, 0, 1 \rangle$
3. $\vec{\mathbf{u}} = u_1\hat{\mathbf{i}} + u_2\hat{\mathbf{j}} + u_3\hat{\mathbf{k}}$
4. $\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}, \vec{\mathbf{v}} \neq 0$ is *direction* of $\vec{\mathbf{v}}$
5. $|\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}| = 1, \vec{\mathbf{v}} \neq 0$
6. $\vec{\mathbf{v}} = |\vec{\mathbf{v}}|\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}, \vec{\mathbf{v}} \neq 0$
7. $\vec{\mathbf{i}} \cdot \vec{\mathbf{j}} = \vec{\mathbf{j}} \cdot \vec{\mathbf{k}} = \vec{\mathbf{i}} \cdot \vec{\mathbf{k}} = 0$
8. $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$
9. $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) = \hat{\mathbf{k}}$
10. $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -(\hat{\mathbf{k}} \times \hat{\mathbf{j}}) = \hat{\mathbf{i}}$
11. $\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) = \hat{\mathbf{j}}$

Dot Product

5. $\vec{\mathbf{u}} \perp \vec{\mathbf{v}} \iff \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 0$
6. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{v}} \cdot \vec{\mathbf{u}}$
7. $(c\vec{\mathbf{u}}) \cdot \vec{\mathbf{v}} = \vec{\mathbf{u}} \cdot (c\vec{\mathbf{v}}) = c(\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})$
8. $\vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} + \vec{\mathbf{w}}) = \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{u}} \cdot \vec{\mathbf{w}}$
9. $\vec{\mathbf{u}} \cdot \vec{\mathbf{u}} = |\vec{\mathbf{u}}|^2$
10. $0 \cdot \vec{\mathbf{u}} = 0$

Cross Product

5. $\vec{\mathbf{u}} \parallel \vec{\mathbf{v}} \iff \vec{\mathbf{u}} \times \vec{\mathbf{v}} = 0$
6. $\overrightarrow{PQ} \times \overrightarrow{PR} \perp \mathcal{B}_{PQR}$
7. $(r\vec{\mathbf{u}}) \times (s\vec{\mathbf{v}}) = (rs)(\vec{\mathbf{u}} \times \vec{\mathbf{v}})$
8. $\vec{\mathbf{u}} \times (\vec{\mathbf{v}} + \vec{\mathbf{w}}) = \vec{\mathbf{u}} \times \vec{\mathbf{v}} + \vec{\mathbf{u}} \times \vec{\mathbf{w}}$
9. $\vec{\mathbf{v}} \times \vec{\mathbf{u}} = -(\vec{\mathbf{u}} \times \vec{\mathbf{v}})$
10. $(\vec{\mathbf{v}} + \vec{\mathbf{w}}) \times \vec{\mathbf{u}} = \vec{\mathbf{v}} \times \vec{\mathbf{u}} + \vec{\mathbf{w}} \times \vec{\mathbf{u}}$
11. $0 \times \vec{\mathbf{u}} = 0$
12. $\vec{\mathbf{u}} \times (\vec{\mathbf{v}} \times \vec{\mathbf{w}}) = (\vec{\mathbf{u}} \cdot \vec{\mathbf{w}})\vec{\mathbf{v}} - (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})\vec{\mathbf{w}}$

Determinant

1. $det\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
2. $\mathbb{A} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
   $det\mathbb{A} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$
   $det\mathbb{A} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_1c_2)$
   $det\mathbb{A} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$

Physics Applications

1. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}}, \vec{\mathbf{D}} = \overrightarrow{PQ}$
2. $|\vec{\mathbf{T}}| = |\vec{\mathbf{r}}||\vec{\mathbf{F}}|sin\theta$
   $\vec{\mathbf{T}}$ is torque vector
   $\vec{\mathbf{r}}$ is length of lever arm
   $\vec{\mathbf{F}}$ is force applied to lever

Notes

- Right hand rule: with thumb along positive z axis, fingers curl from x axis to y axis
- Any vector of length 1 is a unit vector, but $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are standard
- $proj_v\vec{\mathbf{u}}$ is the projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$
- $\vec{\mathbf{v}}$: vector
- $\overrightarrow{PQ}$: line from P to Q
- $\hat{\mathbf{n}}$: unit vector with direction n
- $\mathcal{B}, \mathcal{B}_{PQR}$: Arbitrary plane, plane containing points P, Q, and R
- $\mathbb{A}$: matrix