

Distances & Angles

1. $|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
2. $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
3. $d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|}$, $P \in \vec{L}, \vec{L} \parallel \vec{v}$
4. $d = |\vec{PS} \cdot \frac{\vec{n}}{|\vec{n}|}|$, \vec{n} is normal to plane at point P
5. $\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$

Vector Operations

5. $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
 $|\vec{v}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
6. $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$
7. $k\vec{u} = \langle ku_1, ku_2, ku_3 \rangle$
8. $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$
 $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$
9. $proj_v \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}\right)\frac{\vec{v}}{|\vec{v}|}$
10. $|\vec{u}|\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \vec{u} \cdot \frac{\vec{v}}{|\vec{v}|}$
11. $W = \vec{F} \cdot \vec{D}, \vec{D} = \vec{PQ}$
12. $\vec{u} \times \vec{v} = (|\vec{u}||\vec{v}|\sin\theta)\vec{n}$

Properties

Vector Algebra

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\vec{u} + 0 = \vec{u}$
4. $\vec{u} + (-\vec{u}) = 0$
5. $0\vec{u} = 0$
6. $1\vec{u} = \vec{u}$
7. $a(b\vec{u}) = (ab)\vec{u}$
8. $a(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$
9. $(a + b)\vec{u} = a\vec{u} + b\vec{u}$

Dot Product

5. $\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$
6. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
7. $(c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v}) = c(\vec{u} \cdot \vec{v})$
8. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
9. $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
10. $0 \cdot \vec{u} = 0$

Cross Product

5. $\vec{u} \parallel \vec{v} \iff \vec{u} \times \vec{v} = 0$
6. $(r\vec{u}) \times (s\vec{v}) = (rs)(\vec{u} \times \vec{v})$
7. $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
8. $\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$
9. $(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$
10. $0 \times \vec{u} = 0$
11. $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$

Notes

- $proj_v \vec{u}$ is the projection of \vec{u} onto \vec{v}