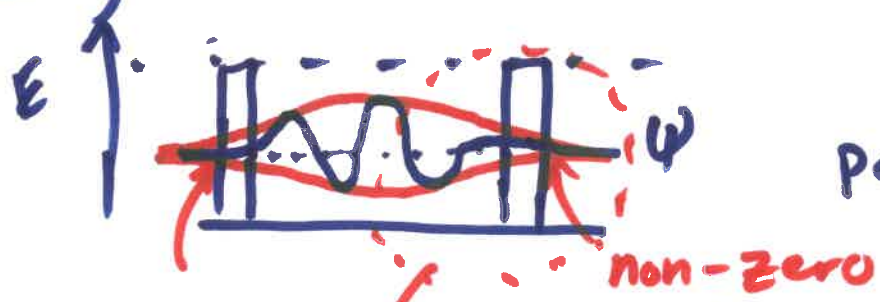
1

Ψ Wavefunction
→ Uncertainty Principle
→ Wave/particle duality
→ Quantisation of Energy
→ Matter ~~of~~ waves
→ states/ state evolution

↓

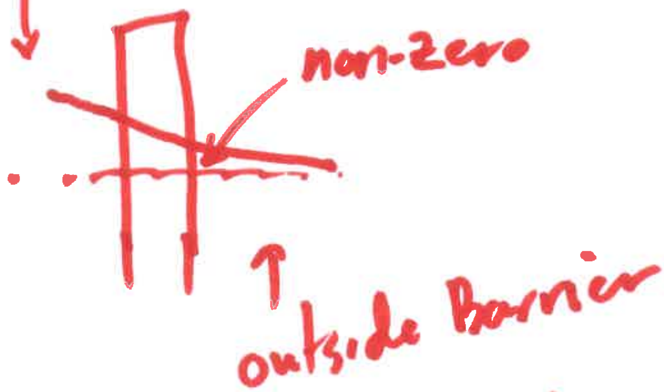
Dirac Notation

Tunnelling



potential

$|\Psi|^2$ $|\Psi|^2$



Scanning Tunnelling Microscope (STM)

② Entanglement

electron - negative charge
- $\frac{1}{2}$ spin

e^- $-\frac{1}{2}$

e^- $\frac{1}{2}$

spin 0

Two particles conserved spin

2 or more particles with a superposition of states

But together

tied together

Outcome or Measurement of one affects the other "instantly" ← EPR

EPR → Einstien, Rosen

Teleportation

∴ entangled particles

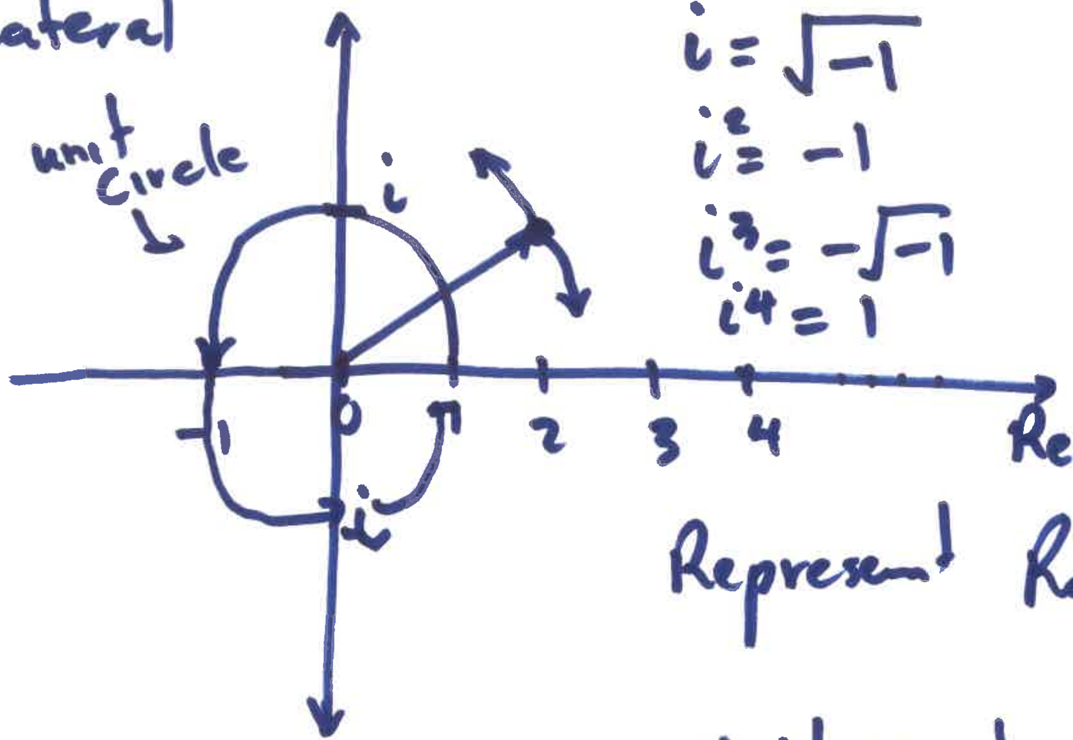
separate them

measure one

instantly collapse.

③ Complex Numbers

Lateral



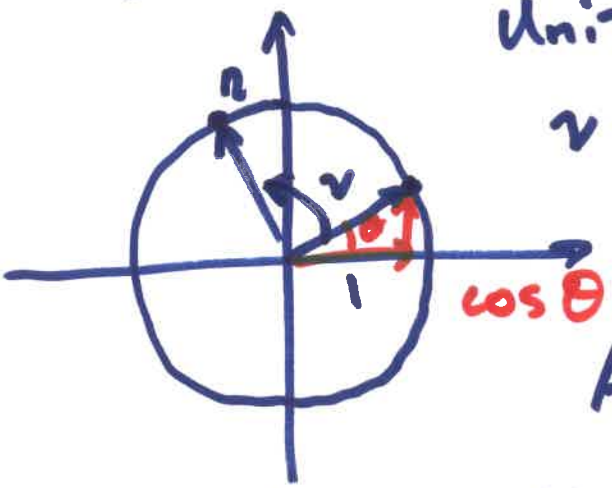
$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -\sqrt{-1}$

$i^4 = 1$

Represent Rotation



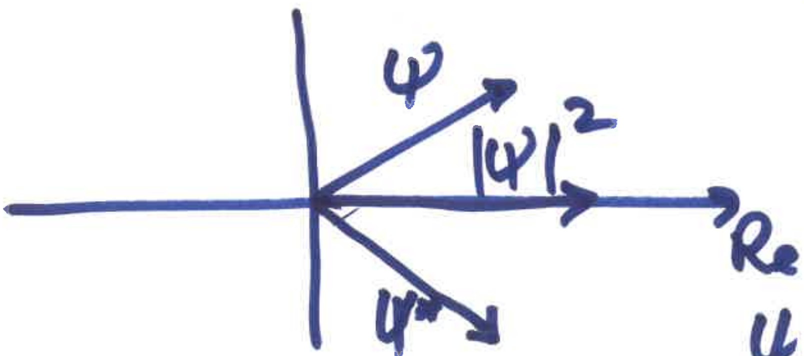
Unit circle

$v = \cos\theta + i\sin\theta$

$Av = \lambda n$

Complex Represent Oscillation
(Rotation)
Scaling

$|\psi|^2$ $\quad \mathbb{R}$



$\psi = a + bi$

$\psi^* = a - bi$

④ Dirac Notation

$\psi \rightarrow$ quantum State

$|\psi|^2 \rightarrow$ Probability Density $= 1$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

Sum

Discrete Sum Σ

$$\int_{-\infty}^{\infty} \psi^*(x)\psi(x)\, dx = 1 \quad ①$$

Operator Q Rotate ψ to a new State

Contraction

$$\int_{-\infty}^{\infty} \psi^*(x)\, Q(x)\, \psi(x)\, dx \quad ②$$

$\psi(x,t) \leftarrow$ time dependent

Vector Conj

discrete function $\psi^* \rightarrow \langle\psi|$ $\qquad$ $|\psi\rangle \leftarrow$ Normal ψ

Separate = no sum

① $\langle\psi|\psi\rangle = 1$ $\qquad$ Sum is implied when together

② $\langle\psi|Q|\psi\rangle = b$

Orientation of vector

shape $(1,N)$ $(N,1)$

(5) $a = (a_0, a_1, \dots, a_{n-1})$ $b = (b_0, b_1, \dots, b_{n-1})$

$$\langle a| = (a_0, a_1, \dots, a_{n-1})$$

$$|b\rangle = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{bmatrix}$$

$\sum_i a_i b_i = \langle a|b\rangle$ vector dot product (inner product)

Orthonormal

Basis Set

Normal

$|i\rangle$ $|j\rangle$ $\langle i|i\rangle = 1$

$\langle j|j\rangle = 1$

Orthogonal dot product $= 0$

$\theta = 90$

dot $= 0$

$\langle i|j\rangle = 0$

$\langle i|j\rangle = \delta_{ij}$ $\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$

Apply Operators Matrix $Q_{mn} = \langle m|Q|n\rangle$

$\langle m|Q|n\rangle a_n = b_n$