1

Quantum Operators

Fixed in notes $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|00\rangle$ lower part vector

Classical bits $\underset{H}{\rightleftarrows}$ Quantum Superpositions

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Why?

$$H|0\rangle = H\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$H|1\rangle = H\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\langle q|q\rangle = 1$ Probability Density

Operators must preserve probabilities

Unitary Operators U (H, X)

$\boxed{\text{Reversibles self adjoint}}$

$$UU^\dagger = I$$

$$U^\dagger = (U^T)^*$$

(2) $U^\dagger = U^{-1}$

$$H(H|0\rangle) = \underbrace{HH}_{I}|0\rangle = |0\rangle$$

reversible

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{NOT}$$

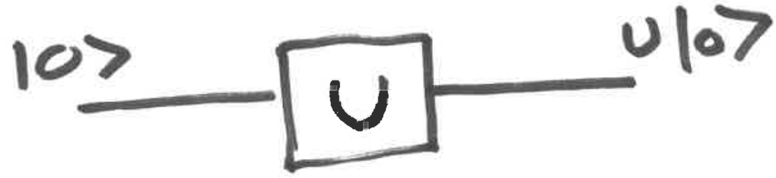
Operator Ω

$$\sum \Omega_{mn} a_n = b_m$$

Projection Matrices $\quad P = |a\rangle\langle b|$

Q Circuits

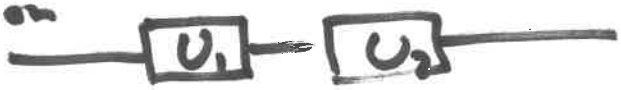
Qubit as a 'wire'



H
X

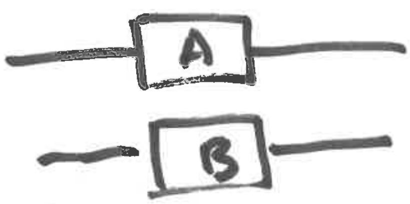
Operators boxes on wire

Composition



$U_1 U_2$

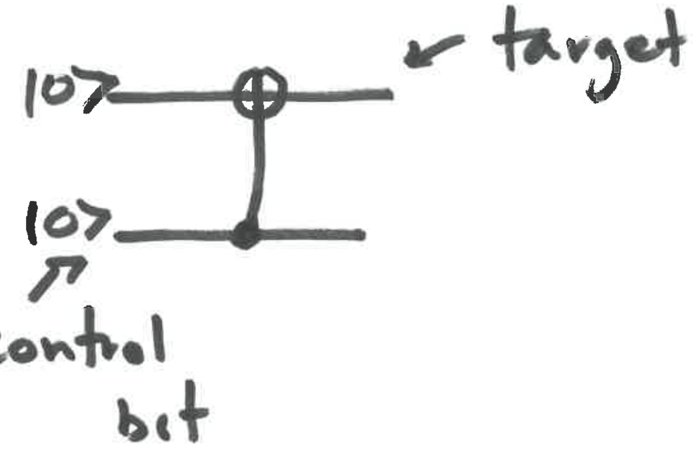
series



$A \otimes B$

multibit

③ CNot



Building Circuits for 1 bit operations

- Operations need to be reversible, self adjoint

Undo

✓1. Identity
✓2. Negation

} uses input variable op $\boxed{1}$

3. Set 0
4. Set 1

} doesn't care about the <u>input</u>. or bit state overwrites

$\boxed{2}$ constant op

$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ X

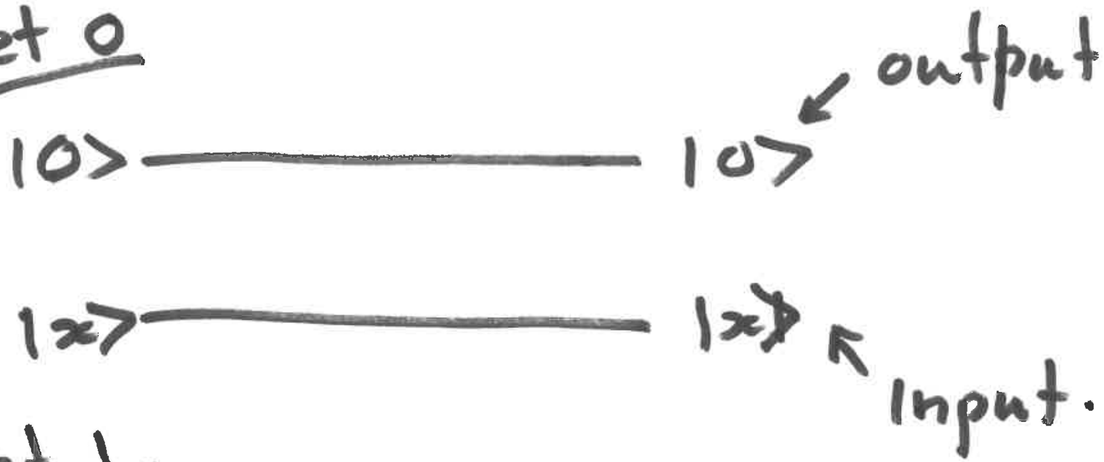
Increase the Dimensions, matrix larger embed matrix into larger Matrix.

Add a second bit to keep track of output. So avoid overwriting result.

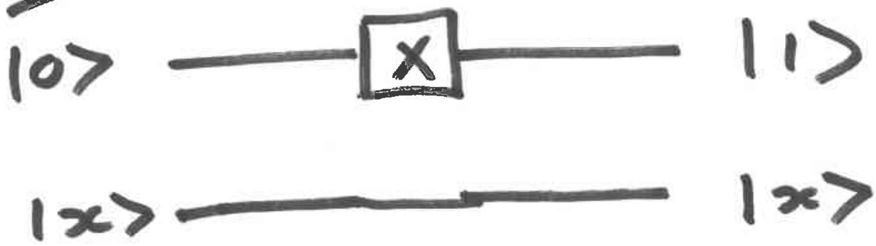
two wires

④ 1-qubit Circuits

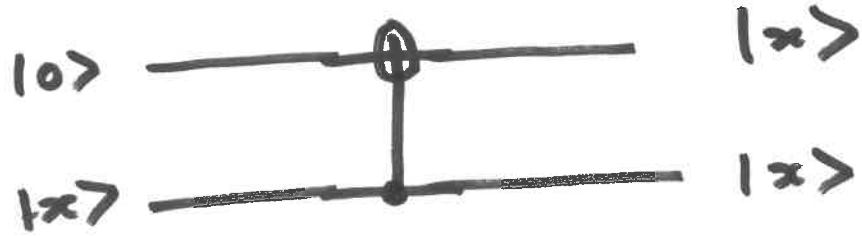
Set 0



Set 1

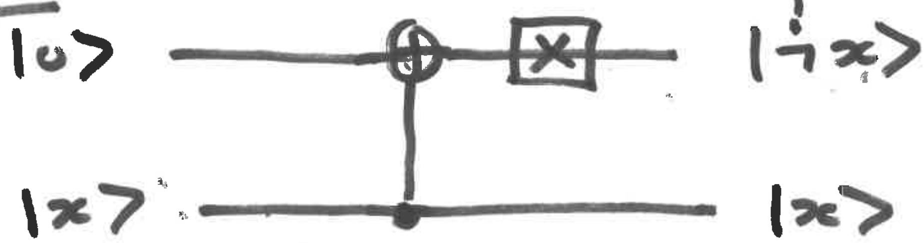


Identity



only time need change output
is when $x = 1$, CNOT

Negation



(5) Deutsch Oracle Problem

Divide a problem into two categories: Make circuit (quantum algorithm) respond to desired category.

$$x \longrightarrow \boxed{BB} \longrightarrow f(x)$$

$f(x)$?

output value

operation?

1-bit, how do you know which of the Four operation was used?

Classically need 2 trys to find out.

However, if we want to find the type/categories of operation Type $\boxed{1}$ or $\boxed{2}$ ~~wh~~ We need only one Query on a Quantum Computer

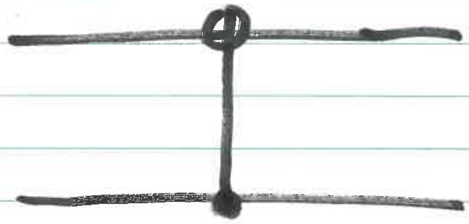
Applies to n-bit aswell ← exponential speed up.

Constant type

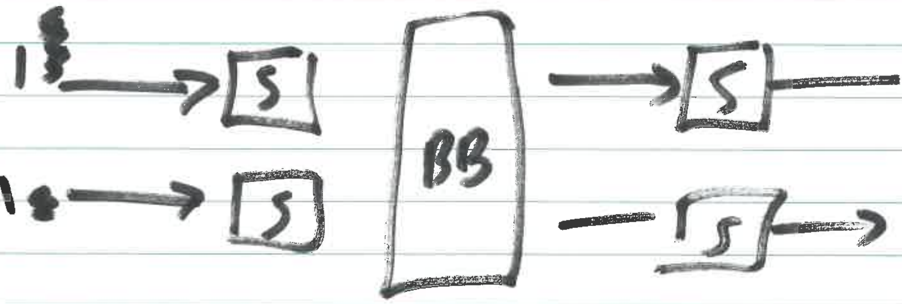
wire aren't joined.

Variable are joined

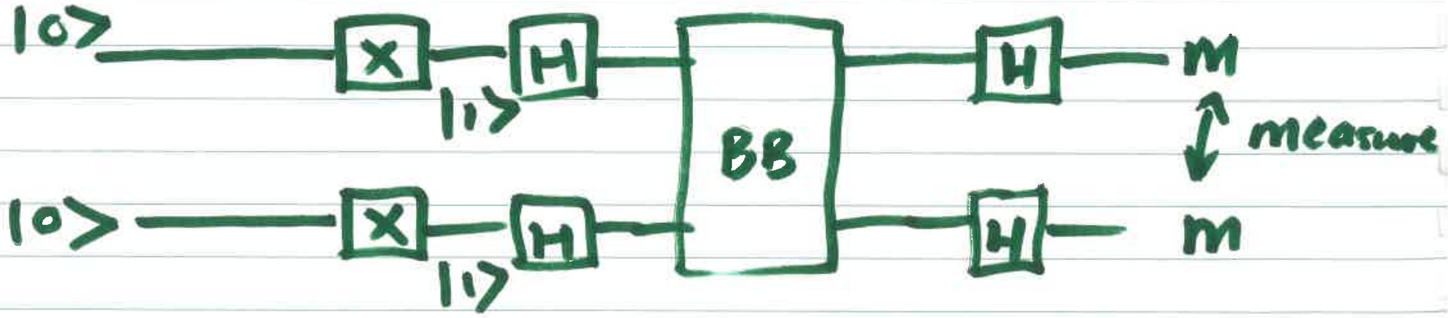
We want to make circuit respond to operation that join~~ed~~ circuits



We also want to use Superposition $\boxed{H}$



Classical output.



$$H|1\rangle = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Two wires

$-H|1\rangle$

$\otimes$ Tensor product

$-H|1\rangle$

$$C\left[\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}\right]$$

$$C\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$|0\rangle$ $\quad$ $|1\rangle$

True only for varible ops

constant ops do nothing!

2 wires $\rightarrow |01\rangle$