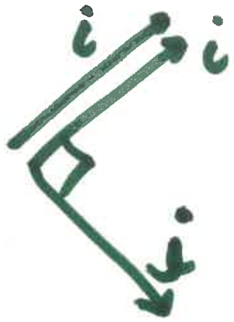
① Qubits

$$\langle i | j \rangle = \delta_{ij}$$



Measurement

$\Downarrow$

Discrete

$\Downarrow$

$\int \to \Sigma$

$$|a\rangle = \sum_{i=0}^{n} \alpha_i |i\rangle$$

$$\alpha_0 |0\rangle + \alpha_1 |1\rangle + \ldots\ldots$$

$$|b\rangle = \sum_{i=0}^{n} \beta_i |i\rangle \qquad \alpha, \beta \in \mathbb{C}$$

$$\langle a| = \sum_{i=0}^{n} \alpha_i^* \langle i|$$

$$\langle b| = \sum_{i=0}^{n} \beta_i^* \langle i|$$

$$\langle a| = (\alpha_0, \alpha_1, \ldots \alpha_n)$$

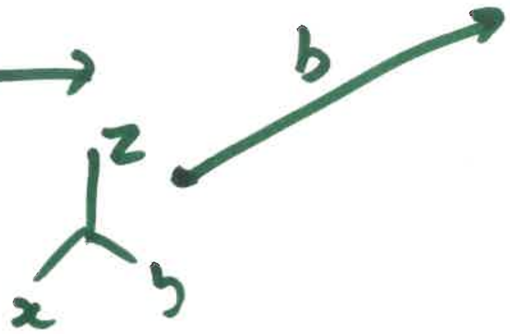
$$|a\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$$

Orthonormal

$$\langle a | b \rangle = \sum_{i=0}^{n} \sum_{j=0}^{n} \alpha_i^* \beta_j \langle i | j \rangle$$

② $\langle a|b\rangle = \sum_{k=0}^{n} \alpha_k^* \beta_k$

$|b\rangle \longrightarrow$



Hyper Sphere

Operators

$$\Omega\,\overset{b}{|a\rangle} = \overset{\tilde{b}}{|b\rangle}$$

matrix   col vector   col vector

rotate/Reflect vector

on unit (n-1)-sphere

n=3 , 2-sphere (ball)

Surface of

Create Operators

$P = |a\rangle\langle b|$ outer product

Tensor Product

$$\begin{pmatrix} \vdots \end{pmatrix} \begin{pmatrix} \cdots \end{pmatrix} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$ matrix

ravel

$$= \begin{pmatrix} \vdots \end{pmatrix}$$ * long vector state

③ Classical Bit

 Switch on/off $|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Classical Operators

1. Identity $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2. Negation
   Not $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3. Set 0 $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$

4. Set 1 $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

2-bit Classical Bit (cbit)

$$|00\rangle = |0\rangle\langle 0|$$

Product of States

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2^K

K - no of bits

$$|a\rangle = \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} \qquad |b\rangle = \begin{pmatrix} u_2 \\ v_2 \end{pmatrix}$$

(4)

$$\begin{bmatrix} \mu_1 \mu_2 & \nu_1 \mu_2 \\ \mu_1 \nu_2 & \nu_1 \nu_2 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} \mu_1 \mu_2 \\ \mu_1 \nu_2 \\ \nu_1 \mu_2 \\ \nu_1 \nu_2 \end{pmatrix} \quad \text{vstack.}$$

Operators - AND, OR

CNOT, negate a bit if other bit is active

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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Qubit

mu

$$|q\rangle = \begin{pmatrix} u \\ v \end{pmatrix}$$

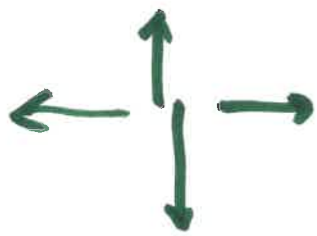
mu

$$\langle q|q\rangle = 1$$

Rather than $u, v \in \{0,1\}$ it is now $u, v \in \mathbb{C}$

Instead of cbit, which was

1-bit 

2-bit 

We generalise to circle

b is in both states (in some proportion)

Still doing digital computation (0,1)

$|0\rangle$, $|1\rangle$ but $|q\rangle$ will collapse to $|0\rangle$, $|1\rangle$

But unit circle gives up more possibilities.

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$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ cbit ✓

$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ half in $|0\rangle$ half in $|1\rangle$

$$|q|^2 = 1/2 + 1/2$$

$$|q\rangle\langle r| = \begin{pmatrix} u_1 u_2 \\ u_1 v_2 \\ v_1 u_2 \\ v_1 v_2 \end{pmatrix}$$

$$|q\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad |r\rangle = |q\rangle$$

$$= |q\rangle\langle q| = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$|q|^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

This system has a $\frac{1}{4}$ prob of collapsing to $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$.

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Quantum Operator

classical bit $\rightleftarrows$ Qubit

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

Hadamard Operator

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Quantum Ops have constraint.

Why Total Probability must be preserved.

$U^\dagger U = I$

$U \rightarrow$ self-adjoint
$\rightarrow$ Unitary