1 Week 3

- If $f(n) = O(g(n))$, there exist constants c_1 and c_2 such that $f(n) \leqslant c_2 \cdot g(n)$ holds for all $n \geqslant c_2$.
- If $f(n) = O(g(n))$, we have $\lim_{n \to \infty} \dfrac{f_1(n)}{g_1(n)} = c =$ some constant c.

2 Week 3 - Extra

- When using 'Direction 1: Constant Finding' setting c_1, always set it to match the coefficent on the LHS so that you can cancel.
- When trying to get a contradiction, try and isolate an $x \cdot c_1$ on the RHS, where $x \in \mathbb{Z}$, such that an expression that contains n is $\leqslant xc_1$
- Make judicious use of the *max* function when adding functions together
- If $f_1(n) + f_2(n) \leqslant c_1 \cdot g_1(n) + c_1' \cdot g_2(n) \leqslant max\{c_1, c_1'\} \cdot (g_1(n) + g_2(n))$, for all $n \geqslant max\{c_2, c_2'\}$.

3 Week 4

Let $f(n)$ be a function that returns a positive value for every integer $n > 0$. We know:

$$f(1) \leqslant c_1$$
$$f(n) \leqslant \alpha \cdot f(\lceil n/\beta \rceil) + c_2 \cdot n^\gamma \text{ for } n \geqslant 2$$

where $\alpha, \beta, \gamma, c_1$ and c_2 are positive constants. Then:

- If $log_b \alpha < \gamma$ then $f(n) = O(n^\gamma)$
- If $log_b \alpha = \gamma$ then $f(n) = O(n^\gamma log(n))$
- If $log_b \alpha > \gamma$ then $f(n) = O(n^{log_\beta (a)})$