# Assignment 2: Verification

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## 1 Part A

Given

$$\begin{array}{rcl} pre & \triangleq & D.len \geqslant max(\{A.len,\ B.len,\ C.len\}) \\ & \wedge & sorted(A)\ \wedge\ sorted(B)\ \wedge\ sorted(C) \end{array}$$

and

$$\begin{array}{rcl} post & \triangleq & D = A \cap B \cap C \\ & \wedge & r \in [0, D.len]\ \wedge\ i \in [0, A.len]\ \wedge\ j \in [0, B.len]\ \wedge\ k \in [0, C.len] \\ & \wedge & (i = A.len \vee j = B.len \vee k = C.len) \end{array}$$

$$\begin{array}{ll} & i, j, k, r, D : [pre, post] \\ \sqsubseteq & \{\text{Composition: middle predicate is } inv\} \\ & i, j, k, r, D : [pre, inv];\ i, j, k, r, D : [inv, post] \end{array}$$

$\because$

$$\begin{array}{rcl} inv & \triangleq & D_{[0,r)} = A_{[0,i)} \cap B_{[0,j)} \cap C_{[0,k)} \\ & & \wedge\ r \in [0, D.len]\ \wedge\ i \in [0, A.len]\ \wedge\ j \in [0, B.len]\ \wedge\ k \in [0, C.len] \end{array}$$

$$\begin{array}{ll} \sqsubseteq & \{\text{Assignment: } pre \Rrightarrow inv[i, j, k, r \backslash 0, 0, 0, 0]\} \\ & i, j, k, r := 0, 0, 0, 0;\ i, j, k, r, D : [inv, post] \end{array}$$

$\because$

$$\begin{array}{rcl} inv[i, j, k, r \backslash 0, 0, 0, 0] & \equiv & D_{[0,0)} = A_{[0,0)} \cap B_{[0,0)} \cap C_{[0,0)} \\ & & \wedge\ 0 \in [0, D.len]\ \wedge\ 0 \in [0, A.len]\ \wedge\ 0 \in [0, B.len]\ \wedge\ 0 \in [0, C.len] \\ & \equiv & \varnothing = (\varnothing \cap \varnothing \cap \varnothing)\ \wedge\ (\text{true}\ \wedge\ \text{true}\ \wedge\ \text{true}\ \wedge\ \text{true}) \\ & \equiv & \varnothing = \varnothing\ \wedge\ \text{true} \\ & \equiv & \text{true} \end{array}$$

$$\begin{array}{ll} \sqsubseteq & \{\text{Strengthen post: } inv \wedge \neg(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \Rrightarrow post\} \\ & i, j, k, r := 0, 0, 0, 0;\ i, j, k, r, D : [inv, inv \wedge \neg(i \neq A.len \vee j \neq B.len \vee k \neq C.len)] \end{array}$$

$\because$

$$inv \wedge \neg(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \Rrightarrow post \quad \equiv \quad inv \wedge (i = A.len \wedge j = B.len \wedge C.len) \Rrightarrow post$$

$\sqsubseteq$ {Repetition Rule}

$i, j, k, r := 0, 0, 0, 0;$
**do** $(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \rightarrow$
$\quad i, j, k, r, D[inv \wedge (i \neq A.len \vee j \neq B.len \vee k \neq C.len),\ inv \wedge (0 \leqslant V < V_0)]$
**od**