Assignment 3: Derivation

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1. (a) n is a **value** parameter. m is a **result** parameter.
   (b) $inv \;\triangleq\; lrun(A, n_0, m)$
   (c) Let

$$\begin{aligned} pre(A, n) &\triangleq lrun(A, n, n+1) \\ inv(A, n, m) &\triangleq lrun(A, n_0, m) \\ post(A, n, m) &\triangleq mrun(A, n_0, m) \end{aligned}$$

by 1(b), and the specification of the procedure. inv, pre and $post$ implicitly capture variables (A, n, m) as parameters from the frame, for syntactic convience.
s.t.

$$\begin{aligned} & n, m : [pre,\ post] \\ \sqsubseteq\ & \{\text{Composition: middle predicate is } inv\} \\ & n, m : [pre,\ inv];\quad n, m : [inv,\ post] \\ \sqsubseteq\ & \{\text{Assignment: } pre \Rrightarrow inv[m \backslash n+1]\} \\ & m := n+1;\quad n, m : [inv,\ post] \end{aligned}$$

$\because$

$$\begin{aligned} inv[m \backslash n+1] &\equiv lrun(A, n_0, m)[m \backslash n+1] \\ &\equiv lrun(A, n_0, n+1) \end{aligned}$$

$\therefore$

$$lrun(A, n, n+1) \;\Rrightarrow\; lrun(A, n_0, n+1)$$

Let

$$guard \;\triangleq\; (m < A.\text{len} \wedge A_{n_0} = A_m)$$

s.t.

$$\begin{aligned} \sqsubseteq\ & \{\text{Strengthen post: } inv \wedge \neg guard \Rrightarrow post\} \\ & m := n+1;\quad n, m : [inv,\ inv \wedge \neg guard] \end{aligned}$$

$\because$

$$\begin{aligned} & inv \wedge \neg guard \;\Rrightarrow\; post \\ \equiv\ & \{\text{Expansion of definitions}\} \\ & lrun(A, n_0, m) \wedge \neg(m < A.\text{len} \wedge A_{n_0} = A_m) \;\Rrightarrow\; mrun(A, n_0, m) \\ \equiv\ & \{\text{Expansion of functions}\} \\ & lrun(A, n_0, m) \wedge \neg(m < A.\text{len} \wedge A_{n_0} = A_m) \;\Rrightarrow\; lrun(A, n_0, m) \wedge (m < A.\text{len} \Rightarrow A_{n_0} \neq A_m) \\ \equiv\ & \{\text{De Morgan's law - negation of conjunction}\} \end{aligned}$$

$$
\begin{array}{ll}
 & lrun(A, n_0, m) \wedge (\neg(m < A.\mathrm{len}) \vee \neg(A_{n_0} = A_m)) \;\Rrightarrow\; lrun(A, n_0, m) \wedge (m < A.\mathrm{len} \Rightarrow A_{n_0} \neq A_m) \\
\equiv & \{P \Rightarrow Q \;\equiv\; \neg P \vee Q\} \\
 & lrun(A, n_0, m) \wedge (\neg(m < A.\mathrm{len}) \vee \neg(A_{n_0} = A_m)) \;\Rrightarrow\; lrun(A, n_0, m) \wedge (\neg(m < A.\mathrm{len}) \vee (A_{n_0} \neq A_m)) \\
\equiv & \{\} \\
 & \mathrm{true}
\end{array}
$$

$$
\begin{array}{ll}
\sqsubseteq & \{\text{Repetition}\} \\
 & m := n + 1; \\
 & \textbf{do } (m < A.\mathrm{len} \wedge A_{n_0} = A_m) \rightarrow \\
 & \qquad n, m : [inv \wedge guard,\; inv \wedge (0 \leqslant V < V_0)] \\
 & \textbf{od}
\end{array}
$$

where

$$
V \;\triangleq\; A.\mathrm{len} \;-\; m
$$

$$
\begin{array}{ll}
\sqsubseteq & \{\text{Assignment: } inv \wedge \neg guard \;\Rrightarrow\; (inv \wedge (0 \leqslant V < V_0))[m \backslash m + 1]\} \\
 & m := n + 1; \\
 & \textbf{do } (m < A.\mathrm{len} \wedge A_{n_0} = A_m) \rightarrow \\
 & \qquad m := m + 1 \\
 & \textbf{od}
\end{array}
$$

$\because$

$$
\begin{array}{rcl}
(inv \wedge (0 \leqslant V < V_0))[m \backslash m + 1] & \equiv & (lrun(A, n_0, m) \wedge (0 \leqslant (A.\mathrm{len} - m) < (???)))[m \backslash m + 1] \\
 & \equiv & lrun(A, n_0, m + 1) \wedge (0 \leqslant (A.\mathrm{len} - (m + 1)) < (???))
\end{array}
$$

TODO

2. $\ell, h : [A.\mathrm{len} > 0,\; mrun(A, \ell, h) \wedge (\forall\, p, q \cdot mrun(A, p, q) \Rightarrow (h - \ell) \leqslant (q - p))]$