

Assignment 2: Verification

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April 12, 2017

1 Part A

Given

$$\begin{aligned} pre &\triangleq D.len \geq \max(\{A.len, B.len, C.len\}) \\ &\quad \wedge \text{sorted}(A) \wedge \text{sorted}(B) \wedge \text{sorted}(C) \end{aligned}$$

and

$$post \triangleq D_{[0,r)} = A \cap B \cap C$$

$$i, j, k, r, D : [pre, post]$$

$$\sqsubseteq \{ \text{Composition: middle predicate is } inv \}$$

$$i, j, k, r, D : [pre, inv]; i, j, k, r, D : [inv, post]$$

where

$$\begin{aligned} inv &\triangleq D_{[0,r)} = A_{[0,i)} \cap B_{[0,j)} \cap C_{[0,k)} \\ &\quad \wedge r \in [0, D.len] \wedge i \in [0, A.len] \wedge j \in [0, B.len] \wedge k \in [0, C.len] \end{aligned}$$

$$\sqsubseteq \{ \text{Assignment: } pre \Rightarrow inv[i, j, k, r \setminus 0, 0, 0, 0] \}$$

$$i, j, k, r := 0, 0, 0, 0; i, j, k, r, D : [inv, post]$$

\therefore

$$\begin{aligned} inv[i, j, k, r \setminus 0, 0, 0, 0] &\equiv D_{[0,0)} = A_{[0,0)} \cap B_{[0,0)} \cap C_{[0,0)} \\ &\quad \wedge 0 \in [0, D.len] \wedge 0 \in [0, A.len] \wedge 0 \in [0, B.len] \wedge 0 \in [0, C.len] \\ &\equiv \emptyset = (\emptyset \cap \emptyset \cap \emptyset) \wedge (\text{true} \wedge \text{true} \wedge \text{true} \wedge \text{true}) \\ &\equiv \emptyset = \emptyset \wedge \text{true} \\ &\equiv \text{true} \end{aligned}$$

$$\sqsubseteq \{ \text{Strengthen post: } inv \wedge \neg(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \Rightarrow post \}$$

$$i, j, k, r := 0, 0, 0, 0; i, j, k, r, D : [inv, inv \wedge \neg(i \neq A.len \vee j \neq B.len \vee k \neq C.len)]$$

∴

$$\begin{aligned}
& inv \wedge \neg(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \Rightarrow post \\
\equiv & inv \wedge (i = A.len \wedge j = B.len \wedge k = C.len) \Rightarrow post \\
\equiv & inv[i, j, k \setminus A.len, B.len, C.len] \Rightarrow post \\
\equiv & D_{[0, r)} = A_{[0, A.len)} \cap B_{[0, B.len)} \cap C_{[0, C.len)} \\
& \wedge r \in [0, D.len] \wedge A.len \in [0, A.len] \wedge B.len \in [0, B.len] \wedge C.len \in [0, C.len]
\end{aligned}$$

□ {Repetition}
 $i, j, k, r := 0, 0, 0, 0;$
do $(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \rightarrow$
 $i, j, k, r, D : [inv \wedge (i \neq A.len \vee j \neq B.len \vee k \neq C.len), inv \wedge (0 \leq V < V_0)]$
od

where

$$\begin{aligned}
V & \triangleq (A.len - i) + (B.len - j) + (C.len - k) \\
& \triangleq (A.len + B.len + C.len) - (i + j + k)
\end{aligned}$$

□ {Selection: $inv \wedge (i \neq A.len \vee j \neq B.len \vee k \neq C.len) \Rightarrow (G_1(i, j) \vee G_2(j, k) \vee G_3(k, i) \vee G_4(i, j, k))$ }
 $i, j, k, r := 0, 0, 0, 0;$
do $(i \neq A.len \vee j \neq B.len \vee k \neq C.len) \rightarrow$
 $TODO$
od

where

$$\begin{aligned}
G_1(i, j) & \triangleq A_i > B_j \\
G_2(j, k) & \triangleq B_j > C_k \\
G_3(k, i) & \triangleq C_k > A_i \\
G_4(i, j, k) & \triangleq (A_i = B_j) \wedge (B_j = C_k)
\end{aligned}$$