Assignment 3: Derivation

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1. (a) n is a **value** parameter. m is a **result** parameter.

   (b) $inv \triangleq lrun(A, n_0, m)$

   (c) Let

$$\begin{aligned} pre(A,n) &\triangleq lrun(A,n,n+1) \\ inv(A,n,m) &\triangleq lrun(A,n_0,m) \\ post(A,n,m) &\triangleq mrun(A,n_0,m) \end{aligned}$$

by 1(b), and the specification of the procedure. inv, pre and $post$ implicitly capture variables (A, n, m) as parameters from the frame, for syntactic convience.
s.t.

$$\begin{aligned} & n, m : [pre,\ post] \\ \sqsubseteq\ & \{\text{Composition: middle predicate is } inv\} \\ & n, m : [pre,\ inv];\quad n, m : [inv,\ post] \\ \sqsubseteq\ & \{\text{Assignment: } pre \Rrightarrow inv[m \backslash n+1]\} \\ & m := n+1;\quad n, m : [inv,\ post] \end{aligned}$$

$\because$

$$\begin{aligned} inv[m \backslash n+1] &\equiv lrun(A,n_0,m)[m \backslash n+1] \\ &\equiv lrun(A,n_0,n+1) \end{aligned}$$

$\therefore$

$$lrun(A,n,n+1) \Rrightarrow lrun(A,n_0,n+1)$$

Let

$$guard \triangleq (m < A.len \wedge A_{n_0} = A_m)$$

s.t.

$$\begin{aligned} \sqsubseteq\ & \{\text{Strengthen post: } inv \wedge \neg guard \Rrightarrow post\} \\ & m := n+1;\quad n, m : [inv,\ inv \wedge \neg guard] \end{aligned}$$

$\because$

$$\begin{aligned} & inv \wedge \neg guard \Rrightarrow post \\ \equiv\ & \{\text{Expansion of definitions}\} \\ & lrun(A,n_0,m) \wedge \neg(m < A.len \wedge A_{n_0} = A_m) \Rrightarrow mrun(A,n_0,m) \\ \equiv\ & \{\text{Expansion of functions}\} \\ & lrun(A,n_0,m) \wedge \neg(m < A.len \wedge A_{n_0} = A_m) \Rrightarrow lrun(A,n_0,m) \wedge (m < A.len \Rightarrow A_{n_0} \neq A_m) \\ \equiv\ & \{\text{De Morgan's law - negation of conjunction}\} \end{aligned}$$

$$
\begin{array}{ll}
& lrun(A, n_0, m) \wedge (\neg(m < A.len) \vee \neg(A_{n_0} = A_m)) \;\Rrightarrow\; lrun(A, n_0, m) \wedge (m < A.len \Rightarrow A_{n_0} \neq A_m) \\
\equiv & \{P \Rightarrow Q \;\equiv\; \neg P \vee Q\} \\
& lrun(A, n_0, m) \wedge (\neg(m < A.len) \vee \neg(A_{n_0} = A_m)) \;\Rrightarrow\; lrun(A, n_0, m) \wedge (\neg(m < A.len) \vee (A_{n_0} \neq A_m)) \\
\equiv & \{\} \\
& \text{true}
\end{array}
$$

$$
\begin{array}{ll}
\sqsubseteq & \{\text{Repetition}\} \\
& \quad m := n + 1; \\
& \quad \mathbf{do}\ (m < A.len \wedge A_{n_0} = A_m) \rightarrow \\
& \qquad\quad n, m : [inv \wedge guard,\ inv \wedge (0 \leqslant V < V_0)] \\
& \quad \mathbf{od}
\end{array}
$$

where

$$
V \;\triangleq\; A.len \;-\; m
$$

$$
\begin{array}{ll}
\sqsubseteq & \{\text{Assignment: } inv \wedge \neg guard \;\Rrightarrow\; (inv \wedge (0 \leqslant V < V_0))[m \backslash m + 1]\} \\
& \quad m := n + 1; \\
& \quad \mathbf{do}\ (m < A.len \wedge A_{n_0} = A_m) \rightarrow \\
& \qquad\quad m := m + 1 \\
& \quad \mathbf{od}
\end{array}
$$