Assignment 3: Derivation

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1. (a) n is a **value** parameter. m is a **result** parameter.

   (b) $inv \triangleq lrun(A, n_0, m)$

   (c) Let

   $$
   \begin{aligned}
   pre(A, n) &\triangleq lrun(A, n, n+1) \\
   inv(A, n, m) &\triangleq lrun(A, n_0, m) \\
   post(A, n, m) &\triangleq mrun(A, n_0, m)
   \end{aligned}
   $$

   by 1.(b), and the specification of the procedure. inv, pre and $post$ implicitly capture variables (A, n, m) as parameters from the frame, for syntactic convience.

   s.t.

   $\quad n, m : [pre, \; post]$
   $\sqsubseteq \quad$ {Composition: middle predicate is inv}
   $\quad n, m : [pre, \; inv]; \quad n, m : [inv, \; post]$
   $\sqsubseteq \quad$ {Assignment: $pre \Rightarrow inv[m \backslash n+1]$}
   $\quad m := n+1; \quad n, m : [inv, \; post]$

   $\because$

   $$
   \begin{aligned}
   inv[m \backslash n+1] &\equiv lrun(A, n_0, m)[m \backslash n+1] \\
   &\equiv lrun(A, n_0, n+1)
   \end{aligned}
   $$

   $\therefore$

   $$ lrun(A, n, n+1) \; \Rightarrow \; lrun(A, n_0, n+1) $$

   Let

   $$ guard \triangleq (m < A.\text{len} \land A_{n_0} = A_m) $$

   s.t.

   $\sqsubseteq \quad$ {Strengthen post: $inv \land \neg guard \; \Rightarrow \; post$}
   $\quad m := n+1; \quad n, m : [inv, \; inv \land \neg guard]$

   $\because$

   $\quad inv \land \neg guard \; \Rightarrow \; post$
   $\equiv \quad$ {Expansion of definitions}
   $\quad lrun(A, n_0, m) \land \neg(m < A.\text{len} \land A_{n_0} = A_m) \; \Rightarrow \; mrun(A, n_0, m)$
   $\equiv \quad$ {Expansion of functions}
   $\quad lrun(A, n_0, m) \land \neg(m < A.\text{len} \land A_{n_0} = A_m) \; \Rightarrow \; lrun(A, n_0, m) \land (m < A.\text{len} \Rightarrow A_{n_0} \neq A_m)$
   $\equiv \quad$ {De Morgan's law - negation of conjunction}

$$lrun(A, n_0, m) \wedge (\neg(m < A.\text{len}) \vee \neg(A_{n_0} = A_m)) \;\Rightarrow\; lrun(A, n_0, m) \wedge (m < A.\text{len} \Rightarrow A_{n_0} \neq A_m)$$

$\equiv \quad \{P \Rightarrow Q \;\equiv\; \neg P \vee Q\}$

$$lrun(A, n_0, m) \wedge (\neg(m < A.\text{len}) \vee \neg(A_{n_0} = A_m)) \;\Rightarrow\; lrun(A, n_0, m) \wedge (\neg(m < A.\text{len}) \vee (A_{n_0} \neq A_m))$$

$\equiv \quad \{\}$

true

$\sqsubseteq \quad \{\text{Repetition}\}$

$m := n + 1;$
do $(m < A.\text{len} \wedge A_{n_0} = A_m) \rightarrow$
   $n, m : [inv \wedge guard,\ inv \wedge (0 \leqslant V < V_0)]$
od

where

$$V \quad \triangleq \quad A.\text{len} \;-\; m$$

$\sqsubseteq \quad \{\text{Assignment: } inv \wedge \neg guard \;\Rightarrow\; (inv \wedge (0 \leqslant V < V_0))[m \backslash m + 1]\}$

$m := n + 1;$
do $(m < A.\text{len} \wedge A_{n_0} = A_m) \rightarrow$
   $m := m + 1$
od

$\therefore$

$(inv \wedge (0 \leqslant V < V_0))[m \backslash m + 1] \quad \equiv \quad (lrun(A, n_0, m) \wedge (0 \leqslant (A.\text{len} - m) < (A.\text{len} - m_0)))[m, m_0 \backslash m + 1, m]$

$\equiv \quad lrun(A, n_0, m + 1) \wedge (0 \leqslant (A.\text{len} - (m + 1)) < (A.\text{len} - m))$

TODO

2.

$$pre(A) \quad \triangleq \quad A.\text{len} > 0$$
$$post(A, \ell, h) \quad \triangleq \quad mrun(A, \ell, h) \wedge (\forall\, p, q \,\cdot\, mrun(A, p, q) \Rightarrow (h - \ell) \leqslant (q - p))$$

by 2. *pre* and *post* implicitly capture variables (A, ℓ, h) as parameters from the frame.

$\ell, h : [pre,\ post]$

$\sqsubseteq \quad \{\text{Composition: middle predicate is } inv\}$

$\ell, h : [pre,\ inv];\ \ \ell, h : [inv,\ post]$

where

$$inv(???) \quad \triangleq \quad \text{TODO}$$

where *inv* implicitly captures varibles (TODO) as parameters from the frame.