Assignment 2: Verification

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1   Part A

Given

$$
\begin{aligned}
pre \quad\triangleq\quad & D.len \geqslant max(\{A.len,\ B.len,\ C.len\}) \\
& \wedge\ sorted(A)\ \wedge\ sorted(B)\ \wedge\ sorted(C)
\end{aligned}
$$

and

$$
\begin{aligned}
post \quad\triangleq\quad & D = A \cap B \cap C \\
& \wedge\ r \in [0, D.len]\ \wedge\ i \in [0, A.len]\ \wedge\ j \in [0, B.len]\ \wedge\ k \in [0, C.len] \\
& \wedge\ (i = A.len\ \vee\ j = B.len\ \vee\ k = C.len)
\end{aligned}
$$

$\quad i, j, k, r, D : [pre, post]$

$\sqsubseteq\quad \{\text{Composition: middle predicate is } inv\}$

$\quad i, j, k, r, D : [pre, inv];\ i, j, k, r, D : [inv, post]$

$\therefore$

$$
\begin{aligned}
inv \quad\triangleq\quad & D_{[0,r)} = A_{[0,i)}\ \cap\ B_{[0,j)}\ \cap\ C_{[0,k)} \\
& \wedge\ r \in [0, D.len]\ \wedge\ i \in [0, A.len]\ \wedge\ j \in [0, B.len]\ \wedge\ k \in [0, C.len]
\end{aligned}
$$

$\sqsubseteq\quad \{\text{Assignment: } pre \Rightarrow inv[i, j, k, r \backslash 0, 0, 0, 0]\}$

$\quad i, j, k, r := 0, 0, 0, 0;\ i, j, k, r, D : [inv, post]$

$\therefore$

$$
\begin{aligned}
inv[i, j, k, r \backslash 0, 0, 0, 0] \quad\equiv\quad & D_{[0,0)} = A_{[0,0)}\ \cap\ B_{[0,0)}\ \cap\ C_{[0,0)} \\
& \wedge\ 0 \in [0, D.len]\ \wedge\ 0 \in [0, A.len]\ \wedge\ 0 \in [0, B.len]\ \wedge\ 0 \in [0, C.len] \\
\equiv\quad & \varnothing = (\varnothing \cap \varnothing \cap \varnothing)\ \wedge\ (\text{true}\ \wedge\ \text{true}\ \wedge\ \text{true}\ \wedge\ \text{true}) \\
\equiv\quad & \varnothing = \varnothing\ \wedge\ \text{true} \\
\equiv\quad & \text{true}
\end{aligned}
$$

$\sqsubseteq\quad \{\text{Strengthen post: } inv \wedge \neg(i \neq A.len\ \vee\ j \neq B.len\ \vee\ k \neq C.len) \Rightarrow post\}$

$\quad i, j, k, r := 0, 0, 0, 0;\ i, j, k, r, D : [inv, inv \wedge \neg(i \neq A.len\ \vee\ j \neq B.len\ \vee\ k \neq C.len)]$

$$\therefore$$

$$inv \wedge \neg(i \neq A.len \ \vee \ j \neq B.len \ \vee \ k \neq C.len) \Rightarrow post \quad \equiv \quad inv \wedge (i = A.len \ \wedge \ j = B.len \ \wedge k = C.len) \Rightarrow post$$
$$\equiv \quad TODO$$

$$\sqsubseteq \quad \{\text{Repetition Rule}\}$$
$$i, j, k, r := 0, 0, 0, 0;$$
$$\mathbf{do} \ (i \neq A.len \ \vee \ j \neq B.len \ \vee \ k \neq C.len) \rightarrow$$
$$i, j, k, r, D : [inv \ \wedge \ (i \neq A.len \ \vee \ j \neq B.len \ \vee \ k \neq C.len), \ inv \ \wedge \ (0 \leqslant V < V_0)]$$
$$\mathbf{od}$$