Assignment 1: Background theory

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1. (a) $y : [\text{ true},\ (x = 0 \Rightarrow y = 0) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x})\ ]$
   (b) $y : [\text{ true},\ (x = 0 \Rightarrow \text{true}) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x})\ ]$
   (c) TODO

2. $$\begin{array}{ll} & x, y : [\text{true},\ x = z^2 \wedge y = z^4] \\ \sqsubseteq & \{\text{Composition}\} \\ & x, y : [\text{true},\ x = z^2];\ x, y : [x = z^2,\ x = z^2 \wedge y = z^4] \\ \sqsubseteq & \{\text{Assignment: true} \Rrightarrow x = z^2[x \backslash z^2]\} \\ & x = z^2;\ x, y : [x = z^2,\ x = z^2 \wedge y = z^4] \\ \sqsubseteq & \{\text{Assignment: } x = z^2 \Rrightarrow x = z^2 \wedge y = z^4[y \backslash x^2]\} \\ & x := z^2;\ y := x^2 \end{array}$$

3. (a) Assuming

$$\begin{aligned} wp(y := 10,\ \text{true}) &\equiv \text{true}[y \backslash 10] \\ &\equiv \text{true} \end{aligned}$$

we can conclude that

$$\begin{aligned} wp(\textbf{if } (x > 0 \ \vee \ y < 10) \rightarrow y := 10 \textbf{ fi},\ \text{true}) &\equiv (x > 0 \ \vee \ y < 10) \ \wedge \\ &\quad\ ((x > 0 \ \vee \ y < 10) \rightarrow wp(y := 10,\ \text{true})) \\ &\equiv (x > 0 \ \vee \ y < 10) \ \wedge \ \text{true} \\ &\equiv (x > 0 \ \vee \ y < 10) \end{aligned}$$

As $y < 10 \Rrightarrow (x > 0 \vee y < 10)$, the Hoare Triple is true.

   (b) Assuming

$$\begin{aligned} wp(x := x + y,\ P[x \backslash x + y]) &\equiv (P[x \backslash x + y])[x \backslash x + y] \\ &\equiv P[x \backslash (x + y) + y] \\ &\equiv P[x \backslash x + 2y] \end{aligned}$$

Using this

$$\{\text{true}[x \backslash x + 2y]\}\ x := x + y\ \{\text{true}[x \backslash x + y]\}$$

holds but

$$\{(x > 8)[x \backslash x + 2y]\}\ x := x + y\ \{(x > 8)[x \backslash x + y]\} \quad \equiv \quad \{x + 2y > 8\}\ x := x + y\ \{x + y > 8\}$$

does not. Therefore, the choice of P determines the validity of the Hoare Triple, due to P having arbitrary. Assuming that P is quantified over all possible predicates, the Hoare Triple does not hold

4. (a)

$$
\begin{array}{ll}
 & y : [y < 10, y > 0] \\
\sqsubseteq & \{\text{Selection: } y < 10 \Rrightarrow (x > 0 \vee y < 10)\} \\
 & \mathbf{if}\ (x > 0 \vee y < 10) \rightarrow y : [(x > 0 \vee y < 10) \wedge (y < 10),\ \ y > 0]\ \mathbf{fi} \\
\sqsubseteq & \{\text{Absorption 1: } (x > 0 \vee y < 10) \wedge (y < 10) = y < 10\} \\
 & \mathbf{if}\ (x > 0 \vee y < 10) \rightarrow y : [y < 10,\ \ y > 0]\ \mathbf{fi} \\
\sqsubseteq & \{\text{Assignment: } y < 10 \Rrightarrow y > 0[y \backslash 10]\} \\
 & \mathbf{if}\ (x > 0 \vee y < 10) \rightarrow y := 10\ \mathbf{fi}
\end{array}
$$

(b)

$$
\begin{array}{ll}
 & y : [y < 10, y > 0] \\
\not\sqsubseteq & \{\text{Selection: } y < 10 \not\Rrightarrow ((x > 0) \wedge (y < 10))\} \\
 & \mathbf{if}\ ((x > 0) \wedge (y < 10)) \rightarrow y : [((x > 0) \wedge (y < 10)) \wedge (y < 10),\ \ y > 0]\ \mathbf{fi}
\end{array}
$$

5.

$$
\begin{array}{ll}
 & w : [P, Q] \\
\sqsubseteq & \{\text{Composition}\} \\
 & w : [P, I]
\end{array}
$$