# Assignment 1: Background theory

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1. (a) $y : [\ \text{true},\ (x = 0 \land y = 0) \lor (y = \frac{y_0}{x} \land x \neq 0)\ ]$
   (b) $y : [\ \text{true},\ (x = 0) \lor (y = \frac{y_0}{x} \land x \neq 0)\ ]$
   (c) TODO

2. $x, y : [\text{true}, x = z^2 \land y = z^4]$
   $\sqsubseteq$ {Composition}
   $x, y : [\text{true}, x = z^2];\ x, y : [x = z^2, x = z^2 \land y = z^4]$
   $\sqsubseteq$ {Assignment: true $\Rrightarrow x = z^2[x \backslash z^2]$}
   $x = z^2;\ x, y : [x = z^2, x = z^2 \land y = z^4]$
   $\sqsubseteq$ {Assignment: $x = z^2 \Rrightarrow x = z^2 \land y = z^4[y \backslash x^2]$}
   $x := z^2;\ y := x^2$

3. (a) Assuming

$$\begin{aligned} wp(y := 10,\ \text{true}) &\equiv \text{true}[y \backslash 10] \\ &\equiv \text{true} \end{aligned}$$

   we can conclude that

$$\begin{aligned} wp(\textbf{if}\ (x > 0\ \lor\ y < 10) \to y := 10\ \textbf{fi},\ \text{true}) &\equiv (x > 0\ \lor\ y < 10)\ \land \\ &\quad ((x > 0\ \lor\ y < 10) \to wp(y := 10,\ \text{true})) \\ &\equiv (x > 0\ \lor\ y < 10)\ \land\ \text{true} \\ &\equiv (x > 0\ \lor\ y < 10) \end{aligned}$$

   As $y < 10 \Rrightarrow (x > 0 \lor y < 10)$, the Hoare triple is true.
   (b) Assuming

$$wp(x := x + y,\ P[x \backslash x + y]) \equiv (P[x \backslash x + y])[x \backslash x + y]$$

   TODO

4. (a) $y : [y < 10, y > 0]$
   $\sqsubseteq$ {Selection: $y < 10 \Rrightarrow (x > 0 \lor y < 10)$}
   **if** $(x > 0 \lor y < 10) \to y : [(x > 0 \lor y < 10) \land (y < 10),\ y > 0]$ **fi**
   $\sqsubseteq$ {Absorption 1: $(x > 0 \lor y < 10) \land (y < 10) = y < 10$}
   **if** $(x > 0 \lor y < 10) \to y : [y < 10,\ y > 0]$ **fi**
   $\sqsubseteq$ {Assignment: $y < 10 \Rrightarrow y > 0[y \backslash 10]$}
   **if** $(x > 0 \lor y < 10) \to y := 10$ **fi**

   (b) $y : [y < 10, y > 0]$
   $\not\sqsubseteq$ {Selection: $y < 10 \not\Rrightarrow ((x > 0) \land (y < 10))$}
   **if** $((x > 0) \land (y < 10)) \to y : [((x > 0) \land (y < 10)) \land (y < 10),\ y > 0]$ **fi**

5. TODO