Assignment 1: Background theory

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1. (a) $y : [\text{ true}, \ (x = 0 \Rightarrow y = 0) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x}) \ ]$
   (b) $y : [\text{ true}, \ (x = 0 \Rightarrow \text{true}) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x}) \ ]$
   (c) TODO

2.
$$
\begin{array}{ll}
 & x, y : [\text{true}, x = z^2 \wedge y = z^4] \\
\sqsubseteq & \{\text{Composition}\} \\
 & x, y : [\text{true}, x = z^2]; \ x, y : [x = z^2, x = z^2 \wedge y = z^4] \\
\sqsubseteq & \{\text{Assignment: true} \Rrightarrow x = z^2[x \backslash z^2]\} \\
 & x = z^2; \ x, y : [x = z^2, x = z^2 \wedge y = z^4] \\
\sqsubseteq & \{\text{Assignment: } x = z^2 \Rrightarrow x = z^2 \wedge y = z^4[y \backslash x^2]\} \\
 & x := z^2; \ y := x^2
\end{array}
$$

3. (a) Assuming

$$
\begin{array}{rcl}
wp(y := 10, \ \text{true}) & \equiv & \text{true}[y \backslash 10] \\
 & \equiv & \text{true}
\end{array}
$$

   we can conclude that

$$
\begin{array}{rcl}
wp(\textbf{if } (x > 0 \ \vee \ y < 10) \rightarrow y := 10 \textbf{ fi}, \ \text{true}) & \equiv & (x > 0 \ \vee \ y < 10) \ \wedge \\
 & & ((x > 0 \ \vee \ y < 10) \rightarrow wp(y := 10, \ \text{true})) \\
 & \equiv & (x > 0 \ \vee \ y < 10) \ \wedge \ \text{true} \\
 & \equiv & (x > 0 \ \vee \ y < 10)
\end{array}
$$

   As $y < 10 \Rrightarrow (x > 0 \vee y < 10)$, the Hoare triple is true.

   (b) Assuming

$$
wp(x := x + y, \ P[x \backslash x + y]) \quad \equiv \quad (P[x \backslash x + y])[x \backslash x + y]
$$

   TODO

4. (a)
$$
\begin{array}{ll}
 & y : [y < 10, y > 0] \\
\sqsubseteq & \{\text{Selection: } y < 10 \Rrightarrow (x > 0 \vee y < 10)\} \\
 & \textbf{if } (x > 0 \vee y < 10) \rightarrow y : [(x > 0 \vee y < 10) \wedge (y < 10), \ y > 0] \textbf{ fi} \\
\sqsubseteq & \{\text{Absorption 1: } (x > 0 \vee y < 10) \wedge (y < 10) = y < 10\} \\
 & \textbf{if } (x > 0 \vee y < 10) \rightarrow y : [y < 10, \ y > 0] \textbf{ fi} \\
\sqsubseteq & \{\text{Assignment: } y < 10 \Rrightarrow y > 0[y \backslash 10]\} \\
 & \textbf{if } (x > 0 \vee y < 10) \rightarrow y := 10 \textbf{ fi}
\end{array}
$$

   (b)
$$
\begin{array}{ll}
 & y : [y < 10, y > 0] \\
\not\sqsubseteq & \{\text{Selection: } y < 10 \not\Rrightarrow ((x > 0) \wedge (y < 10))\} \\
 & \textbf{if } ((x > 0) \wedge (y < 10)) \rightarrow y : [((x > 0) \wedge (y < 10)) \wedge (y < 10), \ y > 0] \textbf{ fi}
\end{array}
$$

5. TODO