

Assignment 3: Derivation

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1. (a) n is a **value** parameter. m is a **result** parameter.
- (b) $inv \triangleq lrun(A, n_0, m)$
- (c) Let

$$\begin{aligned} pre(A, n) &\triangleq lrun(A, n, n + 1) \\ inv(A, n, m) &\triangleq lrun(A, n_0, m) \\ post(A, n, m) &\triangleq mrun(A, n_0, m) \end{aligned}$$

by 1(b), and the specification of the procedure. inv , pre and $post$ implicitly capture variables (A, n, m) as parameters from the frame, for syntactic convenience.

s.t.

$$n, m : [pre, post]$$

$$\sqsubseteq \{ \text{Composition: middle predicate is } inv \}$$

$$n, m : [pre, inv]; \quad n, m : [inv, post]$$

$$\sqsubseteq \{ \text{Assignment: } pre \Rightarrow inv[m \setminus n + 1] \}$$

$$m := n + 1; \quad n, m : [inv, post]$$

\therefore

$$\begin{aligned} inv[m \setminus n + 1] &\equiv lrun(A, n_0, m)[m \setminus n + 1] \\ &\equiv lrun(A, n_0, n + 1) \end{aligned}$$

\therefore

$$lrun(A, n, n + 1) \Rightarrow lrun(A, n_0, n + 1)$$

Let

$$guard \triangleq (m < A.len \wedge A_{n_0} = A_m)$$

s.t.

$$\sqsubseteq \{ \text{Strengthen post: } inv \wedge \neg guard \Rightarrow post \}$$

$$m := n + 1; \quad n, m : [inv, inv \wedge \neg guard]$$

\therefore

$$inv \wedge \neg guard \Rightarrow post$$

$$\equiv \{ \text{Expansion of definitions} \}$$

$$lrun(A, n_0, m) \wedge \neg(m < A.len \wedge A_{n_0} = A_m) \Rightarrow mrun(A, n_0, m)$$

$$\equiv \{ \text{Expansion of functions} \}$$

$$lrun(A, n_0, m) \wedge \neg(m < A.len \wedge A_{n_0} = A_m) \Rightarrow lrun(A, n_0, m) \wedge (m < A.len \Rightarrow A_{n_0} \neq A_m)$$

$$\equiv \{ \text{De Morgan's law - negation of conjunction} \}$$

$$\begin{aligned}
& lrun(A, n_0, m) \wedge (\neg(m < A.len) \vee \neg(A_{n_0} = A_m)) \Rightarrow lrun(A, n_0, m) \wedge (m < A.len \Rightarrow A_{n_0} \neq A_m) \\
\equiv & \{P \Rightarrow Q \equiv \neg P \vee Q\} \\
& lrun(A, n_0, m) \wedge (\neg(m < A.len) \vee \neg(A_{n_0} = A_m)) \Rightarrow lrun(A, n_0, m) \wedge (\neg(m < A.len) \vee (A_{n_0} \neq A_m)) \\
\equiv & \{\} \\
& \text{true}
\end{aligned}$$

$$\begin{aligned}
\sqsubseteq & \{\text{Repetition}\} \\
& m := n + 1; \\
& \mathbf{do} \ (m < A.len \wedge A_{n_0} = A_m) \rightarrow \\
& \quad n, m : [inv \wedge guard, inv \wedge (0 \leq V < V_0)] \\
& \mathbf{od}
\end{aligned}$$

where

$$V \triangleq A.len - m$$

$$\begin{aligned}
\sqsubseteq & \{\text{Assignment: } inv \wedge \neg guard \Rightarrow (inv \wedge (0 \leq V < V_0))[m \setminus m + 1]\} \\
& m := n + 1; \\
& \mathbf{do} \ (m < A.len \wedge A_{n_0} = A_m) \rightarrow \\
& \quad m := m + 1 \\
& \mathbf{od}
\end{aligned}$$

\therefore

$$\begin{aligned}
(inv \wedge (0 \leq V < V_0))[m \setminus m + 1] & \equiv (lrun(A, n_0, m) \wedge (0 \leq (A.len - m) < (???)))[m \setminus m + 1] \\
& \equiv lrun(A, n_0, m + 1) \wedge (0 \leq (A.len - (m + 1)) < (???))
\end{aligned}$$