

Assignment 1: Background theory

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1. (a) $y : [\text{true}, (x = 0 \Rightarrow y = 0) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x})]$
 (b) $y : [\text{true}, (x = 0 \Rightarrow \text{true}) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x})]$
 (c) TODO
2. $x, y : [\text{true}, x = z^2 \wedge y = z^4]$
 $\sqsubseteq \{ \text{Composition} \}$
 $x, y : [\text{true}, x = z^2]; x, y : [x = z^2, x = z^2 \wedge y = z^4]$
 $\sqsubseteq \{ \text{Assignment: } \text{true} \Rightarrow x = z^2[x \setminus z^2] \}$
 $x = z^2; x, y : [x = z^2, x = z^2 \wedge y = z^4]$
 $\sqsubseteq \{ \text{Assignment: } x = z^2 \Rightarrow x = z^2 \wedge y = z^4[y \setminus x^2] \}$
 $x := z^2; y := x^2$

3. (a) Assuming

$$\begin{aligned} wp(y := 10, \text{true}) &\equiv \text{true}[y \setminus 10] \\ &\equiv \text{true} \end{aligned}$$

we can conclude that

$$\begin{aligned} wp(\text{if } (x > 0 \vee y < 10) \rightarrow y := 10 \text{ fi}, \text{true}) &\equiv (x > 0 \vee y < 10) \wedge \\ &\quad ((x > 0 \vee y < 10) \rightarrow wp(y := 10, \text{true})) \\ &\equiv (x > 0 \vee y < 10) \wedge \text{true} \\ &\equiv (x > 0 \vee y < 10) \end{aligned}$$

As $y < 10 \Rightarrow (x > 0 \vee y < 10)$, the Hoare Triple is true.

- (b) Assuming

$$\begin{aligned} wp(x := x + y, P[x \setminus x + y]) &\equiv (P[x \setminus x + y])[x \setminus x + y] \\ &\equiv P[x \setminus (x + y) + y] \\ &\equiv P[x \setminus x + 2y] \end{aligned}$$

Using this

$$\{ \text{true}[x \setminus x + 2y] \} x := x + y \{ \text{true}[x \setminus x + y] \}$$

holds but

$$\{ (x > 8)[x \setminus x + 2y] \} x := x + y \{ (x > 8)[x \setminus x + y] \} \equiv \{ x + 2y > 8 \} x := x + y \{ x + y > 8 \}$$

does not. Therefore, the choice of P determines the validity of the Hoare Triple, due to P having arbitrary. Assuming that P is quantified over all possible predicates, the Hoare Triple does not hold

4. (a) $y : [y < 10, y > 0]$
 $\sqsubseteq \{ \text{Selection: } y < 10 \Rightarrow (x > 0 \vee y < 10) \}$
if $(x > 0 \vee y < 10) \rightarrow y : [(x > 0 \vee y < 10) \wedge (y < 10), y > 0]$ **fi**
 $\sqsubseteq \{ \text{Absorption 1: } (x > 0 \vee y < 10) \wedge (y < 10) = y < 10 \}$
if $(x > 0 \vee y < 10) \rightarrow y : [y < 10, y > 0]$ **fi**
 $\sqsubseteq \{ \text{Assignment: } y < 10 \Rightarrow y > 0[y \backslash 10] \}$
if $(x > 0 \vee y < 10) \rightarrow y := 10$ **fi**
- (b) $y : [y < 10, y > 0]$
 $\not\sqsubseteq \{ \text{Selection: } y < 10 \not\Rightarrow ((x > 0) \wedge (y < 10)) \}$
if $((x > 0) \wedge (y < 10)) \rightarrow y : [((x > 0) \wedge (y < 10)) \wedge (y < 10), y > 0]$ **fi**
5. $w : [P, Q]$
 $\sqsubseteq \{ \text{Composition} \}$
 $w : [P, I]; w : [I, Q]$
 $\sqsubseteq \{ \text{Strengthen Postcondition: } I \wedge \neg(\neg B) \Rightarrow Q \}$
 $w : [P, I]; w : P[I, I \wedge \neg(\neg B)]$
 $\sqsubseteq \{ \text{Repetition} \}$
 $w : [P, I]; \text{do } (\neg B) \rightarrow w : [I \wedge \neg B, I \wedge (0 \leq V < V_0)] \text{ od}$