Assignment 1: Background theory

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1. (a) $y : [\ \text{true},\ (x = 0 \Rightarrow y = 0) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x})\ ]$
   (b) $y : [\ \text{true},\ (x = 0 \Rightarrow \text{true}) \wedge (x \neq 0 \Rightarrow y = \frac{y_0}{x})\ ]$
   (c) TODO

2. $$
   \begin{array}{ll}
   & x, y : [\text{true}, x = z^2 \wedge y = z^4] \\
   \sqsubseteq & \{\text{Composition}\} \\
   & x, y : [\text{true}, x = z^2];\ x, y : [x = z^2, x = z^2 \wedge y = z^4] \\
   \sqsubseteq & \{\text{Assignment: true} \Rrightarrow x = z^2[x \backslash z^2]\} \\
   & x = z^2;\ x, y : [x = z^2, x = z^2 \wedge y = z^4] \\
   \sqsubseteq & \{\text{Assignment: } x = z^2 \Rrightarrow x = z^2 \wedge y = z^4[y \backslash x^2]\} \\
   & x := z^2;\ y := x^2
   \end{array}
   $$

3. (a) Assuming

   $$
   \begin{array}{rcl}
   wp(y := 10,\ \text{true}) & \equiv & \text{true}[y \backslash 10] \\
   & \equiv & \text{true}
   \end{array}
   $$

   we can conclude that

   $$
   \begin{array}{rcl}
   wp(\textbf{if}\ (x > 0\ \vee\ y < 10) \rightarrow y := 10\ \textbf{fi},\ \text{true}) & \equiv & (x > 0\ \vee\ y < 10)\ \wedge \\
   & & ((x > 0\ \vee\ y < 10) \rightarrow wp(y := 10,\ \text{true})) \\
   & \equiv & (x > 0\ \vee\ y < 10)\ \wedge\ \text{true} \\
   & \equiv & (x > 0\ \vee\ y < 10)
   \end{array}
   $$

   As $y < 10 \Rrightarrow (x > 0 \vee y < 10)$, the Hoare triple is true.

   (b) Assuming

   $$
   \begin{array}{rcl}
   wp(x := x + y,\ P[x \backslash x + y]) & \equiv & (P[x \backslash x + y])[x \backslash x + y] \\
   & \equiv & P[x \backslash (x + y) + y] \\
   & \equiv & P[x \backslash x + 2y]
   \end{array}
   $$

   TODO

4. (a) $$
   \begin{array}{ll}
   & y : [y < 10, y > 0] \\
   \sqsubseteq & \{\text{Selection: } y < 10 \Rrightarrow (x > 0 \vee y < 10)\} \\
   & \textbf{if}\ (x > 0 \vee y < 10) \rightarrow y : [(x > 0 \vee y < 10) \wedge (y < 10),\ y > 0]\ \textbf{fi} \\
   \sqsubseteq & \{\text{Absorption 1: } (x > 0 \vee y < 10) \wedge (y < 10) = y < 10\} \\
   & \textbf{if}\ (x > 0 \vee y < 10) \rightarrow y : [y < 10,\ y > 0]\ \textbf{fi} \\
   \sqsubseteq & \{\text{Assignment: } y < 10 \Rrightarrow y > 0[y \backslash 10]\} \\
   & \textbf{if}\ (x > 0 \vee y < 10) \rightarrow y := 10\ \textbf{fi}
   \end{array}
   $$

(b)
$$
\begin{array}{ll}
 & y : [y < 10, y > 0] \\
\not\sqsubseteq & \{\text{Selection: } y < 10 \not\Rightarrow ((x > 0) \wedge (y < 10))\} \\
 & \textbf{if } ((x > 0) \wedge (y < 10)) \rightarrow y : [((x > 0) \wedge (y < 10)) \wedge (y < 10),\ y > 0] \textbf{ fi}
\end{array}
$$

5. TODO