Assignment 3: Derivation

A run in an array is a non-empty subarray whose values are all the same

$$\mathit{run}(A,i,j) \mathrel{\widehat{=}} (0 \le i < j \le A.\mathrm{len}) \wedge (\forall p,q : [i,j) \cdot A_p = A_q)$$

A left run in an array is a run in the array that cannot be extended further to the left

$$\mathit{lrun}(A,i,j) \mathrel{\widehat{=}} \mathit{run}(A,i,j) \wedge (i > 0 \Rightarrow A_{i-1} \neq A_i)$$

A maximal run in an array is a left run in the array that cannot be extended further to the right.

$$\mathit{mrun}(A,i,j) \mathrel{\widehat{=}} \mathit{lrun}(A,i,j) \wedge (j < A.\mathrm{len} \Rightarrow A_i \neq A_j)$$

Answer the following questions:

1. A procedure *FindMaxRun*(n,m) finds the maximal run in A beginning at the provided index n, where n is required to be the beginning of a left run. The specification of this procedure is

   $$n,m : [\mathit{lrun}(A,n,n+1), \mathit{mrun}(A,n_0,m)]$$

   Provide an implementation of the procedure by

   (a) deciding on whether n and m are **value**, **result** or **value result** parameters

   (b) suggesting a suitable invariant for deriving code for this procedure using one of the patterns for finding an invariant from the lectures

   (c) using your invariant to derive the procedure's code.

2. A program to find indices ℓ and h such that $A_{[\ell,h)}$ is a shortest maximal run in the non-empty array A is specified as follows.

   $$\ell,h : [A.\mathrm{len} > 0, \mathit{mrun}(A,\ell,h) \wedge (\forall p,q \cdot \mathit{mrun}(A,p,q) \Rightarrow (h-\ell) \le (q-p))]$$

   Suggest a suitable invariant for deriving code for this procedure using one of the patterns for finding an invariant from the lectures. Use your invariant to derive the program's code to *loop level*, i.e., once you have determined the specification of the loop's body you need not refine it further. However, all refinement steps of code outside the loop body must be shown and justified.

   Hint: Your program may call the procedure *FindMaxRun* to help establish the loop invariant before entering the loop.

   Note: For those using Latex, ℓ can be produced by \ell when in a maths environment.

Marking

The assignment is worth 20% of your final grade.

The marks for each question are as follows:

1.(a) — 2 marks
1.(b) — 2 marks
1.(c) — 6 marks

2. Invariant — 2 marks
   Derivation — 8 marks

A mark of zero will be given for work with little or no academic merit.

Late submission

The submission of this assignment by the due date is the sole responsibility of the student. Students should not leave assignment preparation until the last minute and must plan their workloads to meet the deadline. It is your responsibility to manage your time effectively.

Assessment items received after the due date will receive a mark of zero unless you have been approved to submit the assessment item after the due date as set out in the Electronic Course Profile.

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http://www.itee.uq.edu.au/itee-student-misconduct-including-plagiarism