MATH3202 - Integer Programming - Section A

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Sets

J	Set of juices
G	Set of gourmet juices, where $G \subsetneq J$
F	Set of types of fruit
D	Set of types of deliverable fruit[1], where $D \subsetneq F$
Q	Set of quarters

Data

p_{jf}	Proportion $\in \mathbb{N}_{[0,1]}$ of fruit $f \in F$ in juice $j \in J$
c_f	Cost (\$/kL) of local fruit $f \in F$
d_{jq}	Anticipated ability to sell kL of juice $j \in J$ in quarter $q \in Q$
b_q	Demand of kL of orange juice in Brisbane in quarter $q \in Q$
g_j	Whether a juice $j \in J$ is gourmet, g_j is 1 if g_j is gourmet
r	Cost (\$/kL) of reconstituted orange juice
s	Sell price (\$/kL) of any juice $j \in J$
l	Truck delivery size (kL) of any fruit $f \in F$

Variables

x_{jq}	Number of kL of juice $j \in J$ produced in quarter $q \in Q$
t_{fq}	Number of trucks delivering a given fruit $f \in F$ in quarter $q \in Q$
g_{jq}	Whether juice $j \in G$ is produced in quarter $q \in Q$

Objectives

$$\max \sum_{j \in J} \sum_{q \in Q} \left(x_{jq} \cdot s - \ x_{jq} \cdot \left(\sum_{f \in F} p_{jf} \cdot c_f \right) \right)$$

$$\max \left(\sum_{j \in J} \sum_{q \in Q} x_{jq} \cdot s \right) - \left(\sum_{f \in D} \sum_{q \in Q} t_{fq} \cdot l \cdot c_f \right) - \left(\sum_{j \in J} \sum_{q \in Q} x_{jq} \cdot p_{jo} \cdot r \right)$$

where $o \in F$, where o represents Orange

Constraints

$$\begin{aligned}
x_{jq} &\geq 0 && \forall j \in J,\ \forall q \in Q && (1)\\
t_{fq} &\geq 0 && \forall f \in F,\ \forall q \in Q && (2)\\
g_{jq} &\in \{0,1\} && \forall j \in J,\ \forall q \in Q && (3)\\
x_{jq} &\leq d_{jq} && \forall j \in J,\ \forall q \in Q && (4)\\
\sum_{j \in J} x_{jq} p_{jo} &\leq b_q && \forall q \in Q && (5)\\
\sum_{j \in J} x_{jq} p_{jf} &\leq t_{fq} l && \forall f \in D,\ \forall q \in Q && (6)\\
\sum_{j \in G} g_{jq} &= 2 && \forall q \in Q && (7)\\
x_{jq} &\leq g_{jq} d_{jq} && \forall j \in G,\ \forall q \in Q && (8)\\
g_{jq} + g_{j(next(q))} &\geq 1 && \forall j \in G,\ \forall q \in Q \setminus \{l\} && (9)
\end{aligned}$$

where $o \in F$, where o represents Orange, where $l \in Q$, where l is the last quarter, where $next$ yields the successive quarter for some $q \in Q \setminus \{l\}$.

- Constraints (1) and (2) are basic non-negativity constraints on x and t.
- Constraint (3) ensures g is a binary variable.
- Constraint (4) ensures the production of juice does not exceed the anticipated ability to sell that juice
- Constraint (5) ensures that the amount of orange juice is limited by the demand we noted for our original shipping schedule into Brisbane
- Constraint (6) ensures that each quantity of juice is limited by the supply of its constituent fruits by trucks
- Constraint (7) ensures that we only produce two gourmet juices each quarter
- Constraint (8) binds our decision variable choosing which gourmet juice we choose to produce, by reiterating constraint (4) when production is enabled, and capping production at 0 when disabled
- Constraint (9) ensures that no gourmet juice is out of production for more than one quarter in a row, by ensuring that adjacent quarters have at least 1 decision variable enabled

Sets

L Set of locations

Data

c_{ft} Cost ($) of traveling from location $f \in L$ to location $t \in L$

Variables

t_{ft} Decision to travel from location $f \in L$ to location $t \in L$

Objectives

$$\max \sum_{f \in L} \sum_{t \in L} t_{ft} c_{ft}$$

Constraints

$$\sum_{f \in L} t_{ft} = 1 \qquad \forall t \in L \tag{10}$$

$$\sum_{t \in L} t_{ft} = 1 \qquad \forall f \in L \tag{11}$$

$$t_{ft} + t_{tf} \leq 1 \qquad \forall f \in L,\ \forall t \in L \tag{12}$$

- Constraint (10) ensures that each location is departed from only once
- Constraint (11) ensures that each location is arrived at only once
- Constraint (12) ensures that there are no two location loops