

PHIL3110 - Assignment 1

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Part A

Problem 1

$$1. \quad \frac{\frac{Pa \vee Qa^{(1)} \quad (Pa \vee Qa) \rightarrow \perp}{\perp} \quad \frac{\perp}{\neg(Pa \vee Qa)} \quad (1) \ (\neg I)}{\neg Pa \wedge \neg Qa} \quad (DM-1)$$

De Morgan's Law 1

$$\frac{\frac{\frac{\varphi^{(1)}}{\varphi \vee \psi} \quad \neg(\varphi \vee \psi)}{\perp} \quad (1) \ (\neg I) \quad \frac{\frac{\frac{\psi^{(2)}}{\varphi \vee \psi} \quad \neg(\varphi \vee \psi)}{\perp} \quad (2) \ (\neg I)}{\neg\varphi \wedge \neg\psi}}$$

$$2. \quad \frac{Qa \vee Ra^{(3)} \quad \frac{Qa^{(1)} \quad Qa \rightarrow Pa}{Pa} \quad \frac{Ra^{(2)} \quad Ra \rightarrow Pa}{Pa}}{\frac{Pa}{(Qa \vee Ra) \rightarrow Pa} \quad (3) \ (\rightarrow I)} \quad (1) \ (2) \ (\vee E)$$

Problem 2

1.

$$Q^{\mathcal{M}} = \{m_1\}$$

$$T^{\mathcal{M}} = \{\langle m_1, m_1 \rangle, \langle m_1, m_2 \rangle, \langle m_2, m_2 \rangle\}$$

2. Distressingly, \mathcal{L} does not define any constant symbols, nor does \mathcal{M} provide interpretations of constant symbols in \mathcal{M} .

Thus

$$\mathcal{M} \not\models \exists x \neg Txx$$

However assuming \mathcal{M}^+ , where \mathcal{M}^+ is the expanded model \mathcal{M} , where $m^{\mathcal{M}} = m$ for all $m \in M$, we see that

$$M \models \exists x \neg Txx$$

as

$$\langle m_3^{\mathcal{M}}, m_3^{\mathcal{M}} \rangle \notin T^{\mathcal{M}}$$

3. No, as \mathcal{M} does not define any constant symbols $\mathcal{M} \not\models \exists x \varphi$ for some arbitrary φ (as x will bind no constant symbols), and thus

$$\mathcal{M} \not\models \exists x \forall y (Qy \leftrightarrow Tyx)$$

Assuming \mathcal{M}^+ ,

$$\mathcal{M}^+ \not\models \exists x \forall y (Qy \leftrightarrow Tyx)$$

By fixing x to $m_1^{\mathcal{M}}$, we see that

$$\forall y \cdot y \in Q^{\mathcal{M}} \leftrightarrow \langle y, m_1^{\mathcal{M}} \rangle \in T^{\mathcal{M}}$$

as

$$\begin{aligned} m_1^{\mathcal{M}} &\in Q^{\mathcal{M}} \text{ and } \langle m_1^{\mathcal{M}}, m_1^{\mathcal{M}} \rangle \in T^{\mathcal{M}} \\ m_2^{\mathcal{M}} &\notin Q^{\mathcal{M}} \text{ and } \langle m_2^{\mathcal{M}}, m_1^{\mathcal{M}} \rangle \notin T^{\mathcal{M}} \\ m_3^{\mathcal{M}} &\notin Q^{\mathcal{M}} \text{ and } \langle m_3^{\mathcal{M}}, m_1^{\mathcal{M}} \rangle \notin T^{\mathcal{M}} \end{aligned}$$

Part B

Problem 3

1. (a) *BHMB* is not a good sandwich
 (b) $((B)JB)HB$ is a good sandwich
 (c) *BHBMBJM* is not a good sandwich
2. **Definition 1** A sandwich is good iff there is some stage n such that sandwich $\in \text{Stage}(n)$ where:
 - $\text{Stage}(0) = \{B\}$
 - $\text{Stage}(n+1)$ is the set of φ such that either:
 - (a) $\varphi \in \text{Stage}(n)$
 - (b) φ is of the form ψMB , ψHB , or ψJB , where $\psi \in \text{Stage}(n)$
3. Make A the set of all n such that for each $\psi \in \text{Stage}(n)$, ψ does not contain any two instances of the same ingredient adjacently.

Base

$0 \in A$. As $\text{Stage}(0) = \{B\}$, all $\varphi \in \text{Stage}(0)$ do not contain any two instances of the same ingredient adjacently. We can also see that all $\varphi \in \text{Stage}(0)$ are terminated by B

Induction Step

Suppose $n \in A$.

Lemma 1 Since $n \in A$, and every $\varphi \in \text{Stage}(n)$ is of the form B , ψMB , ψHB , or ψJB , for $\psi \in \text{Stage}(m)$ for some $m < n$, all φ are terminated by B .

Lemma 2 Since $n \in A$, every $\varphi \in \text{Stage}(n)$ does not contain any two instances of the same ingredient adjacently.

For each $\varphi \in \text{Stage}(n+1) \setminus \text{Stage}(n)$, recalling the definition for $\text{Stage}(n+1)$, φ is of the form ψMB , ψHB , or ψJB , for some $\psi \in \text{Stage}(n)$

Given

- all $\varphi \in \text{Stage}(n)$ are terminated by B (Lemma 1)
- all $\varphi \in \{MB, HB, JB\}$ don't begin with B , and are not themselves adjacent ingredients
- Lemma 2

all $\varphi \in \text{Stage}(n+1) \setminus \text{Stage}(n)$ do not contain two instances of the same ingredient adjacently.

Thus, $n+1 \in A$. Then by induction, we see that every $n \in A$.

0.1 Problem