ASSIGNMENT 2

Problem 1. (10 marks) Let A, B and C be sets such that $A \precsim B$ and $B \precsim C$. Is it the case that $A \precsim C$? If so, prove it; otherwise provide a counterexample.

Problem 2. (20 Marks) Complete the following:

(1) Note that $\varphi_1 := \exists x\, x = x$ says that there is at least one object. Show that there are sentences, φ_n, which say that for any natural number n, there are at least n objects.

(2) Let T be a theory which has a model of cardinality n for every natural number n; i.e., T has models of arbitrarily large finite cardinality. Let S be the theory consisting of the sentences in T and every sentence φ_n as described above; i.e,

$$S = T \cup \{\varphi_n \mid n \in \omega\}.$$

Show that every finite subset Δ of T has a model.

(3) Use the result of the previous step and the compactness theorem to show that T has an infinite model.

Problem 3. (20 marks) Describe a strategy for a Turing machine and provide an algorithm using streamlined notation which starts with a block of n-many 1's and outputs a block of $2n$-many 1's after the original block with a single blank space separating them. The machine should start under the left most 1 and finish under the leftmost 1.

Thus, for example, if the inital state of the tape and machine is as follows:

	1	1	1	1																			
	$\hat{q}_0$																						

.

The machine should halt in the following state:

	1	1	1	1		1	1	1	1	1	1	1	1										
	$\hat{q}_{Halt}$																						

.

Assume that the tape is infinite to the right, but has only one square to the left of the initial state of the head of the machine.

Important: Submit the code for this machine as a .txt file that will run on the Turing machine simulator (http://morphett.info/turing/turing.html). Submit the code via email to my address with the subject heading: PHIL3110Turing. Also submit a printout of the code with the rest of your submission.

Problem 4. (10 marks) (Recursive Sets) Suppose A, B, C are recursively enumerable subsets of the natural numbers such that every natural number is in one and only one of the sets A, B and C. Using Church's thesis if required, show that A, B and C are recursive.