PHIL3110 - Assignment 1

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April 16, 2018

Part A

Problem 1

1. $$\dfrac{\dfrac{Pa \lor Qa^{(1)} \qquad (Pa \lor Qa) \to \bot}{\bot}}{\neg(Pa \lor Qa)}\ (1)\ (\neg I)$$

2. $$\dfrac{\dfrac{Qa \lor Ra^{(3)} \qquad \dfrac{Qa^{(1)} \qquad Qa \to Pa}{Pa} \qquad \dfrac{Ra^{(2)} \qquad Ra \to Pa}{Pa}}{Pa}\ (1)\ (2)\ (\lor E)}{(Qa \lor Ra) \to Pa}\ (3)\ (\to I)$$

Problem 2

1. $$Q^{\mathcal{M}} = \{m_1\}$$
$$T^{\mathcal{M}} = \{\langle m_1, m_1\rangle, \langle m_1, m_2\rangle, \langle m_2, m_2\rangle\}$$

2. Distressingly, $\mathcal{L}$ does not define any constant symbols, nor does $\mathcal{M}$ provide interpretations of constant symbols in $\mathcal{M}$.

   Thus

   $$\mathcal{M} \not\models \exists x\, \neg Txx$$

   However assuming $\mathcal{M}^+$, where $\mathcal{M}^+$ is the expanded model $\mathcal{M}$, where $m^{\mathcal{M}} = m$ for all $m \in M$, we see that

   $$M \models \exists x\, \neg Txx$$

   as

   $$\langle m_3^{\mathcal{M}}, m_3^{\mathcal{M}}\rangle \notin T^{\mathcal{M}}$$

3. No, as $\mathcal{M}$ does not define any constant symbols $\mathcal{M} \not\models \exists x\, \varphi$ for some arbitrary φ (as x will bind no constant symbols), and thus

   $$\mathcal{M} \not\models \exists x \forall y\, (Qy \leftrightarrow Tyx)$$

   Assuming $\mathcal{M}^+$,

   $$\mathcal{M}^+ \not\models \exists x \forall y\, (Qy \leftrightarrow Tyx)$$

   By fixing x to $m_1^{\mathcal{M}}$, we see that

   $$\forall y \cdot y \in Q^{\mathcal{M}} \leftrightarrow \langle y, m_1^{\mathcal{M}}\rangle \in T^{\mathcal{M}}$$

as

$$
\begin{aligned}
&m_1^{\mathcal{M}} \in Q^{\mathcal{M}} \text{ and } \langle m_1^{\mathcal{M}}, m_1^{\mathcal{M}} \rangle \in T^{\mathcal{M}} \\
&m_2^{\mathcal{M}} \notin Q^{\mathcal{M}} \text{ and } \langle m_2^{\mathcal{M}}, m_1^{\mathcal{M}} \rangle \notin T^{\mathcal{M}} \\
&m_3^{\mathcal{M}} \notin Q^{\mathcal{M}} \text{ and } \langle m_3^{\mathcal{M}}, m_1^{\mathcal{M}} \rangle \notin T^{\mathcal{M}}
\end{aligned}
$$