COSC3500 Forum Presentation

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22nd of October

Outline

Description

The task was to create a stock-standard, 2-dimensional gravitational n-body simulator.

All bodies were to be assumed to be point masses. The simulation was to be accurate, maintaining a constant total energy, and exhibiting phenomena such as apsidal precession.

Demo

Integration I

$$F = G\frac{m_1 m_2}{r^2}$$

$$\begin{aligned} a_i &= F(x_i) \\ v_{i+1} &= v_i + a_i \, \Delta t \end{aligned}$$

Integration II

Dehen and Read note that the Euler method ‘performs very poorly in practice’, further noting that ‘errors are proportional to Δt^2’. They contrast it with the second-order *Leapfrog* symplectic integrator, which is ‘heavily used in collisionless N-body applications’.

$$\begin{aligned} x_i &= x_{i-1} + v_{i-1/2}\,\Delta t \\ a_i &= F(x_i) \\ v_{i+1/2} &= v_{i-1/2} + a_i\,\Delta t \end{aligned}$$

which only requires a single acceleration calculation per every two half timesteps

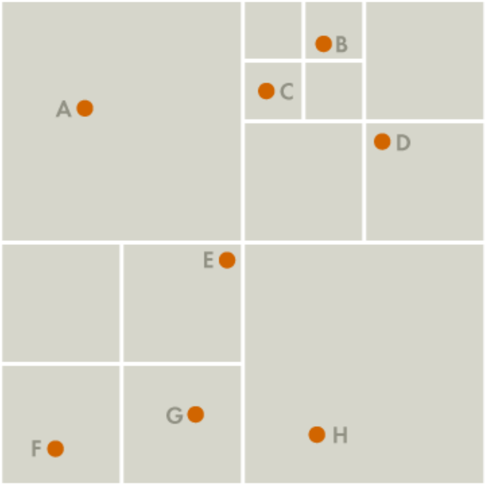
Integration III

and a 'kick-drift-kick' form

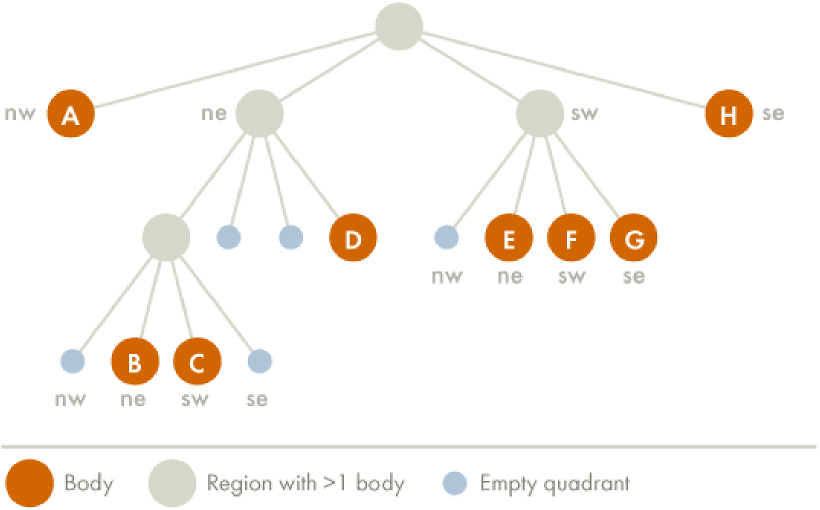
$$\begin{aligned}
v_{i+1/2} &= v_i + a_i \frac{\Delta t}{2} \\
x_{i+1} &= x_i + v_{i+1/2} \Delta t \\
v_{i+1} &= v_{i+1/2} + a_{i+1} \frac{\Delta t}{2}
\end{aligned}$$

that is stable with variable timstepping, but incurs an additional acceleration calculation per every two half timesteps.

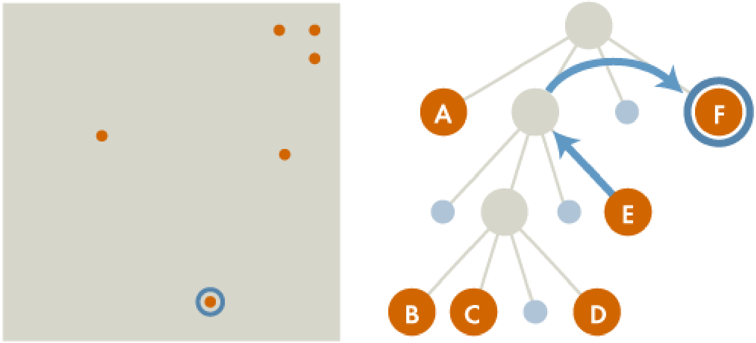
Barnes-Hut I



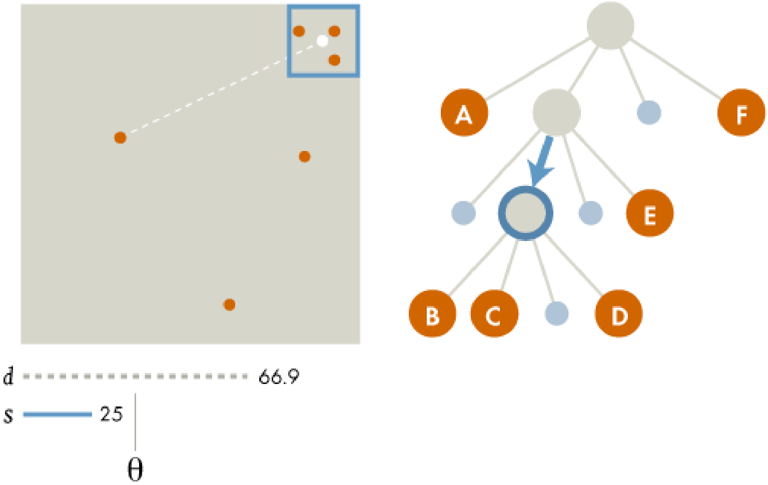
Barnes-Hut II



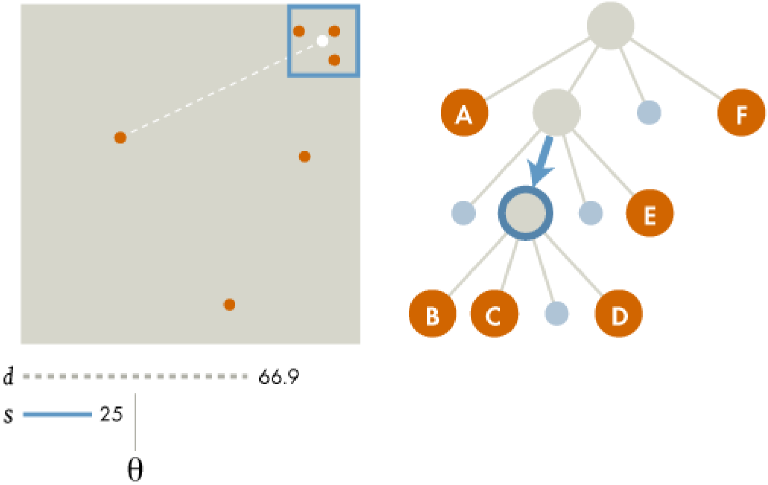
Barnes-Hut III

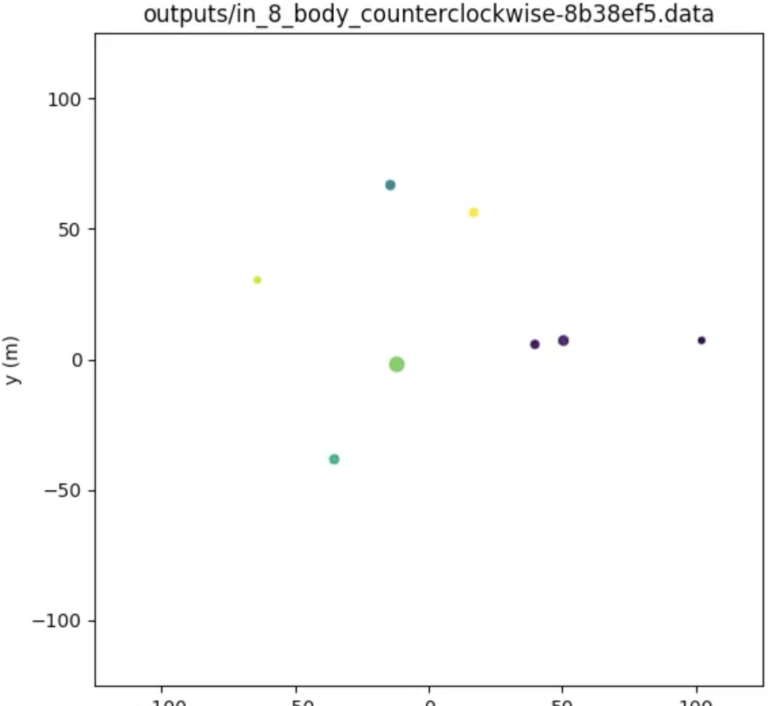


Barnes-Hut IV



Barnes-Hut V

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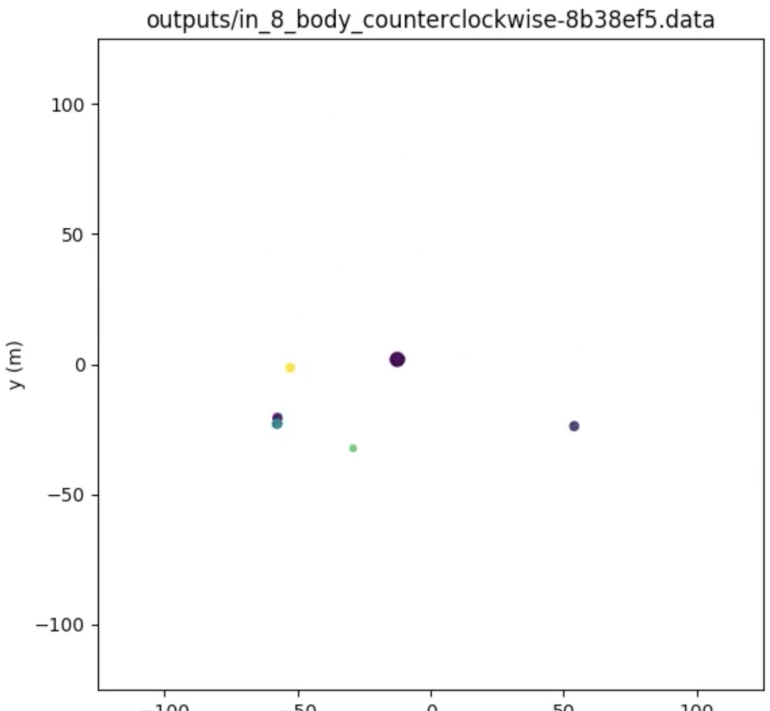
100
50
0
−50
−100
y (m)
−100
−50
0
50
100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y (m)
100
50
0
−50
−100
0
50
100

Readable Mathematics

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n} \sum_{i}^{n} X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.