Maxwell Jung
9319674

Homework 0

CMPSC 165B
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1. Inner Product

   $a = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$

   $a \cdot b = (1 \cdot 3) + (2 \cdot -5) + (4 \cdot 1) = -3$

2. Matrix Multiplication

   $A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix}$

   A and B cannot be multiplied because the number of columns from A does not match the number of rows from B and vice versa.

   $A^\intercal = \begin{bmatrix} 1 & 2 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$

   $A^\intercal \times B = \begin{bmatrix} 1 & 2 \\ 4 & -1 \\ -3 & 3 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix}$

   $= \begin{bmatrix} (1)(-2) + (2)(0) & (1)(0) + (2)(-1) & (1)(5) + (2)(4) \\ (4)(-2) + (-1)(0) & (4)(0) + (-1)(-1) & (4)(5) + (-1)(4) \\ (-3)(-2) + (3)(0) & (-3)(0) + (3)(-1) & (-3)(5) + (3)(4) \end{bmatrix} = \begin{bmatrix} -2 & -2 & 13 \\ -8 & 1 & 16 \\ 6 & -3 & -3 \end{bmatrix}$

   $rank(\begin{bmatrix} -2 & -2 & 13 \\ -8 & 1 & 16 \\ 6 & -3 & -3 \end{bmatrix}) = 2$ because $-5 \begin{bmatrix} -2 \\ -8 \\ 6 \end{bmatrix} - 8 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = 2 \begin{bmatrix} 13 \\ 16 \\ -3 \end{bmatrix}$.

   $\begin{bmatrix} -2 \\ -8 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ are linearly independent. Therefore, $rank(\begin{bmatrix} -2 & -2 & 13 \\ -8 & 1 & 16 \\ 6 & -3 & -3 \end{bmatrix}) = 2$.

   $B \times A^\intercal = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$

   $= \begin{bmatrix} (-2)(1) + (0)(4) + (5)(-3) & (-2)(2) + (0)(-1) + (5)(3) \\ (0)(1) + (-1)(4) + (4)(-3) & (0)(2) + (-1)(-1) + (4)(3) \end{bmatrix} = \begin{bmatrix} -17 & 11 \\ -16 & 13 \end{bmatrix}$

   $rank(\begin{bmatrix} -17 & 11 \\ -16 & 13 \end{bmatrix}) = 2$ because $det(\begin{bmatrix} -17 & 11 \\ -16 & 13 \end{bmatrix}) = (-17 \cdot 13) - (-16 \cdot 11) \neq 0$ and non-zero determinant indicates a full-rank matrix.

3. Juvenile Delinquency

   1. $P(\text{delinquent}) = \frac{33+64}{33+64+343+360} = \frac{97}{800} = 12.125\%$
   2. $P(\text{delinquent}|\text{boy scout}) = \frac{33}{33+343} \approx 8.78\%$

3. since $P(\text{delinquent}) \neq P(\text{delinquent}|\text{boy scout})$, the status of boy scout and juvenile delinquency are dependent.

4. Euclidean projection

   1. x^* is essentially the vector in Ω that has the smallest Euclidean distance from vector y. Therefore, if $y = 1.1$, and $\Omega = \mathbb{N}$, the closet vector is $x^* = 1$ with a distance of 0.1. All other natural numbers have distance greater than 0.1.
   2. As mentioned earlier, x^* is the vector in Ω that's closest to vector y in Euclidean distance.

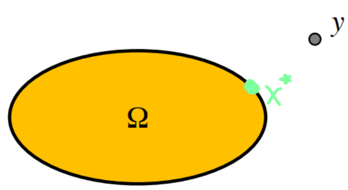

Figure 1: x^* for a given y

5. Python Numpy

```python
import numpy as np

a1 = np.array([1,2,4])
b1 = np.array([3,-5,1])

a2 = np.array([[1,4,-3],
               [2,-1,3]])
b2 = np.array([[-2,0,5],
               [0,-1,4]])

print(f'inner product of {a1} and {b1} is {a1 @ b1}')
print(f'inner product of {a2.T} and {b2} is {a2.T @ b2}')
print(f'inner product of {b2} and {a2.T} is {b2 @ a2.T}')
```

```
inner product of [1 2 4] and [ 3 -5  1] is -3
inner product of [[ 1  2]
 [ 4 -1]
 [-3  3]] and [[-2  0  5]
 [ 0 -1  4]] is [[-2 -2 13]
 [-8  1 16]
 [ 6 -3 -3]]
inner product of [[-2  0  5]
```

```
[ 0 -1  4]] and [[ 1  2]
[ 4 -1]
[-3  3]] is [[-17  11]
[-16  13]]
```