Fall 2022 CS165B: Introduction to Machine Learning – Self Assessment

Due: **Friday, Sep 30th, 11:59 pm PST**

This class is math orientated, which requires you to understand some basics of linear algebra, probability theory, and numerical optimization. This class also requires you to complete machine problems (MPs) using PYTHON and to write your homework using LATEX. Please use the following questions to check if you have enough background knowledge. This homework will only be graded on a binary basis but will count toward your final grade.

1. Given two column vectors $a = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$, what is the value of the inner product of the two vectors? Compute by hand and show the intermediate steps.

2. What is matrix multiplication? Can you multiply two arbitrary matrices? Given two matrices $A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix}$, can you multiply the two matrices? How about A^T and B? When multiplication is possible, compute by hand and show the intermediate steps. What is the *rank* of the resulting matrix?

3. In a study of Boy Scouts and Juvenile Delinquency[1], researchers have obtained the following statistics from past years:

Boy Scout	Delinquent	
	Yes	No
Yes	33	343
No	64	360

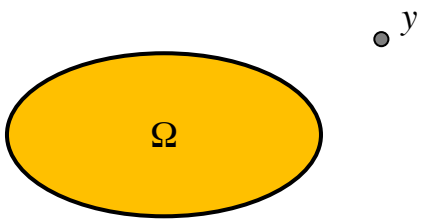
   1. What is the probability of juvenile delinquent?
   2. Given a boy scout, what is the probability of juvenile delinquent?
   3. Is the status of boy scout independent of juvenile delinquent?

4. An *Euclidean projection* of a d-dimensional point $y \in \mathbb{R}^d$ to a set Ω is given by the following optimization problem:

$$x^* = \arg\min_x \|x - y\|_2^2, \quad \text{subject to: } x \in \Omega \tag{1}$$

where Ω is the *feasible set*, $\|\cdot\|_2$ is the ℓ_2 norm of a vector, and $x^* \in \mathbb{R}^d$ is the projected vector.

   1. What is x^* if $y = 1.1$ and $\Omega = \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers.
   2. Locate the x^* in the following picture:



5. Use PYTHON with NUMPY/SCIPY package to verify your solution in Questions 1 and 2.

[1] https://onlinecourses.science.psu.edu/stat504/node/103