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Homework 5

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1. $Q = \begin{bmatrix} 0 & 0_d^T \\ 0_d^T & I_d \end{bmatrix}$

$$u^T Q u = \begin{bmatrix} u_0 & u_1 & u_2 & \dots & u_d \end{bmatrix} \begin{bmatrix} 0 & 0_d^T \\ 0_d^T & I_d \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_d \end{bmatrix}$$

$$= \begin{bmatrix} u_0 & u_1 & u_2 & \dots & u_d \end{bmatrix} \begin{bmatrix} 0 \\ u_1 \\ u_2 \\ \vdots \\ u_d \end{bmatrix}$$

$$= \sum_{i=1}^{d} u_i^2 \geq 0$$

2. $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

   $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, y_1 = -1$

   $x_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, y_2 = -1$

   $x_3 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, y_3 = +1$

   equation 1: $-b \geq 1$

   equation 2: $w_2 - b \geq 1$

   equation 3: $-2w_1 + b \geq 1$

   equation (-1): $b \leq -1$

   equation (1+2): $w_2 \geq 0$

   equation (1+3): $w_1 \leq -1$

   $\frac{1}{2}w^T w = \frac{1}{2}(w_1^2 + w_2^2) \geq \frac{1}{2}$

   $w^* = \begin{bmatrix} w_1 = -1 \\ w_2 = 0 \end{bmatrix}$

   $b* = -1$

   margin: $\frac{1}{||w||} = \frac{1}{1} = 1$

Homework 5

3. (a) QP solver on Question 2 dataset (previous problem). Output is in the form $\begin{bmatrix} b^* \\ w^* \end{bmatrix}$

hw5 > q3a.py > ...

```python
import numpy as np
from cvxopt import matrix, solvers

X = np.array([[0, 0], [0, -1], [-2, 0]])
y = np.array([-1, -1, 1]).reshape(-1, 1)

dim = X.shape[1]  # dimensionality
num = X.shape[0]  # sample size

Q = np.identity(dim+1)
Q[0, 0] = 0
p = np.zeros((dim+1, 1))
A = np.array(np.concatenate((y, y * X), axis=1)).astype(float)
c = np.ones((num, 1))

sol = solvers.qp(P=matrix(Q), q=matrix(p), G=matrix(-A), h=matrix(-c))
print(sol['x'])

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL GITLENS: VISUAL FILE HISTORY JUPYTER

```
PS C:\Users\Maxwell Jung\UCSB\CS 165B\hw5> python q3a.py
     pcost       dcost       gap    pres   dres
 0:  2.5606e-01  4.4983e-01  8e-01  1e+00  6e-16
 1:  3.8605e-01  4.8351e-01  4e-02  2e-01  4e-16
 2:  5.0271e-01  4.9877e-01  4e-03  6e-17  1e-15
 3:  5.0021e-01  4.9981e-01  4e-04  1e-16  1e-16
 4:  5.0003e-01  4.9997e-01  6e-05  3e-17  1e-16
 5:  5.0000e-01  5.0000e-01  8e-06  3e-16  6e-16
 6:  5.0000e-01  5.0000e-01  1e-06  3e-16  2e-16
 7:  5.0000e-01  5.0000e-01  2e-07  9e-18  5e-16
Optimal solution found.
[-1.00e+00]
[-1.00e+00]
[ 3.93e-04]
```

Figure 1: Toy Dataset QP Solver using CVXOPT package

(b) QP solver execution time: