Introduction to Spatial Data

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Course Outline

- Introduction – types of spatial data, goals of analysis
- Exploratory data analysis – plotting, variograms
- Modeling – Gaussian Processes (GP), spatial prediction (kriging)
- Estimation – variogram fitting, spatial regression and GLM
- Bayesian modeling – Metropolis Hastings, Gibbs sampler
- Large data – computing challenges, efficient alternatives

More about the course

- Evaluation – presenting a paper on large scale spatial analysis
- Materials available on `https://github.com/abhirupdatta/spatial-course-2017`
- Texts for reference:
  - (Main) Banerjee, S., Carlin, B. P., and Gelfand, A. E. (2014), Hierarchical Modeling and Analysis for Spatial Data, Boca Raton, FL: Chapman and Hall/CRC, 2nd ed (BCG)
  - Cressie, N. A. C. and Wikle, C. K. (2011), Statistics for spatio-temporal data, Hoboken, NJ: Wiley, Wiley Series in Probability and Statistics

What is spatial data?

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- Any data with some geographical information

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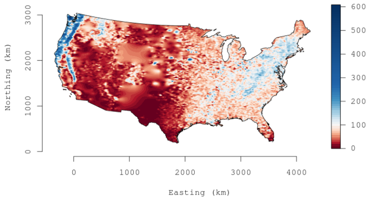
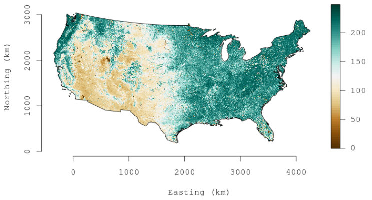
- Example: US forest biomass data



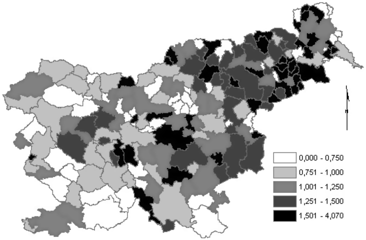
(a) US forest biomass data



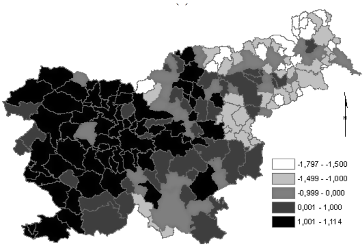
(b) NDVI (predictor)

What is spatial data?

- Any data with some geographical information
- Example: Slovenia stomach cancer data



(a) Standardized cancer incidence



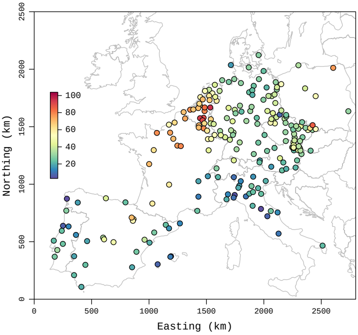
(b) Socioeconomic score (predictor)

What is spatial data?

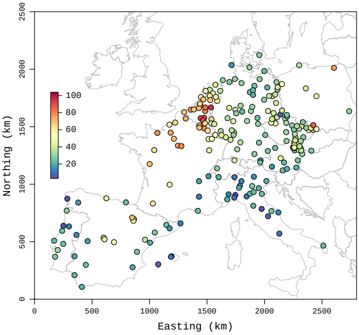
- Any data with some geographical information
- Common sources of spatial data: climatology, forestry, ecology, environmental health, disease epidemiology, real estate marketing etc
  - have many important predictors and response variables,
  - are often presented as maps,
- Other examples where the space need not be the space on earth:
  - Neuroimaging (data for each voxel in the brain)
  - Genetics (position along a chromosome)

Spatio-temporal Data

- Data from multiple timepoints at some or each of the locations
- Example: European air pollution data



(a) PM_{10} levels in March, 2009



(b) PM_{10} levels in June, 2009

Types of spatial data

- Three broad categories

Types of spatial data

- Point-referenced data
  - Each observation is associated with a location (point)
  - Data represents a sample from a continuous spatial domain
  - Also referred to as *geocoded* or *geostatistical* data

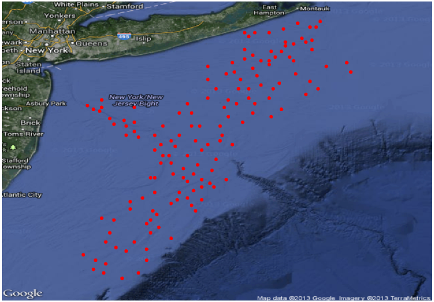


Figure: Locations of scallops abundance data

Types of spatial data

- Areal data
  - Each observation is associated with a region like state, county etc.
  - Usually a result of aggregating point level data
  - The spatial information is represented in terms of a graph depicting the relative orientation of the regions

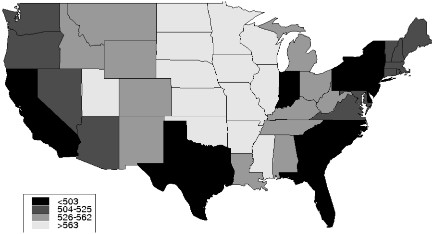


Figure: SAT scores across the 48 contiguous states in the US

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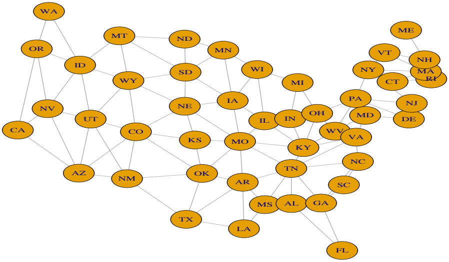


Figure: Graph for US state map

Types of spatial data

- Point pattern data
  - The locations are viewed as “random”
  - Need not have variables at locations, just the pattern of points
  - Interest in the pattern of occurrences of an event like disease incidence, species distribution, crimes etc.

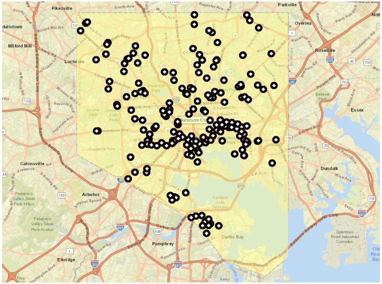


Figure: Locations of robberies in Baltimore in February 2017

Types of spatial data

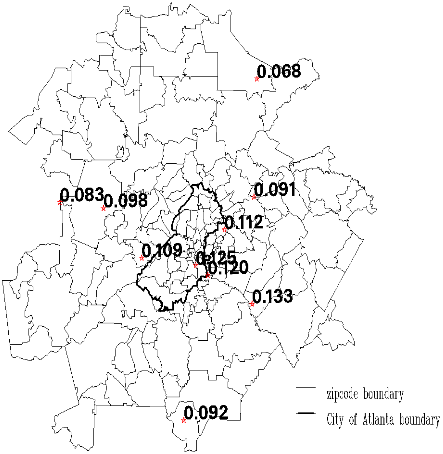


Figure: Areal and point-referenced data: Atlanta zip codes and 8-hour maximum ozone levels (ppm) at 10 sites, July 15, 1995

Point-referenced data

- Point-level modeling refers to modeling of spatial data collected at locations referenced by coordinates (e.g., lat-long, Easting-Northing).
- Data from a spatial process $\{Y(\mathbf{s}) : \mathbf{s} \in D\}$, D is a subset in Euclidean space.
- Example: $Y(\mathbf{s})$ is a pollutant level at site $\mathbf{s}$
- Conceptually: Pollutant level exists at all possible sites
- Practically: Data will be a partial realization of a spatial process – observed at $\{\mathbf{s}_1, \ldots, \mathbf{s}_n\}$
- Statistical objectives: Inference about the process $Y(\mathbf{s})$; predict at new locations.
- Remarkable: Can learn about entire $Y(\mathbf{s})$ surface. The key: Structured dependence

Plotting point referenced data

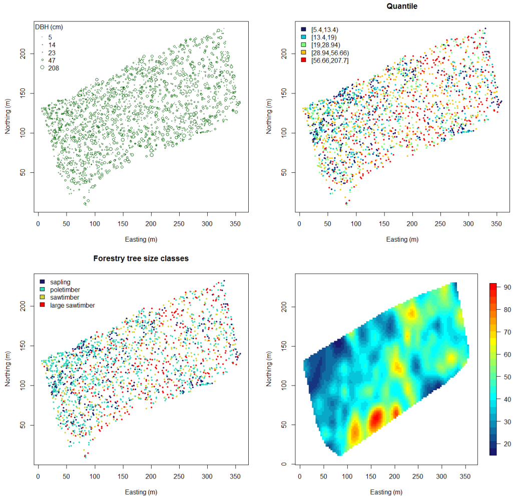
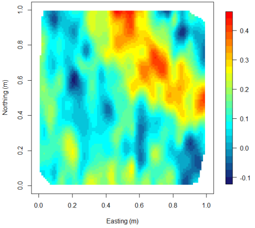


Figure: Western Experimental Forest (WEF) inventory data on diameter at breast height (DBH) of plants

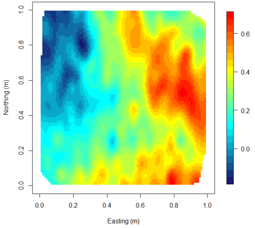
What's so special about spatial?

- A typical setup: Data observed at n locations $\{\mathbf{s}_1, \ldots, \mathbf{s}_n\}$
- At each $\mathbf{s}_i$ we observe the response $y(\mathbf{s}_i)$ and a $p \times 1$ vector of covariates $\mathbf{x}(\mathbf{s}_i)'$
- Linear regression model: $y(\mathbf{s}_i) = \mathbf{x}(\mathbf{s}_i)'\beta + \epsilon(\mathbf{s}_i)$
- $\epsilon(\mathbf{s}_i)$ are iid $N(0, \tau^2)$ errors
- Although the data is spatial, this is an ordinary linear regression model
- $y = (y(\mathbf{s}_1)', y(\mathbf{s}_2)', \ldots, y(\mathbf{s}_n)')'$; $X = (\mathbf{x}(\mathbf{s}_1), \mathbf{x}(\mathbf{s}_2), \ldots, \mathbf{x}(\mathbf{s}_n))'$
- Inference: $\hat{\beta} = (X'X)^{-1}X'Y \sim N(\beta, \tau^2(X'X)^{-1})$
- Prediction at new location $\mathbf{s}_0$: $\widehat{y(\mathbf{s}_0)} = \mathbf{x}(\mathbf{s}_0)'\hat{\beta}$
- Does this always suffice or we need any thing specialized method for such data?

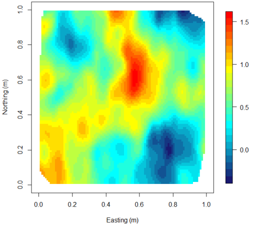
Exploratory data analysis



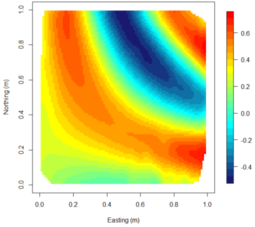
(a) Data 1



(b) Data 2



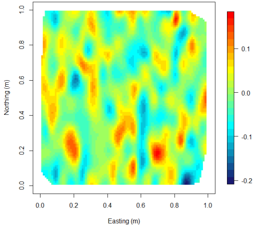
(c) Data 3



(d) $x(s)$

Residual plots

- Linear regression: $y(\mathbf{s}_i) = \beta_0 + x(\mathbf{s}_i)\beta_1 + \epsilon(\mathbf{s}_i)$



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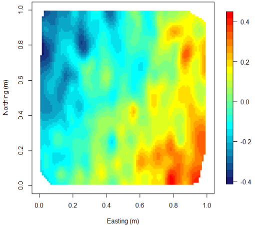
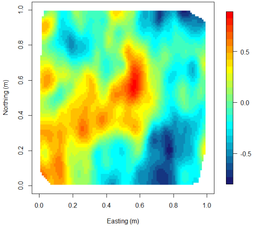


(b) Data 2



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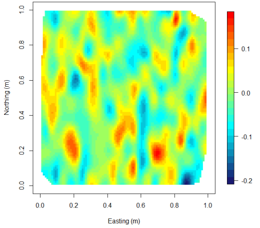

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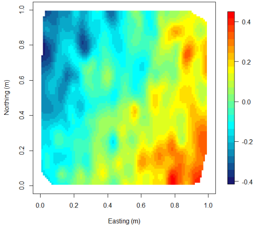
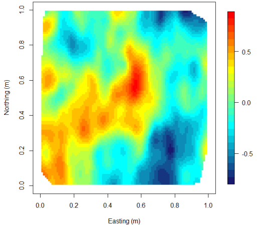


(b) Data 2



(c) Data 3

- Strong residual spatial pattern in datasets 2 and 3
- The covariate $x(\mathbf{s})$ does not explain all spatial variation in $y(\mathbf{s})$

More EDA

- Besides eyeballing residual surfaces, how to do more formal EDA to identify spatial pattern ?

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First law of geography: *"Everything is related to everything else, but near things are more related than distant things."* – Waldo Tobler

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- The residual surface seems continuous
- If a spatial surface $Y(\mathbf{s})$ is continuous then $(Y(\mathbf{s}+\mathbf{h}) - Y(\mathbf{s}))^2 \to 0$ as $||\mathbf{h}|| \to 0$
- In general $(Y(\mathbf{s}+\mathbf{h}) - Y(\mathbf{s}))^2$ increasing with $||\mathbf{h}||$ will imply a spatial correlation

Empirical variogram

- Plot $(Y(\mathbf{s}_i) - Y(\mathbf{s}_j))^2$ as function of $||\mathbf{s}_i - \mathbf{s}_j||$ for all i, j

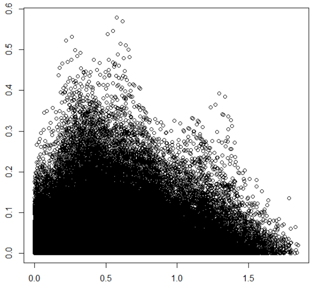


Figure: Points cloud for data 1

Empirical variogram

- Binning: Grid up the t space into intervals $I_1 = (0, t_1)$, $I_2 = (t_1, t_2)$, and so forth, up to $I_K = (t_{K-1}, t_K)$. Representing t values in each interval by its midpoint, we define:

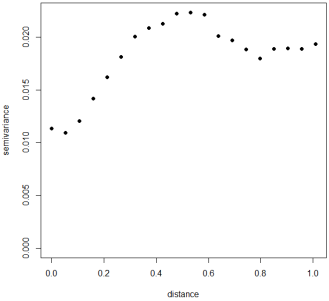
$$N(t_k) = \{(\mathbf{s}_i, \mathbf{s}_j) : \|\mathbf{s}_i - \mathbf{s}_j\| \in I_k\}, k = 1, \ldots, K.$$

- Empirical Variogram:

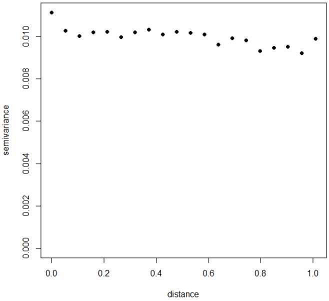
$$\gamma(t_k) = \frac{1}{|N(t_k)|} \sum_{\mathbf{s}_i, \mathbf{s}_j \in N(t_k)} (Y(\mathbf{s}_i) - Y(\mathbf{s}_j))^2$$

- Semivariogram $=$ 0.5$\times$Variogram

Empirical variogram: Data 1



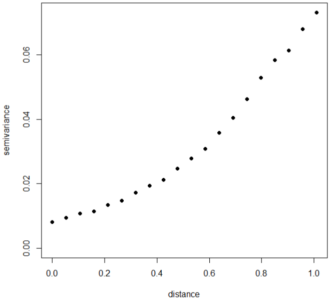
(a) Data 1: y



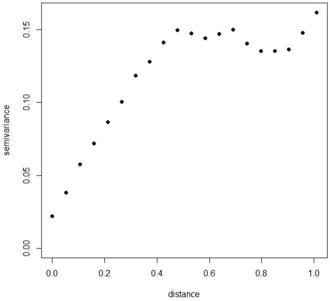
(b) Data 1: residuals

- Residuals display little spatial variation

Empirical variograms: Data 2 and 3



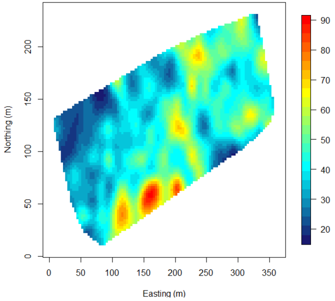
(c) Data 2: residuals



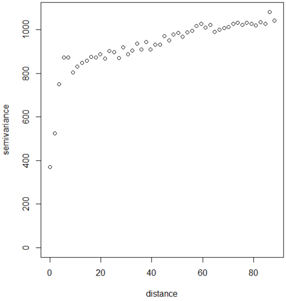
(d) Data 3: residuals

- Variogram of the residuals points to spatial variation

EDA for WEF data



(a) DBH



(b) Variogram of DBH

- Regression model: DBH $\sim$ Species

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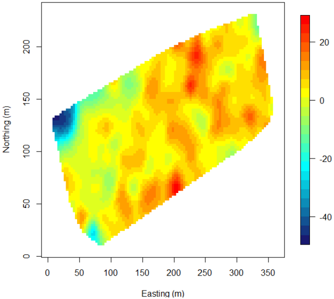
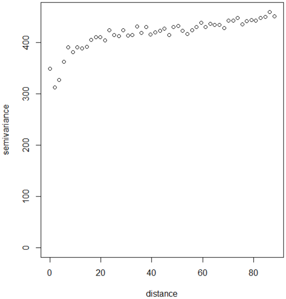


(c) DBH residuals

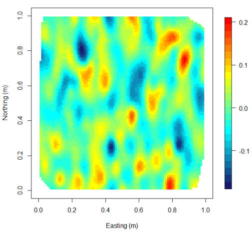


(d) Variogram of DBH residuals

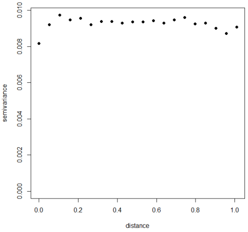
- Surface plot and variogram of residuals point to spatial variation

Using the locations

- Linear regression with the co-ordinates added as regressors:
  $y(\mathbf{s}_i) = \beta_0 + x(\mathbf{s}_i)\beta_1 + s_{ix}\beta_2 + s_{iy}\beta_3 + \epsilon(\mathbf{s}_i)$



(a) Residuals for data 2



(b) Empirical variogram

- The linear model for the co-ordinates explains most of the spatial variation in dataset 2

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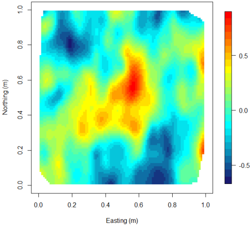
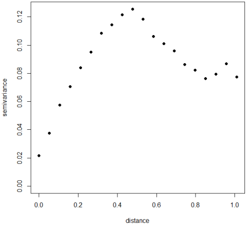


(a) Residuals for data 3



(b) Empirical variogram

- Dataset 3 still exhibits strong spatial correlation

Using the locations

- Linear model for the co-ordinates often does not suffice
- More general model: $y(\mathbf{s}_i) = \beta_0 + x(\mathbf{s}_i)\beta_1 + w(\mathbf{s}_i) + \epsilon(\mathbf{s}_i)$
- How to choose the function $w(\cdot)$?
- Since we want to predict at any location over the entire domain, this choice will amount to choosing a surface $w(\mathbf{s})$
- How to do this ?

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- How to do this ? Answer in next class

References

- BCG book chapters 1.1, 2.1.4, 2.5
- US forest biomass data: Datta, A., Banerjee, S., Finley, A. O., and Gelfand, A. E. (2016). Hierarchical nearestneighbor gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111(514):800-812.
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- Scallops data: BCG book figure 1.11(b)
- US SAT score data: BCG book Fig 4.1
- Baltimore robbery map: http://maps.baltimorepolice.org/flexviewer/
- Atlanta ozone data: BCG book figure 1.3
- WEF data: *spBayes* package in *R*