3.2-1

$f(n), g(n)$ are monotonic increasing $\vdash$ $f(n)+g(n)$ is monotonic incr, $f(g(n))$ is monotonic incr

1) Prove $f(m)+g(m) \leq f(n)+g(n)$, $m \leq n$

$f(m_1) \leq f(n_1)$ where $m_1 \leq n_1$

$g(m_2) \leq f(n_2)$ where $m_2 \leq n_2$

$\vdash$ Let $a = m_1 = m_2$

Let $b = n_1 = n_2$

$f(a)+g(a) \leq f(b)+g(b)$

QED

$f(x) \leq f(y)$

$g(x) \leq g(y)$

$\vdash$ Assume: $f(g(a)) \leq f(g(b))$

Prove: $f(g(a+1)) \leq f(g(b+1))$

2)

$f(g(a)) \leq f(g(b))$

$f(g(a) \leq g(b))$

Let $c = a$, $d = b$

$c \leq d$

$f(g(c) \leq g(d))$

Let $q = g(c)$, $r = g(d)$

$q \leq r$ by $g(x) \leq g(y)$

$f(q \leq r)$

$f(q) \leq f(r)$

= TRUE ✓