~~Show that for any real constants a & b, where b>0~~

~~$(n+a)^b = \Theta(n^b)$~~

~~Base case: a=0~~

~~Subs $(n+0)^b = \Theta(n^b)$~~

~~$n^b = \Theta(n^b)$~~

~~defn Θ: $c_1 n^b \le n^b \le c_2 n^b$~~

~~Let $c_1 = 1$, $c_2 = 1$~~

~~Lemma: $\Theta(n^b) / n^b \ge 1$~~

~~$\Theta(n^b) \ge n^b$~~

$\boxed{3.1\text{-}2}$ Prove $\forall a, b \in \mathbb{R}, b > 0$

$(n+a)^b = \Theta(n^b)$

Base $b=1$

$(n+a)^1 = \Theta(n^1)$

$0 \le c_1 n \le n+a \le c_2 n$

Let $c_1 = 0$, $n_0 = |a|$, $c_2 = 2$

$0 \le 0 \cdot n \le n + a \le 2n$

If a is negative, n will be at least |a|, in which case $|a| + a = 0$

Since n must be $\ge |a|$, $n+a$ will never exceed $2n$. Therefore, $2n$ is always $\ge n+a$

Inductive Case

Assume $(n+a)^b = \Theta(n^b)$

Prove $(n+a)^{b+1} = \Theta(n^{b+1})$

$(n+a)^{b+1}$

defn power $= (n+a)^1 \cdot (n+a)^b$

Induct $= (n+a)^1 \cdot \Theta(n^b)$

Induct $= \Theta(n^1) \cdot \Theta(n^b)$

Defn Θ $= \Theta(n \cdot n^b)$

Pow $= \Theta(n^{b+1})$

QED