3.2-7

$\phi = \frac{1+\sqrt{5}}{2}$ $\quad \hat{\phi} = \frac{1-\sqrt{5}}{2}$

Base Case: $i = 0$

$F_0 = \frac{\phi^0 - \hat{\phi}^0}{\sqrt{5}}$

$0 = 0$ ✓

Base Case: $i = 1$

$F_1 = \frac{\phi - \hat{\phi}}{\sqrt{5}}$

$1 = \frac{\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)}{\sqrt{5}}$

$1 = \frac{\frac{1}{2}(1-1+\sqrt{5}+\sqrt{5})}{\sqrt{5}}$

$1 = \frac{1/2\,(2\sqrt{5})}{\sqrt{5}}$

$1 = 1$ ✓

Lemma 1

$\phi + 1 = \phi^2$

$\frac{1+\sqrt{5}}{2} + \frac{2}{2}$

$= \frac{3+\sqrt{5}}{2}$

$= \left(\frac{1+\sqrt{5}}{2}\right)^2$

$= \phi^2$

Lemma 2

$\hat{\phi} + 1 = \hat{\phi}^2$

$\frac{1-\sqrt{5}}{2} + \frac{2}{2}$

$= \frac{3-\sqrt{5}}{2}$

$= \left(\frac{1-\sqrt{5}}{2}\right)^2$

$= \hat{\phi}^2$

Inductive: Assume $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$

Prove $F_{i+1} = \frac{\phi^{i+1} - \hat{\phi}^{i+1}}{\sqrt{5}}$

defn Fib $\quad F_{i+1} = F_i + F_{i-1}$

Inductive Step $= \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}} + \frac{\phi^{i-1} - \hat{\phi}^{i-1}}{\sqrt{5}}$

Arithmetic $= \frac{1}{\sqrt{5}}\left[\phi^i - \hat{\phi}^{i-1} + \phi^{i-1} - \hat{\phi}^{i-1}\right]$

Group terms and factor out $= \frac{1}{\sqrt{5}}\left[\left(\phi^{i-1}(\phi+1)\right) - \left(\hat{\phi}^{i-1}(\hat{\phi}+1)\right)\right]$

Lemma 1 and Lemma 2 $= \frac{1}{\sqrt{5}}\left[\left(\phi^{i-1} \cdot \phi^2\right) - \left(\hat{\phi}^{i-1} \cdot \hat{\phi}^2\right)\right]$

Arithmetic $= \frac{\phi^{i+1} - \hat{\phi}^{i+1}}{\sqrt{5}}$ QED