A Locally Active Memristor and Its Application in a Chaotic Circuit

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***Abstract*—In this brief, we propose a novel locally active memristor based on a voltage-controlled generic memristor model and use the analysis methods of standard nonlinear theory to analyze its characteristics and illustrate the concept of local activity via the dc *v-i* loci of memristor and non-volatile memory via the power-off plot of memristor. A chaotic attractor is observed with a simple nonlinear circuit that only includes three circuit elements in parallel: 1) a nonlinear locally active memristor; 2) a linear passive inductor; and 3) a linear passive capacitor. Then, we analyze the dynamical characteristics of the above circuit and show complex bifurcation behaviors.**

***Index Terms*—Local activity, non-volatile, memristor, nonlinear circuit, bifurcation behaviors.**

I. Introduction

LOCALLY-ACTIVE systems can generate complex behaviors and rich dynamics. Locally-active circuit element is essential for a nonlinear dynamical system to maintain oscillating and amplify small signals [1]. As a novel memory device, memristor and locally-active memristor were proposed by Chua. Since then, researchers begin to study the locally-active memristor. Although it is not commercially available yet, its potential value has attracted many researchers. Nowadays, there is a lot of research on memristor. However, there is not much research on the quantitative analysis methods about the nonlinear characteristics of memristor. Our objective here is to propose a novel memristor model, analyze its nonlinear characteristics and conclude that the memristor is not only non-volatile [2] but also locally active. Then, we show the memristor's pinched hysteresis loop at different frequencies. In order to verify the memristor is locally active, we design a nonlinear circuit system using the memristor and two other linear energy storage elements in parallel, observe whether the circuit is oscillation and provide some quantitative analysis method. In particular, the nonlinear circuit we designed is the simplest possible circuit that can exhibit chaotic attractors [5]. We show the simulation results with two different initial states and explain the physical significance of locally-active. Moreover, the dynamical behavior of circuit associated with initial states illustrates the emergence of extreme multistability [3]. Multistability reveals a rich diversity of stable states of a dynamical system and might have important consequences in the reproducibility of some chemical reactions [4]. The contributions in this brief are as follows. We propose a novel locally-active memristor, design a chaotic circuit and provide the quantitative analysis methods about the causes of the chaotic attractor.

The structure of this brief is organized as follows: we first present a generic memristor model based upon Chua's Unfolding principle [6], analyze the locally-active characteristic by observing the DC *v-i* loci of the memristor, judge the local stability (instability) of the equilibrium point via drawing the driving point (DP) characteristic of the memristor, analyze whether the memristor is non-volatile memory via the Power-Off plot (POP) of memristor and show the pinched hysteresis loop of memristor (Section II). Then we design a nonlinear circuit using the proposed memristor, discuss the circuit topology and equations, and observe the oscillation status and attractor types of different initial states (Section III). Finally, some numerical description for the dynamical characteristic of system is given (Section IV). All simulation results are observed from the MALTAB platform.

II. Generic Memristor Model

Chua [2] classified all memristors into three classes including ideal memristor, generic memristor and extended memristor and presented the mathematical definitions of memristors. The ideal memristor is the simplest and most practical model, like the publication of HP's paper [7], reporting a 2-terminal device that the form of State-Dependent Ohm's law is an ideal memristor. The extended memristor is an extended form of ideal memristor and Chua and Kang [8] generalized the memristor concept to a much broader class of nonlinear dynamical systems and named it as memristive systems. The mathematical definition of generic memristor model is shown below.

$$y = g(x)u \tag{1}$$

$$dx/dt = f(x, u) \tag{2}$$

When the input (output) of system is assumed to be in voltage (current) form, i.e., $u = v_m(y = i_m)$, the memristor is the voltage-controlled. When $u = i_m(y = v_m)$, the memristor is the current-controlled. Chua [6] unfold the memristor's state equations in order to develop a more precise quantitative mode of memristor and the mathematical concept of the unfolding voltage-controlled memristor is following, where x is a scalar

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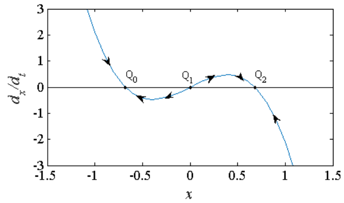


Fig. 1. POP of the voltage-controlled generic memristor.

and all parameters are called memristor unfolding parameters.

$$dx/dt = f(x, v_m) = a_1x + \ldots + a_m x^m + b_1 v_m + \ldots + b_n v_m^n + \sum_{j,k=1}^{p,r} c_{jk} x^j v_m^k \quad (3)$$

Now, we use the trial and error method to derive the voltage-controlled generic memristor, i.e.,

$$dx/dt = a_1x + a_3x^3 + b_1v_m + c_{11}xv_m \quad (4)$$
$$i_m = W(x)v_m = k(x^2 - x - 1)v_m \quad (5)$$

where v_m, i_m and x denote the voltage, current and state variable of memristor. Equations (4) and (5) respectively are the state evolution function and memductance function of memristor. We also use the trial and error method to deduce these values of the unfolding parameters in equations (4) and (5), $a_1 = 1.8, a_3 = -3.9, b_1 = 1.4, c_{11} = -1.5$ and $k = 1$. The memristor's model is

$$dx/dt = 1.8x - 3.9x^3 + 1.4v_m - 1.5xv_m \quad (6)$$
$$i_m = W(x)v_m = (x^2 - x - 1)v_m \quad (7)$$

We analyze its nonlinear characteristics to prove that the memristor is non-volatile and locally active.

A. Power-Off Plot of the Memristor

The non-volatile memristor can remember its most recent memristance (memductance) state when the power is shut off [9]. The Power-Off Plot (POP) is one of the dynamic routes that the rate of change (*dx/dt*) in (6) vs. x plane with the input voltage v_m setting to zero, that is $dx/dt|_{v_m=0} = f(x, 0)$. With reference to [2, Sec. 9], the non-volatile memristor theorem is proposed by Chua that states a memristor with a scalar state variable x is non-volatile if its POP intersects the x-axis at 2 or more points with a negative slop. Therefore, the non-volatile memory of memristor can be determined by the POP technique, like the method used in [10].

The POP of the above memristor (obtained by setting $v_m = 0$) in equation (6) is given by

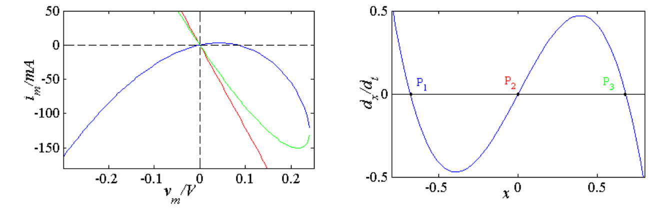
$$dx/dt|_{v_m=0} = f(x, 0) = 1.8x - 3.9x^3 \quad (8)$$

The plot is shown in Fig. 1. It has three intersections with the x-axis that are three equilibrium points located at $x = -0.6794, x = 0$, and $x = 0.6794$, respectively, where $dx/dt = 0$. The dynamic route in Fig. 1 identifies that the equilibrium points Q_0 and Q_2 are asymptotically stable, whereas the equilibrium points Q_1 is unstable. Hence, for different initial state $x(0)$, the memristor can exhibit one of two asymptotically stable equilibrium states, namely

$$\begin{aligned} x = x(Q) &= -0.6794, \ \textit{if}\ x(0) < 0 \\ &= 0.6794, \ \textit{if}\ x(0) \geq 0 \end{aligned} \quad (9)$$

These two equilibrium states give rise to the following two corresponding stable conductance

$$\begin{aligned} W(x(Q_0)) &= (-0.6794)^2 + 0.6794 - 1 = 0.141, \ \textit{if}\ x(0) < 0 \\ &= (0.6794)^2 - 0.6794 - 1 = -1.218, \ \textit{if}\ x(0) \geq 0 \end{aligned} \quad (10)$$



(a) (b)

Fig. 2. (a) DC V_m-I_m loci of the memristor defined by (6) and (7). (b) DP characteristic of the memristor for $v_m = 0$.

Since the two stable values of the memductance W can be used to represent the binary state 0 or 1, this memristor is said to be a non-volatile memory [2] because the memductance $W = 0.141$S or -1.2178S is retained for all times $t > 0$, until an appropriate external voltage signal is applied to switch the equilibrium state to the other stable state.

Equation (9) implies that the state $x(t)$ is different with initial state $x(0)$. Hence, the memristor of equations (6) and (7) is endowed with a non-volatile memory.

B. DC V_m-I_m Loci of the Memristor

With reference to [2, Sec. 10], we know that the DC Voltage-Current characteristic (DC V_m-I_m loci) is a smooth curve consisting of a set of points and a sufficient test to claim that the memristor is locally-active. These points of DC V_m-I_m loci can be derived in steps [11], [12] depending on the equations (6) and (7) and finally be plotted in Fig. 2. We can determine whether the memristor is locally active by observing DC V_m-I_m loci. The same analysis method about locally-active memristor is used in [13] and [14].

From equation (6), we can observed that the equation is a third-order homogeneous equation of x, hence, the state equation (6) has at most three equilibrium points for any value of v_m and under the equation (7) has at most three values of i_m for any v_m (See Fig. 2 (a), the blue, the green and the red curve refer to three different state variables of the memristor). The memristor voltage range was $v_m \in$[-0.3, 0.24] V, corresponding to the state interval $x \in$[-0.8397, 0.7015] and the current range $i_m \in$[-290.259, 369.198] mA. For the purpose of clarity, the current range of Fig. 2 (a) was limited in $i_m \in$[-200, 50] mA.

From Fig. 2 (a), we can find that there exist three values of i_m corresponding to any value of v_m except $v_m = 0$. Observe that for the equation (7) we have the input $v_m = 0$, and the output value $i_m = 0$ always, however, the state interval x has three different values those respectively are $X_1 = -0.6794, X_2 = 0$, and $X_3 = 0.6794$. The memductance $W(x)$ of memristor is

$$W(x) = x^2 - x - 1 \quad (11)$$

Namely, with regard to the point $v_m = 0, i_m = 0$, the memristor has three different memductances those are respectively $W(X_1) = 0.1409$ (the blue curve), $W(X_2) = -1.2178$ (the green curve) and $W(X_3) = -1.0$ (the red curve). A careful examination of Fig. 2 (a) reveals that the green curve and the red curve are negative slope, and the blue one contain a small region with a negative slop. Hence, the memristor is locally active and can be designed to amplify small signals [6].

For different value of v_m, the rate of evolution of the state x named *dx/dt* can be plotted versus the state x itself (e.g., see the Fig. 2 (b) for $v_m = 0$), namely the well-known driving point characteristic (DP plot) [11] which is a powerful graphical tool from nonlinear systems theory [13]. Local stability of nonlinear system can be studied via the number and moving loci of DP characteristic crosses the horizontal axis. If the DP characteristic crosses the horizontal axis from top-left to bottom-right (e.g., see the points P_1 and P_3 in Fig. 2 (b)), the equilibrium point is locally asymptotically stable, otherwise the equilibrium point is unstable (e.g., see the point P_2 in Fig. 2 (b)). Under the state interval of interest (6) has most three equilibria for $v_m = 0$ and with reference to the curve in Fig. 2 (b), the three equilibrium points of system are $X_1 = -0.6794$, $X_2 = 0$ and $X_3 = 0.6794$, i.e., the abscissas of points $P_1 = (-0.6794, 0)$, $P_2 = (0, 0)$ and $P_3 = (0.6794, 0)$. The state dynamic theory above elucidates that the equilibrium points P_1 and P_3 are locally asymptotically stable, and the equilibrium points P_2 is locally unstable. With reference to Fig. 2 (a), the blue curve (passive memductance region) and the green one (negative memductance region) are stable, otherwise, the red curve (negative memductance region) is unstable when $v_m = 0$. Hence, depending on the initial state $x(0)$, the memristor can exhibit one of the two stable memductance states. That is to say, the solution $x(t)$ starting from $x(0) \geq 0$ must tend to $x = X_3 = 0.6794$ as $t \to \infty$, meanwhile the memductance of memristor is negative and the memristor is locally-active. On the contrary, if $x(0) < 0$, the solution $x(t)$ will tend to $x = X_1 = -0.6794$ as $t \to \infty$, the memductance of memristor is positive and the memristor is locally-passive. Hence, only when we set $x(0) \geq 0$, the nonlinear system including the presented memristor may be locally-active system that can manifest the complex dynamical behaviors for example chaos [1]. The detailed simulation result will be given below (Section III). The state variable x denotes the flux φ in voltage-controlled memristor. In a memristor emulator, we can obtain flux φ through an integrator and a voltage source, like the equivalent circuit in [15] that is $\varphi(t) = \int_{-\infty}^{t} u(\tau)d\tau$. If we set $u(t) > 0$, we can get $x(0) = \varphi(0) > 0$.

C. Pinched Hysteresis Loop of the Memristor

Any two-terminal device exhibiting a pinched hysteresis loop which always passes through the origin in the voltage-current plane when driven by any periodic input current source, or voltage source, with zero DC component is called a memristor [2]. Thus the pinched hysteresis loop is always used as the characteristic fingerprint to identify memristor [16]. Moreover, the pinched hysteresis loop should shrink to a single-valued function when the frequency tends to infinity [17].

Fig. 3 shows the pinched hysteresis loops of the above generic memristor at different frequencies when it is driven by a sinusoidal single with amplitude $A = 1V$ (e.g., Fig. 3 (a) shows the curve at frequency $f = 0.2\ Hz$, Fig. 3 (b) shows the curve at frequency $f = 1.2\ Hz$). By combining Fig. 3(a) and (b), we obtain that the pinched hysteresis loop of memristor shrink to a single-valued function as we increase the frequency of input single. Notice that Fig. 3 (a) shows that the memristor has negative memductance region and Fig. 3 (b) degenerates into a linear negative memductance.

In conclusion, we analyze the generic voltage-controlled memristor we proposed in this brief using the nonlinear theory, such as POP, DC v-i loci and DP characteristic and find that the memristor is non-volatile and locally active. Then, we plot the pinched hysteresis loop of the generic memristor at different frequencies. Moreover, with regard to the DC v-i loci and DP characteristic, we can find that there exists a stable locally active equilibrium point [18] in the memristor. Therefore, the memristor has the capability to make up nonlinear dynamical system and maintain oscillating. The next section will present the nonlinear system, analyze the circuit topology structure and some simulation results.

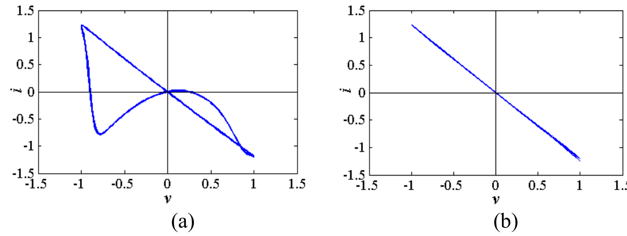


Fig. 3. The simulate results measured with voltage-controlled memristor driven by a sinusoidal voltage signal with amplitude $A = 1V$ at different frequencies. (a) Frequency $f = 0.2Hz$ (b) Frequency $f = 1.2Hz$.

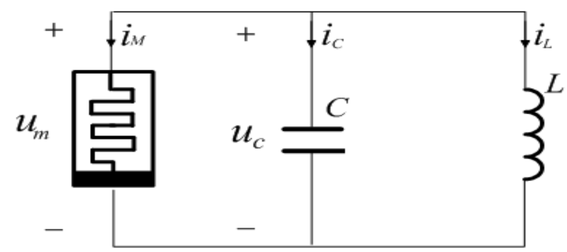


Fig. 4. The schematic of simplest chaotic circuit.

III. Chaotic Oscillation in a Simplest Nonlinear Circuit With Locally-Active Memristor

Consider a three-element simplest circuit that only includes the proposed memristor, an inductor and a capacitor in parallel shown in Fig. 4.

We take the state variable of memristor x, the capacitor voltage u_C and the inductor current i_L as the state variables and the circuit dynamic is described by the equation (12):

$$\begin{cases} \frac{du_c}{dt} = \frac{-1}{C}\left(k\left(x^2 - x - 1\right)u_C + i_L\right) \\ \frac{di_L}{dt} = \frac{1}{L}u_C \\ \frac{dx}{dt} = a_1 x + a_3 x^3 + b_1 u_C + c_{11} u_C x \end{cases} \tag{12}$$

It has been proved that the locally active characteristic of memristor can determine whether the nonlinear system is oscillating [1]. That is to say, only the memristor is locally-active, the chaotic system in Fig. 4 can maintain oscillating, and if the memristor is locally-passive, no oscillation would develop in the circuit. In other words, the condition of nonlinear system to oscillate is that there exists a locally-active element in the system. This explains why in the corresponding parameters with different initial states, the nonlinear system can generate two different states of oscillation and non-oscillation and different attractor types [19].

The locally active characteristic is related to the cell equilibrium points, thus we should calculate the equilibrium points of system defined by equation (12) firstly. The equilibrium point is obtained by setting $du_C/dt = di_L/dt = dx/dt = 0$ in equation (12). With regard to equation (12), the equilibrium point is only related to the parameter a_1 and a_3, and has nothing to do with other parameters. Therefore, if the memristor is determined, and then the equilibrium points of system (12)

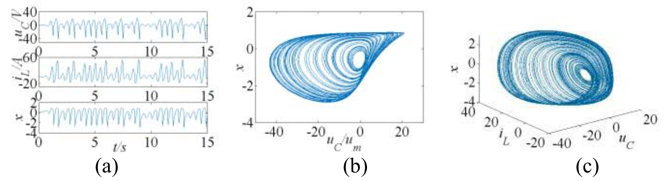


Fig. 5. The simulation results of circuit in Fig. 4 with initial state (0.1 0.1 0.2). (a) Time waveforms of the capacitor voltage u_C, the inductor current i_L and the state of memristor x. (b) The memristor's voltage u_m versus the state x of chaotic attractor. (c) The 3D attractor of chaotic circuit.

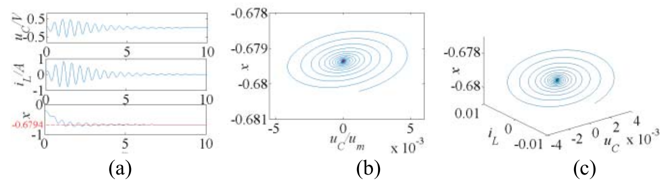


Fig. 6. The simulation results of circuit in Fig. 4 with initial state (0.1 0.1 -0.2). (a) Time waveforms of the capacitor voltage u_C, the inductor current i_L and the state of memristor x. (b) The memristor's voltage u_m versus the state x of stable point attractor. (c) The 3D attractor of chaotic circuit.

is fixed regardless of the capacitance C and the inductance L. We set equations (6) and (7) as the memristor of system (12), then solve the equation (12) and can get the equilibrium points E_1 (0 0 -0.6794), E_2 (0 0 0) and E_3 (0 0 0.6794). Notice that, the equilibrium points of system (12) E_1, E_2 and E_3 is corresponded to the equilibrium points of memristor defined by (6) and (7) P_1, P_2 and P_3 in Fig. 2 (b). Since the point P_1, P_3 are locally asymptotically stable and point P_2 is unstable for memristor, the state $x(t)$ will tend to P_1 and P_3 with different initial state $x(0)$ (the proof is in Section II-B). With reference to Fig. 2, the equilibrium point P_3 (the point P_1) is the locally-active (locally-passive) operating point of the memristor, thus the equilibrium points E_3 (equilibrium points E_1) is corresponded to the locally-active (locally-passive) memristor.

Next, the simulation results will show that only the memristor is locally-active, the nonlinear system is oscillating. With reference to the circuit of Fig. 4, we set C to 130mF, L to 50mH, and the initial state to (0.1 0.1 0.2). The time domain waveforms and the attractor plot obtained by simulating equation (12) are shown in Fig. 5.

However, under the same parameters of circuit, we change the initial state to (0.1 0.1 -0.2). The time domain waveforms and the attractor are shown in Fig. 6.

Comparing Fig. 5 and Fig. 6, we can observe that the initial state $x(0)$ can determine whether the circuit in Fig. 4 maintain oscillating. In Fig. 5, we set the initial state to (0.1 0.1 0.2). Therefore, $x(0)$ is larger than zero, thus $x(t)$ will tend to $x = 0.6794$ as $t \to \infty$, corresponding to the negative memductance or locally-active region. This is the reason that the nonlinear system can maintain oscillating. However in Fig. 6, the initial state is (0.1 0.1 -0.2) and $x(0)$ is smaller than zero, thus $x(t)$ will tend to $x = -0.6794$ as $t \to \infty$ (see the red dashed line in Fig. 6 (a)). Notice that the memductance of memristor is positive and the memristor lies in locally-passive region. The nonlinear system cannot maintain oscillation agreeing with the simulation observations. The simulation results show that the memristor's voltage u_m versus the state x stabilize to a point (0, -0.6794) in Fig. 6 (b).

The corresponding equilibrium point, the memductance and the locally-active characteristic of memristor, stability of equilibrium point, the Lyapunov exponent and attractor type of the different initial states are reported in Table I.

TABLE I
NONLINEAR CHARACTERISTIC OF DIFFERENT INITIAL STATES

Initial state	(0.1 0.1 0.2)	(0.1 0.1 -0.2)
Equilibrium point	(0 0 0.6794)	(0 0 -0.6794)
Memductance	negative	positive
Characteristic of memristor	locally-active	locally-passive
Eigenvalues	λ_1= -3.6005 λ_2= 4.68+11.49i λ_3= 4.68-11.49i	λ_1= -3.6005 λ_2= -0.54+12.39i λ_3= -0.54-12.39i
Lyapunov exponent	(1.26 0 -10)	(-0.52 -0.56 -3.6)
Attractor type	chaotic attractor	stable point attractor

In conclusion, according to the simulation results, we know that locally-active memristor is essential to generate oscillation and chaos for nonlinear system. Furthermore, when the system is chaotic, the memristor will mostly be in the locally-active region. In this brief, the locally-active characteristic of memristor defined by equations (6) and (7) is controlled by the initial state $x(0)$. Thus, we can control the oscillatory state of nonlinear circuit in Fig. 4 by changing the initial state $x(0)$.

IV. NONLINEAR DYNAMICAL ANALYSIS METHOD OF CHAOTIC CIRCUIT

This section shows that the nonlinear system described in equation (12) has complex bifurcation behaviors (e.g., periodic trajectory, period-doubling trajectory and chaotic attractor) and provides dynamical analysis methods including stability of equilibrium point, Lyapunov exponent and bifurcation diagram to study the characteristic of nonlinear system.

A. Stability of Equilibrium Point

In Section III, the simulation results have demonstrated that using the initial state near the equilibrium point $E_3(E_1)$, the nonlinear system can exhibit chaotic attractor (stable point attractor). Next, we analyze the stability of equilibrium point via the eigenvalues of Jacobian matrix at equilibrium point.

We know that the equilibrium points are E_1 (0 0 -0.6794), E_2 (0 0 0) and E_3 (0 0 0.6794) and that E_2 (corresponded to P_2) is unstable, thus we study the Jacobian matrix at equilibrium point E_1 and E_3. Notice that, the memristor of nonlinear system (12) we used is defined by equations (6) and (7).

The Jacobian matrix at $E_1$is easily derived as

$$J_1 = \begin{bmatrix} -0.141/\mathrm{C} & -1/\mathrm{C} & 0 \\ 1/\mathrm{L} & 0 & 0 \\ 2.4191 & 0 & -3.6005 \end{bmatrix} \tag{13}$$

The according characteristic equation is expressed as

$$-(3.6005 + \lambda)(\lambda^2 + 0.141\lambda/C + 1/LC) = 0 \tag{14}$$

From equation (14), we can easily find one eigenvalue is $\lambda_1 = -3.6005$. What's more $\lambda^2 + 0.141\lambda/C + 1/LC$ is a quadratic function of λ, has only a single intersection with vertical axis $1/LC$, and the abscissa of the minimum is $\lambda = -0.0705/C$, where C and L denote capacitance and inductance, we know that $1/LC > 0$, $-0.007.5/C < 0$ and the function $\lambda^2+0.141\lambda/C+1/LC$ has no intersection with λ axis in $\lambda > 0$. Hence, the eigenvalues of equation (14) are all negative real roots or complex roots with negative real part and the system is stable and cannot oscillate in equilibrium point E_1 regardless of the capacitance C and inductance L. The conclusion also verifies the correctness of Fig. 6 and is consistent with the conclusion proposed in Section II.

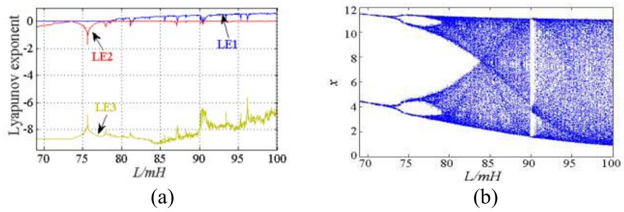


Fig. 7. (a) Lyapunov exponent spectra of bifurcation parameter L. (b) Bifurcation diagram of bifurcation parameter L.

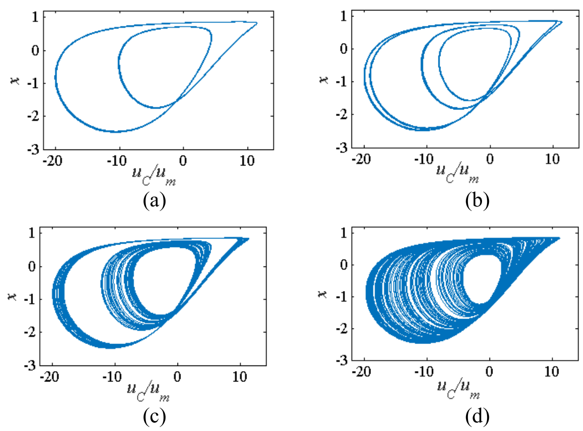


Fig. 8. The attractor evolutions with parameter L: (a) $L = 70mH$ (b) $L = 76mH$ (c) $L = 79mH$ (d) $L = 87mH$.

Next, system (12) is linearized at E_3, The Jacobian matrix is

$$J_2 = \begin{bmatrix} 1.2178/C & -1/C & 0 \\ 1/L & 0 & 0 \\ 0.3809 & 0 & -3.6005 \end{bmatrix} \tag{15}$$

The according characteristic equation is

$$-(3.6005 + \lambda)(\lambda^2 - 1.2178\lambda/C + 1/LC) = 0 \tag{16}$$

We also find that there is one eigenvalue $\lambda_1 = -3.6005$. However the differences are the function $\lambda^2 - 1.2178\lambda/C$ $+1/LC$ and the abscissa of the minimum $\lambda = 0.6089/C > 0$. Hence, the eigenvalues are all positive real roots or complex roots with positive real part except for λ_1 and the system can generate oscillating in equilibrium point E_3 like Fig. 5.

B. Dynamical Properties of Bifurcation Parameters

With regard to the nonlinear system in Fig. 4, the inductance L and capacitance C can determine the dynamic behavior of system (12) and lead to the bifurcation. Hence, they can be named as bifurcation parameter. Next, we take the parameter L as an example to study the influence of the dynamic characteristic of system about the bifurcation parameter. In the following simulation, we set equations (6) and (7) as the memristor of system (12) and the initial state is (0.1 0.1 0.2).

We set C to $300mF$. When L is gradually increased from $69mH$ to $100mH$, the Lyapunov exponent spectra and bifurcation diagram are shown in Fig. 7 (a) and Fig. 7 (b).

As shown in Fig. 7, the state of system evolves from periodic oscillation to chaotic oscillation by period-doubling bifurcation with the increase of circuit parameter L. In the region [90 91]mH, the system goes through a cycle window. The attractor evolutions of the system with bifurcation parameter L are shown in Fig. 8, where the values of L are $70mH$, $76mH$, $79mH$, and $87mH$.

Some numerical evidences of chaos are provided in this section, such as the stability of equilibrium point, the Lyapunov exponent and bifurcation diagram. Also, the simulation results of Section III have been verified.

V. Conclusion

A novel voltage-controlled generic memristor that is locally-active and non-volatile is proposed and utilized for the realization of chaotic oscillating circuit. By observing the DC V_m-I_m loci and DP characteristic of the memristor, we find there is a stable equilibrium point in the negative memductance region. In order to verify the locally active characteristic of memristor and explain the physical significance, we show the simulation results using MATLAB of different initial states. Some numerical evidences of chaos also have been provided.

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