Maxx Wilson
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ASE 389 HW1

Question 1A

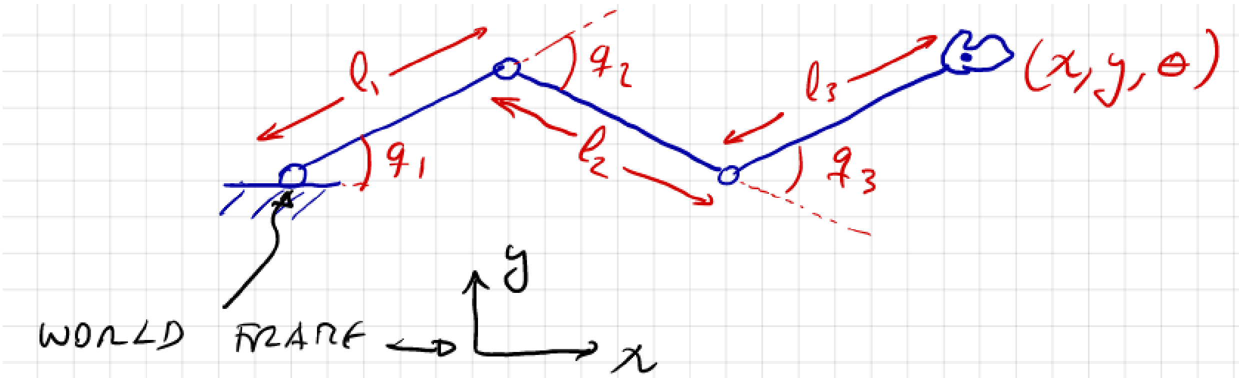

Figure 1. Forward Kinematics for Three-Link Planar Arm in 2D

Given joint angles and link parameters, the position of the end-effector can be derived using trigonometry, where the cosine and sine of each links angle with respect to the world frame will yield the XY coordinates of its respective joint, leading up to the End Effector.

$$x = l_1 cos(q_1) + l_2 cos(q_1 + q_2) + l_3 cos(q_1 + q_2 + q_3)$$

$$y = l_1 sin(q_1) + l_2 sin(q_1 + q_2) + l_3 sin(q_1 + q_2 + q_3)$$

$$\theta = q_1 + q_2 + q_3$$

Question 1B

With the forward kinematics formulated above, partial derivatives can be taken with respect to the joint angles to form a Jacobian which maps joint velocities to end-effector velocities.

$$J_p = \frac{\partial(x, y)}{\partial(q_1, q_2, q_3)}$$

$$J_p = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \end{bmatrix}$$

$$\frac{\partial \mathrm{x}}{\partial \mathrm{q}_1} = -l_1 sin(q_1) - l_2 sin(q_1 + q_2) - l_3 sin(q_1 + q_2 + q_3)$$

$$\frac{\partial x}{\partial q_2} = -l_2 sin(q_1 + q_2) - l_3 sin(q_1 + q_2 + q_3)$$

$$\frac{\partial x}{\partial q_3} = -l_3 sin(q_1 + q_2 + q_3)$$

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$$\frac{\partial y}{\partial q_1} = l_1 cos(q_1) + l_2 cos(q_1 + q_2) + l_3 cos(q_1 + q_2 + q_3)$$

$$\frac{\partial y}{\partial q_2} = l_2 cos(q_1 + q_2) + l_3 cos(q_1 + q_2 + q_3)$$

$$\frac{\partial y}{\partial q_3} = l_3 cos(q_1 + q_2 + q_3)$$

$$J_p = \begin{bmatrix} -l_1 sin(q_1) - l_2 sin(q_1 + q_2) - l_3 sin(q_1 + q_2 + q_3) & -l_2 sin(q_1 + q_2) - l_3 sin(q_1 + q_2 + q_3) & -l_3 sin(q_1 + q_2 + q_3) \\ l_1 cos(q_1) + l_2 cos(q_1 + q_2) + l_3 cos(q_1 + q_2 + q_3) & l_2 cos(q_1 + q_2) + l_3 cos(q_1 + q_2 + q_3) & l_3 cos(q_1 + q_2 + q_3) \end{bmatrix}$$

Question 1C

In similar fashion to the End-Effector position, a Jacobian can also be derived to relate joint angular velocities to end-effector angular velocity. The simple relation between them yields the trivial Jacobian shown below.

$$J_R = \frac{\partial(\theta)}{\partial(q_1, q_2, q_3)}$$

$$J_R = \begin{bmatrix} \frac{\partial\theta}{\partial q_1} & \frac{\partial\theta}{\partial q_2} & \frac{\partial\theta}{\partial q_3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Question 2

Firstly, a function to produce cubic trajectories implemented. This enhanced the control laws of each of the following problems by including feedforward acceleration and a non-zero velocity setpoint.

Let q be a generalized coordinate that can represent translational or angular position.

Given $q_0, q_f, \dot{q}_0, \dot{q}_f, t_o, t_f$, a cubic polynomial can be fit using the following relationship:

$$\begin{bmatrix} {t_o}^3 & {t_o}^2 & t_o & 1 \\ 3 * {t_o}^2 & 2 * t_o & 1 & 0 \\ {t_f}^3 & {t_f}^2 & t_f & 1 \\ 3 * {t_f}^2 & 2 * t_f & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} q_0 \\ \dot{q}_0 \\ q_f \\ \dot{q}_f \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} {t_o}^3 & {t_o}^2 & t_o & 1 \\ 3 * {t_o}^2 & 2 * t_o & 1 & 0 \\ {t_f}^3 & {t_f}^2 & t_f & 1 \\ 3 * {t_f}^2 & 2 * t_f & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} q_0 \\ \dot{q}_0 \\ q_f \\ \dot{q}_f \end{bmatrix}$$

$$q^{des}(t) = at^3 + bt^2 + ct + d$$

$$\dot{q}^{des}(t) = 3at^2 + 2bt + c$$

$$\ddot{q}^{des}(t) = 6at + 2b$$

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

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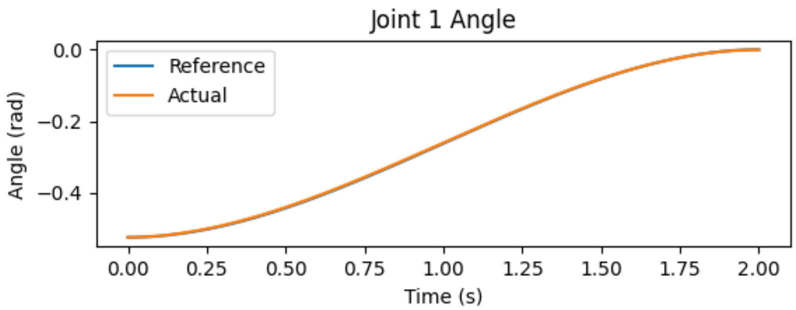
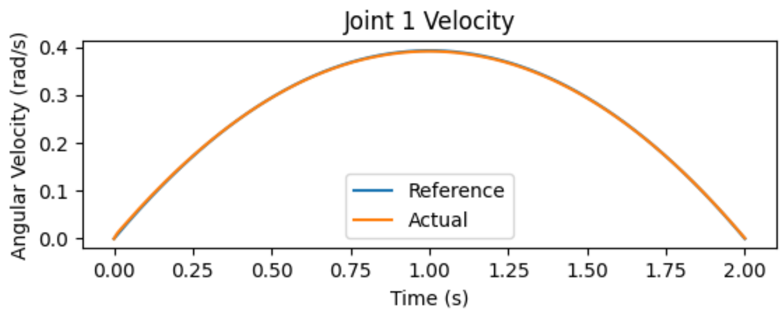
Control Law

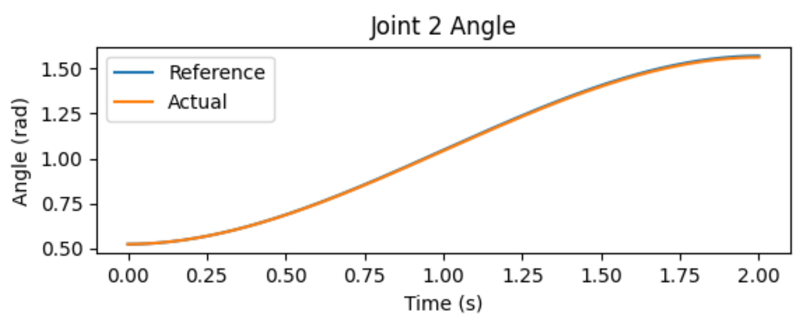
$$\ddot{q}^{ref} = \ddot{q}^{des(t)} + k_p * \left(q^{des}(t) - q(t)\right) + k_d * \left(\dot{q}^{des(t)} - \dot{q}(t)\right)$$

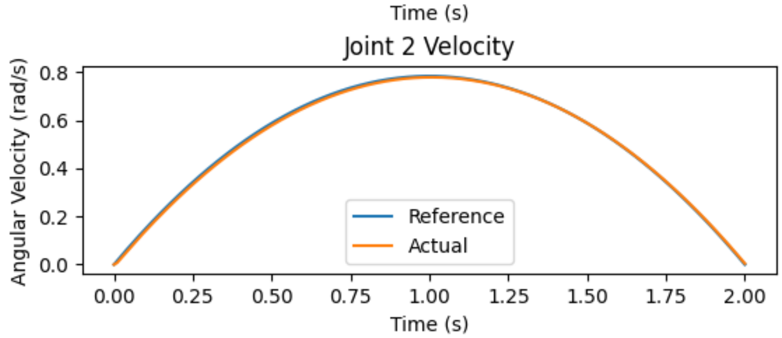
$$\tau_{\text{joints}} = A(q)\ddot{q}^{ref} + b\left(q(t), \dot{q}(t)\right)$$

Results









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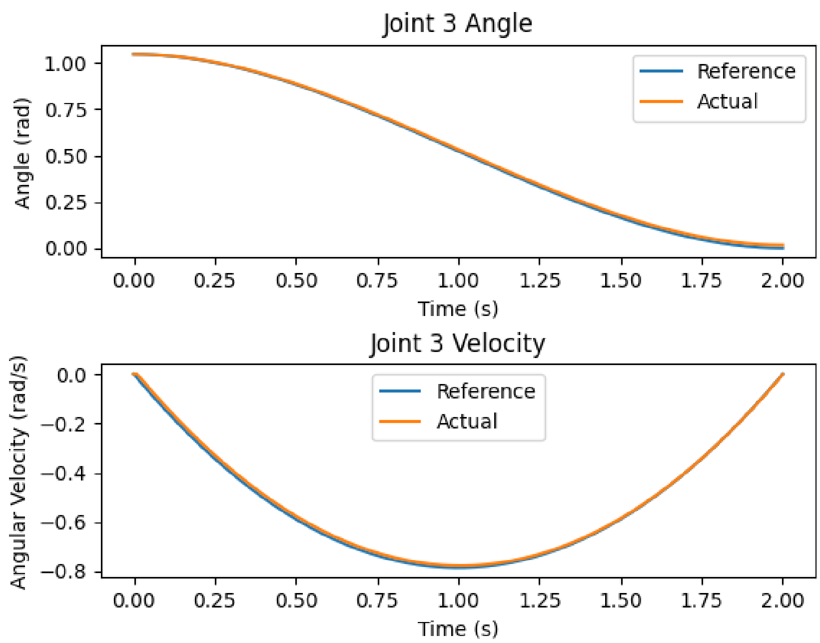

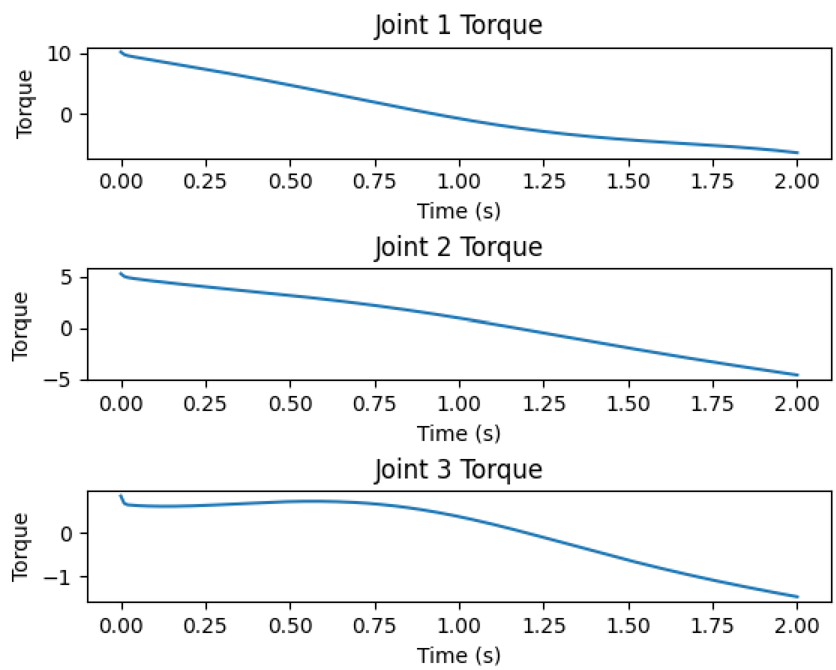

For joint space control, the described control strategy tracks the reference trajectories with minimal error. The only discrepancies are caused by joint friction at the initial motion and are shown by the slight dip in the torques around initial time.

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Question 3

As the trajectory generation is agnostic to the type of coordinate (angular vs translational), the same trajectory functions were used with different initial conditions to generate motions in operational space.

Operational Space Control Law

$$X = \begin{bmatrix} \theta \\ x \\ y \end{bmatrix}$$

$$\ddot{X}^{ref} = \ddot{X}^{des}(t) + k_p * \left(X^{des}(t) - X(t)\right) + k_d * \left(\dot{X}^{des}(t) - \dot{X}(t)\right)$$

Converting Operational Space Acceleration to Joint Space

$$J = \frac{\partial(\theta, x, y)}{\partial(q_1, q_2, q_3)}$$

$$\dot{X} = J\dot{q}$$

$$\frac{d}{dt}(\dot{X}) = \frac{d}{dt}(J\dot{q})$$

$$\ddot{X} = J\ddot{q} + \dot{J}\dot{q}$$

$$\ddot{q} = J^{+}(\ddot{X} - \dot{J}\dot{q})$$

Joint Space Control Law

$$\ddot{q}^{ref}(t) = J^{+}(\ddot{X}^{ref}(t) - \dot{J}\dot{q})$$

$$\tau_{joints} = A(q)\ddot{q}^{ref} + b\left(q(t), \dot{q}(t)\right)$$

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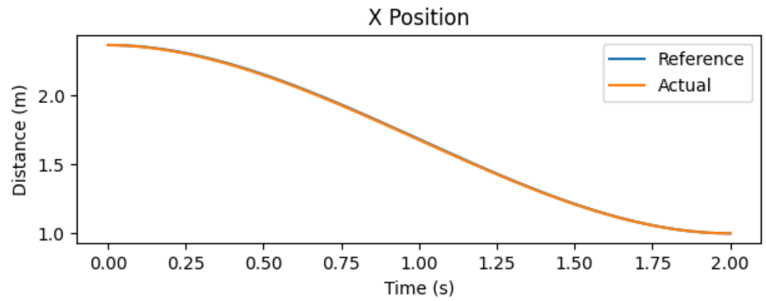
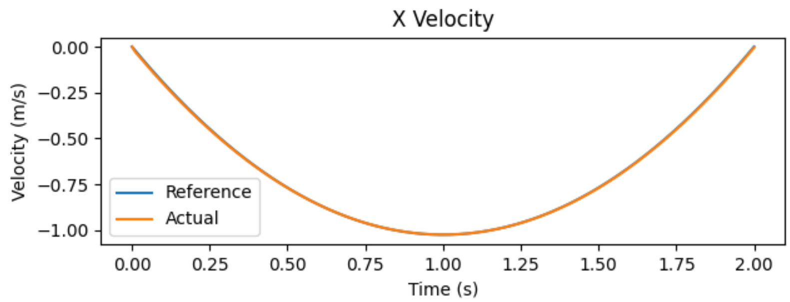
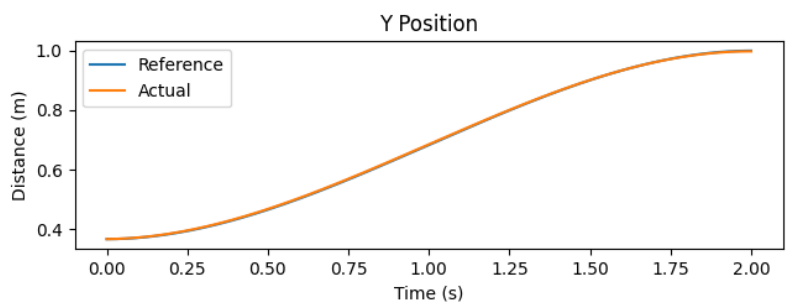
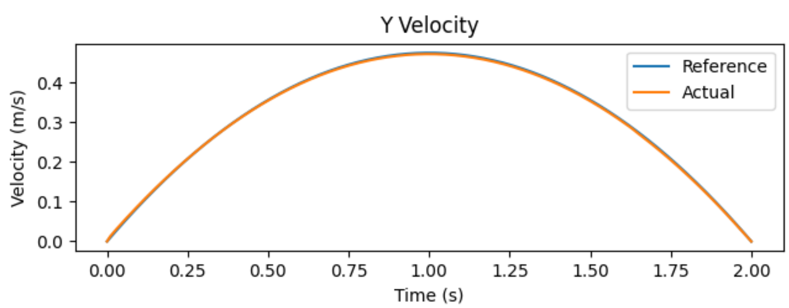
Results









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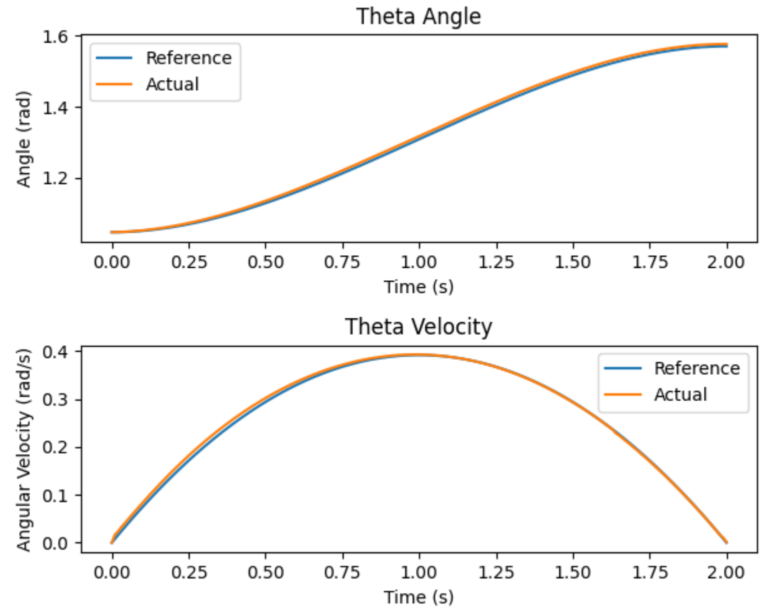

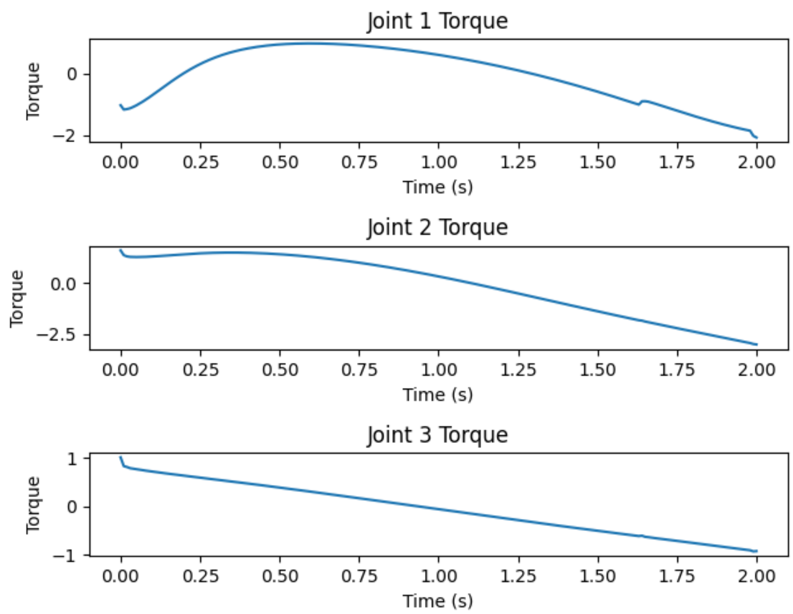

Like the previous results, the control law results in exceptional trajectory tracking for all three operational space commands. Joint torques are smooth with small deviations where the feedback control accounted for model deviations.

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Question 4

It is desired to control two separate tasks in a hierarchy such that Task 1 is fully accomplished and errors in Task 2 can be minimized without interfering with the primary task. This will be done by projecting control of Task 2 into the Null Space of Task 1. The demonstration will command a vertical angle to the end-effector (Task 1) and zero angles to each joint (Task 2) to show how the error is distributed amongst the three joints.

Control Law

$$X_1 = \theta$$

$$X_2 = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$\tau = J_1^T F_1 + J_{2/1}^T F_{2/1} + b\big(q(t), \dot{q}(t)\big)$$

Main Task

$$J_1 = \frac{\partial(\theta)}{\partial(q_1, q_2, q_3)} = [1 \quad 1 \quad 1]$$

$$M_1 = (J_1 A^{-1} J_1^T)^{-1}$$

$$\ddot{X}_1^{ref} = \ddot{X}_1{}^{des}(t) + k_{p1} * \left(X_1{}^{des}(t) - X_1(t)\right) + k_{d1} * \left(\dot{X}_1{}^{des}(t) - \dot{X}_1(t)\right)$$

$$F_1 = M_1(\ddot{X}_1^{ref} - \dot{J_1}\dot{q})$$

Main Task Null Space

$$\bar{J}_1 = A^{-1} J_1^T M_1$$

$$N_1 = I - \bar{J}_1 J_1$$

Posture Task

$$J_2 = I$$

$$J_{2/1}^* = J_2 N_1 = N_1$$

$$M_{2/1} = \left(J_{2/1} A^{-1} J_{2/1}^T\right)^+$$

$$\ddot{X}_2^{ref} = \ddot{X}_2{}^{des}(t) + k_{p2} * \left(X_2{}^{des}(t) - X_2(t)\right) + k_{d2} * \left(\dot{X}_2{}^{des}(t) - \dot{X}_2(t)\right)$$

$$F_{2/1} = M_{2/1}(\ddot{X}_2^{ref})$$

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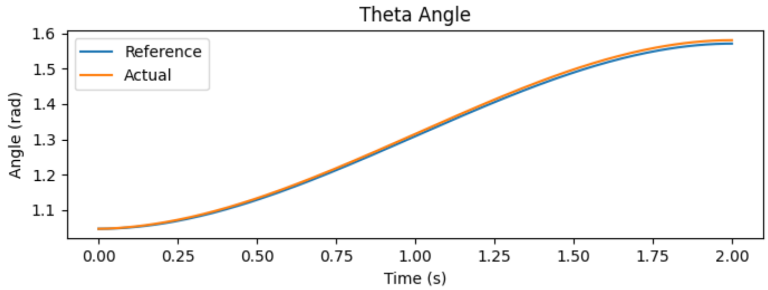
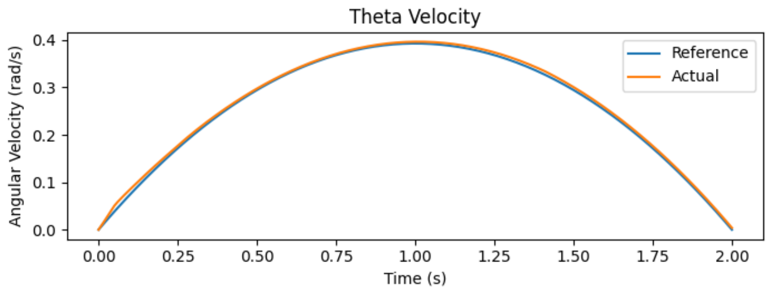
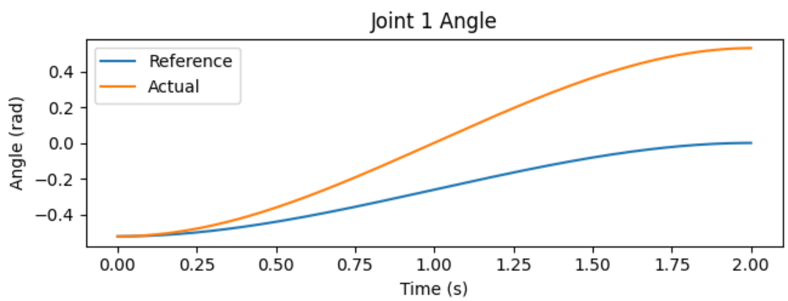
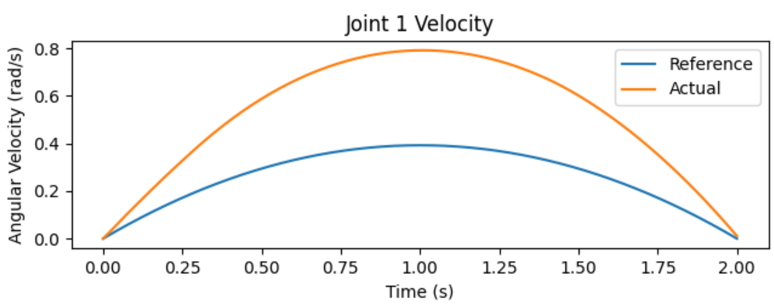
Results









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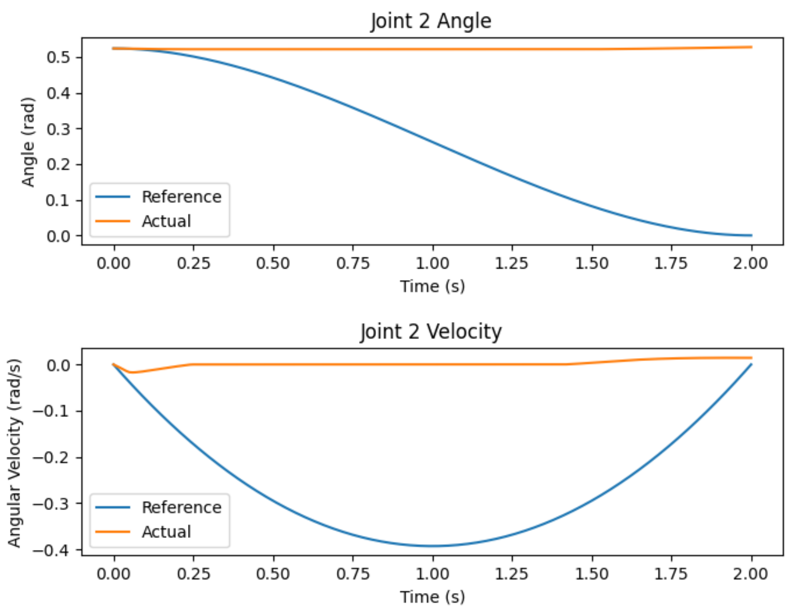

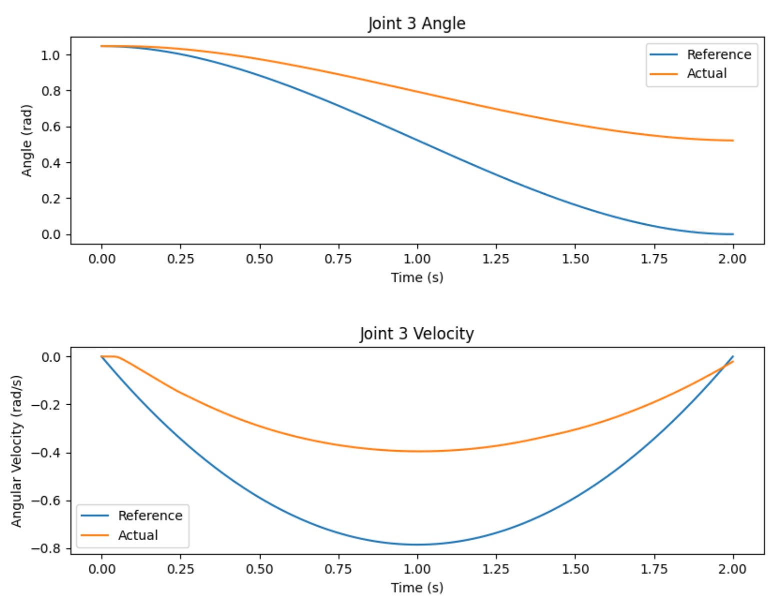

The resulting plots show that the primary task is accomplished completely, and the secondary posture task minimizes the total error by distributing it over the three joints roughly equally. The end effector ends at 90° (as commanded) and the three joints are each around 30°.

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Question 5

For obstacle avoidance, normal operational space control will be used until the end effector comes within a threshold distance of the obstacle. At this point, the operational space control will be projected into the null space of a repulsion field function such that it can continue moving parallel to the obstacle while maintaining a safe distance.

End Effector Projection on to Obstacle

Let the obstacle be a line defined by two points, p_1 and p_2, and the end effector position be p_3.

$$a = p_2 - p_1$$

$$b = p_3 - p_1$$

$$p_{proj} = p_1 + \frac{aa^T}{a^T a} b$$

Repulsion Barrier Function

$$d_{obs} = p_{proj} - p_3$$

$$\hat{d}_{obs} = \frac{d_{obs}}{||d_{obs}||}$$

$$V_{obs} = ||d_{obs} - \beta_{safety}\hat{d}_{obs}||^2$$

Obstacle Avoidance Task Control Law

$$J_{obs} = \left(R_g^o\right)^{-1} \cdot\ S_{obs} \cdot R_g^o \cdot J_p$$

$$S_{obs} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_o = \left(J_{obs} A^{-1} J_{obs}^T\right)^{-1}$$

It is easier to formulate the control law for obstacle avoidance as an acceleration vector in the obstacle frame, and then to rotate it into the global frame. This is because the gradient aligns with the normal vector to the obstacle in the obstacle frame. The derivative term is added in the global frame to avoid an unnecessary transform on the velocity.

$$a_{obs}^o = -K_{p,o} \cdot \nabla_p V_{obs} = -K_{p,o} \begin{bmatrix} 0 \\ 2 \cdot ||d_{obs} - \beta_{safety}|| \end{bmatrix}$$

$$a_{obs}^g = R_o^g \cdot a_{obs}^o + K_{d,o}\dot{x}$$

$$F_{obs} = M_o\left(a_{obs}^g - \dot{J_{obs}}\dot{q}\right)$$

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Operational Space Control Law

$$\bar{J}_{obs} = A^{-1} J_{obs}^T M_{obs}$$

$$N_{obs} = I - \bar{J}_{obs} J_{obs}$$

$$J_{p/o} = J_p N_{obs}$$

$$M_{p/o} = \left(J_{p/o} a^{-1} J_{p/o}^T\right)^+$$

$$\ddot{X}^{ref} = \ddot{X}^{des}(t) + K_{p,p} \cdot \left(X^{des}(t) - X(t)\right) + K_{d,p} \cdot \left(\dot{X}^{des}(t) - \dot{X}(t)\right)$$

$$F_{p/o} = M_{p/o}\left(\ddot{X}^{ref} - \dot{J}_{p/o}\dot{q}\right)$$

Posture Task

It was found that as only two dimensions (X and Y) were commanded in the operational space, there was an extra, unregulated degree-of-freedom (manifested in the angular velocity of the joints). A third posture task was added to make the system more stable and the results more consistent. Posture was projected into the null space of the operational space task (within the null space of the repulsion field).

$$\bar{J}_{p/o} = A^{-1} J_{p/o}^T M_{p/o}$$

$$N_{p/o} = \left(I - \bar{J}_{p/o} J_{p/o}\right) \cdot N_{obs}$$

$$J_{q/p/o} = I \cdot N_{p/o} = N_{p/o}$$

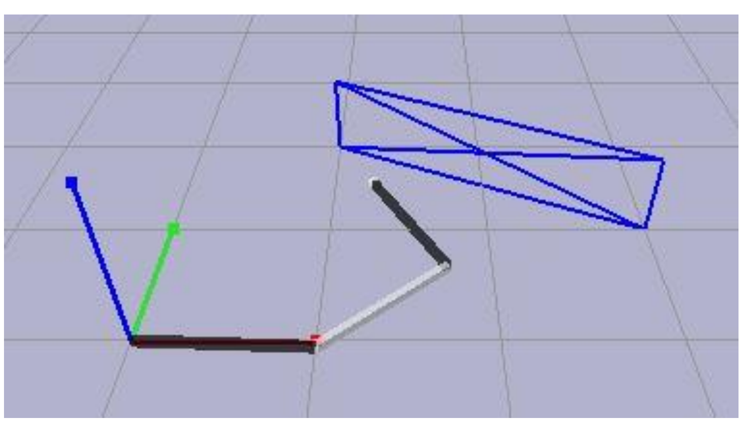
$$M_{q/p/o} = \left(J_{q/p/o} A^{-1} J_{q/p/o}^T\right)^+$$

$$F_{q/p/o} = M_{q/p/o}\left(-K_{p,q} \cdot q - K_{d,q} \cdot \dot{q}\right)$$

Final Control Law

$$\tau = J_{obs}^T F_{obs} + J_{p/o}^T F_{p/o} + J_{q/p/o}^T F_{q/p/o}$$

Results



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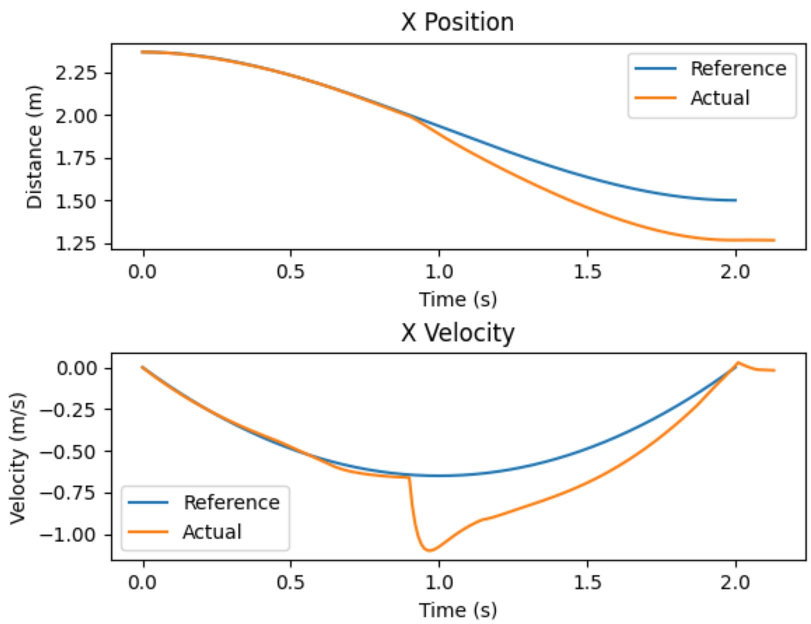

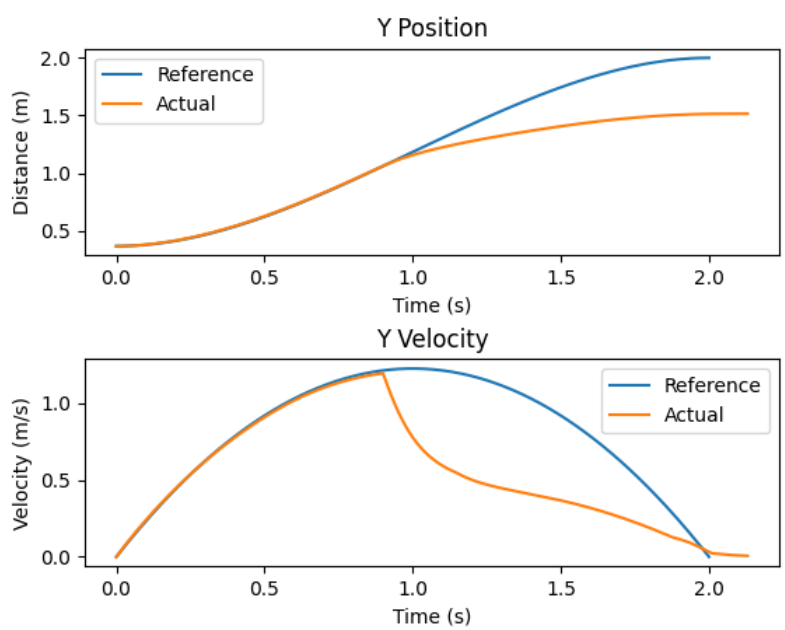

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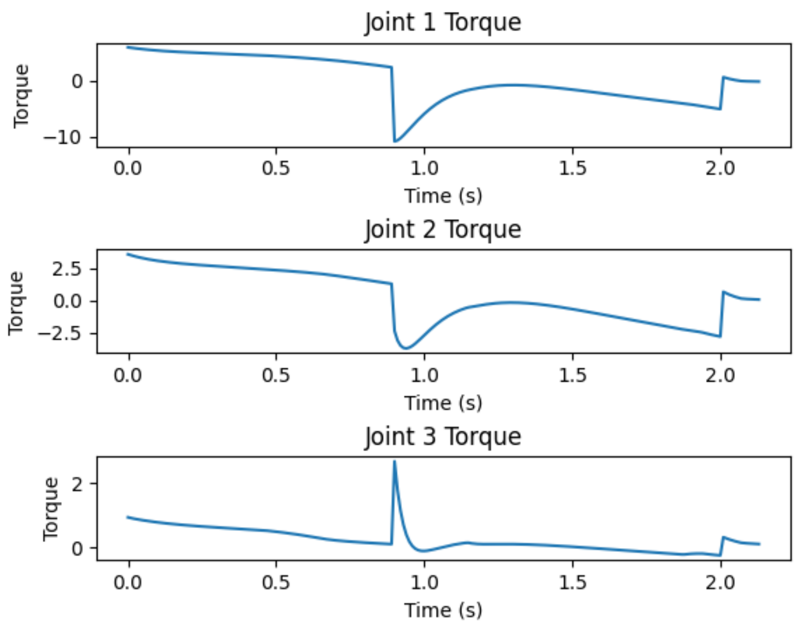

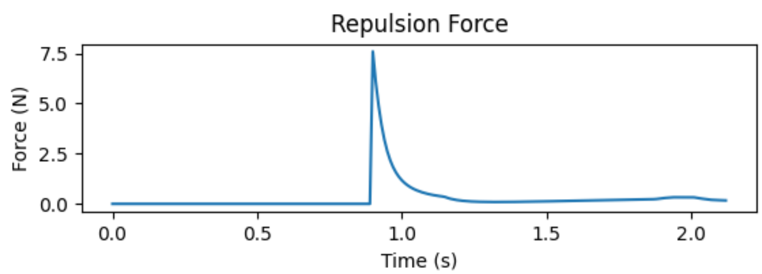

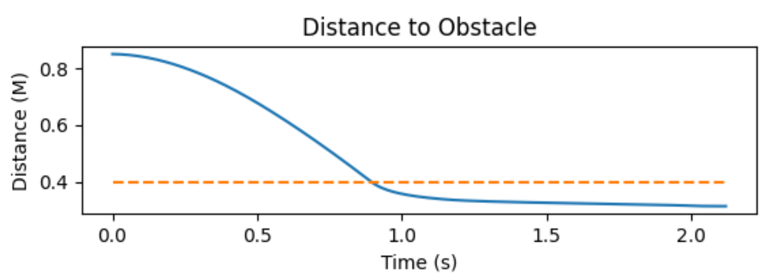

The implemented task hierarchy was successful in minimizing operational space and posture error while avoiding the obstacle. The repulsion force comes into effect when the end effector crosses the safety threshold, diverting the end effector trajectory. The control law for the obstacle repulsion is purely feedback and so requires no prior trajectory generation. The operational space trajectories are projected into the null space such that error is minimized without prior knowledge of the obstruction.