Human-Centered Robotics HW3

Maxx Wilson

Analytical Solution of DCM Dynamics

Given DCM dynamics, compute the analytical solution of the DCM Trajectory

$$\dot{\xi} = \frac{1}{b}(\xi - r_{vrp}) = \frac{\xi}{b} - \frac{r_{vrp}}{b}$$

$$\xi(t) = f(\xi_0, r_{vrp}, t)$$

Solution

Apply Laplace Transform to first order ODE

$$L[\dot{\xi}] = L[\frac{\xi}{b} - \frac{r_{vrp}}{b}]$$

The initial condition ξ_0 appears from the $\dot{\xi}$ term. Additionally, the Laplace transform of the constant, r_{vrp}, yields an s in the denominator.

$$\xi \cdot s - \xi_0 = \frac{\xi}{b} - \frac{r_{vrp}}{b \cdot s}$$

Solving for ξ,

$$\xi(s) = \xi_0 \frac{1}{s - \frac{1}{b}} + r_{vrp} \frac{-\frac{1}{b}}{s(s - \frac{1}{b})}$$

Applying known Inverse Laplace Transforms, we can convert this equation to the time domain.

$$\boxed{\xi(t) = \xi_0 \cdot e^{t/b} + r_{vrp}(1 - e^{t/b})}$$

Initial DCM Point for i^{th} step

Objective: Find initial DCM point for each step.

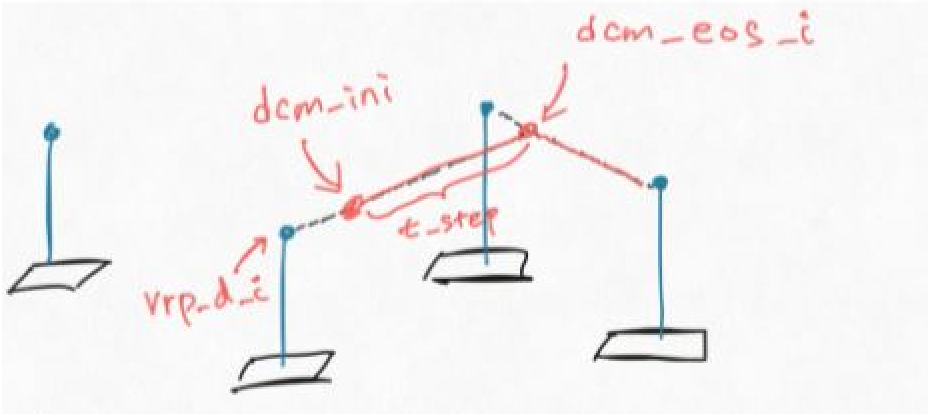

Figure 1: DCM Trajectory

The initial DCM point is a function of the end point, the virtual repellent point, and the step duration. The objective is to obtain the following equation.

$$\xi_{ini,i} = f(\xi_{eos,i}, r_{vrp}, t_{step})$$

Solution

Evaluated at t_{step}, the DCM function will return the end point. Given this condition, we could substitute in $\xi eos, i$ and t_{step} to solve for the required initial condition, ξ_0. It is also equivalent to evaluate the function with negative time and the endpoint as the initial condition, shown below.

$$\xi_{eos,i} = \xi_{ini,i} \cdot e^{t_{step}/b} + r_{vrp}(1 - e^{t_{step}/b})$$

$$\xi_{ini,i} \cdot e^{t_{step}/b} = \xi_{eos,i} - r_{vrp}(1 - e^{t_{step}/b})$$

$$\xi_{ini,i} = (\xi_{eos,i} - r_{vrp}(1 - e^{t_{step}/b})) \cdot e^{-t_{step}/b}$$

$$\boxed{\xi_{ini,i} = \xi_{eos,i} \cdot e^{-t_{step}/b} + r_{vrp}(1 - e^{-t_{step}/b})}$$

Double Support Phase

Find boundary conditions for the double support phase

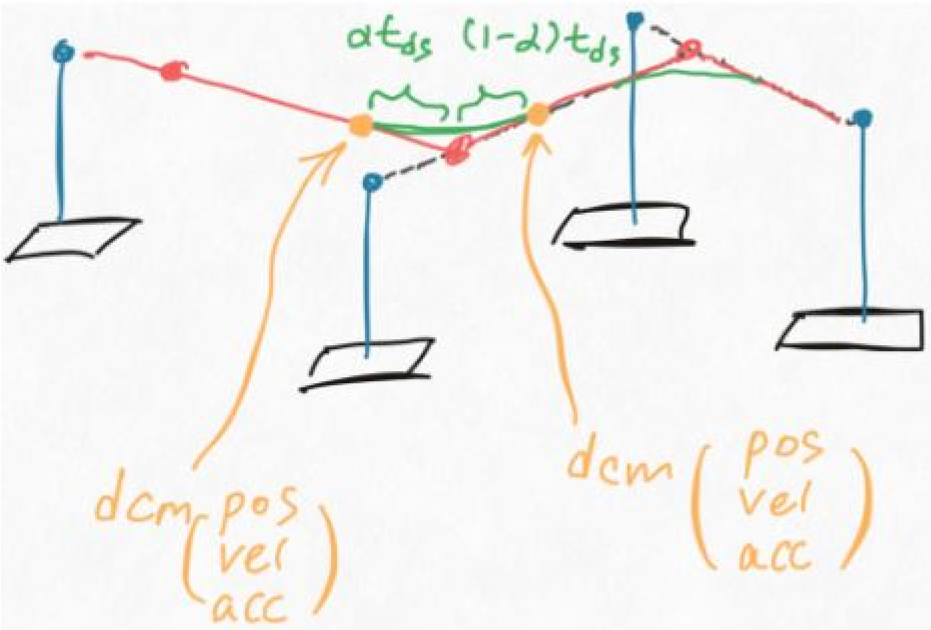

Figure 2: Polynomial interpolation used for double support phase

To account for the double support phase during step transitions, a polynomial interpolation is introduced which is dependent on the phase duration and an alpha gain. Initial and final conditions for the polynomial are calculated from the original DCM equation. Both equations reference the initial point of the next step, as it is identical to the end point of the previous step.

$$\xi_{ini,ds,i} = \xi_{eos,i-1} \cdot e^{-t_{ds,ini}/b} + r_{vrp,i-1} \cdot (1 - e^{-t_{ds,ini}/b})$$
$$\dot{\xi}_{ini,ds,i} = (\xi_{ini,ds,i} - r_{vrp,i-1})/b$$
$$\ddot{\xi}_{ini,ds,i} = \dot{\xi}_{ini,ds,i}/b$$

$$\xi_{eos,ds,i} = \xi_{ini,i} \cdot e^{t_{ds,end}/b} + r_{vrp,i} \cdot (1 - e^{t_{ds,end}/b})$$
$$\dot{\xi}_{eos,ds,i} = (\xi_{eos,ds,i} - r_{vrp,i})/b$$
$$\ddot{\xi}_{eos,ds,i} = \dot{\xi}_{eos,ds,i}/b$$

Results

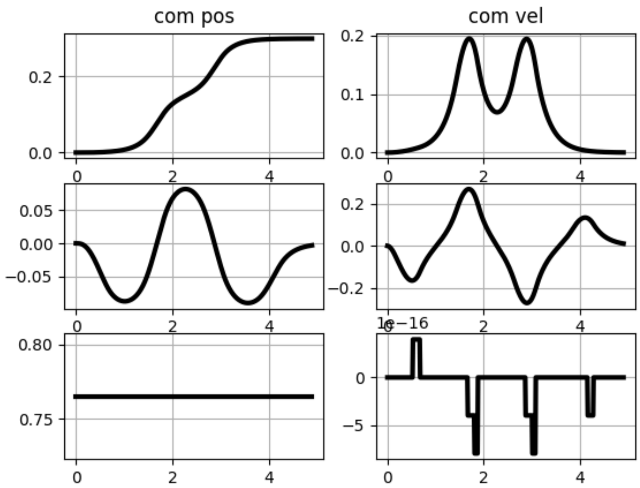

Figure 3: Position and Velocity of Center of Mass

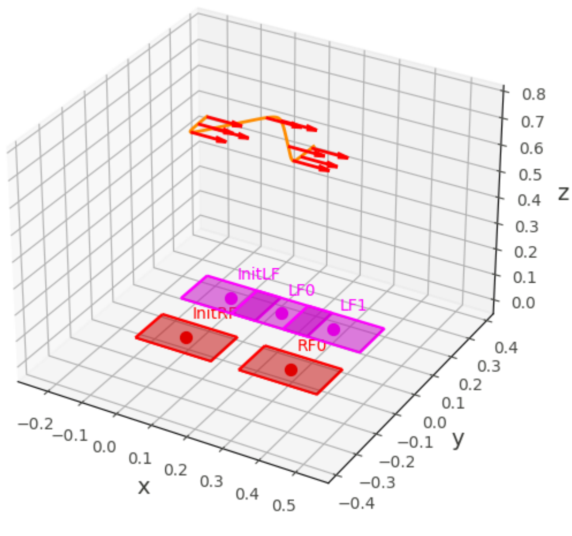

Figure 4: 3D DCM Point Trajectory with Footsteps

Backwards Walking

To enable backwards walking, the only change that must be made is in the desired footsteps inputted into the planner. Results are shown in the figures below, where the footstep order has been reversed and the footsteps move backwards from the initial stance. Notice that the initial footsteps are now at the front. The red arrows represent the body orientation.

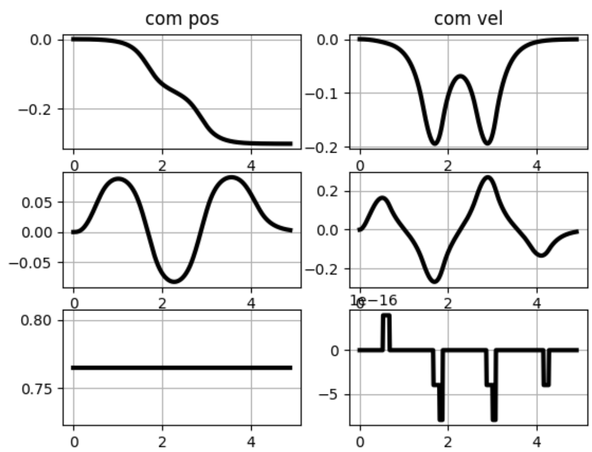

Figure 5: Position and Velocity of Center of Mass

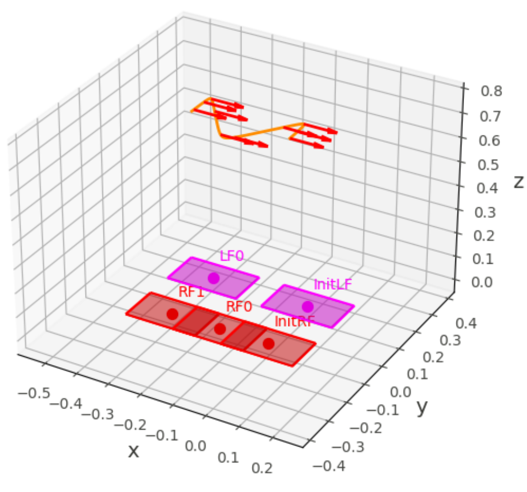

Figure 6: 3D DCM Point Trajectory with Backwards Footsteps