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EMOTIONS IN PROBLEM SOLVING

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Emotions are important part of non-routine problem solving. A positive disposition to mathematics has a reciprocal relationship with achievement, both enhancing the other over time. In the process of solitary problem solving, emotions have a significant role in self-regulation, focusing attention and biasing cognitive processes. In social context, additional functions of emotions become apparent, such as interpersonal relations and social coordination of collaborative action. An illustrative case study presents the role of emotions in the problem solving process of one 10-year old Finnish student when he is solving an open problem of geometrical solids. The importance of emotions should be acknowledged also in teaching. Tasks should provide optimal challenge and feeling of control. The teacher can model the appropriate enthusiasm and emotion regulation. Joking and talking with a peer are important coping strategies for students.

Emotion, problem solving, coping

INTRODUCTION

Problem solving competence is given high priority in curriculum documents (OECD, 2003) and especially in mathematics, problem solving is considered essential (Schoenfeld, 1992). In this presentation we do not consider mathematical routine tasks as problems. Instead, a mathematical task is a problem for a person only if he or she does not immediately know how to solve it.

The role of affective elements in mathematical problem solving has been widely acknowledged. Already Polya (1957) addresses determination and hope (p. 93) in his short dictionary of heuristics, mentioning also the necessity to become familiar with all emotions related to the problem solving process. More explicitly the role of affective variables was elaborated in the 1980's, when Mason, Burton, and Stacey (1982), Schoenfeld (1985), McLeod (1988), Goldin (1988, 2000) and Cobb, Yackel & Wood (1989) all gave a significant role for affect in their analysis of mathematical problem solving.

The first part of this presentation will review non-cognitive aspects of non-routine problem solving behaviour. This discussion will be divided into two sections, the first focussing on students' relatively stable emotional traits that may influence the problem solving behaviour and the other part on the continuously changing emotional states that occur during the problem solving process. The second part of the presentation will provide an illustrative example of one student's emotions and coping while engaged in problem solving in classroom setting. The last part of the presentation will focus on the different ways how the teacher can enhance the desirable emotional climate in a problem solving lesson.

In the literature, there are several definitions for emotions stemming from three distinct traditions. The Darwinian research tradition focuses on the evolutionary and biological foundation of human emotions. Freudian tradition focuses on the role of emotions in psychopathology. The cognitive tradition focuses on the interplay of emotions and cognitions. Despite the separate histories, these traditions agree upon some aspects. There is a general agreement that emotions consist of three processes: physiological processes that regulate the body, subjective experience that regulates behaviour and expressive processes that regulate social coordination. Moreover, most emotion theories agree that emotions are closely related to personal goals, that they have an important role in human coping and adaptation. Furthermore, they perceive a physiological reaction to make emotions that different from cognition. Emotion theories are different in the number of emotions they identify, the degree of consciousness they attribute to emotions, and in how they perceive the relation between emotion and cognition. (Hannula, 2004)

Some emotion theories (e.g. Lazarus, 1991) identify a large number of different emotions based on the different social scenarios and cognitive appraisals related to the emotion, while some other emotion theories (e.g. Buck, 1999; Ekman, 1971; 1992) identify a small number of basic emotions that differ in their physiology, and the different cognitive appraisals and social scenarios are seen as external (though closely related) to the emotion. Moreover, some theorists have attempted to organize emotions based on the dimensions of arousal and valence (e.g. Lang, 1995).

In this paper, we identify different emotions along the tradition of basic emotions. These basic emotions include at least happiness, sadness, fear, anger, and disgust (Buck, 1999; Power & Dalgleish, 1997, p. 110). Further basic emotions include some of the self-regulative emotions, where best candidates seem to be surprise, curiosity and confusion (Lehman, D'Mello & Person, 2008; Goldin, 2000). Furthermore, the list should include also further social emotions such as shame (Pekrun & Stephens, 2010), which has a characteristic physiological expression (blushing), and attachment and submission (Buck, 1999), which have important influence on interpersonal relationships. Other emotions (e.g., gratitude, pride, contempt etc.) are seen as basic emotions in a characteristic situation or with a characteristic target or, in some cases, as blends of two or more basic emotions. For example, anxiety can be considered as fear for failing on what one is doing, and hopelessness can be considered as anticipatory sadness when failure is seen to be inevitable. The more complex emotions are based on the basic emotions and characterised largely by the related cognitions (Buck, 1999; Power & Dalgleish, 1997). Although we adopt the approach of basic emotions, we use more specific

vocabulary for these emotions when the social scenario or cognitive appraisal plays a significant role. Although some researchers define moods as a separate concept, we do not discuss them separately, because we consider them here as low-intensity emotions (Pekrun & Stephens, 2010).

In this paper, we adopt the view by Damasio (1999) regarding three stages of consciousness of emotions:

I separate three stages of processing along a continuum: *a state of emotion*, which can be triggered and executed nonconsciously; *a state of feeling*, which can be represented nonconsciously; and *a state of feeling-made-conscious*, i.e., known to the organism having both emotion and feeling. (p. 37; as cited in Schlöglmann, 2003)

Student disposition and mathematical competence

It is widely acknowledged that the problem solver's overall disposition (attitudes, beliefs, values, motivational orientations) influences how successful the solver will be in their attempts to solve the problem. There are several different theories that focus on different aspects of the disposition. Usually these studies have focused on the relationship between a specific affective trait (anxiety, attitude, motivation or beliefs) and achievement in mathematics. However, there is good reason to assume that these findings apply in a similar way also to the more specific relation between these affective traits and achievement in problem solving.

Perhaps the first approach to explore the connection between disposition and mathematical performance were the numerous studies on mathematics anxiety and its negative correlation with mathematics performance (for a meta-analysis, see Hembree, 1990). Similar relationship was confirmed between attitude towards and achievement in mathematics (McLeod, 1992).

Motivation research has several theoretical approaches and use of terminology is sometimes confusing (Murphy & Alexander 2000; Niemivirta 2004, 10; Pintrich 1994). However, there is conclusive empirical evidence for a positive correlation between motivation and performance (e.g. Middleton and Spanias, 1999). The more value the student gives to solving a particular problem, the more persistent the student is in his or her effort. More specifically, it has been confirmed that intrinsic motivation (interest in the task) is more productive than extrinsic motivation due to rewards (Middleton & Spanias, 1999). In the achievement goal theory, a positive relationship has been found between mastery goal orientation and achievement (Friedel, Cortina, Turner & Midgley 2007; Midgley, Kaplan, Middleton, Maehr, Urdan, Anderman, Anderman & Roeser 1998). Results concerning performance goal orientation and achievement have been less consistent. Some have identified negative learning behavior, while other results indicate performance orientation to lead to positive learning behavior and achievement (Freeman 2004; Midgley et al. 1998.)

Bandura's theory of self-efficacy is looking at the connection between self-efficacy beliefs and actual performance in a specific domain (Bandura & Schunk 1981). Numerous studies have confirmed the high correlation between the two (e.g. Lee, 2009).

Although survey studies indicate a clear correlation between mathematics-related affect and achievement, it has been more difficult to confirm the direction of causality. Ma and Kishor (Ma & Kishor, 1997a, b; Ma, 1999) have summarized much of that research in their meta-analyses. In one of these studies, they synthesized 113 survey studies of the relationship between attitude towards (=liking) mathematics and achievement in mathematics. The causal direction of the relationship was found to be stronger from attitude to achievement (Ma & Kishor, 1997a). However, there has been criticism of studies that do not use a longitudinal design (see Ma & Xu, 2004). Minato and Kamada (1996) reviewed eight studies that had used a cross-lagged panel correlation technique (a longitudinal design) in order to synthesize findings on the causal relationship between attitude towards mathematics and achievement in mathematics. In most of the studies, there was no predominance of either attitude or achievement. In the few instances that predominance was found, the causal direction was from attitude to achievement. However, Ma and Xu (2004) found a contrasting result with a larger and more representative sample. According to their study, the dominant causal relationship is from achievement to attitude. Taken together these studies suggest a reciprocal rather than unidirectional causality between achievement and affect. Such reciprocal relationship has been identified between self-efficacy and achievement in mathematics across countries (Williams and Williams, 2010).

There are also studies that address more specifically how student's beliefs influence the choices they make. Schoenfeld (1985) concluded that students' beliefs were an important determinant of their problem solving success. He observed that most students were likely to give up if the problem was not solved in five minutes, and concluded that students possessed a belief that mathematics problems can be solved in five minutes or less.

As a summary we can conclude that there is strong evidence for the correlation between student disposition towards mathematics and their actual performance in mathematics. High achievement is related to liking mathematics and determination to do well. Moreover, the relationship is reciprocal, indicating that changes in either can lead to changes in the other. Moreover, beliefs about mathematics and problem solving may direct the problem solver to explore venues that support innovative solutions or they may suggest staying on more familiar paths.

Emotional states and problem solving

Another perspective looks more specifically the different choices the student makes throughout the problem solving process and the role of emotions in it. Regardless of the overall disposition, all problem solvers encounter positive and negative emotions that influence their solution process (e.g. Schoenfeld, 1985; McLeod, 1992; Goldin, 2000). In fact, emotions are an essential part of the problem solver's self-regulation (Goldin, 2000; Hannula, 2006; Carlson & Bloom, 2005; Malmivuori, 2006). Moreover, problem solving takes place by social beings in the complexity of the learning environment where multiple goals need to be addressed (Hannula, 2006; Goldin et al., 2011).

Next, we will review some studies that have explored which emotions are present in the learning context and more specifically in problem solving. It has been suggested that six basic

emotions (anger, sadness, fear, disgust, happiness, and surprise; Ekman 1972, 1992) would be rare in learning context (Vogel-Walcutt, Fiorella, Carper, & Schatz, 2012). However, as these basic emotions relate to social coordination, they should be more frequent in collaborative learning settings. Pekrun has identified a number of emotions that appear frequently during the learning process and three dimensions in their taxonomy: valence, activation, and object focus (Pekrun & Stephens, 2010). The variety of emotions in their twelve categories can be alternatively categorized under basic emotions: the negative activating emotions represent three different basic emotions: anger (anger, frustration), fear (anxiety), and shame (shame) while all deactivating negative emotions (boredom, hopelessness, sadness, and disappointment) are variants of sadness. The activating positive emotions (enjoyment, hope, anticipatory joy, joy, pride, and gratitude) are all variants of the basic emotion of happiness while the positive deactivating emotions (relaxation, anticipatory relief contentment, and relief) seem not to be basic emotions, but rather lack of any emotions or removal of a negative emotion (relief).

Emotions that have been identified to appear frequently during solitary problem solving (logic problems) are curiosity, boredom, frustration, confusion and happiness while occurrences of anxiety, contempt, eureka, anger, disgust, fear, sadness, and surprise were found to be rare. However, the study design included an incorrect feedback to 25 % of the responses, which might explain the high frequency of curiosity, frustration and confusion. (Lehman, D'Mello, and Person, 2008). In solitary mathematical problem solving, emotions identified to have importance in the self-regulative processes include curiosity, puzzlement, bewilderment, frustration, pleasure, elation, satisfaction, anxiety, and despair (DeBellis and Goldin 2006). When mathematical problem solving has been studied in the mathematics homework context, researchers have identified the following emotions to appear in the mother-child interaction: tension, distress/dismay, frustration, sadness, boredom/apathy, anger/disgust, contempt, positive interest, affection/caring, joy/pleasure, humor, and pride (Else-Quest, Hyde, & Hejmadi, 2008). If we analyze the emotions in these lists according to a basic emotions framework, we can expect to see most of the basic emotions, when observing problem solving: curiosity to begin with and happiness if the problem is solved. In case the student struggles with the problem or cannot solve the problem there might be sadness, confusion, fear, anger or disgust, depending the attribution the student gives for the lack of progress.

There is a general agreement that emotions are functional and that they have an important role in human adaptation to different situations and learning. This applies also to mathematical problem solving. However, dynamic theories about the role of emotions in the process of problem solving are at the moment at their preliminary stages (Lehman et. al., 2008, Goldin, Epstein, Schorr & Warner, 2011)

Emotions serve three fundamentally different functions in human self-regulation: physiological, psychological and social. The first function is physiological adaptation, where the most clear example is the 'fight of flight'-response to surprising threatening stimulus. The emotion triggers release of adrenaline, which prepares the body physically to fight or alternatively to escape. (Power & Dalglish, 1997) This functional aspect of emotions is

relevant to learning situations in the sense that most emotions have physiological reactions as a side effect. For example, fear may influence the physiology in a way that is detrimental for optimal cognitive functioning in a test situation.

The second role of emotions is in the psychological self-regulation through influence on cognitive processing. Just as fear or anger have clear consequences in the physiological adaptation; surprise and curiosity have clear influence on attentional processes and memory. It is now well established that emotions direct attention and bias cognitive processing. For example, fear (anxiety) directs attention towards threatening information and sadness (depression) biases memory towards a less optimistic view of the past (Power and Dalgleish 1997; Linnenbrink and Pintrich 2004).

It has been suggested (Forgas 2008) that the positive emotions would promote the more inductive, bottom-up thinking while the negative emotions would promote the more deductive, top-down thinking. Although there is not yet sufficient evidence to conclude that to be case for all positive and negative emotions, it seems that positive emotions would facilitate the creative aspects of problem solving, while the negative emotions would facilitate reliable memory retrieval and performance of routines, which also are essential in certain phases of problem solving (Pekrun & Stephens, 2010). More intensive levels of mathematics anxiety seem to be exclusively detrimental for problem solving and the suggested mechanism is the overloading of working memory as the subject is preoccupied with one's math fears and anxieties (Ashcraft and Krause, 2007; Rubinstein and Tannock, 2010).

Moreover, emotions not only bias memory retrieval, but also function as a 'fixative' in the storage to long term memory. A recent study has identified that activity in the amygdala during an Aha! experience is a strong predictor of which solutions will remain in long-term memory (Ludmer, Dudai and Rubin 2011).

Emotions serve also a third adapting function in the social coordination of a group, which brings forth new types of emotions. The close relation between emotions and the social interaction is well acknowledged (e.g. Forgas, 2008, Pekrun and Stephens, 2010). Previously Cobb et al. (1989) identified that students' emotions are related to two types of problems in collaborative problem: mathematical problems and cooperation problems. Hannula (2005) identified three different social functions for emotions in collaborative problem solving:

1. emotions concerning interpersonal relationship needs and goals (e.g. sadness due to exclusion),
2. emotions concerning individual learning goals when their cause is attributed to peers (e.g. gratitude for help), and
3. emotions concerning social coordination of individual goals (e.g. anger when own idea has been rejected by others).

The interpersonal relationship needs and goals are not specific to mathematics or even learning, but they should not be ignored when we analyze student emotions in the classroom. The emotions that relate to individual learning goals, on the other hand, are rather

straightforward extensions of self-regulative emotions in solitary problem solving. The third category deserves further elaboration.

In the coordination of collaborative behavior, emotions can be expressed and interpreted unconsciously. Emotions may also be used consciously in power games or as means to solve communication problems. Furthermore, emotions may be interpreted consciously, when they become subject to reflection and re-evaluation. Emotional communication in the coordination of collaborative problem solving behavior can be very powerful, one example being shared cognitive intimacy (Hannula 2005, c.f. Williams 2002):

In shared cognitive intimacy, students enter an intimate interaction with each other and with a task. This intimacy is indicated by one student frequently continuing or completing the other student's utterances and occasionally by both speaking in unison. It is an example of a situation where students can achieve their cognitive and social goals simultaneously. This dual intimacy with peers and mathematics is rewarding for the students and, furthermore, it can be an extremely useful tool for enhancing the classroom climate. One problem with this kind of intimacy is that sometimes it may exclude other students. (Hannula 2005, p. 35)

The same study identified also a number of strategies that a student may use to 'saving their face' when problem solving does not seem successful (Hannula 2005). Of these strategies devaluing of the task and expressing lack of effort may be detrimental for the development of productive social norms for collaboration.

A study on mother and child emotions during mathematics homework supports the hypothesis that emotions are both an influence on and an outcome of mathematics performance. Moreover, it provides some interesting qualitative data on the dynamics of emotions while doing homework, suggesting a reciprocal relation where either mother or child may initiate important changes in the emotional state of the other. (Else-Quest, Hyde, & Hejmadi, 2008).

Emotions and problem solving in the school context

So far, most research on mathematics-related affect has been done using surveys, which has provided strong evidence for the theories regarding trait-type emotions (although good longitudinal studies are too few). Research on the role of emotions in cognitive processes has mainly been conducted qualitatively, and while such studies are good for identifying important concepts and their relations, they seldom are rigorous enough for testing theories. Quantitative studies, on the other hand, are mainly conducted in laboratory settings and the ecological validity of research findings must be tested in realistic school settings (Pekrun & Stephens, 2010).

Perhaps the first to report the role of emotions both in the cognitive and social domain were Cobb et. al (1989). Their study challenged the previous observations of emotionally neutral or 'flat' classrooms, illustrating instead a lively classroom:

Children frequently jumped up and down, hugged each other, and rushed off to tell the teacher when they solved a particularly challenging problem. Significantly, the positive

emotional acts occurred when the children completed personally challenging tasks or constructed mathematical relationships. (Cobb et al., 1989, p. 61)

In their framework, social norms provide a framework for interpreting the individual emotions (Cobb et al., 1989).

Another approach has been to report case studies of student's emotions while solving problems in a mathematics classroom (e.g. Evans, Morgan & Tsatsaroni, 2006; Hannula, 2003; Op 't Eynde & Hannula, 2006; Williams, 2002).

However, Goldin, Epstein, Schorr, and Warner (2011) have suggested that there is a need for 'mid-level' concepts between in-depth qualitative studies and 'high-level' concepts like norms. In a qualitative study of an US inner-city middle school, they identified a number of behavioral patterns that integrate students' affective and social interactions, which they call *engagement structures*. Each engagement structure is characterized by ten aspects, which include goal, behavioral and emotional pattern, self-talk and interaction with beliefs. These engagement structures could be also described as behavioral scripts that relate to a specific goal or desire. (Goldin, Epstein, Schorr, & Warner, 2011)

Emotions are ubiquitous in human interaction and, therefore, it is extremely challenging to extract the influence of teacher's emotional support in research design. However, in computer-based learning environments it is easier to study the influence of emotional or motivational support. Kim and Hodgen (2012) found out that even a minimal six minute video aimed at improving student emotion regulation was able to produce significant influence on university students' academic emotions in an on-line remedial mathematics course.

As said earlier, emotions serve the three purposes of physiological adaptation, psychological regulation, and social coordination. Although emotions are functional for the human species, not all emotional reactions are functional in classroom context. For example, anger provides functional physiological adaptation to overcoming obstacles when solving a physical problem (e.g. moving a heavy object). Yet, it is usually less functional when solving cognitive challenges collaboratively. Moreover, the physiological aspect of the emotion (e.g. adrenaline or endorphins) may have effects beyond the duration of the emotional event and such emotional residual may interfere with following emotional episodes. Such unwanted side effects of emotional reactions could be considered as emotional 'noise' confusing smooth emotional communication.

Therefore, especially in the schools context, emotions need to be regulated (De Corte et al., 2011; Fried, 2012). Emotion regulation refers to

the ways individuals influence which emotions they have, when they have them, and how they experience and express these emotions. (Gross, 1998, p. 275)

There are different ways to categorize emotion regulation strategies. The target of emotion regulation can be attention, emotion-relevant knowledge, or body manifestations of emotion, their psychological function may be oriented towards needs, persons or goals, and both antecedent and response-focused strategies are possible (Fried, 2012). Emotion regulation

that mainly focuses on reducing negative emotions or their effects is often called coping. The coping strategies that Flemish high-school students report using when facing difficult mathematics test, homework or lesson are (in order of frequency) active coping (i.e. effort), joking and acceptance, social-emotional coping (i.e. seeking social support), abandoning and negation, religion and – rarely – alcohol and drug use (De Corte et. al., 2011). In addition to coping, it is important to consider the emotional regulation that occurs before emotional reaction through choice of goals and selective attention.

AN ILLUSTRATIVE EXAMPLE: AN 11-YEAR OLD STUDENT SOLVING AN OPEN PROBLEM

In this section, we shall analyze one student's emotions in the course of one lesson. The case will illustrate different functions of emotions.

The context

The analysis is based on a video recording of a research class that participates in a project, where one mathematics lesson each month is used to solve an open-ended problem and the participating eight teachers meet once a month with project research team to discuss using open-ended problems in teaching (Näveri, Pehkonen, Hannula, Laine & Heinilä, 2011). This analysis will focus on one particular 10-year old male student (Tomiⁱ) and his interaction with his teacher and his two peers sitting next to him (Arto and Eetu). One video camera was used for recording the overall progress of six students. We do not have a full coverage of Tomi's behavior during this lesson. However, due to his position in the middle of the group, he appears on the video most of the time.

The analytical framework

For the analysis of this case, we shall use the emotion coding scheme by Else-Quest, Hyde, and Hejmadi (2008), which was developed to analyze mother-child interaction during homework. This coding scheme is based on two emotion coding schemes: Ekman's FACS system (Ekman, Friesen & Hager, 2002), which identifies six basic emotions from facial expressions and Gottman's SPAFF system (Levenson & Gottman, 1983), which identifies 16 emotions in dyadic interactions based on facial expressions as well as other verbal and nonverbal expressions of emotions. This coding scheme was chosen for this analysis, as it has been tested and found useful in context where both self-regulative and social aspects of emotions are essential. The coding scheme includes 13 emotions: tension, distress/dismay, frustration, sadness, boredom/anxiety, anger/disgust, contempt, positive interest, affection/caring, joy/pleasure, humor, pride, and off-task. The coding scheme provides a number of markers for each emotion. Table 1 provides markers for two of the emotions. For details, see Else-Quest, Hyde, and Hejmadi (2008).

Table 1. Two examples of emotions and their markers used for the analysis.

Emotion	Markers
Joy/Pleasure	High-fives, smiling (lip corners up, raised cheeks, outer brow down), exclamations (e.g., "Wow!" or "Cool!")

Pride	Sitting upright, "showing off", similar facial expression as joy/pleasure, but antecedent event is achievement
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The case

The open problem of the lesson was to construct models of different solids using given manipulatives (peas and cocktail sticks) and to record the number of edges and vertices of each solid. In the first phase, the students were asked to find out different solids that they can construct using no more than 12 sticks. When student could not find any further solids with 12 sticks, they were permitted to use more sticks (14, 16, etc.).

The lesson began by the teacher giving instructions to the class and showing one exemplary model of a shape to the students. Each student was given their own pile of sticks, and Tomi, Arto, and Eetu received one cup of peas to share, located on Tomi's desk. In the transcript, the time starts to run from the moment students have received the peas and sticks, and they begin their construction.

Time

00:00 {Tomi, Arto, and Eetu all start building a cube as their first solid.} ⁱⁱ

01:39 {Tomi accidentally hurts his hand into a sharp stick.} Ow, sshiit! {He pulls his hand away in a reflex-like motion. Emotion: distress/dismay.}

Here, Tomi experienced pain and cursed because of it, indicating that he did experience anger. However, no further consequences could be observed. It is likely that Tomi was little more careful when attaching the next pea, but the incident did not distract him from his work nor did other students react to it. However, a similar incident happened later in the lesson, when he encountered a rotten pea. Then his face indicated disgust, and his work was interrupted until he had cleaned the rotten pea away. The incident also attracted the attention of Arto and Eetu, who interrupted their own work to examine the cause of disgust.

These two incidents illustrate how the 'primitive', self-protective role of emotions can intervene problem solving. In both cases they were given priority (the first one was like a reflex), but at least for Tomi, the distraction was short. However, students who are more timid and prone to anxiety, might not overcome the emotional distraction as easily as Tomi.

01:45 {Tomi is still building the cube. Emotion (momentarily): tension. After that he speaks some off-topic with Arto and Eetu.}

02:38 {The cube is more than half ready. Tomi leans close to it and points at each of the sticks (vertexes) and also to each vertex that has not yet been constructed. Arto and Eetu speak off-topic past Tomi, who ignores it. Emotion: positive interest.}

02:44 Tomi: This will be exactly 12! {The tone of voice indicates excitement, perhaps also pride.}

In this episode Tomi was effectively regulating his attention when focusing on the task and he expressed elation when realizing that the construction fulfills the requirements of the task.

Hannula

Specific to this episode was that these emotions related to the individual goals of Tomi and perhaps for that reason they did not well fit the markers of the coding scheme. The tension in the beginning was not of unpleasant nature; rather it just reflected the intensity of concentration. Later, Tomi expressed his excitement verbally, perhaps wishing to share the emotion with peers, but they were focusing on their individual work and they did not react.

- 03:07 Tomi: {Arto is picking pea from the cup, as he has done several times.} Why are you taking mine! {Turns his face to look Arto directly in the eyes, at the same time moving his hand on the desk sharply towards Arto, perhaps as to mark 'his' space. Then moves his hand towards Arto's hand, as if trying to stop him from taking peas. Clearly, he is challenging Arto, although the specific emotion is not clear.}
- 03:10 Arto: Because we don't have [own]ⁱⁱⁱ.
- 03:11 Tomi: [Ah, right.] We don't have. {Covers his face with both hands.}
- 03:13 Arto: What are you freaking out! {Smiling}
- 03:16 Eetu: {Tomi takes hand off his face, is blushing, smiles meekly. Emotion: shame.} For real, Tomi! Laughable. Someone's insane here. One freaks out. 'Why are you taking mine!' {pitched voice}. {Arto smiles widely and giggles, also Tomi laughs shyly, still blushing}
- 03:26 Tomi: I forgot that we'd have our own {in a meek voice}
- 03:30 Eetu: You forgot that we'd have our own?
- 03:32 Tomi: {Inaudible}

This second episode was clearly about social coordination, use of shared resources but also about the social relationships and reputation. First, Tomi reacted to defend his 'rightful territory'. However, as he realized that peas were a shared resource he was very embarrassed, which is in this case very expressive: he gave rapidly up his challenge, covered his face, blushed, and even the tone of his voice changed. Such a reaction was perhaps the most reliable apology to make, as blushing cannot be faked. Eetu was emphasizing the incident, making fun of Tomi, who in this situation was vulnerable. However, all three laugh at the end, indicating that Tomi did not feel the attack too serious. This incident was also noticed by two other students who were seen on the video, turning to look at the three boys a little bit worried first, and at least one of them was smiling at Eetu's ridiculing of Tomi. Also in this case the coding scheme was insufficient, providing a category for neither of the main emotions, although the challenge could be classified under tension.

Also this incident was passed with little effect on students' problem solving. There was clearly some emotional residual for Tomi, and possibly this incident influenced his goals in the future episodes (he was ambitious, wanting to produce something special).

All three boys finished their cubes, Tomi made swiftly also a tetrahedron.

- 03:36 Tomi: {to Arto} What are you doing? {Looks at the constructions of Arto and Eetu.}

Hannula

- 04:21 Arto: For real, don't do exactly same as me.
- 04:26 Tomi: I don't even know what you intend to do.
- 05:00 Tomi: You are doing the same as me. All these [cubes] are identical.
- 05:12 Tomi: {Builds a 'house' of the cube and the tetrahedron. Emotion: Humour.}
Haha haha, look! Ha! {Arto looks at the 'house' and smiles.}
- 05:21 Tomi: What to do now? {Emotion: boredom.}
- 05:44 Tomi: What would I do now? {Tomi leans back, slouching on his chair, soon sits again more upright, then puffs air out with a sound and fidgets with sticks. Emotion: boredom.}
- 06:04 Tomi: {to teacher} I don't get it. {Emotion: distress/dismay. Teacher's response is inaudible.}
- 06:08 Tomi: Can they be something totally own? {Emotion: Joy/pleasure. Tomi begins to construct a new model.}

Teacher: They can be your own. As long as there are not more than 12 sticks.

Tomi had difficulties to continue after the two first solids. He realized that they had done identical cubes and Arto warned him from doing the same as he. Tomi used joking as a coping strategy, but symptoms of boredom become soon clearly visible. He did not like the situation, and sought support from his teacher. Something that the teacher said gave him an idea to work with.

For the sake of brevity and to keep the red line visible, only part of Tomi's interactions is reported in the next section. Tomi built a 'flat' shape of two attached squares.

- 08:30 Tomi: Teacher! See, ladders! {Shows his construction to teacher, whose reaction cannot be seen, plausibly he is occupied with some other students. Arto is looking at Tomi's 'ladders' and both boys are laughing. Emotion: humour.}
- 08:40 Eetu: What is that?
- 08:42 Tomi: {Emotion: changes rapidly to neutral.} I don't know. This just came out like this.
- 08:50 Tomi: {To teacher, showing his 'ladders', smiling. However, emotion is not joy/pleasure.} Does this count?
- 08:54 Teacher: That's not a solid yet. It's on a plane, in a way. Think if you could continue it.
{Tomi makes a new type of construction.}
- 09:40 Tomi: Look! {Emotion: joy/pleasure.}
- 09:47 Tomi: Well, teacher! Is this one?
- 09:52 Teacher: See, it is still slightly open here. {Pointing.}
- 09:53 Tomi: Hmph! {Emotion: frustration.}

{Tomi adds one stick to his construction, ending up with a model that is not a polyhedron. It is relatively complex model and it is ambiguous which solid it would represent. However, it could be interpreted as a model of a solid whose some surfaces are curved.}

10:02 Tomi: Well is this now? {Emotion: Joy/pleasure. Teacher is with another student.}

10:27 Tomi: {to Arto} Hey look! {Expressed amazement}. Diamond. {Emotion: joy/pleasure.}

11:14 Tomi: Teacher! Come and look. I will show.

11:18 Tomi: It's a diamond. A perfect diamond. {Funny voice.}

11:28 {Teacher arrives. Tomi's emotion: Looking for reinforcement?}

11:32 Teacher: Is it a solid now? {The teacher does not accept the solution and his nonverbal communication seems to converse that to Tomi.}

11:33 Tomi: No.

11:34 Teacher: Think! {Turns and leaves.}

11:35 Tomi: {Twists the model a little.} Now it is! {Smiling. Emotion not clear. Teacher turns to look, smiles gently, his expression communicating something like "you can't fool me".}

11:45 Tomi: I don't want to demolish this. {Pulls the model apart. Emotion: sadness.}

11:47 Tomi: A dog! {Arto gasps, pretending to be amazed.}

11:48 Eetu: Where's a dog? {First Arto, then also Tomi and Eetu start laughing. It is not clear, but Tomi might be slightly blushing again. Emotion: Humour.}

12:20 Tomi: Now I got it!

12:45 Tomi: Hey teacher, this is certainly a solid! {Hits his desk with both hands, fists clenched. Emotion: frustration.}

{Tomi makes a new shape, which also has 'curved' surfaces.}

13:52 Tomi: Is this, then? And this? {Emotion is not clear. Although he gives a social smile, the emotional feeling is more like sadness.}

13:58 Teacher: You really got [interesting.]

13: 59 Tomi: [And look!] This is a house. No, a tent.

14:06 Teacher: It has to be closed from all directions. {Turns and leaves.}

14:10Tomi: Now I will close it from all directions. I will cram if need be. {Emotion: Anger/disgust.}

Here it became apparent that Tomi considered his teacher as the criterion for acceptable solutions. Although he himself got pleasure from inventing his models, his joy proved to be repeatedly premature as his teacher's judgment rejected the attempts. Joking was the all-around coping strategy for Tomi. Although Tomi showed great resilience, his negative

emotions grow stronger as rejections were repeated. The expressions of anger and sadness and the related self-talk were not directed at anyone. Yet, there was a feeling of display attached to them. Perhaps the display of negative emotions in a theatrical fashion was a way to regulate the expression of emotions to be socially acceptable. Moreover, Tomi's emotional communication with his teacher seemed to develop towards increasingly angling for sympathy.

This analysis provides an example of the rich variety of different functions of emotions in mathematical problem solving in classroom setting. More specifically, the task was open and the duration of problem solving was long, here we have analyzed less than half of the time Tomi spent working on the task. One thing that is apparent here is the presence of emotions and student's capacity to regulate them in a productive manner.

CONCLUSIONS AND IMPLICATIONS FOR TEACHING

The main lesson to learn from the research on emotions in problem solving is that emotions are an essential part of problem solving. Some emotions direct attention and intuition and are functional, perhaps necessary, in the process of successful problem solving. Emotions play an important part also in learning from the experience of both successful and unsuccessful problem solving. Moreover, emotions influence the formation and development of mathematics-related motivation, attitudes, and beliefs. As some emotions seem to be more beneficial to learning outcomes than others, teachers and curriculum developers need to pay attention to the student emotions.

What are the characteristics of a classroom that promotes optimal emotional climate? It has been shown that the teacher enthusiasm (Frenzel et al., 2009) and the chosen method of teaching (Schukajlow, Leiss, Pekrun, Blum, Müller, & Messner, 2011) can have an influence on student emotions.

Several schools have implemented programs to enhance students' social and emotional learning and aim to promote an emotionally healthy learning environment. Specific goals for the program are, for example, that students acquire core competencies to recognize and manage emotions, set and achieve positive goals, appreciate the perspectives of others, establish and maintain positive relationships, make responsible decisions, and handle interpersonal situations constructively. According to a meta-analysis, universal school-based social-emotional development programs have beneficial effects on positive social behavior, problem behaviors, and academic performance (Durlak, Weissberg, Dymnicki, Taylor & Schellinger, 2011).

When students are engaged and face an optimal challenge, they can experience flow (Csikszentmihalyi & Csikszentmihalyi, 1992; Williams, 2002). Unfortunately present school seems to rarely provide experiences of flow. Quite the contrary, a specific and persistent problem is that classrooms are often emotionally flat, and boredom (i.e. temporary feelings of low-arousal and unpleasant emotions induced by environmental factors) is one of the most frequently experienced emotions. (Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010; Nett, Goetz, & Hall, 2011; Vogel-Walcutt, Fiorella, Carper, & Schatz, 2012). Classroom environment seems to be a critical determinant of students' boredom experiences indicating

that instructional approaches can influence the amount of boredom in the class (Nett et al., 2011). Specifically, subjective experience of control decreases the level of boredom (Pekrun et al., 2010).

Pekrun and Stephens (2010) claim that except for research on test anxiety, there is little research concerning the effects of the task and the environment on academic emotions. Based on Pekrun's control-value theory and results regarding test anxiety, they suggest the following characteristics to have positive effects on student emotions:

- students perceive value in the task and have a feeling of control,
- there is a match between task demands and student competence,
- tasks and learning environments meet individual needs,
- teacher is enthusiastic,
- student autonomy is supported,
- emphasis on mastery goals,
- positive feedback, and
- positive consequences of achievement.

Because of the functional aspect of emotions in self-regulation and learning, there should be space for emotions in the classroom. Even the negative emotions related to failures seem to have an important role in creating intuitions (Immordino-Yang & Faeth, 2010). However, sometimes emotions need to be regulated. There are three possible perspectives to emotion regulation in the classroom: teacher regulation of student emotions; student and teacher self-regulation of own emotions, and students' regulation of peer emotions (Fried, 2012).

The teacher can influence the students' emotion regulation through modeling emotion regulation strategies. Through teacher modeling, students learn to recognize and understand the role of different emotions in problem solving, and they also learn about the different ways to control the experience and expression of these emotions. The teacher may also provide more direct support through controlling student emotions, although a teaching style where students are encouraged to think for themselves (autonomy supportive) is more effective in helping students develop their emotion regulation. (Fried, 2012)

Perhaps more important than focusing directly on students' emotion regulation, is to develop such social norms in the classroom that encourage students to regulate their own and peer emotions. Also joking and seeking peer support serve important function in coping with frustration or other negative emotions (De Corte et al., 2011). Firstly, expressive environments have been found to support development of emotion regulation strategies. If such expressive environments have also a feeling of community, where students feel belonging to the classroom, they can more easily assimilate external regulation they observe into the self. (Fried, 2012)

Cobb et al. (1989) also emphasized the relationship between social norms and emotions. In the classroom they observed, engagement in mathematical activity was the goal and therefore, even weaker students experienced and expressed positive emotions as they participated in group activities and whole class discussions. More specifically, they did not

observe a single event over the semester, where a student would have become frustrated and given up the task.

Hannula (2006) suggested that open approach (e.g. Nohda, 2000) would provide opportunities to meet student needs of autonomy, competence and belonging. An open approach would fulfill many of the criteria suggested by Pekrun and Stephens (2010). In the ongoing research project (Näveri et al., 2011) we explore the overall effect of open problems for mathematics related affect and achievement. It is clear already from our preliminary analysis that open problems bring a lot of emotions in the classroom, including flow. Moreover, we have identified that in a good open task there are different levels of complexity that the students can choose from.

One key element is to develop social norms in the class that ensure safe space for explorations and a feeling of playfulness in the lesson. The appropriate climate of a lesson can be further supported through presenting problems in a humorous form, and introducing them to the class in a humorous light (Shmakov & Hannula, 2010). The modes of working with the problems should provide opportunities for students to reach their social goals of belongingness and the teacher should be prepared to provide emotion scaffolding for those students who are not yet able to cope with moments of frustration. Finding the solution should provide a feeling of accomplishment and pride for all students. This can be best achieved through focus on the process, and by highlighting important variation in ideas and perceiving unsuccessful attempts as important learning opportunities.

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References

- Ashcraft M. H. & Krause, J.A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review* 14, 243–248.
- Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21(2), 132–144.
- Bandura, A. & Schunk, D. H. (1981). Cultivating competence, self-efficacy and intrinsic interest through proximal self-motivation. *Journal of Personality and Social Psychology* 41 (3), 586–598.
- Brock, L. L., Nishida, T. K., Chiong, C., Grimm K. J., & Rimm-Kaufman, S. E. (2008). Children's perceptions of the classroom environment and social and academic performance: A longitudinal analysis of the contribution of the responsive classroom approach, *Journal of School Psychology* 46, 129–149.
- Buck, R. (1999). The Biological Affects: A Typology. *Psychological Review* 106 (2), 301–336.
- Carlson, M., & Bloom, I. (2005). The cyclic nature of problem solving: an emergent multidimensional problem-solving framework. *Educational Studies in Mathematics* 58 (1), 45–75.
- Chapman, O. (2003). Facilitating peer interactions in learning mathematics: Teachers' practical knowledge. In M. J. Haines & A. B. Fuglestad (Eds.), *Proc. 28th Conf. of the Int. Group for the Psychology of Mathematics Education*, (Vol. 2, pp. 191–198). Bergen, Norway: PME.

- Csikszentmihalyi, M. & Csikszentmihalyi, I. (1992). (Eds.) *Optimal experience: Psychological studies of flow in consciousness*. Cambridge: Press Syndicate of the University of Cambridge.
- Damasio, A. R. (1999). *The Feeling of What Happens*. New York: Harcourt Brace & Company.
- DeBellis, V.A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: a representational perspective. *Educational Studies in Mathematics* 63(2), 131-147.
- De Corte, E., Depaepe, F., Op 't Eynde, P. & Verschaffel, L. (2011). Students' self-regulation of emotions in mathematics: an analysis of meta-emotional knowledge and skills. *ZDM – The international Journal on Mathematics Education* 43 (4), 483-496.
- Durlak, J.A., Weissberg, R. P., Dymnicki, A.B., Taylor, R.D & Schellinger, K..B. (2011). The Impact of Enhancing Students' Social and Emotional Learning: A Meta-Analysis of School-Based Universal Interventions. *Child Development* 82 (1), 405–432.
- Ekman, P. (1972). Universals and cultural differences in facial expression of emotion. In J. K. Cole, (ed.) *Nebraska symposium on motivation*, pp. 207-283. Lincoln, USA: University of Nebraska Press.
- Ekman, P. (1992). An argument for basic emotions. *Cognition and Emotion*.6, 169–200.
- Ekman, P., Friesen, W. V., & Hager, J. C. (2002). *Facial Action Coding System*. Salt Lake City: A Human Face.
- Else-Quest, N. M., Hyde, J. S., & Hejmadi, A. (2008). Mother and child emotions during mathematics homework. *Mathematical Thinking and Learning* 10, 5–35.
- Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics* 63 (2), 209–226.
- Forgas, J.P. (2008). Affect and cognition. *Perspectives on Psychological Science* 3 (2), 94–101.
- Freeman, K. E. (2004). The significance of motivational culture in schools serving african american adolescents: a goal theory approach. In P. R. Pintrich & M. L. Maehr (Eds.) *Advannces in motivation and achievement. Volume 13. Motivating students, improving schools: The legacy of Carol Midgley*, pp. 65–95. The Netherlands: Elsevier Jai.
- Frenzel, A.C., Goetz, T., Lüdtke, O., Pekrun, R. & Sutton, R.E. (2009) Emotional Transmission in the Classroom: exploring the relationships between teacher and student enjoyment, *Journal of Educational Psychology*, 101(3), 705-716.
- Fried, L. (2012). Teaching teachers about emotion regulation in the classroom. *Australian Journal of Teacher Education* 36 (3), 117 – 127.
- Friedel, J. M., Cortina, K. S., Turner, J. C. & Midgley, C. (2007). Achievement goals, efficacy beliefs and coping strategies in mathematics: The roles of perceived parent and teacher goal emphases. *Contemporary Educational Psychology*, 32, 434–458.
- Goldin, G. A. (1988). Affective representation and mathematical problem solving. In M. J. Behr, C. B. Lacampagne & M. M. Wheeler (Eds.), *Proceedings of the 10th Annual Meeting of PME-NA* (pp. 1-7). DeKalb: Northern Illinois University, Department of Mathematics.
- Goldin, G. A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning*, 2(3), 209-219.
- Goldin, G. A., Epstein, Y. M., Schorr, R. Y. & Warner, L. B. (2011) . Beliefs and engagement structures: behind the affective dimension of the mathematical learning. *ZDM – The international Journal on Mathematics Education* 43 (4), 547-560.
- Grecu, J. (2008). *Fundamente Metodice Ale Utilizarii Umorului În Procesul De Pregatire A Cadrelor Didactice Pentru Predarea Limbii Engleze* [The Methodological Fundamentals of Humor Application in the Process of English Teachers' Training] A doctoral thesis in Pedagogy. Universitatea pedagogica de stat "Ion Creanga". Chisinau, Moldova.

- Gross, J. J. (1998). The emerging field of emotion regulation: an integrative review. *Review of General Psychology* 2, 271–99
- Hannula, M. S. (2004). Affect in mathematical thinking and learning. *Acta universitatis Turkuensis B* 273. Finland: University of Turku.
- Hannula, M. S. (2005). Shared cognitive intimacy and self-defence: two socio-emotional processes in problem solving. *Nordic studies on Mathematics Education* 1, 25–41
- Hannula, M. S. (2006). Motivation in Mathematics: Goals reflected in emotions. *Educational Studies in Mathematics* 63 (2), 165–178.
- Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education* 21, 33–46.
- Immordino-Yang, M. H. & Faeth, M. (2010). The role of emotion and skilled intuition in learning. In D. Sousa (Ed.), *Mind, brain, and education: Neuroscience implications for the classroom*. (pp. 67–81). Washington, DC: American Psychological Association.
- Kim, C.M. & Hodges, C. B. (2012). Effects of an emotion control treatment on academic emotions, motivation and achievement in an online mathematics course. *Instructional Science* 40, 173–192.
- Lang, P. J. (1995). The emotion probe: Studies of motivation and attention. *American Psychologist* 50 (5), 372–385
- Lazarus, R. S. (1991). *Emotion and Adaptation*. Oxford, NY: Oxford University Press.
- Lee, J. 2009. Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. *Learning and Individual Differences* 19, 355–365.
- Lehman, B., D’Mello, S., & Person, N. (2008). All alone with your emotions: An analysis of student emotions during effortful problem solving activities. Paper presented at the Workshop on Emotional and Cognitive issues in ITS at the Ninth International Conference on Intelligent Tutoring Systems. Accessed April 15, 2012 at <http://141.225.218.248/web-cslwebroot/emotion/files/lehman-affectwkshp-its08.pdf>.
- Linnenbrink, E. A., & Pintrich. P. R. (2004). Role of affect in cognitive processing in academic contexts. In D. Y. Dai and R. J. Sternberg, (eds.), *Motivation, emotion, and cognition; Integrative perspectives on intellectual functioning and development*, pp. 57–88. Mahwah, NJ: Lawrence Erlbaum.
- Ludmer, R., Dudai, Y. & Rubin. N. (2011). Uncovering camouflage: Amygdala activation predicts long-term memory of induced perceptual insight. *Neuron* 69 (5), 1002–1014.
- Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education* 30, 520–541.
- Ma, X., & Kishor. N. (1997a). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analyses. *Journal for Research in Mathematics Education* 28 (1), 26–47.
- Ma, X., & Kishor. N. (1997b). Attitude toward self, social factors, and achievement in mathematics: A meta-analytic review. *Educational Psychology Review* 9, 89–120.
- Ma, X, & Xu. J. (2004). Determining the Causal Ordering between Attitude toward Mathematics and Achievement in Mathematics. *American Journal of Education* 110 (May), 256–280.
- Malmivuori, M. L. (2006). Affect and self-regulation. *Educational Studies in Mathematics* 63 (2), 149–164.
- Mason, J., Burton, L. & Stacey, K. (1982). *Thinking mathematically*. New York: Addison Wesley.
- McDonough, A., & Clarke, D. (2002). Describing the practice of effective teachers of mathematics in the early years. In N. A. Pateman, B. J. Doherty, & J. Zilliox (Eds.), *Proc. 27th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 261–268). Honolulu, USA: PME.

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- McLeod, D. B. (1988). Affective issues in mathematical problem solving: Some theoretical considerations. *Journal for Research in Mathematics Education*, 19, 134-141
- McLeod, D. B. (1992). Research on affect in mathematics education: a reconceptualization. In D.A.Grouws, (ed.), *Handbook of Research on Mathematics Learning and Teaching*, pp. 575-596. New York: MacMillan.
- Middleton, J. A. & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education* 30, 65-88.
- Midgley, C., A. Kaplan, M. Middleton, M. L. Maehr, T. Urdan, L. H Anderman, et al. (1998). The Development and Validation of Scales Assessing Students' Achievement Goal Orientations. *Contemporary Educational Psychology* 23, 113-131.
- Minato, S., and T. Kamada. (1996). Results on research studies on Causal predominance between achievement and attitude in junior high school mathematics of Japan. *Journal for Research in Mathematics Education* 27, 96-99.
- Murphy, P. K., & Alexander. P. A. (2000). A motivated Exploration of Motivation Terminology. *Contemporary educational psychology* 25, 3-53.
- Näveri, L., Pehkonen, E., Hannula, M.S., Laine, A. & Heinilä, L. (2011). Finnish elementary teachers' espoused beliefs on mathematical problem solving. In: B. Rösken & M. Casper (eds.), *Current State of Research on Mathematical Beliefs XVII. Proceedings of the MAVI-17 Conference*, pp. 161-171. University of Bochum, Germany.
- Nett, U. E., Goetz, T., & Hall, N. C. (2011). Coping with boredom in school: An experience sampling perspective. *Contemporary Educational Psychology* 36, 49-59.
- Niemivirta, M. (2004). *Habits of Mind and Academic Endeavors. The Correlates and Consequences of Achievement Goal Orientation*. Department of Education, Research Report 196. Helsinki: Helsinki University Press. Doctoral thesis.
- Nohda, N. (2000). Teaching by open approach methods in Japanese mathematics classroom. In: T. Nakahara and M. Koyama (eds.), *Proc. 24th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 39-53). Hiroshima, Japan: PME.
- OECD (2003). The PISA 2003 Assessment Framework – Mathematics, Reading, Science and Problem Solving Knowledge and Skills. Paris: OECD.
- Op 't Eynde, P. & Hannula, M. S. (2006). The case study of Frank. *Educational Studies in Mathematics* 63, 123-129.
- Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). Accepting emotional complexity: A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics* 63 (2), 193-207.
- Pekrun, R., Goetz, T., Daniels, L. M., Stupnisky, R. H., & Perry, R. P. (2010). Boredom in achievement settings: Exploring control-value antecedents and performance outcomes of a neglected emotion. *Journal of Educational Psychology* 102 (3), 531 - 549.
- Pekrun, R. & Stephens, E. J. (2010). Achievement emotions: A control value approach. *Social and Personality Psychology Compass* 4/4, 238-255.
- Pintirch, P. R. (1994). Continuities and Discontinuities: Future Directions for Research in Educational Psychology. *Educational Psychologist*, 29, 137-148.
- Polya, G. (1957). *How to Solve It: A New Aspect of Mathematical Method*. Princeton, NJ: Princeton University Press.
- Power, M. & Dalgleish. T. (1997). *Cognition and Emotion; From order to disorder*. UK: Psychology Press.

- Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic emotions in students' self-regulated learning and achievement: A program of quantitative and qualitative research. *Educational Psychologist*, 37(2), 91–106.
- Rubinsten, O. & Tannock, R. (2010). Mathematics anxiety in children with developmental dyscalculia. *Behavioral and Brain Functions* 6 (46)
- Schlöglmann, W. (2003). Can neuroscience help us better understand affective reactions in mathematics learning? In M. A. Mariotti (Ed.) *Proceedings of Third Conference of the European Society for Research in Mathematics Education*, 28 February - 3 March 2003, Bellaria, Italia. Accessed April 10, 2012 at <http://ermeweb.free.fr/CERME3/Groups/TG2/TG2_schloeglmann_cerme3.pdf>
- Schoenfeld, A. H. (1985). *Mathematical Problem Solving*. San Diego: Academic Press.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: MacMillan.
- Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, R. (2011). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational Studies in Mathematics* 79 (2), 215–237.
- Shmakov, P. & Hannula, M. S. (2010). Humour as means to make mathematics enjoyable. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.) *Proceedings of CERME 6, January 28th-February 1st 2009, Lyon France*, pp. 144 – 153. INRP 2010 <downloaded 15.6.2010: www.inrp.fr/editions/cerme6>
- Vogel-Walcutt, J. J., Fiorella, L., Carper, T. & Schatz, S. (2012) The definition, assessment, and mitigation of state boredom within educational settings: a comprehensive review. *Educational Psychology Review* 24, 89–111.
- Williams, G. (2002). Associations between mathematically insightful collaborative behaviour and positive affect. In A. D. Cockburn & E. Nardi (eds.) *Proc. 26th Conf. of the Int. Group for the Psychology of Mathematics Education*, (Vol. 4 pp. 402-409). Norwich, UK: PME.
- Williams, T., & Williams, K. (2010). Self-efficacy and performance in mathematics: Reciprocal determinism in 33 nations. *Journal of Educational Psychology* 102 (2), 453-466.

ⁱ All names are pseudonyms

ⁱⁱ Curly brackets { } indicate observations and interpretations based on the video.

ⁱⁱⁱ Square brackets [] indicate overlapping talk.