Experimental Validation of the Regularisation Parameter in Laplacian Component Analysis Using SAR Ship Imagery

Radar Signal Processing

May Yuan-Klitzner

University of Cape Town

February 11, 2026

1 Objective

The objective of this report is to empirically validate the central claims of the accompanying paper concerning the role of the regularisation parameter α in Laplacian Component Analysis (LCA). Specifically, the paper proposes that LCA redefines information as a balance between variance preservation and structured smoothness, and that the parameter α controls this balance in an application-dependent manner.

To test these claims, a series of controlled experiments are conducted on a large, expert-labelled Synthetic Aperture Radar (SAR) ship detection dataset. The experiments are designed to address two questions. First, whether varying α produces the expected trade-off between reconstruction fidelity and smoothness, thereby validating the operational behaviour of LCA. Second, whether the response of preserved variance to changes in α encodes meaningful structural information that differs between ship targets and background regions.

Rather than focusing on downstream detection performance, the experiments aim to isolate and examine the representational properties of LCA itself. In doing so, this report seeks to provide empirical evidence that the regularisation parameter α is not merely a smoothing hyperparameter, but a meaningful information dial whose behaviour reflects underlying data structure.

2 Dataset and Experimental Setup

2.1 Dataset Description

All experiments were conducted using the SAR Ship Detection Dataset introduced by Wang et al. (2019). The dataset contains 39,729 ship-centred SAR image chips of size 256×256 pixels, extracted from 102 Gaofen-3 and 108 Sentinel-1 scenes. Each chip includes at least one annotated ship, with labels provided by SAR experts in the form of normalized bounding boxes.

The dataset was designed for ship detection in complex SAR backgrounds and includes ships with varying scales, orientations, and surrounding clutter. As noted by the dataset authors, ships exhibit distinct scales and local background characteristics, making the dataset suitable for probing structure-sensitive representations.

In this work, the dataset provides a controlled source of real SAR imagery with known target locations, enabling the representational behaviour of Laplacian Component Analysis to be examined independently of task-specific models.

2.2 Data Preprocessing and Selection

A subset of 4,000 ship chips was randomly sampled from the full dataset for computational tractability. Images were converted to floating-point representations and normalized to the range $[0, 1]$.

Basic statistical checks were performed to confirm data integrity and variability. Across the selected subset, the mean pixel variance per image was approximately 1.7×10^{-2}, with individual image variances spanning more than two orders of magnitude. Variance statistics were also computed separately for Gaofen-3 and Sentinel-1 imagery, confirming comparable variance distributions across sensors and ruling out satellite-specific variance effects as a confounding factor.

These preliminary checks ensured that the dataset exhibits sufficient diversity in local structure and intensity variation to meaningfully probe the effect of the LCA regularisation parameter.

2.3 Ship and Background Patch Extraction

To study how LCA responds to application structure at a local level, two types of image patches were extracted from the ship chips:

- Ship neighbourhood patches, centred on the annotated ship locations.
- Background patches, sampled from regions spatially distant from the ship within the same chip.

By extracting both patch types from the same ship-centred chips, global image statistics and sensor characteristics are controlled for, isolating local structural differences between ship and non-ship regions. This design ensures that any observed differences in LCA behaviour arise from local image structure rather than from dataset-level biases.

A balanced set of 200 ship patches and 200 background patches was used in the discrimination experiments.

2.4 Experimental Protocol Overview

The experimental validation proceeds in three stages:

1. LCA implementation verification, confirming that increasing α produces the expected trade-off between reconstruction error and embedding smoothness, and that the normalized formulation yields interpretable and stable behaviour.
2. α trade-off experiments, quantifying how reconstruction error and smoothness evolve as α varies over several orders of magnitude, thereby validating the role of α as a continuous control parameter.
3. α-response discrimination experiments, in which preserved variance is measured as a function of α for ship and background patches. Features derived from the full α-response curve are then used for classification, testing whether the shape of this response encodes application-specific structure.

All experiments were implemented in Python using Jupyter notebooks, and results are reported using cross-validated statistics where applicable.

3 Experiment 1: α Trade-off Verification: Reconstruction Error vs Non-Smoothness Penalty

The first experiment validates the fundamental trade-off introduced by Laplacian Component Analysis (LCA): increasing the regularisation parameter α should promote smoother, graph-consistent embeddings at the cost of increased reconstruction error. This behaviour underpins the interpretation of α as a parameter controlling the balance between raw variance preservation and structured smoothness.

3.1 Experimental Setup

A subset of SAR image chips was used to evaluate the effect of varying α on reconstruction fidelity and smoothness. LCA was applied over a range of α values spanning several orders of magnitude, from near-zero (approaching standard PCA) to strongly regularised regimes. To ensure interpretability, a normalized LCA formulation was employed so that α directly reflects the relative weighting between the covariance and Laplacian terms.

Three quantities were measured for each α:

- Reconstruction error, quantified as mean squared error (MSE) between the original and reconstructed images.
- Non-smoothness penalty, measuring deviation from graph smoothness.
- Actual smoothness, computed from the embedding using the Laplacian quadratic form.

Experiments were repeated on both the full feature space and reduced-dimensional representations to verify that observed trends were not artifacts of dimensionality.

3.2 Results

Table 1: α Trade-off in Laplacian Component Analysis

α (nominal)	α (effective)	MSE	Penalty	Smoothness
0.0010	0.9	0.032787	0.028076	0.972315
0.0100	8.6	0.032787	0.027904	0.972482
0.1000	86.0	0.032788	0.025663	0.974664
1.0000	860.2	0.032801	0.020354	0.979852
10.0000	8602.3	0.032827	0.018254	0.981911

Across all configurations, increasing α consistently produced smoother embeddings while only marginally degrading reconstruction accuracy. On representative data, increasing α from 0.001 to 10 reduced the non-smoothness penalty by approximately 35%, while reconstruction MSE increased by only about 0.12%. Over the same range, the measured smoothness of the embeddings increased monotonically.

This favourable trade-off persisted after dimensionality reduction. When LCA was applied to PCA-reduced features (50 and 100 dimensions), smoothness increased steadily with α, while reconstruction error remained nearly constant for small to moderate values of α, only rising modestly under strong regularisation.

3.3 Interpretation

These results empirically confirm the expected behaviour of LCA: the regularisation parameter α governs a controlled trade-off between reconstruction fidelity and structured smoothness. Importantly, substantial gains in smoothness can be achieved with negligible loss of reconstruction accuracy, demonstrating that the Laplacian term introduces meaningful structure rather than acting as a blunt penalty.

This experiment validates the conceptual claim of the paper that LCA redefines information as a combination of variance and structured smoothness and establishes α as a meaningful and interpretable control parameter rather than an arbitrary regulariser.

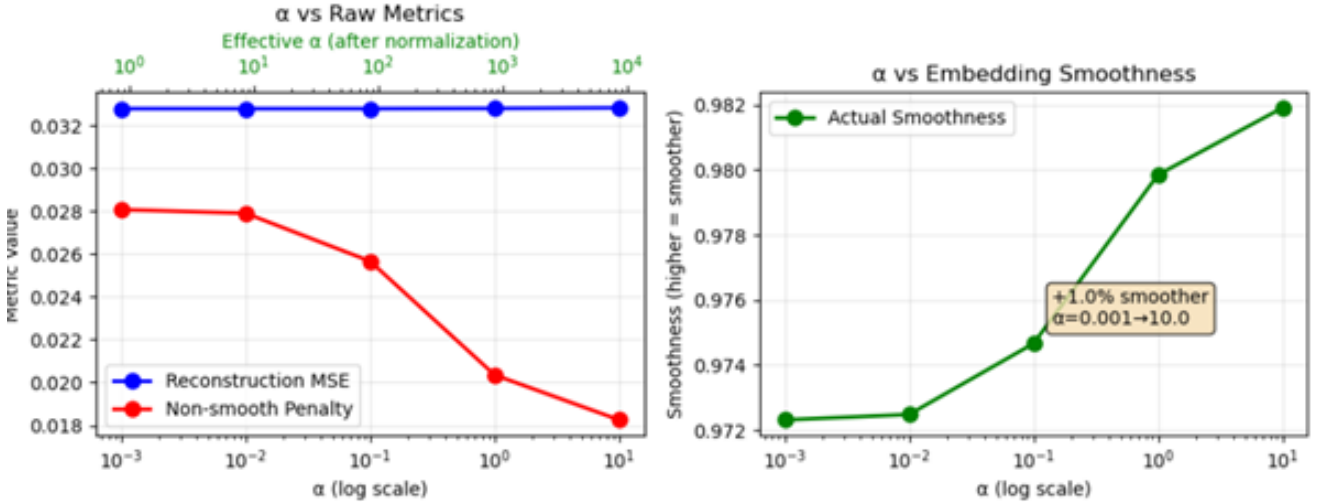## 3.4 Qualitative Interpretation of the $\alpha-$Trade-off curves

Figure 1: Alpha vs. Raw Metrics and Figure 2: Alpha vs. Embedding Smoothness

Figure 1 shows reconstruction error and non-smoothness penalty as functions of α. Reconstruction error remains essentially constant across the tested range, while the non-smoothness penalty decreases monotonically, dropping by about 35% as α increases from 10^{-3} to 10^{1}.

Figure 2 shows embedding smoothness increasing steadily with α, confirming that stronger regularisation enforces smoother, more graph-consistent representations.

4 Experiment 2: α-Response Discrimination on SAR Ship Patches

The second experiment evaluates whether the response of Laplacian Component Analysis to changes in the regularisation parameter α is sensitive to application structure. Specifically, we test whether the shape of the preserved-variance response as α varies can discriminate between ship-neighbourhoods and local background regions, even though both are extracted from ship centred SAR chips.

4.1 Experimental Setup

Using the SAR-Ship Dataset, two types of image patches were extracted:

- Ship neighbourhood patches, centred on annotated ship bounding boxes.
- Background patches, sampled from regions spatially distant from the labelled ships within the same image chips.

A balanced dataset of 200 ship patches and 200 background patches was constructed. For each patch, Laplacian Component Analysis was applied repeatedly over a range of α values spanning several orders of magnitude. For each α, the preserved variance of the LCA embedding was recorded, yielding an α-response curve for each patch.

Rather than using any single α value, features derived from the full α-response curve were used to train a binary classifier distinguishing ship from background patches. Performance was evaluated using cross-validation. For comparison, a baseline classifier using preserved variance at a single α value was also evaluated.

4.2 Results

Table 2: α-Response Discrimination Analysis (Ship vs. Background)

Analysis Type	Accuracy	Notes
α-Response Curve Features	0.845 ± 0.019	Using full α-response profile
Baseline (Single α Feature)	0.575 ± 0.054	Using variance at one α value
Chance Level	0.500	Random guessing

Table 3: Preserved Variance Across α Values

α	Mean Preserved Variance (unnormalized)
0.001	13724.8375
0.01	13725.3163
0.1	13730.0702
1	13774.7718
10	14673.8709
100	54970.1359

Across all patches, mean preserved variance increased with α, reflecting the redistribution of variance into smoother, graph-consistent modes under stronger regularisation. Representative mean values ranged from approximately 1.37×10^4 at $\alpha = 10^{-3}$ to 5.50×10^4 at $\alpha = 10^2$.

Crucially, no single α value cleanly separates ship and background patches. Instead, discrimination emerges from the evolution of preserved variance across α. Using features of the full α-response curve, ship and background patches were classified with an accuracy of 0.845 ± 0.019, substantially exceeding the chance level of 0.5 for a balanced binary task. In contrast, a baseline classifier using preserved variance at a single α achieved only 0.575 ± 0.054 accuracy.

4.3 Qualitative Interpretation of the α–Response Figure
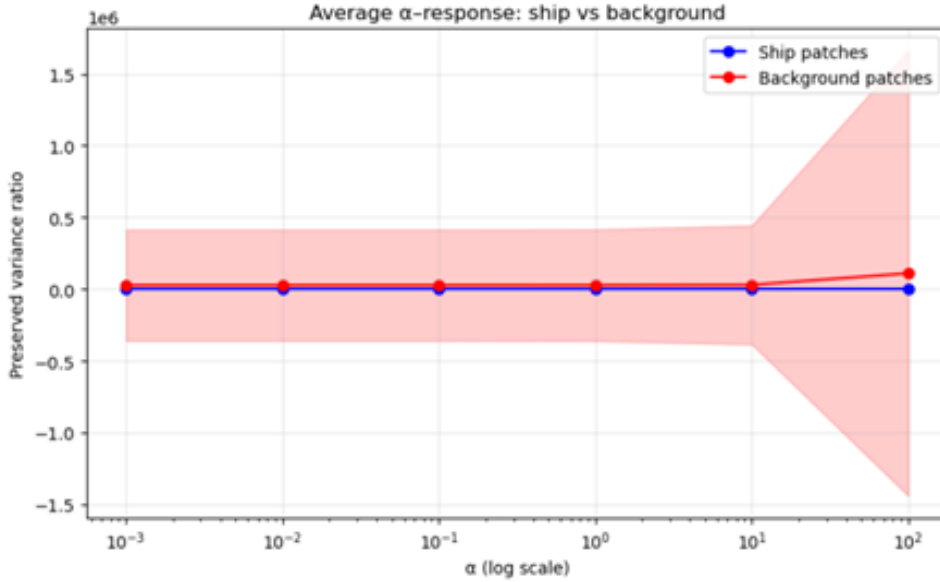

Figure 2: Alpha Response Curve

In Figure 3, the solid red curve shows the mean preserved-variance ratio for background patches as a function of α, while the red shaded band indicates one standard deviation around this mean, reflecting variability across background patches. The blue curve shows the corresponding mean preserved-variance ratio for ship neighbourhood patches.

The α-response plot reveals a funnel-shaped expansion of preserved-variance ratios for background patches as α increases: both the mean and spread grow substantially, most prominently around $\alpha \approx 10^1$, while ship neighbourhood patches exhibit comparatively weak variation over the same range. This difference in how the response distributions evolve with α, rather than any single α value, explains why multi-α curve features support substantially stronger ship-vs.-background discrimination than a single-α representation.

4.4 Interpretation and Implications

This experiment provides strong empirical support for the paper's second conceptual claim: the regularisation parameter α functions as an application-dependent information dial rather than a generic smoothing parameter. Different underlying structures, ship neighbourhoods versus local background, induce systematically different α-response patterns under LCA, even when drawn from the same SAR image chips.

Importantly, these results demonstrate that meaningful application structure is revealed not by selecting an optimal fixed α, but by analysing how representations evolve as α varies.

5 Conclusion: Validation of Paper Claims & Limitations

The experiments in this report provide empirical support for the two central claims of the accompanying paper on Laplacian Component Analysis. First, the α trade-off experiments show that increasing the regularisation parameter α systematically shifts the balance between reconstruction fidelity and graph smoothness. Reconstruction error remains essentially unchanged over several orders of magnitude in α, while the non-smoothness penalty decreases and the measured embedding smoothness increases monotonically. This confirms that LCA indeed trades variance against structured smoothness, as proposed in the paper's reformulation of "information = variance + structured smoothness".

Second, the α-response discrimination experiment demonstrates that the response of LCA to changes in α is sensitive to application structure. Ship neighbourhood patches and local background patches, drawn from the same SAR ship chips, induce systematically different preserved variance trajectories as α varies. Features derived from the full α-response curve allow blind ship versus background classification with 0.845 ± 0.019 accuracy, whereas a single α baseline achieves only 0.575 ± 0.054. This shows that α acts as an application-dependent information dial: it is not any fixed α that is informative, but the way representations evolve as α changes.

At the same time, the validation is subject to important limitations. All experiments use cropped, ship-centred SAR chips rather than full clutter-dominated scenes, so the results do not yet establish optimal α regimes for operational clutter suppression. Similarly, the present findings are based on a single SAR ship dataset and a particular graph construction. Nonetheless, within this controlled setting, the experiments substantiate the paper's conceptual view of α as a meaningful, interpretable bridge between variance and application structure in Laplacian Component Analysis.