

# Perceive With Confidence: Statistical Safety Assurances for Navigation with Learning-Based Perception

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1           **Abstract:** Rapid advances in perception have enabled large pre-trained models  
2            to be used out of the box for transforming high-dimensional, noisy, and partial  
3            observations of the world into rich occupancy representations. However, the reli-  
4            ability of these models and consequently their safe integration onto robots remains  
5            unknown when deployed in environments unseen during training. In this work,  
6            we address this challenge by rigorously quantifying the uncertainty of pre-trained  
7            perception systems for object detection via a novel calibration technique based on  
8            conformal prediction. Crucially, this procedure guarantees robustness to distribu-  
9            tion shifts in states when perceptual outputs are used in conjunction with a plan-  
10           ner. As a result, the calibrated perception system can be used in combination with  
11           any safe planner to provide an end-to-end statistical assurance on safety in un-  
12           seen environments. We evaluate the resulting approach, *Perceive with Confidence*  
13           (PwC), with experiments in simulation and on hardware where a quadruped robot  
14           navigates through previously unseen indoor, static environments. These experi-  
15           ments validate the safety assurances for obstacle avoidance provided by PwC and  
16           demonstrate up to 40% improvements in empirical safety compared to baselines.

17           **Keywords:** Uncertainty quantification, occupancy prediction, robot navigation

## 18           1 Introduction

19           How can we decide if the outputs of a given perception system are sufficiently reliable for safety-  
20           critical robotic tasks such as autonomous navigation? Significant strides in perception over the past  
21           few years have enabled large pre-trained models to be used out of the box [1] for tasks such as *oc-  
22           cupancy prediction*, which serves as a fundamental building block for navigation. However, current  
23           pre-trained models are still not reliable enough for safe integration into many real-world robotic  
24           systems. Despite being trained on vast amounts of data, these systems can often fail to generalize  
25           to novel environments [2, 3, 4]. In this paper, we ask: *how can we leverage the power of large  
26           pre-trained occupancy prediction models while providing safety assurances for robot navigation?*

27           Consider a legged robot tasked with navigating in a cluttered environment such as a home, office, or  
28           warehouse (Figure 1). A typical navigation pipeline for such a system consists of two modules: (i) a  
29           perception module that detects obstacles, and (ii) a planner that produces collision-free trajectories  
30           assuming accurate perception. However, there are two challenges associated with obtaining reliable  
31           outputs from the perception module. First, the environments in which we deploy our robots will  
32           be *unseen* during training, and thus require *generalization* to new obstacle geometries, appearances,  
33           and other environmental factors. Second, *closed-loop deployment* of the perception system in con-  
34           junction with a planner causes a shift in the distribution of *states* (e.g., relative locations to obstacles)  
35           that are visited by the robot. Since the robot’s planner influences future states, the robot may view  
36           obstacles from unfamiliar relative poses (Figure 1) and cause the perception system to fail.

37           In this paper, we address these challenges by performing rigorous *uncertainty quantification* for  
38           the outputs of a pre-trained perception system in order to achieve reliably safe (i.e., collision-free)  
39           navigation. We utilize techniques from *conformal prediction* [5] in order to lightly process the  
40           outputs of a pre-trained obstacle detection system in a way that provides a *formal assurance* on  
41           correctness: with a user-specified probability  $1 - \epsilon$ , the processed perceptual outputs will correctly

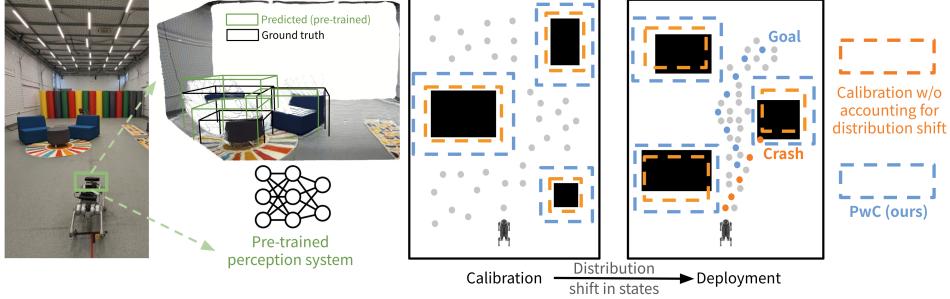


Figure 1: PwC lightly processes the outputs of a pre-trained perception system (green bounding boxes) using conformal prediction in order to ensure a bounded misdetection rate despite *any* distribution shift in states (gray dots). The calibrated perception system (blue boxes) paired with a non-deterministic filter and a safe planner provide an end-to-end statistical assurance on safety in new test environments.

42 detect obstacles in a *new* environment. To enable this, we assume access to a modest-sized (e.g.,  $|\cdot| =$   
43 400) dataset of environments that are representative of deployment environments with ground-truth  
44 obstacle annotations, and use these for *calibrating* the outputs of the perception system. Crucially,  
45 we propose a novel calibration technique that ensures robustness of the perception system to *any*  
46 *closed-loop distribution shift in states*. Hence, the calibrated outputs can be used in conjunction with  
47 *any* safe planner to provide an end-to-end statistical assurance on safety in new static environments  
48 with a user-specified threshold  $1 - \epsilon$ . To the best of our knowledge, this is the first work to calibrate  
49 a given black-box perception system in a way that ensures robustness to closed-loop distribution  
50 shifts in order to provide end-to-end statistical assurances on safe navigation.  
51 Our framework, *Perceive with Confidence* (PwC), is evaluated with experiments in simulation and  
52 hardware on the Unitree Go1 quadruped navigating in indoor environments with objects that are  
53 unseen during calibration (Figure 1). We validate PwC’s ability to provide end-to-end statistical  
54 assurances on collision avoidance, while also providing up to 40% increase in safety with only  
55 modest reductions in task completion rates compared to baselines that use the pre-trained perception  
56 model directly, fine-tune it on the calibration dataset, or utilize conformal prediction for uncertainty  
57 quantification but do not account for closed-loop distribution shift.

## 58 2 Problem Formulation and Overview

59 **Dynamics and environments.** Suppose that the dynamics of the robot are described by  $s_{t+1} =$   
60  $f_E(s_t, a_t)$ , where  $s_t \in \mathcal{S}$  is the robot’s state at time-step  $t$ ,  $a_t \in \mathcal{A}$  is the action, and  $E \in \mathcal{E}$  is the  
61 *environment* that the robot operates in during a given episode. We primarily focus on navigation  
62 with static obstacles; in this context, the environment  $E$  specifies the locations and geometries of  
63 objects. We assume that environments that the robot will be deployed in are drawn from an *unknown*  
64 distribution  $\mathcal{D}_{\mathcal{E}}$ , e.g., a distribution over possible rooms that the robot may be deployed in. We  
65 will make no assumptions on this distribution besides the ability to sample a finite dataset  $D =$   
66  $\{E_1, \dots, E_N\}$  of i.i.d. environments from  $\mathcal{D}_{\mathcal{E}}$ .

67 **Sensor and perception system.** The robot is equipped with a sensor  $\sigma : \mathcal{S} \times \mathcal{E} \rightarrow \mathcal{O}$  that provides  
68 observations  $o_t = \sigma(s_t, E)$  (e.g., depth images) based on the robot’s state and environment. We  
69 assume access to a pre-trained perception model  $\phi : \mathcal{O} \rightarrow \mathcal{Z}$ , which processes raw sensor observa-  
70 tions into an occupancy representation of the environment. In this paper, we work with perception  
71 models for obstacle detection that output 3D bounding boxes. The representations  $(z_0, \dots, z_t)$  up  
72 to the current time-step are aggregated into an overall representation  $m_t \in \mathcal{M}$  (e.g., a map).

73 **Policy.** The representation  $m_t$  is used by a planning algorithm in order to produce actions. Denote  
74 the resulting end-to-end policy that utilizes a perception model  $\phi$  by  $\pi^{\phi} : \mathcal{O}^{t+1} \rightarrow \mathcal{Z}^{t+1} \rightarrow \mathcal{M} \rightarrow$   
75  $\mathcal{A}$ , which maps histories of sensor observations to actions.

76 **Safety and task performance.** Let  $C_E^{\text{safe}}$  be a cost function that captures safety (e.g., obstacle  
77 avoidance). Specifically, let  $\mathcal{S}_{0,E}$  denote the allowable set of initial conditions in environment  $E$ .  
78 Then,  $C_E^{\text{safe}}(\pi^{\phi}) \in \{0, 1\}$  assigns a cost of 0 if policy  $\pi^{\phi}$  maintains safety from any initial state

79  $s_0 \in \mathcal{S}_{0,E}$  when deployed over a given time horizon in environment  $E$ , and a cost of 1 otherwise.  
80 An additional cost function  $C_E^{\text{task}}$  can be used to capture task performance (e.g., time to reach a goal).

81 **Goal: statistical safety assurance.** Our goal is to provide a statistical assurance on safety for the  
82 end-to-end policy  $\pi^\phi$ . We propose a procedure that uses a finite dataset  $D$  of environments in order  
83 to produce a *calibrated* perception system  $\bar{\phi} : \mathcal{O} \xrightarrow{\phi} \mathcal{Z} \xrightarrow{\rho} \mathcal{Z}$ . Our approach is modular: outputs of  
84 the calibrated perception system may be used with *any* safe planner (cf. Section 4) to ensure:

$$C_{\mathcal{D}_{\mathcal{E}}}^{\text{safe}}(\pi^{\bar{\phi}}) := \mathbb{E}_{E \sim \mathcal{D}_{\mathcal{E}}} \left[ C_E^{\text{safe}}(\pi^{\bar{\phi}}) \right] \leq \epsilon, \quad (1)$$

85 for a user-specified safety tolerance  $\epsilon$ , while also post-processing outputs from  $\phi$  as lightly (i.e.,  
86 non-conservatively) as possible in order to allow the robot to optimize task performance.

### 87 3 Offline: Calibrating the Perception System

88 In this section, we describe our approach to the uncertainty quantification of a pre-trained perception  
89 system. We focus on the challenges highlighted in Section 1: providing statistical assurances on safe  
90 generalization to novel environments and ensuring that the offline calibration procedure is robust to  
91 shifts in the distribution of states induced by the online implementation of the planner.

#### 92 3.1 Misdetection Rate and Closed-Loop Distribution Shift

93 We focus on perception systems that output bounding boxes that predict the locations of objects in  
94 the environment. As an example, Figure 1 (left) shows one such real-world environment wherein  
95 the union  $A$  of the black boxes denotes the ground-truth locations of the chairs. Let  $B_s$  denote the  
96 union of the green bounding boxes predicted by the perception system  $\phi$  from robot state  $s \in \mathcal{S}$ .  
97 Since the environment in which the robot is deployed may contain partially occluded objects that  $\phi$   
98 was not explicitly trained on, the perception system's outputs may be inaccurate.

99 **Closed-loop distribution shift.** In addition to this challenge of generalization, we highlight another  
100 challenge that any uncertainty quantification method for perception must tackle. Suppose we fix a  
101 policy  $\pi^\phi$  (that uses perception system  $\phi$ ) and collect a dataset of observations in different calibration  
102 environments from the states that result from applying  $\pi^\phi$ . We can use ground-truth bounding boxes  
103 in these environments to produce a calibrated perception system  $\bar{\phi}$  with a statistical assurance on  
104 correctness for the distribution of observations induced by  $\pi^\phi$ . However, if we now apply the policy  
105  $\pi^{\bar{\phi}}$  using the *calibrated* perception system  $\bar{\phi}$ , the resulting distribution of states will be *different*  
106 from the distribution that forms the calibration dataset, thus invalidating the statistical assurance.  
107 We refer to this challenge as *closed-loop distribution shift*, which is similar to challenges that arise  
108 in offline reinforcement learning [6] and imitation learning [7].

109 Our key idea for tackling closed-loop distribution shift is to use a *policy-independent* misdetec-  
110 tion cost,  $\bar{C}_E$ , which considers worst-case errors across *all* states in an environment<sup>1</sup>,  $\bar{C}_E(\phi) :=$   
111  $\max_{s \in \mathcal{S}} \mathbb{1}_{A \not\subseteq B_s}$ . We will present a calibration procedure that allows us to bound this misdetection  
112 cost with high probability in a new environment, and thus guarantee the correctness of the calibrated  
113 perception system independent of the robot policy using conformal prediction (CP).

#### 114 3.2 Calibration Procedure

115 **Dataset.** We assume access to a dataset of  $N$  i.i.d. environments  $D = \{E_1, \dots, E_N\} \sim \mathcal{D}_{\mathcal{E}}$  (cf.  
116 Section 2). In each environment,  $E_i$ , we have access to the union  $A_i$  of the ground-truth bounding  
117 boxes of all the objects in the environment and the unions  $B_{s,i}$  of the predicted bounding boxes  
118 generated by the pre-trained perception system  $\phi$  from each state  $s \in \mathcal{S}$ . We construct the calibration  
119 dataset either using real-world environments or create simulation environments using real-world  
120 data [8, 9, 10] to ensure that the calibration dataset is representative of deployment environments.

<sup>1</sup>It would be infeasible to consider *all* possible states in an environment. In practice, we use a sampling-based motion planner and consider a fixed set of samples for our calibration that could be used by any planner.

121 **Calibration.** In each calibration environment  $E_i$ , we find the inflation  $\Delta_{q_i}$  of the bounding box  
 122 predictions  $B_{s,i}$  so as to ensure that all the ground-truth boxes are fully enclosed by the inflated  
 123 boxes, i.e.,  $A \subseteq B_{s,i} + \Delta_{q_i}, \forall s \in \mathcal{S}$ . Here,  $B_{s,i} + \Delta_{q_i}$  refers to the inflation of each bounding box  
 124 in the union  $B_{s,i}$  by  $2q_i$  along each dimension. We define the *non-conformity score* for environment  
 125  $E_i$  to be the minimum required inflation in that environment (background on CP in Appendix A):

$$U_i = \min_{q_i} q_i \text{ s.t. } A_i \subseteq B_{s,i} + \Delta_{q_i}, \forall s \in \mathcal{S}. \quad (2)$$

126 Observe that  $U_i \leq 0 \implies A_i \subseteq B_{s,i}, \forall s \in \mathcal{S}$  and a growing  $U_i$  signals a worse performance of  
 127 the pre-trained perception system. We compute the nonconformity scores for all our i.i.d. sampled  
 128 environments  $\{E_1, \dots, E_N\}$ . Hence, the following guarantee holds for the non-conformity score,  
 129  $U_{\text{test}}$ , in a new environment,  $E_{\text{test}}$ , with probability  $1 - \delta$  over the sampling of the calibration dataset,

$$\mathbb{P}[U_{\text{test}} \leq \hat{q}_{1-\epsilon} | U_1, \dots, U_N] \geq \text{Beta}_{N+1-v,v}^{-1}(\delta), \quad v := \lfloor (N+1)\hat{\epsilon} \rfloor, \quad (3)$$

130 where,  $\text{Beta}_{N+1-v,v}^{-1}(\delta)$  is the  $\delta$ -quantile of the Beta distribution, and we use a modified  $\hat{\epsilon}$  for  
 131 calibration to achieve the desired  $1 - \epsilon$  coverage, i.e., we compute the associated quantile  $\hat{q}_{1-\epsilon}$  as  
 132 the  $\lceil (N+1)(1-\hat{\epsilon}) \rceil^{\text{th}}$  largest value of all the non-conformity scores collected during calibration.<sup>2</sup>

133 **Proposition 1** Consider the calibrated perception system  $\bar{\phi}$  that modifies every bounding box output  
 134 of the perception system  $\phi$  by scaling the predicted bounding boxes as  $\bar{B} = B + \Delta_{\hat{q}_{1-\epsilon}}$ . With  
 135 probability  $1 - \delta$  over the sampling of the dataset used for calibration, the calibrated perception  
 136 system,  $\bar{\phi}$ , is guaranteed to have an  $\epsilon$ -bounded misdetection rate on new test environments:

$$\mathbb{E}_{E_{\text{test}} \sim \mathcal{D}_{\mathcal{E}}} \left[ \bar{C}_{E_{\text{test}}}(\bar{\phi}) | U_1, \dots, U_N \right] \leq \epsilon. \quad (4)$$

137 The above proposition (proof in Appendix B) gives us a formal assurance on the correctness of the  
 138 perception system *independent of the robot's policy*. As we describe below, the calibrated perception  
 139 can thus be combined with *any* safe planner to bound the collision rate to  $\epsilon$ .

## 140 4 Online: Perception and Planning

141 We now focus on the online imple-  
 142 mentation of the method described  
 143 in Section 3 to reduce conservatism  
 144 when used in conjunction with a safe  
 145 planner. In general, a safe planner  
 146 takes into account the dynamics of  
 147 the robot and produces plans in the  
 148 state space  $\mathcal{S}$ . We call  $\mathcal{X}$  the  
 149 configuration space of the robot (e.g.,  
 150  $x$ - $y$  location for a point). For any  
 151 given environment  $E$ , we partition  $\mathcal{X}$   
 152 into three sub-spaces (Figure 2a): the  
 153 known free space  $\mathcal{X}^{\text{free}}$ , known occu-  
 154 pied space  $\mathcal{X}^{\text{occ}}$ , and unknown space  $\mathcal{X}^{\text{unknown}}$ .

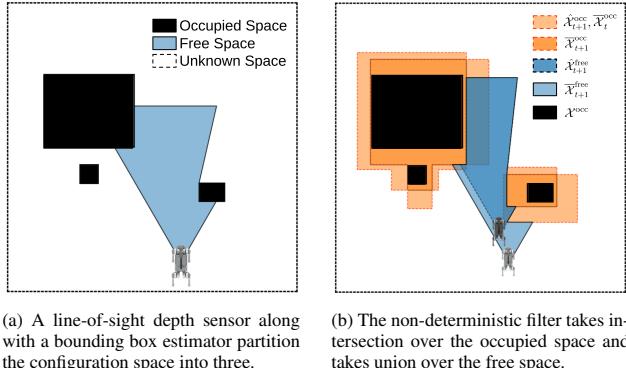


Figure 2

155 **Non-deterministic filter.** We utilize the assurance obtained from Section 3 to implement a *non-*  
 156 *deterministic filter* [11, Ch. 11.2.2] which shrinks the occupied space and grows the known free  
 157 space over time. Suppose the robot's perceived partition (i.e., map) of the configuration space  $\mathcal{X}$   
 158 at time  $t$  is  $m_t := (\hat{\mathcal{X}}_t^{\text{free}}, \hat{\mathcal{X}}_t^{\text{occ}}, \hat{\mathcal{X}}_t^{\text{unknown}})$ . At a new time step  $t+1$ , the robot returns a new  
 159 set of bounding box predictions,  $\hat{\mathcal{X}}_{t+1}^{\text{occ}}$ . The filter then updates the new perceived occupied space  
 160 with  $\hat{\mathcal{X}}_{t+1}^{\text{occ}} = \hat{\mathcal{X}}_t^{\text{occ}} \cap \hat{\mathcal{X}}_{t+1}^{\text{occ}}$ . We then compute the new estimate of free space  $\hat{\mathcal{X}}_{t+1}^{\text{free}}$  based on  $\hat{\mathcal{X}}_{t+1}^{\text{occ}}$ ,

<sup>2</sup>In practice, we choose the calibration threshold  $\hat{\epsilon}$  such that the dataset conditional guarantee (3) achieves the desired  $(1 - \epsilon)$ -coverage with probability  $1 - \delta = 0.99$  over the sampling of the calibration dataset.

161 considering occlusions and limited field of view. Figure 2b shows the non-deterministic filter applied  
 162 for one instance. The new perceived free space is updated with  $\bar{\mathcal{X}}_{t+1}^{\text{free}} = \bar{\mathcal{X}}_t^{\text{free}} \cup \hat{\mathcal{X}}_{t+1}^{\text{free}}$ .

163 The non-deterministic filter pairs effectively with our method in Section 3 for two key reasons: 1) it  
 164 mitigates the conservatism of our bounding box expansion by intersecting  $\bar{\mathcal{X}}_t^{\text{occ}}$ , rapidly reducing its  
 165 size even if the initial prediction with CP bounds appears generous; and 2) Prop. 1 ensures that with  
 166 high probability in a new test environment,  $\bar{\mathcal{X}}_t^{\text{free}}$  never intersects the true occupied space  $\mathcal{X}^{\text{occ}}$ . We  
 167 demonstrate the rapid expansion of known free space in Figure 3 for our simulated setup (Sec. 5).

168 **Safe planning.** With our formal assurance on the estimated free space  $\bar{\mathcal{X}}_t^{\text{free}}$ , we can utilize *any* safe  
 169 planner [12, 13, 14] to ensure end-to-end safety, as long as the planner includes a safety filter that  
 170 takes into account the robot’s dynamics in order to reject potentially unsafe actions with the assump-  
 171 tion of known state and static (but unknown) environment [15, Corollary 1.4]. For our simulation  
 172 and hardware experiments, we use the safe planner proposed in [16], which enforces an inevitable  
 173 collision set (ICS) constraint [17]. We describe implementation details in Appendix D.

174 **Proposition 2** *For any user-specified safety tolerance  $\epsilon$ , the calibrated perception system  $\bar{\phi}$  in  
 175 Proposition 1 combined with any safe planner that chooses actions based on the outputs of the  
 176 non-deterministic filter ensures the end-to-end safety for the overall policy  $\pi^{\bar{\phi}}$ :*

$$C_{\mathcal{D}_{\mathcal{E}}}^{\text{safe}}(\pi^{\bar{\phi}}) := \mathbb{E}_{E \sim \mathcal{D}_{\mathcal{E}}} \left[ C_E^{\text{safe}}(\pi^{\bar{\phi}}) \right] \leq \epsilon, \quad (5)$$

177 where  $C_E^{\text{safe}}(\pi^{\bar{\phi}})$  is the cost function for safety from Section 2.

178 This result (proved in Appendix E) is a direct consequence of the formal assurance on the calibrated  
 179 perception system that ensures correctness from *any* state in a new test environment (sampled i.i.d.  
 180 from the same distribution as the calibration environments) with probability  $1 - \epsilon$  over environments.

## 181 5 Simulated Experiments: Vision-Based Navigation

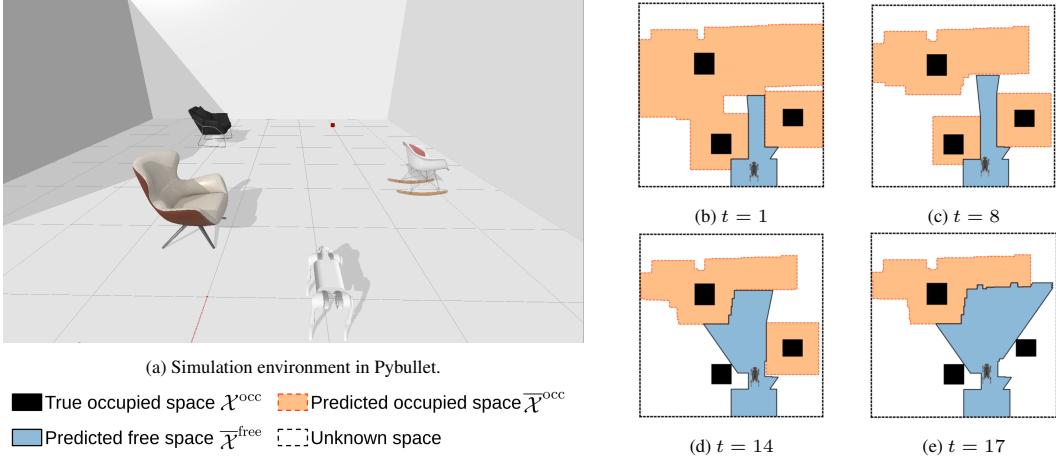


Figure 3: Simulation and non-deterministic filter updates. (a) An example environment in simulation. (b - d) The robot begins with large occupied space predictions due to the inflation obtained through offline calibration (Section 3). After a few updates, the predicted occupied space  $\bar{\mathcal{X}}^{\text{occ}}$  shrinks significantly.

182 We evaluate our approach for vision-based navigation in the PyBullet simulator [18] using a diverse  
 183 set of chairs from the 3D-Front dataset [10]. We use the 3DETR end-to-end transformer model [19]  
 184 as our pre-trained perception system.

185 **Baselines.** We compare our approach (*Perceive with Confidence* — PWC) to three baselines to illus-  
 186 trate its effectiveness in achieving a user-specified safety rate. First, we consider the most common  
 187 approach of directly using the outputs of the perception system [19] in our planning pipeline. We  
 188 call this baseline **3DETR**. Next, we consider the common practice of fine-tuning the outputs of the

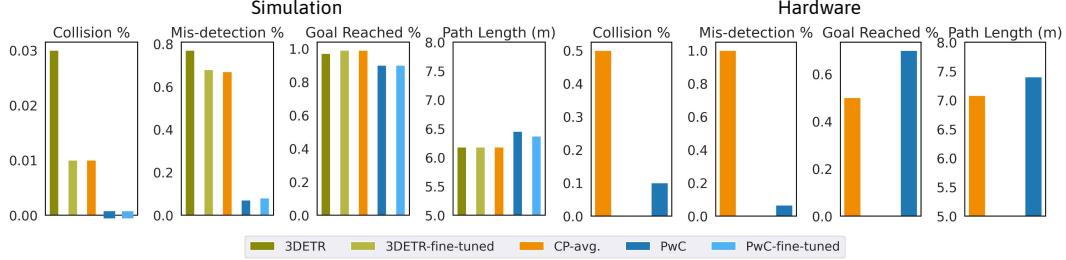


Figure 4: **(Left)** Results for the simulated experiments described in Section 5. Simulations are across 100 new environments with 1 - 5 chairs. **(Right)** Results for the hardware trials described in Section 6. Experiments are across 30 different chair configurations with 4-8 chairs present in each configuration. Here the path length is averaged only for successful trials for both PwC and CP-avg. due to the varying goal locations.

189 perception system using a small dataset of task-representative environments  $D_{\text{tune}}$  (cf. Section F.1).  
 190 We call this perception system **3DETR-fine-tuned**. Lastly, we perform calibration using confor-  
 191 mal prediction; however, instead of accounting for the closed-loop distribution shift, we bound the  
 192 misdetection rate averaged across environments *and* states (similar to [20], which does not utilize  
 193 conformal prediction, but quantifies expected errors in a perception system for a pre-defined distri-  
 194 bution of states). We refer to this baseline as **CP- avg**. We consider two variations of our approach  
 195 for comparison to the above baselines. First, we refine 3DETR outputs using our calibration pro-  
 196 cedure described in Section 3. We call this approach **PwC**. Second, the 3DETR outputs are fine-tuned  
 197 and calibrated using split conformal prediction as described in Appendix F.1; we call this approach  
 198 **PwC-fine-tuned**. Details regarding calibration and the planner setup are provided in Appendix G.

199 **Results: Misdetection Rate.** We first compare our method,  
 200 PwC, to the baseline CP-avg that is also calibrated using confor-  
 201 mal prediction but without accounting for the closed-loop  
 202 distribution shift. We compare the misdetection rate, i.e.,  
 203 whether obstacles in the scene are classified as free space at  
 204 any point during a trial. We vary the allowable misdetection  
 205 bound  $\epsilon$  for each method, and compute the rate of misdetec-  
 206 tions in 100 test environments. As seen in Figure 5, our method  
 207 is guarantees a rate of misdetection lower than the threshold  $\epsilon$   
 208 while CP-avg violates this threshold for every  $\epsilon$  considered.

209 **Results: Collision Rate.** We compare PwC to the baselines  
 210 in 100 new environments drawn from the same distribution as  
 211 calibration environments. Figure 3 illustrates one such test en-  
 212 vironment and the evolution of the free space in this environment using PwC. Figure 3 shows that  
 213 though the initial calibrated perception system outputs are inflated, the non-deterministic filter is able  
 214 to expand the predicted free space in a few time steps and ensure that the robot can navigate without  
 215 unnecessary conservatism. The results are summarized in Figure 4 and the metrics for success and  
 216 failure are described in Appendix G. We observe that our proposed approaches, PwC and PwC-fine-  
 217 tuned, have no collisions in any environments. While the robot reaches the goal in a slightly lower  
 218 percentage of environments compared to baselines, we emphasize that ours is the only approach that  
 219 is able to ensure a low, statistically guaranteed misdetection rate across test environments.

220 To further illustrate the effect of misdetections on safety, we  
 221 consider a different distribution of environments wherein we  
 222 randomly place a *single* chair in the straight line path be-  
 223 tween the initial position of the robot and the goal. For a  
 224 safety threshold  $1 - \epsilon = 0.85$ , we compare PwC, CP-avg,  
 225 and 3DETR. The results are provided in Figure 6 for 100 new  
 226 test environments, wherein the goal is reached if the robot nav-  
 227 igates to within 2 m of the goal. In these environments, the  
 228 desired safety rate is not met by the baselines while our approach  
 229 is still statistically guaranteed to be safe.

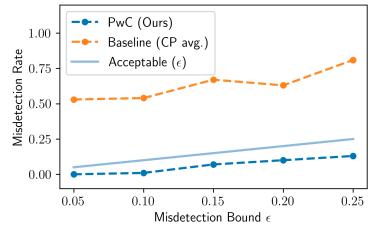


Figure 5: As we relax the confidence threshold by increasing  $\epsilon$ , the misdetection rate increases but remains bounded for PwC. The baseline method has a misdetection rate much higher than acceptable.

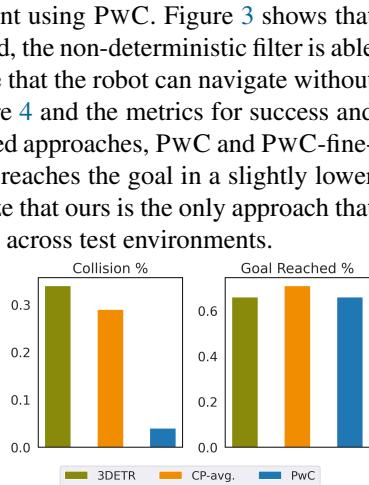


Figure 6: A comparison between the collision rates of different perception systems that use the same safe planner.

230 We provide additional simulation results that illustrate the effects of 1) closed-loop distribution  
 231 shifts on safety in Appendix G.2 wherein PwC is robust to an increase in the level of closed-loop  
 232 distribution shift while the baseline, CP-avg., is not which leads to higher collision rates for CP-avg.  
 233 and 2) the tradeoff in different partition sizes for fine-tuning using split-CP in Appendix F.1.2.

## 234 6 Hardware Validation: Vision-Based Quadruped Navigation

235 We now validate the end-to-end statistical safety assurance of our approach on a quadrupedal hard-  
 236 ware platform. As in our simulation setup in Section 5, the robot is tasked with navigating to a goal  
 237 location while avoiding different chairs placed in varying configurations across a 8m x 8m room. We  
 238 utilize the perception system calibrated in simulation with a guaranteed safety rate of  $1 - \epsilon = 0.85$ ,  
 239 and compare our PwC method against CP-avg. (defined in Section 5) across 30 different physical  
 240 environments (60 trials total). See Appendix H for more details about the hardware setup.

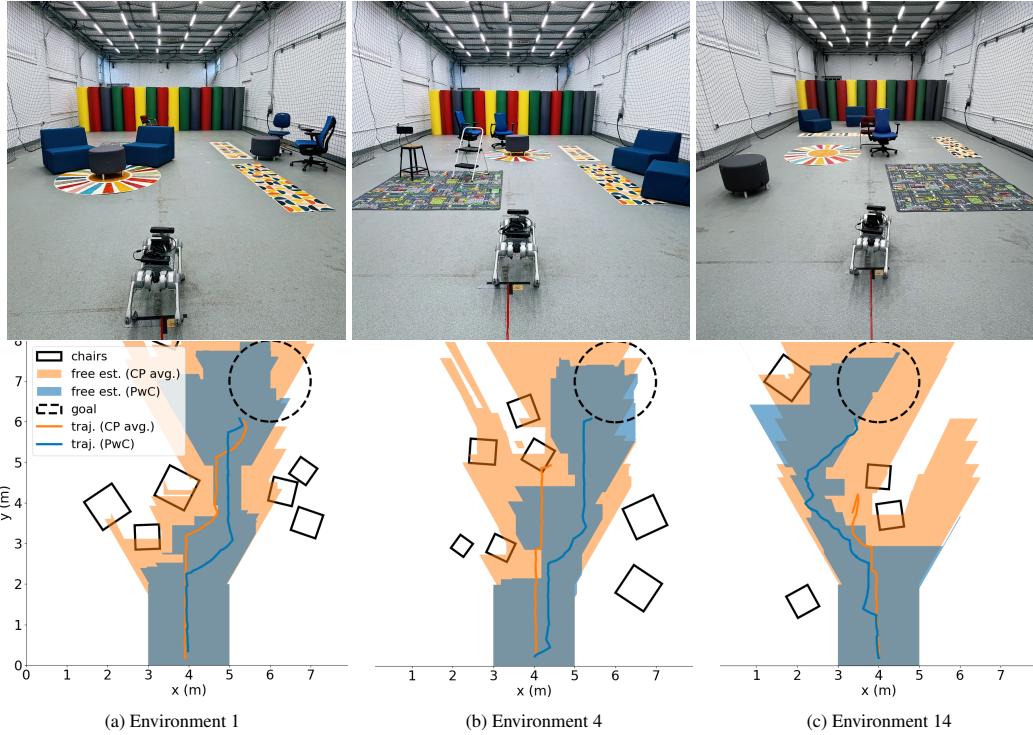


Figure 7: (Top) The physical layouts of the example hardware trails. (Bottom) A bird's-eye view of the estimated free spaces (shaded regions), and the trajectories performed by the robot (solid lines) with our method (blue) and the baseline (orange). In all three trials, PwC is able to successfully navigate to the goal through narrow paths (in Environment 1) and occluded areas/goal (in Environment 3). Baseline approach, CP-avg., misdetects free space in all environments leading to collisions in Environments 2 and 3.

241 **Results.** For PwC, we used the  $\hat{q}_{0.85} = 0.73$ m threshold found in simulation to inflate the pre-  
 242 dicted bounding boxes returned from 3DETR in order to achieve 85% confidence that our robot  
 243 will remain safe in new environments. We summarize key statistics of PwC compared against CP-  
 244 avg. ( $\hat{q}_{0.85} = 0.02$ ) across 30 different environments in Figure 4 (right). Importantly, our trials  
 245 demonstrate that our confidence bound holds on hardware in real environments and without being  
 246 too conservative. PwC was safe through 90% of the trials and also had comparable path length to  
 247 the baseline. Meanwhile, the baseline struggled in the real environments by having misdetections in  
 248 each trial and colliding with a chair in half of the trials. See Figure 7 for trajectories and free space  
 249 estimations through several environments with narrow spaces, occluded chairs, and occluded goals.  
 250 The supplementary video contains full example trials.

251 PwC's low misdetection rate and higher success rate in these trials emphasize the efficacy of the  
 252 bounding box inflation provided by CP paired with the non-deterministic filter. This pairing, in a

253 principled way, inflates the (potentially poor) bounding box detections to properly capture obstacles  
254 but quickly shrinks the occupied space with the filter such that the robot can still navigate effectively.

## 255 7 Related Work

256 **Safe planning.** Collision avoidance is a crucial goal in autonomous navigation. Safe planning  
257 methods typically rely on the assumption that the robot has perfect knowledge of its state and envi-  
258 ronment [15]. Recent approaches have allowed for occlusion [16, 21, 22, 23] or accounted for losing  
259 sight of a previously tracked object [24], but still require either perfect detection of seen objects or  
260 bounded sensor noise. Such assumptions are impractical for learning-based perception modules that  
261 can fail catastrophically in new environments.

262 **Formal assurances for perception-based control.** Proposed methods include control barrier func-  
263 tions (CBFs) [25, 26], verification methods on neural networks (NNs) [27, 28], and other learning-  
264 based methods [28, 29, 30, 20, 31, 32, 33, 34]. However, these works either do not guarantee gener-  
265 alization to novel environments [27, 28], or ignore closed-loop distribution shifts [31, 20], or require  
266 end-to-end training and a good prior [32, 33, 34], or demand usage/design of specific controllers  
267 [25, 26, 29]. Some also make strong assumptions on the perception system [35, 36] that are unreal-  
268 istic for deployment. In contrast, our method does not need any of the above, and is lightweight and  
269 modular, allowing for the use of any downstream safe planners to ensure end-to-end safety.

270 **Conformal prediction.** Conformal prediction (CP) [5, 37, 38] is an uncertainty quantification  
271 framework particularly suitable for robotics applications [39, 40, 41, 42] where learned modules  
272 are deployed in environments drawn from unknown distributions. In this work, we focus on provid-  
273 ing uncertainty quantification for the perception system, which usually involves high-dimensional  
274 inputs and closed-loop distribution shifts. Prior works [41, 20, 43, 44] either provide guarantees for  
275 a single environment, assume known environments, or do not account for closed-loop distribution  
276 shifts. To the best of our knowledge, this is the first work to obtain end-to-end safety assurances for  
277 the perception and planning system in new environments while being robust to closed-loop distribu-  
278 tion shifts and amenable to changes in the planner parameters.

## 279 8 Discussion and Conclusions

280 We presented a modular framework for rigorously quantifying the uncertainty of a pre-trained per-  
281 ception model in order to provide an end-to-end statistical safety assurance for perception-based  
282 navigation tasks. Notably, our statistical assurance holds for generalization to new environmental  
283 factors (e.g., new obstacle geometries and configurations) and allows for the distribution shift of  
284 states that may occur during closed-loop deployment of the perception system with the planner. Our  
285 simulation and hardware experiments validated the theoretical safety assurances provided by PwC,  
286 while demonstrating significant empirical improvements in safety compared to baseline approaches  
287 that do not consider closed-loop distribution shift.

288 **Limitations and future work.** One limitation of our work is the assumption of static obstacles. As  
289 a future direction, we are interested in quantifying uncertainty in both the state of agents moving  
290 in the environment and predictions of their *semantic labels* (e.g., “pedestrian” vs. “bicyclist”), and  
291 utilizing game-theoretic planning techniques that account for the uncertainty in the agents’ current  
292 state and future motion. Additionally, the inflation of bounding boxes we acquire from CP intro-  
293 duces some conservatism. We outline an extension to our approach in Appendix F to address this  
294 challenge by utilizing more general occupancy representations beyond bounding boxes, e.g., scene  
295 completion networks [45], which produce voxel-wise occupancy confidences. Constructing differ-  
296 ent non-conformity score functions that incorporate confidences from a pre-trained model could  
297 also potentially reduce conservatism. Lastly, we are interested in uncertainty quantification for per-  
298 ception models that support tasks beyond point-to-point navigation, e.g., calibrating the outputs of  
299 multi-modal foundation models for language-instructed navigation where we ensure accurate detec-  
300 tion. We expect that rigorous uncertainty quantification is a necessary step towards fully leveraging  
301 the power of large foundation models [1] while safely integrating them into future robotic systems.

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