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June 4, 2023

Let $k_0 = 20$ and $x = (x_1, x_2) \in \mathbb{R}^2$. The equation is

$$-\Delta u - k(x)^2 u = f, \quad \text{in } \mathbb{R}^d, \tag{1}$$

$$\left| \frac{\partial u}{\partial r} - \mathrm{i}ku \right| = o(r^{\frac{1-d}{2}}), \quad r \to \infty, \tag{2}$$

where $r = |x|$,

$$f(x) = \begin{cases} \frac{1}{\sqrt{\pi} a} e^{-\left(\frac{|x-(0.5,0)|}{a}\right)^2}, & |x-(0.5,0)| < 0.5 \\ 0, & otherwise, \end{cases} \quad \text{with } a = \frac{\pi}{2k_0}, \tag{3}$$

and

$$k(x) = \begin{cases} \frac{k_0}{1.5}, & -0.5 \le x_1 \le 0, -0.25 \le x_2 \le 0.25, \\ k_0, & otherwise. \end{cases} \tag{4}$$

$|x-(0.5,0)|$ means $\sqrt{(x_1-0.5)^2 + (x_2-0)^2}$.

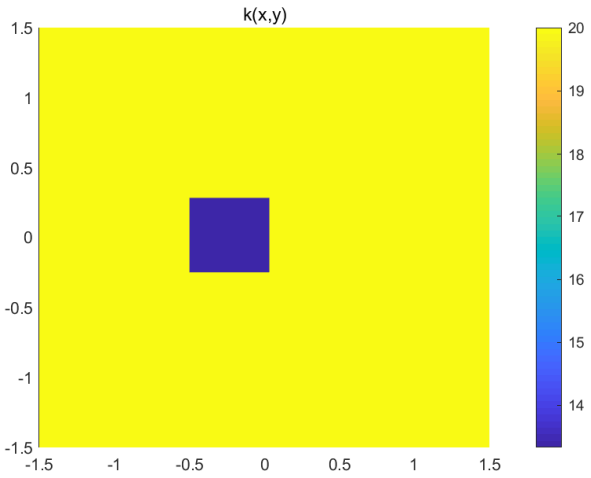

Figure 1: $k(x)$

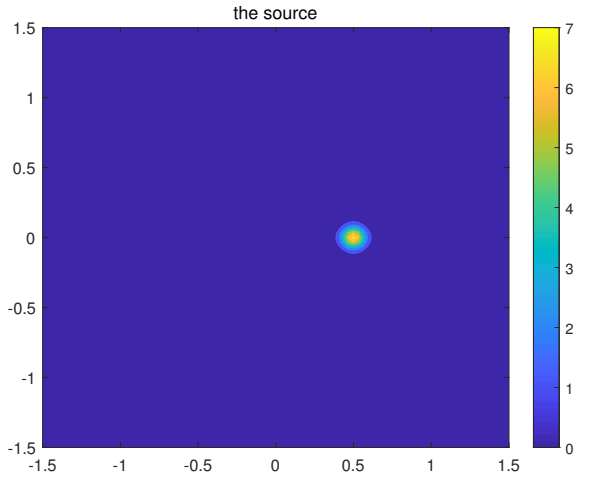

Figure 2: $f(x)$

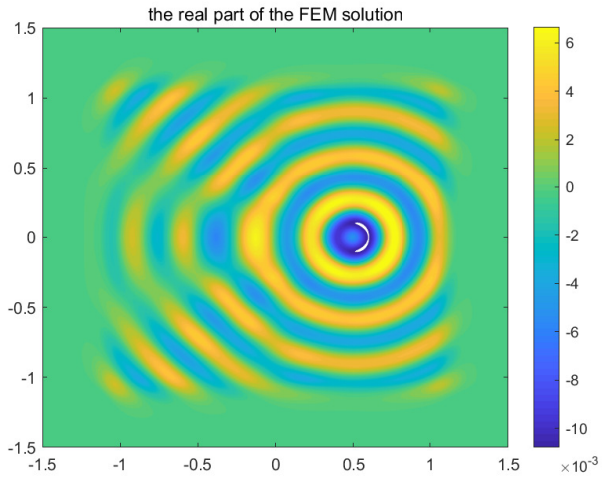

Figure 3: The part in $(-1,1)\times(-1,1)$ is the real part of the solution.

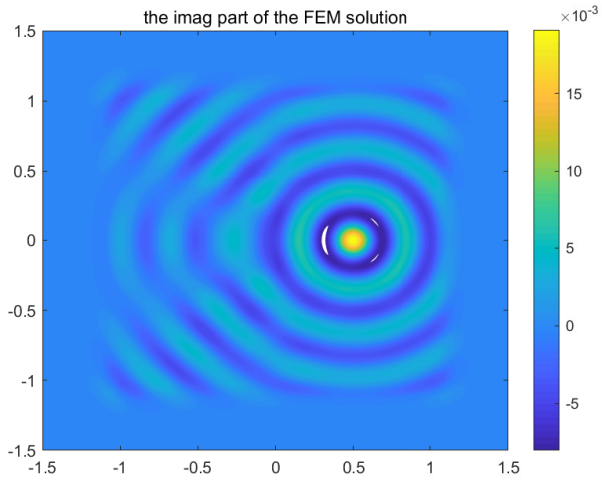

Figure 4: The part in $(-1,1)\times(-1,1)$ is the imaginary part of the solution.