Market Structure and Inequality

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Abstract

Over the past fifty years, there has been a contemporaneous increase in labor market concentration and wealth inequality in the United States. Labor market concentration can lead to increased employer labor-market power which can worsen wage and wealth inequality. However, the worsening inequality can also increase employer labor-market power since workers with less resources will be in worse bargaining positions with potential employers. We study how these two phenomena interact by developing a novel structural model of a frictional labor market with granular, strategic firms and heterogeneous agents subject to a borrowing constraints. When bargaining for wages, households near the borrowing constraint take a significant wage haircut compared to if they had more savings or insurance. The haircut increases as labor market concentration increases. This highlights the importance of sources insurance when studying labor market monopsony power. We the calibrate a partial equilibrium version of the model for every U.S. commuting zone and industry pair. Finally, the paper reevaluates labor market policies including the minimum wage, unemployment insurance, and earned income tax credit.

Keywords: *Market power, monopsony, wealth inequality, borrowing constraints*
JEL Codes: *E24, E21, C63*

1 Introduction

Recent studies indicate that the distribution of wealth has shifted significantly over time, with a notable increase in inequality (**??**). For instance, estimates from **?** indicate that the share of wealth held by the top 1% in the United States rose from approximately 25% in 1980 to over 40% in recent years. Rising inequality and stagnant wages have led to concerns about whether the American dream is still alive or if a significant portion of the population may no longer be able to earn enough to support themselves adequately.

This increase in wealth inequality has coincided with another long-term trend in the U.S. economy: growing labor market concentration, with fewer employers controlling a larger share of employment.[1] In markets dominated by a few large employers, workers have fewer outside options, giving employers greater market power to offer lower wages. This impact is especially pronounced for low-wage workers, whose limited resources make it difficult to wait for better job opportunities.

The goal of this study is twofold: First, to quantify the impact of rising employer market power, driven by increased concentration, on growing wealth inequality. Second, to highlight the importance of factoring in this relationship when formulating labor market policies. To achieve this, we develop a quantitative model in which risk-averse agents, who can be either employed or unemployed, make saving and consumption decisions subject to a borrowing constraint while searching for jobs in a market with large firms. We calibrate our model to the U.S. economy to quantify the role of concentration in shaping the wealth distribution and perform several policy counterfactuals.

Our modeling framework builds on **?**, henceforth referred to as JNS, who incorporate granular firms into a canonical random search and bargaining model DMP. Like JNS, large firms in our model can exert greater market power because workers have fewer outside options in terms of alternative jobs, and hence firms can pay lower wages. We extend their model in two

[1]**?** documents a decline in the number of firms per worker in the U.S. economy, both nationally and within labor markets segmented by industry and location. Several papers have also documented changes in the employment-based Hirschman-Herfindahl Index (HHI); see **?** for references.

important dimensions. First, workers are risk-averse and have the ability to borrow or save using a single risk-free asset. Second, firms are strategic when they post vacancies. (Note the benefits of doing this here and how it changes conclusions).

In our model, workers who are borrowing-constrained get paid lower wages due to their outside option being worse. When there are more firms in the market, two things happen: (1) Firms post more vacancies so less risk and hence less saving, this narrows the asset distribution (if everyone saves less...), (2) since firms

2 Model

In this section, we build upon the random search model presented in **?**, henceforth referred to as JNS. As in JNS, workers in our model apply to jobs distributed across a finite number of firms, wages are Nash bargained, and granular employers exhibit market power by not competing with themselves.

The key difference in our framework is that workers are risk-averse and have the ability to borrow or save using a single risk-free asset. This addition enables us to study the interplay between wealth distribution and market structure. Additionally, we endogenize the firms' vacancy decisions by replacing the "free-entry" condition in JNS with a corresponding optimization condition for granular firms.

Ultimately, we will add on-the-job search, individuals savingin capital and equity. But for now, we start with the simple model, where these elements are missing to ensure that the comparative statics from this model go in the direction as in data.

2.1 Setup

Time is continuous and there is no aggregate uncertainity. Within a labor market, there is a unit measure of infinitely lived workers and M granular firms, each differing in productivity denoted by $z_i \in \{z_1, z_2, \dots z_M\}$. Since firms are granular, each firm i controls a positive measure of vacancies v_i.

Workers can be either employed or unemployed. Employed workers exogenously separate from their employer at rate δ. Workers are homogeneous in terms of productivity, and when employed at firm i, a worker produces z_i units of the final good. Thus, workers are identical ex ante, but differ ex post due to different labor market histories.

2.1.1 Matching

The rate at which workers encounter job openings is described by a constant returns to scale matching function, $m(u, v)$, where u is the unemployment rate and $v = \sum_{i=1}^{M} v_i$. Job offers for unemployed workers arrive at the Poisson rate $\lambda \equiv m(u, v)/u$ and vacancies meet a worker at the rate $q \equiv m(u,v)/v$. Additionally, since each vacancy encounters workers at the same rate, by law of large numbers the rate at which an unemployed worker encounters firm i is given by $\lambda\pi_i$, where $\pi_i = v_i/v$. Verify.

When an unemployed worker encounters an open vacancy, wages are bargained as outlined below. The bargained wages never exceed the firm's marginal productivity, so a firm never rejects any matches. Unemployed workers, however, may decline offers if the value of rejecting the offer outweighs the value of working for the offered wage. If the worker rejects an offer from a firm, the firm bars the worker from it's future job opportunities for the duration of the worker's current unemployment spell. Like JNS, we do not allow a worker to be permanently barred from a firm they reject, to maintain tractability.

2.1.2 *Workers*

Workers maximize their lifetime utility given by

$$\mathbb{E}_0 \int e^{-\rho t} u(c_t) dt,$$

where ρ is the subjective rate of time preference and c_t is consumption at time t. $u(.)$ is... Workers are impatient in that the subjective rate of time preference exceeds the risk free rate.

We will focus on a stationary equilibrium, and hence we drop the time subscripts.

Unemployed workers receive a flow value of b, while employed workers receive their contracted wages. Workers make consumption decisions both when employed and unemployed. Assets are subject to a lower bound $\underline{a} \leq 0$ and evolve according to

$$\dot{a} = ra + \omega - c,$$

where a is the current asset level, r is the risk free interest rate, and ω is the household's labor income or unemployment income. We assume $\underline{a} = -b/r$.

Let $W(a,w)$ denote the expected present value for an individual employed with assets a and a wage w, and let $U(a)$ represent the expected present value of being unemployed with asset level a. Given that the worker chooses consumption to maximize utility subject to the asset evolution equation and a borrowing constraint, we can express the value of employment as follows:

$$\rho W(a,w) = \max_c \; u(c) + \partial_a W(a,w)[ra + w - c] + \delta[U(a) - W(a,w)] \tag{1}$$

Throughout the paper, we use $\partial_x Y$ as a short-hand notation to denote $\partial Y/\partial x$.

Now consider the problem of a worker who is currently unemployed with assets a. This worker will accept a job offer from firm i if the value of being employed at this firm is greater than the value of rejecting it. Hence, we can express the value of being unemployed as follows:

$$\rho U(a) = \max_c \; u(c) + \partial_a U(a)[ra + b - c] + \sum_{i=1}^{M} \lambda_i \left(\max\{W_i(a), \tilde{U}_i(a)\} - U(a) \right) \tag{2}$$

Here, $W_i(a) \equiv W(a, w_i(a))$, where $w_i(a)$ denotes the bargained wage between firm i and a worker with assets a and $\tilde{U}_i(a)$ denotes the value of unemployment for a worker excluded from firm j, which is given by,

$$\rho \tilde{U}_j(a) = \max_c \; u(c) + \partial_a \tilde{U}_j(a)[ra + b - c] + \sum_{i \neq j} \lambda_i \left(\max\{W_i(a), \tilde{U}_i(a)\} - \tilde{U}_j(a) \right) \tag{3}$$

2.1.3 Wage Determination

Denote $J_i(w)$ as the value of a filled job to firm i at w and V_i as the value of an empty vacancy. Then Nash-bargained wages at firm i for worker with assets a are determined by:

$$w_i(a) \in \arg\max_w \left(W(a,w) - \tilde{U}_i(a) \right)^{\beta} \left(J_i(w) - V_i \right)^{1-\beta}$$

Firm and worker take each other's threat points $\tilde{U}_i(a)$ and V_i, and value of unemployment $U(a)$ as given. Given the value functions, note that $w_i(a)$ will be a function of $U(a), \tilde{U}_i(a)$ and V_i.

2.1.4 Firm Optimization

Value of a filled job to firm i at w:

$$rJ_i(w) = z_i - w + \delta[-J_i(w)]$$

Firm's value from an empty vacancy:

$$rV_i = -c(v_i) + q(u,v)\left(\int_{\underline{a}}^{\infty} J_i(w_i(a))\tilde{g}_i^u(a)\,\mathrm{d}a - V_i\right)$$

where $c(v_i)$ is flow cost of posting a vacancy that depends on total number of vacancies posted by firm i (justify) and $\tilde{g}_i^u(a)$ (change notation) is the proportion of workers who are unemployed but not excluded from firm i and have assets a.

Note that we can write V_i as:

$$V_i = \frac{-c(v_i)(r+\delta) + q(u,v)\int_{\underline{a}}^{\infty}[z_i - w_i(a)]\tilde{g}_i^u(a)\,\mathrm{d}a}{(r+\delta)(r+q(u,v))}$$

Firms choose vacancies to maximize their steady-state profits given by $\Pi_i = V_i v_i$. We assume that while making this choice, firms understand that their vacancy choice impacts the rate at which empty vacancies meet workers $q(u,v)$ and also impacts the contracted wages $w_i(a)$ through changes in their outside option V_i while bargaining.[2] However, the firm takes the unemployment rate u, the vacancies of other firms $\boldsymbol{v}_{-i}$, and workers' value of unemployment $U(a)$ and their outside option while bargaining $\tilde{U}_i(a)$, as well as the steady-state distribution of workers g as given.

Hence, firm i's vacancy-choice problem is given by:

$$\max_{v_i} \; \Pi_i(v_i | u, \boldsymbol{v}_{-i}, \tilde{U}_i, U, g)$$

Write this more clearly and intuitively.

2.2 Computation

Detail how the model is solved. (For example like section 3.7 in Krussell et al 2010)

[2] Note that from the firm's perspective $w_i(a)$ depends on $U(a), \tilde{U}_i(a)$ and V_i.

3 Numerical Examples

In this section, we illustrate comparative statics from our model by utilizing several numerical illustrations. For each of the examples, we use the following values for the parameters unless otherwise mentioned.

3.1 Size-based market power

1. Figure (1 x 3): Three values of $z_L < z_M < z_H$ on x-axis. On y-axis:

   - Panel A: Share/Number of vacancies
   - Panel B: Wages
   - Panel C: Markdown

   Do for 2 and 3 firms like Figure 3 in BHM-og. Then, we can talk about what happens to the market with the addition of the additional high-productivity firm.

2. Figure (1 x 2): Wages as functions of assets for different firms. Panel A: 3 firm case, Panel B: 2 firm case. (Keep scale same)

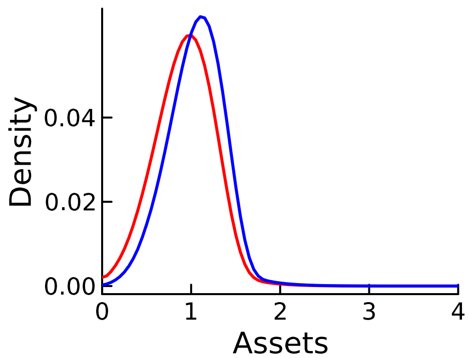
3. Figure (1 x 2): Panel A: Consumption policy functions for unemployed (for 2 and 3 firm case), Panel B: Saving policy functions for unemployed (for 2 and 3 firm case)

4. (Just want to see this first) Consumption policy functions for when employed at firm 1, 2, and 3 at some fixed low wage and some high wage.

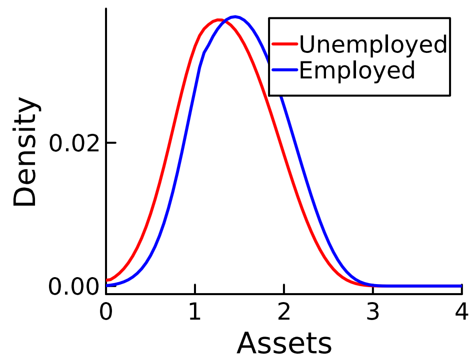
5. Figure (1 x 2): Same as 2 but for saving policy functions

6. Asset distribution of unemployed workers and employed workers: Panel A (3 firm case), Panel B (2 firm case)

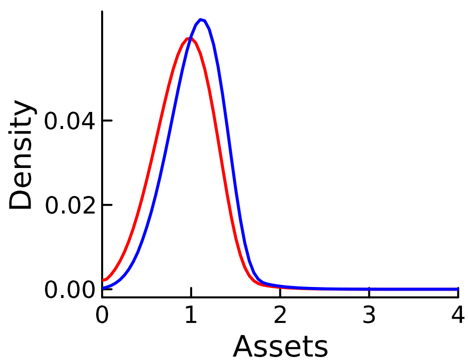
Figure 1: Impact of increasing number of firms



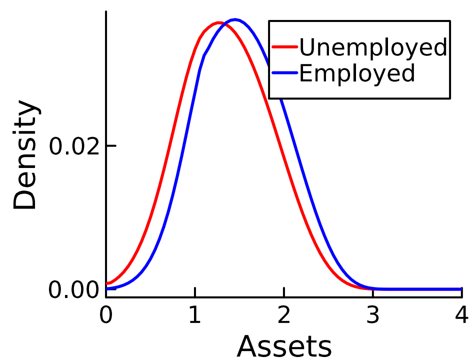
(a) Wages



(b) Unemployment

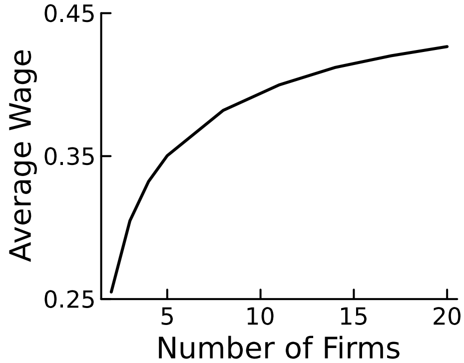


(c) Vacancies

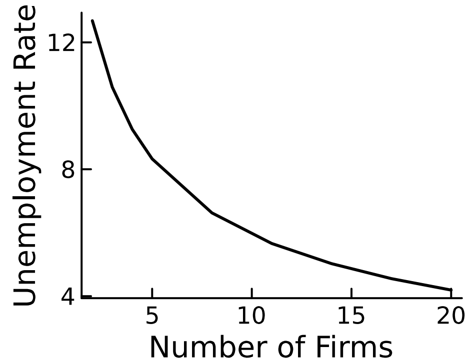


(d) Firm Profits

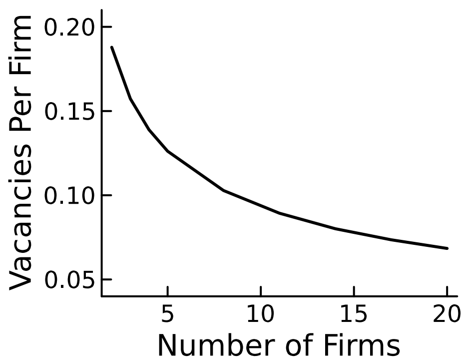
Figure 2: Impact of increasing number of firms



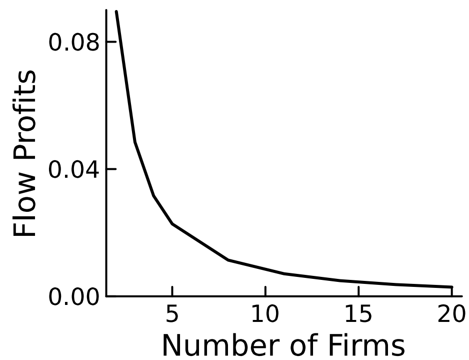
(a) Wages



(b) Unemployment



(c) Vacancies



(d) Firm Profits

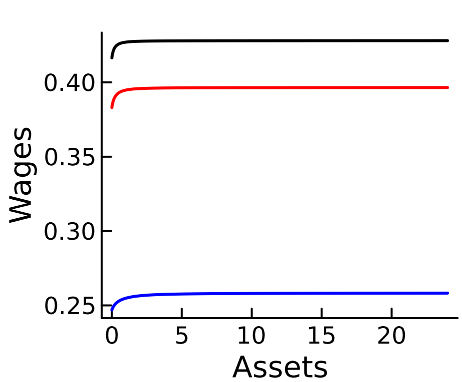
3.2 Varying the number of firms

The first thing we analyze in the model is that given a homogenous productivity firms what is the impact of an increase in the number of firms. In our model, with more number of firms, firms will pay higher wages, as the outside option of workers is better during bargainin in form of other firms jobs. At the same time, firms will post more vacancies as there are more firms but also the number of vacancies per firm will be higher because they don't strategically post less as each firm is small to create congestion or significantly improve their position in bargaining by doing so.

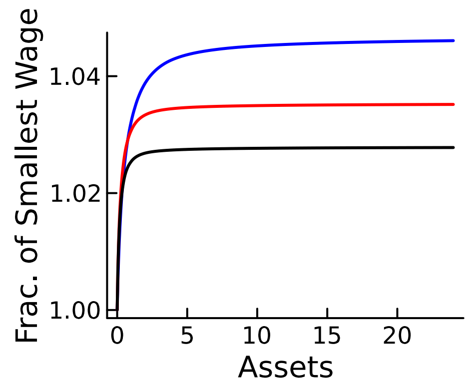
1. Figure (2 x 2): Number of firms on x-axis. On y-axis:

    - Panel A: Unemployment rate

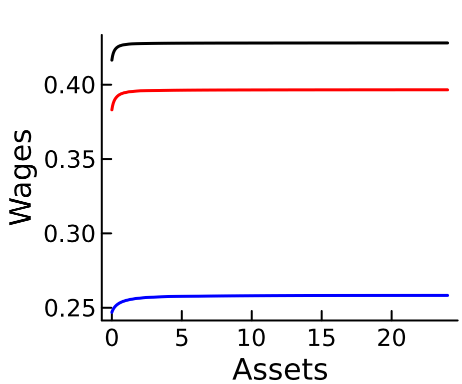
FIGURE 3: IMPACT OF INCREASING NUMBER OF FIRMS



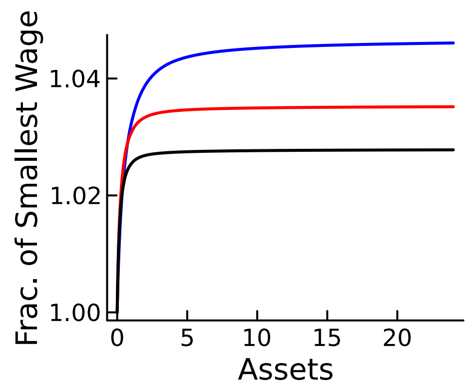
(A) WAGES



(B) UNEMPLOYMENT



(C) VACANCIES



(D) FIRM PROFITS

- Panel B: Average wages (with bars for min-max? see if this looks good)
- Panel C: Average assets/consumption (which is useful?) (with bars for min-max)
- Panel D: Firm profits

2. Figure (2 x 2): Assets on on x-axis. On y-axis:

   - Panel A: Wages
   - Panel B: Consumption function (for unempl and empl)
   - Panel C: Savings function (for unempl and empl)
   - Panel D: Firm value from hiring worker with assets a

   In these figures just do for say 3 values of number of firms. For eg. if we are doing experiments for 2-10 firms. Then plot for num of firms = 2, 5, 10 or something or just for 2 and 10 if that looks better.

3.3 Unemployment insurance

Just want to see first. Plot (i) unemployment rate, (ii) average wages, as a function of b varying the number of firms.

3.4 Minimum wage

Just want to see first. Plot unemployment rate and average wages as a function of number of firms with and without minimum wage.

3.5 Other experiments

- Rasing β: plot asset distribution and w(a) in two cases
- Raising δ: plot asset distribution and w(a) in two cases
- Adding expense shocks
- What else?

Parameter	Description	Value	Target/Source
	Externally Calibrated		
γ	Coefficient of relative risk aversion	2	
ρ	Discount rate	0.05	
r	Interest rate	0.03	
α	Elasticity of the matching function	0.5	
	Internally Calibrated		
A	Efficiency of the matching function		UE rate
δ	Job destruction rate		Unemployment rate
b	Unemployment flow value		$0.45 \times$ wages
$\{z_j\}_{j=1}^{J}$	Levels of the productivity distribution		Wage distribution
$f_m(z)$	Mass of the productivity distribution	–	BDS
β	Worker's bargaining power		$Corr(w, a) \mid$ firm size
σ	Vacancy posting cost parameter		

4 Quantifying the Theory

We calibrate the model to quantitatively study the impact of concentration on inequality and conduct policy counterfactuals. In Section 4.1, we outline our calibration strategy for determining the parameters. **??** presents the computed policy functions. Finally, in **??**, we validate the model by comparing model-generated moments at chosen parameters to their empirical counterparts.

4.1 Calibration

We match long-run averages of empirical moments for the US economy over the period XX–XX to the moments implied by the stationary equilibrium of our model. We assume the economy consists of a set of labor markets, each defined as a combination of Metropolitan Statistical Areas (MSAs) and 2-digit NAICS sectors.[3] The model is solved and simulated separately for each labor market. Whenever the corresponding empirical moment can be measured at the market level, we select market-specific parameters and index markets by m. A unit of time is defined as one month.

[3]This choice is driven by data availability.

Preferences. The preferences over consumption are represented by the Constant Relative Risk Aversion (CRRA) utility function,

$$u(c) = \begin{cases} \dfrac{c^{1-\gamma}-1}{1-\gamma}, & \gamma > 0 \\ \log(c), & \gamma = 1, \end{cases}$$

where γ is the coefficient of relative risk aversion. In the baseline calibration, we set $\gamma = 2$. The workers' discount rate is set to 5 percent, and the risk-free interest rate is set to 3 percent.

Matching technology. The matching function takes the Cobb-Douglas form and is given by $m(u_m, v_m) = Au_m^{\alpha} v_m^{1-\alpha}$, where A represents the matching efficiency, and α denotes the elasticity of matches with respect to the stock of unemployed workers. The match elasticity is set to the standard value of 0.5.[4] The matching efficiency parameter, A, is pinned down by the aggregate monthly UE rate of 0.45 (**?**).

Separation rate (δ). We utilize the aggregate UE rate of 0.45 with the steady-state unemployment rate of 5.7 percent from **?** to pindown δ, which is assumed to be the same across all markets.

Flow value of unemployment (b). We calibrate b to equal 0.45 replacement rate of wages for individual workers. (Replacement rates available here.)

Firm productivity distribution. In our model, a firm's productivity is closely tied to its size. Since productivity is unobservable, we utilize data available by firm size to proxy for the firm-productivity distribution. Due to the lack of detailed measures of firm size in publically available data, we assume firm productivity takes five values corresponding to the five firm size bins consistently observable in the SIPP and BDS. This allows us to quantify the relationship between wages and assets within each firm bin using SIPP data and get the share of firms in each bin from the BDS. [5] While the shares for five productivity types are directly observable from the BDS, the five productivity values are pinned down by the distribution of wages, conditional on assets within each bin.

[4] This might be a bit problematic. Also, I think A needs to be free.

[5] Note that technically, the smallest firm size bin in SIPP is "less than 20", but it is "less than 26" in the BDS data. Alternatively, we can do 4 bins instead of 5.

Worker's bargaining power (β). A worker's wages in the model depend on their outside option, which depends on their assets. Conditional on firm type, a higher β would lead to a lower correlation between assets and wages for identical workers. Essentially, if workers have higher bargaining power, they are able to get a higher share of the firm surplus and the relationship between their outside option and wages is weakened. We target the correlation between assets and wages to pin down β.

Vacancy posting cost. Flow cost of posting a vacancy is given by the iso-elastic function $c(v_i) = v_i^{1+\sigma}/(1+\sigma)$. The cost parameter σ determines the distribution of vacancies across firms. In particular, keeping else constant, a higher σ means vacancies are more concentrated at high-productivity firms. [Check this in code]. Could target HHI, but can't find an aggregate one yet. Once we add capital to the model, we also target the labor share of income.

4.2 Validation and results

In progress...

5 Results and Policy Counterfactuals

In progress...

6 Conclusion

A Model Appendix

A.1 Reservation wage/productivity

Reservation wage for firm i for asset a worker $w_i^r(a)$ is characterized by:

$$W(a, w_i^r(a)) = \tilde{U}_i(a)$$

If in equilibrium, $z_i > z_j$ implies $v_i > v_j$ and $w_i(a) > w_j(a)$ for all a, then $\tilde{U}_i(a) < \tilde{U}_j(a)$. Hence, we would have that $w_i^r(a) < w_j^r(a)$. High-productivity firms are more sought after. This implies that there is a threshold $z_r(a)$, such that the worker accepts all offers from firms with $z > z_r(a)$ in the equilibrium. If firms ordered by productivity $z_1 < z_2 < \ldots < z_M$ then denote $i_r(a) = \arg\min_i \{z_i | z_i > z_r(a)\}$. In this case we can write the value functions as:

$$\rho U(a) = \max_{c \geq 0} \; u(c) + \sum_{i=i_r(a)}^{M} \lambda_i \left[W_i(a) - U(a)\right]$$

$$\text{s.t.} \quad \dot{a} = ra + b - c$$

Also, intuitively, lower asset workers should have lower $i_r(a)$?

A.2 Optimal policy functions

A worker with asset level a and flow income ω can be in one of the following $2M + 1$ states,

$$\chi \in \{u_0, \tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_M, e_1, e_2, \ldots, e_M\}.$$

Here, u_0 represents the state of being unemployed and not excluded from any firm, $\tilde{u}_i$ represents the state of being unemployed and excluded from firm i, and e_i represents the state of being employed at firm i.

Denote the optimal consumption and saving policy functions by $c(a, \omega, \chi)$ and $s(a, \omega, \chi)$, which are characterized by the following equations:

$$u'(c(a, b, u_0)) = \partial_a U(a)$$

$$u'(c(a, b, \tilde{u}_i)) = \partial_a \tilde{U}_i(a) \quad \text{for } i = 1, 2, \ldots, M$$

$$u'(c(a, w, e_i)) = \partial_a W(a, w) \quad \text{for } i = 1, 2, \ldots, M$$

Note that $\omega = b$ in all of the unemployed states. Also, the consumption policy function in all employed states is identical. Finally, the optimal job-offer acceptance function, $I_i(a)$, is defined as follows:

$$I_i(a) = \begin{cases} 1, & W_i(a) \geq \tilde{U}_i(a) \\ 0, & \text{otherwise} \end{cases}$$

A.3 Steady-state distribution

Let $g(a, \omega, \chi)$ denote the steady-state joint distribution over assets, flow income, and labor market states. Let $g_j^u(a)$ represent the proportion of unemployed workers available to firm j (i.e., not in firm j's excluded set), defined as $g_j^u(a) = g(a, b, u_0) + \sum_{i \neq j} g(a, b, u_i)$. Also, note that the steady-state unemployment rate can be written as:

$$u = \sum_{i=0}^{M} \int g(a, b, u_i) \, \mathrm{d}a$$

The following Kolmogorov Forward (or Fokker-Planck) equations characterize the steady-state distribution:

- Unemployment without exclusion:

$$-\frac{d}{da}[s(a, b, u_0) g(a, b, u_0)] - g(a, b, u_0) \sum_{i=1}^{M} \lambda_i + \delta \sum_{i=1}^{M} \int g(a, w, e_i) \, \mathrm{d}w = 0$$

  For $\omega \neq b$, $g(a, \omega, u_0) = 0$.

- Unemployment with exclusion, for $j = 1, 2, \ldots, M$,

$$-\frac{d}{da}\left[s(a, b, \tilde{u}_j) g(a, b, \tilde{u}_j)\right] - g(a, b, \tilde{u}_j) \sum_{i \neq j} \lambda_i + \lambda_j [1 - I_j(a, b)] \left(g(a, b, u_0) + \sum_{i \neq j} g(a, b, \tilde{u}_i) \right) = 0$$

  For $\omega \neq b$, $g(a, \omega, u_j) = 0$ for $j = 1, 2, \ldots, M$.

- Employment, for $j = 1, 2, \ldots, M$,

$$-\frac{d}{da}\left[s(a, \omega, e_j) g(a, \omega, e_j)\right] - \delta g(a, \omega, e_j) + \lambda_j I_j(a, b) \left(g(a, b, u_0) + \sum_{i \neq j} g(a, b, u_i) \right) = 0$$

A.4 Comparison to Free-Entry Condition

If firms small enough, reasonable to assume firm solves $\max_{v_i} \Pi_i(v_i|V_i)$. This corresponds to free entry condition:

$$\int \frac{z_i - w_i(a)}{r+\delta} \cdot \tilde{g}_i^u(a) \, \mathrm{d}a = \frac{c(v_i)}{q(u,v)}$$

A.5 Wage Determination

$$\rho W(a,w) = u(c_e(a,w)) + \partial_a W(a,w)[ra + w - c_e(a,w)] + \delta[U(a) - W(a,w)]$$

$$W(a,w) = \frac{u(c_e(a,w)) + u'(c_e(a,w))[ra + w - c_e(a,w)] + \delta U(a)}{\rho + \delta}$$

By envelope theorem (is this correct?),

$$\partial_w W(a,w) = \frac{u'(c_e(a,w))}{\rho+\delta}$$

$$w_i(a) \in \arg\max_w \left(W(a,w) - \tilde{U}_i(a)\right)^{\beta} \left(J_i(w) - V_i\right)^{1-\beta}$$

FOC:

$$\beta\left(J_i(w) - V_i\right)\partial_w W(a,w) + (1-\beta)\left(W(a,w) - \tilde{U}_i(a)\right)\partial_w J_i(w) = 0$$

For firm's optimization, derivative of w with respect to v_i will involve taking the derivative of $W(a,w)$ with respect to w. While taking the later derivative, firm doesn't take $c_e(a,w)$ as given.

A.6 Aggregation

Note that the assumed matching technology implies that the contact rate $\lambda_m = A(v_m/u_m)^{1-\alpha}$. According to the model the UE transition rate is given by $\lambda_m \times P_m$, where P_m is the probability that a match is formed conditional on meeting. The aggregate UE rate then would be:

$$A \cdot \frac{\sum_m (v_m/u_m)^{1-\alpha} \times P_m \times u_m}{u}$$

So, matching efficiency A would be pinned by the aggregate UE rate.

The aggregate steady-state unemployment rate would be:

$$u = \sum_m \frac{\lambda_m P_m}{\lambda_m P_m + \delta} \pi_m^L$$

This will help pin δ.

B Extensions

B.1 Unemployment flow depends on past wages

Value of being employed with assets a and wages w:

$$\begin{aligned} \rho W(a,w) = \max_c \; & u(c) + \partial_a W(a,w)[ra + w - c] \\ & + \delta[U(a,b(w)) - W(a,w)] \end{aligned} \tag{4}$$

Value of being unemployed with assets a and flow income b:

$$\begin{aligned} \rho U(a,b) = \max_c \; & u(c) + \partial_a U(a,b)[ra + b - c] \\ & + \sum_{i=1}^{M} \lambda_i \left(\max\{W(a, w_i(a,b)), \tilde{U}_i(a,b)\} - U(a,b) \right) \end{aligned} \tag{5}$$

Here, $w_i(a,b)$ denotes bargained wage at firm i for an unemployed worker with assets a and flow income b.

Value of being unemployed and excluded from firm j with assets a and flow income b:

$$\begin{aligned} \rho \tilde{U}_j(a,b) = \max_c \; & u(c) + \partial_a \tilde{U}_j(a,b)[ra + b - c] \\ & + \sum_{i \neq j} \lambda_i \left(\max\{W(a, w_i(a,b)), \tilde{U}_i(a,b)\} - \tilde{U}_j(a,b) \right) \end{aligned} \tag{6}$$

Let $g(a, \omega, \chi)$ denote the steady-state distribution over assets, flow income ω and labor market state χ which can take one of the following $2M + 1$ values:

$$\chi \in \{u_0, \tilde{u}_1, \ldots, \tilde{u}_M, e_1, \ldots, e_M\}$$

$s(a, \omega, \chi)$ the denotes the optimal saving policy function. Let $I_i(a,b)$ denote the offer acceptance policy function.

Kolmogorov Forward equations:

Unemployment without exclusion:

$$-\frac{d}{da}\left[s(a,\omega,u_0)g(a,\omega,u_0)\right]-g(a,\omega,u_0)\sum_{i=1}^{M}\lambda_i+\delta\sum_{i=1}^{M}\int_{w:\omega=b(w)}g(a,w,e_i)\,\mathrm{d}w=0$$

Unemployment with exclusion, for $j=1,2,\ldots,M$,

$$-\frac{d}{da}\left[s(a,\omega,\tilde{u}_j)g(a,\omega,\tilde{u}_j)\right]-g(a,\omega,\tilde{u}_j)\sum_{i\neq j}\lambda_i+\lambda_j[1-I_j(a,b)]\left(g(a,\omega,u_0)+\sum_{i\neq j}g(a,\omega,\tilde{u}_i)\right)=0$$

Employment, for $j=1,2,\ldots,M$,

$$-\frac{d}{da}\left[s(a,\omega,e_j)g(a,\omega,e_j)\right]-\delta g(a,\omega,e_j)+\lambda_j\int_{b:w_j(a,b)=\omega}I_j(a,b)\left(g(a,b,u_0)+\sum_{i\neq j}g(a,b,u_i)\right)\mathrm{d}b=0$$

B.2 Worker heterogeneity

B.3 On-the-job search

B.4 Capital and equity