Market Structure and Inequality

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Abstract

Over the past fifty years, there has been a contemporaneous increase in wealth inequality and labor market concentration in the United States. In this project, we study how these two phenomena interact by developing a novel structural model of a frictional labor market with large firms and risk-averse workers who make consumption and savings decisions. We use the model to quantify the role of increased labor market concentration in driving long-term shifts in wealth distribution and to offer novel insights about labor market policies, such as minimum wage and unemployment insurance.

Keywords: *Market power, monopsony, wealth inequality, borrowing constraints*

JEL Codes: *E24, E21, C63*

1 Introduction

Recent studies indicate that the distribution of wealth has shifted significantly over time, with a notable increase in inequality (Piketty, 2014). The share of wealth held by the top 1% in the United States rose from approximately 25% in 1980 to over 40% in recent years (Saez and Zucman, 2016). This increase in wealth inequality has coincided with another long-term trend: rising concentration in the labor market. For instance, the share of employment held by the top-4 firms in the retail sector more than doubled, increasing from 12% in the 1980s to about 30% by 2010 (Autor et al., 2017). In concentrated markets, workers have limited outside options, giving employers greater market power to suppress wages. This effect can be especially pronounced for workers with constrained financial resources, as it limits their ability to wait for better job opportunities. As a result, labor market concentration can contribute to wealth inequality both by exacerbating wage suppression for workers lower on the wealth distribution and by redirecting the profits from reduced wages to wealthier shareholders.

In this paper, we quantify the impact of rising employer concentration on wealth inequality and evaluate the role of labor market policies, such as minimum wage and unemployment insurance, given this dynamic. To achieve this, we develop a rich quantitative model and calibrate it to reflect trends in U.S. labor markets, defined by industry and location.

Specifically, we embed a job search model with large firms into a model of consumption and savings. In our model, workers are risk-averse and can borrow or save by holding two types of risk-free assets: capital, which is used as an input in production, and equity in firms' profits. Workers face a borrowing constraint and can save to partially insure themselves against unemployment risk.[1] Job search is random, with both unemployed and employed workers sampling job offers from firms with different productivity levels. On-the-job search creates a job ladder where workers gradually transition to more productive firms, deriving higher and higher value from successive employment matches.

Wages in our model are determined through strategic bargaining between firms and workers.

[1]Essentially, we are combining a job search model with the Bewley-Huggett-Aiyagari (BHA) framework. See Bewley (1980), Huggett (1993), and Aiyagari (1994).

Lower-asset unemployed workers, particularly those constrained by borrowing limits, have worse fallback options, which weakens their position during bargaining and leads to lower negotiated wages. Finally, because our model includes a finite number of firms rather than an infinite continuum of atomistic firms, these firms also exert market power in two additional ways. First, through a novel channel that we introduce: firms facing vacancy posting costs strategically post vacancies to control market tightness and strengthen their position in wage bargaining. Second, as in Jarosch et al. (2024), workers negotiating with a firm that dominates a significant share of the market have worse outside options, giving large firms greater ability to pay wages below the workers' marginal productivity.

To calibrate the model, we represent the U.S. economy as a collection of labor markets, each represented by a combination of Metropolitan Statistical Areas (MSAs) and 2-digit NAICS sectors. We use data from the Survey of Income and Program Participation (SIPP) to estimate the joint empirical distribution of wages and assets within different firm-size bins. Additionally, we utilize the Business Dynamics Statistics (BDS) to calculate the share of firms within each firm-size bin across various markets for the years 1990–2022. These shares act as a measure of market concentration, reflecting how the presence of large firms varies across markets and evolves over time. We calibrate the model to match the 1990 economy and then examine how changes in firm size have influenced wealth inequality in the present day.

Our framework positions us well to provide unique insights into three key policies. The first is the optimal minimum wage. In our framework, low-wage workers who are close to their borrowing constraint face greater employer market power, which suggests that our estimate of the optimal minimum wage might be higher than previous estimates. Second, given the importance of a worker's ability to insure against risk in the wage bargaining process, we will examine the impact of social welfare policies on the low-wage labor market. These insights will help us determine the optimal mix of minimum wage (a tax borne by employers) and social welfare (a tax on society as a whole). Third, we will examine unemployment insurance (UI). In the context of our model, higher market concentration leads to reduced vacancy postings and, hence, lower job arrival rates and increased unemployment risk. As a result, unemployment insurance may play a more significant role in highly concentrated markets.

Related Literature. Our paper builds on several strands of the literature. It is the first to embed a labor search model with granular firms into a Bewley-Huggett-Aiyagari (BHA) economy (Bewley, 1980; Huggett, 1993; Aiyagari, 1994), allowing us to examine the role of market concentration in a setting with unemployment risk and limited insurance. While other studies, such as Krusell et al. (2010) and Lise (2013), incorporate random search models, and Chaumant and Shi (2022) directed search, none of these allow for large firms hiring multiple workers.

Standard approaches to modeling market power in the labor market have traditionally followed the monopsony framework of Robinson (1933) (e.g., Card et al. (2018), Berger, Herkenhoff, and Mongey (2022), Lamadon, Mogstad, and Setzler (2022), MacKenzie (2019), Haanwinckel (2023), and Yeh, Macaluso, and Hershbein (2022)). The core idea in these models is that large firms "underhire" and "underpay" relative to the perfectly competitive benchmark. The job search model here builds Jarosch, Nimczik, and Sorkin (2024) (henceforth JNS), who depart from this standard approach and incorporate granular firms into a canonical random search and bargaining model, the Diamon-Mortensen-Pissaradis (DMP) model. In their model, firm size impacts the bargaining position of workers, affecting wages rather than employment quantities.

We extend JNS by reintroducing elements of traditional monopsony, where firms act strategically. Specifically, firms internalize congestion effects and strategically post vacancies to influence market tightness and strengthen their bargaining position in wage negotiations. This restores the connection between market concentration and employment quantities, as in classic monopsony models, while still allowing for unemployment risk. Additionally, we expand the JNS framework by incorporating on-the-job search, similar to Bagga (2023).

Finally, we also contribute to the literature on optimal minimum wage. This is the first paper that can study the role of minimum wage as an insurance.

2 Model

In this section, we build upon the random search model presented in Jarosch et al. (2024), henceforth referred to as JNS. As in JNS, workers in our model apply to jobs distributed across a finite number of firms, wages are Nash bargained, and granular employers exhibit market power by not competing with themselves. The key difference in our framework is that workers are risk-averse and have the ability to borrow or save using a single risk-free asset. This addition enables us to study the interplay between wealth distribution and market structure. Additionally, we endogenize the firms' vacancy decisions by replacing the "free-entry" condition in JNS with a corresponding optimization condition for granular firms.

Ultimately, we will incorporate on-the-job search and allow individuals to save in both capital and firm equity. However, for now, we begin with a simplified model to ensure that the comparative statics, without these additional elements, align with the patterns observed in the data.

2.1 Setup

Time is continuous and there is no aggregate uncertainity. Within a labor market, there is a unit measure of infinitely lived workers and a $M \in \mathbb{N}$ number oh granular firms, each differing in productivity denoted by $z_i \in \{z_1, z_2, \ldots z_M\}$. Since firms are granular, each firm i controls a positive measure of vacancies v_i.

Workers can be either employed or unemployed. Employed workers exogenously separate from their employer at rate δ. Workers are homogeneous in terms of productivity, and when employed at firm i, a worker produces z_i units of the final good. Thus, workers are identical ex ante, but differ ex post due to different labor market histories.

2.1.1 Matching

The rate at which workers encounter job openings is described by a constant returns to scale matching function, $m(u, v)$, where u is the unemployment rate and $v = \sum_{i=1}^{M} v_i$. Job offers for unemployed workers arrive at the Poisson rate $\lambda \equiv m(u, v)/u$ and vacancies meet a worker at the rate $q \equiv m(u,v)/v$. Additionally, since each vacancy encounters workers at the same rate, by law of large numbers the rate at which an unemployed worker encounters firm i is given by $\lambda \pi_i$, where $\pi_i = v_i/v$.

When an unemployed worker encounters an open vacancy, wages are bargained as outlined below. The bargained wages never exceed the firm's marginal productivity, so a firm never rejects any matches. Unemployed workers, however, may decline offers if the value of rejecting the offer outweighs the value of working for the offered wage. If the worker rejects an offer from a firm, the firm bars the worker from it's future job opportunities for the duration of the worker's current unemployment spell. Like JNS, we do not allow a worker to be permanently barred from a firm they reject, to maintain tractability.

2.1.2 Workers

Workers maximize their lifetime utility given by

$$\mathbb{E}_0 \int e^{-\rho t} u(c_t) dt,$$

where ρ is the subjective rate of time preference and c_t is consumption at time t. We will focus on a stationary equilibrium, and hence we drop the time subscripts.

Unemployed workers receive a flow value of b, while employed workers receive their contracted wages. Workers make consumption decisions both when employed and unemployed. Assets are subject to a lower bound $\underline{a} \leq 0$ and evolve according to

$$\dot{a} = ra + \omega - c,$$

where a is the current asset level, r is the risk free interest rate, and ω is the household's labor income or unemployment income. We assume $\underline{a} = 0$.

Let $W(a,w)$ denote the expected present value for an individual employed with assets a and a wage w, and let $U(a)$ represent the expected present value of being unemployed with asset level a. Given that the worker chooses consumption to maximize utility subject to the asset evolution equation and a borrowing constraint, we can express the value of employment as follows:

$$\rho W(a,w) = \max_c \; u(c) + \partial_a W(a,w)[ra + w - c] + \delta[U(a) - W(a,w)] \tag{1}$$

Throughout the paper, we use $\partial_x Y$ as a short-hand notation to denote $\partial Y / \partial x$.

Now consider the problem of a worker who is currently unemployed with assets a. This worker will accept a job offer from firm i if the value of being employed at this firm is greater than the value of rejecting it. Hence, we can express the value of being unemployed as follows:

$$\rho U(a) = \max_c \; u(c) + \partial_a U(a)[ra + b - c] + \sum_{i=1}^{M} \lambda_i \left(\max\{W_i(a), \tilde{U}_i(a)\} - U(a) \right) \tag{2}$$

Here, $W_i(a) \equiv W(a, w_i(a))$, where $w_i(a)$ denotes the bargained wage between firm i and a worker with assets a and $\tilde{U}_i(a)$ denotes the value of unemployment for a worker excluded from firm j, which is given by,

$$\rho \tilde{U}_j(a) = \max_c \; u(c) + \partial_a \tilde{U}_j(a)[ra + b - c] + \sum_{i \neq j} \lambda_i \left(\max\{W_i(a), \tilde{U}_i(a)\} - \tilde{U}_j(a) \right) \tag{3}$$

2.1.3 *Wage Determination*

Denote $J_i(w)$ as the value of a filled job to firm i at w and V_i as the value of an empty vacancy. Then Nash-bargained wages at firm i for worker with assets a are determined by:

$$w_i(a) \in \arg\max_w \left(W(a,w) - \tilde{U}_i(a) \right)^{\beta} \left(J_i(w) - V_i \right)^{1-\beta}$$

Firm and worker take each other's threat points $\tilde{U}_i(a)$ and V_i, and value of unemployment $U(a)$ as given.

2.1.4 *Firm Optimization*

Value of a filled job to firm i at w:

$$rJ_i(w) = z_i - w + \delta[-J_i(w)]$$

Firm's value from an empty vacancy:

$$rV_i = -c(v_i) + q(u,v)\left(\int_{\underline{a}}^{\infty} J_i(w_i(a))\tilde{g}_i^u(a)\,\mathrm{d}a - V_i\right)$$

where $c(v_i)$ is flow cost of posting a vacancy that depends on total number of vacancies posted by firm i and $\tilde{g}_i^u(a)$ is the proportion of workers who are unemployed but not excluded from firm i and have assets a.

Note that we can write V_i as:

$$V_i = \frac{-c(v_i)(r+\delta) + q(u,v)\int_{\underline{a}}^{\infty}[z_i - w_i(a)]\tilde{g}_i^u(a)\,\mathrm{d}a}{(r+\delta)(r+q(u,v))}$$

Firms choose vacancies to maximize their steady-state profits given by $\Pi_i = V_i v_i$. We assume that while making this choice, firms understand that their vacancy choice impacts the rate at which empty vacancies meet workers $q(u,v)$ and also impacts the contracted wages $w_i(a)$ through changes in their outside option V_i while bargaining.[2] However, the firm takes the unemployment rate u, the vacancies of other firms $\boldsymbol{v}_{-i}$, and workers' value of unemployment $U(a)$ and their outside option while bargaining $\tilde{U}_i(a)$, as well as the steady-state distribution of workers g as given. Hence, firm i's vacancy-choice problem is given by:

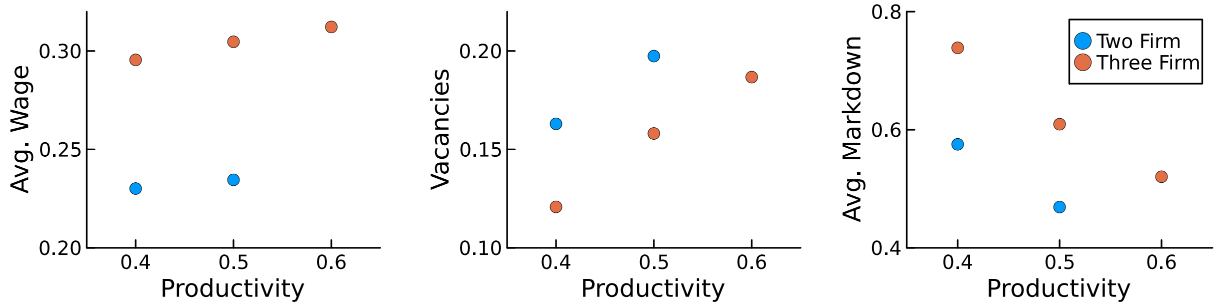
$$\max_{v_i}\ \Pi_i(v_i|u, \boldsymbol{v}_{-i}, \tilde{U}_i, U, g)$$

2.2 Computation

To compute the stationary equilibrium, we define an equilibrium residual function and use a nonlinear equation solver to find the equilibrium aggregates. The input to this function is the number of vacancies per firm type $\{v_i\}$, the value of a vacancy per firm type $\{V_i\}$, the unemployment rate u, and the number of unemployed people who are excluded from firm i $\{\tilde{u}_i\}$. Applying the number of vacancies and unemployment rates in the matching function, we get the job match rates. In this model, the wages bargained between firm and household are a function of the firm's productivity and value of vacancy and household assets. We need to

[2] Note that from the firm's perspective $w_i(a)$ depends on $U(a), \tilde{U}_i(a)$ and V_i.

FIGURE 1: SIZE-BASED MARKET POWER



solve for this simultaneously with the household value functions. With an initial guess of the value functions, we solve for the wage function. Then, we updated the value functions. We continue this loop until both value and wage functions have converged. We then compute the distribution of households and household aggregates. Taking the household aggregates, we compute the residual values for the equilibrium conditions. The root of this nonlinear function are solved using a standard non-linear solver.

3 NUMERICAL EXAMPLES

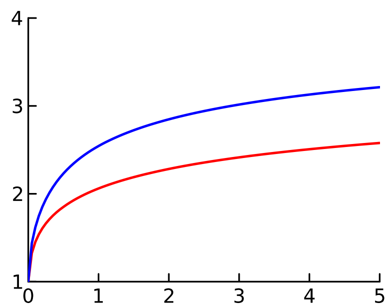
In this section, we illustrate comparative statics from our model by utilizing several numerical illustrations. For each of the examples, we use the following values for the parameters unless otherwise mentioned.

3.1 *Size-based market power*

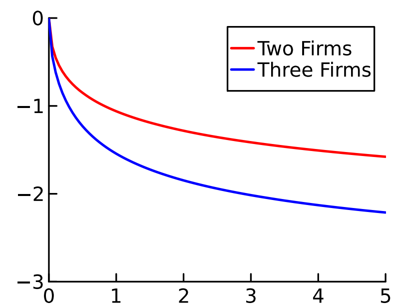
3.2 *The Impact of Labor Market Concentration*

To examine the effect of labor market concentration, we vary the number of equally productive firms in a market. Figure 3 shows how market outcomes change as the number of firms in the market increase. First, as the number of firms increase, the average wage in the market increases. This is because, with more firms, the outside option of workers is better during bargaining in form of other firms jobs. Under our baseline parameterization, the wage gain is substantial: going from a market with two firms to a market with five firms results in a twenty

FIGURE 2: SIZE-BASED MARKET POWER



(A) CONSUMPTION



(B) SAVINGS

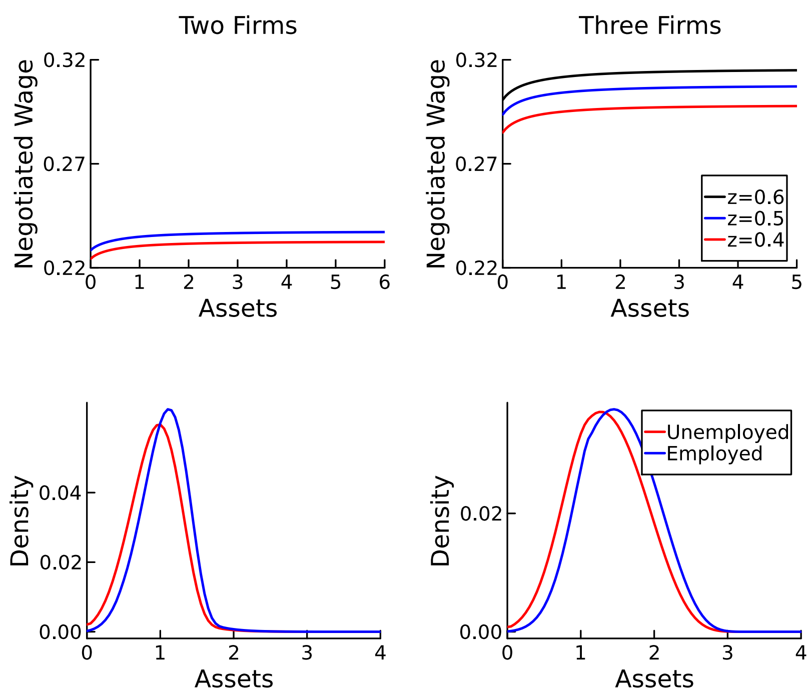
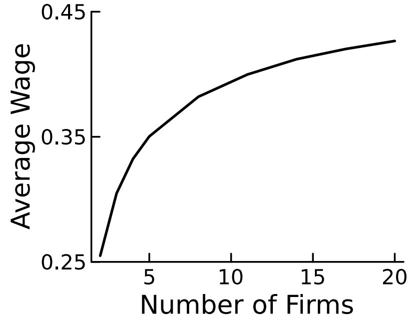
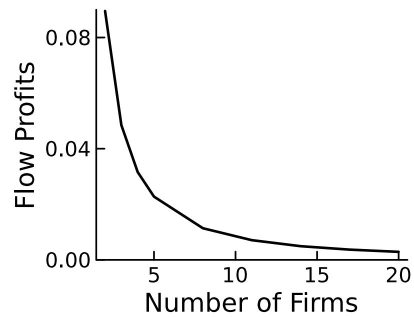


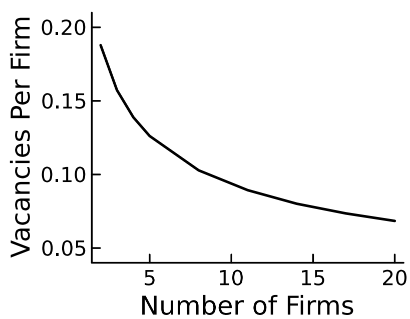
FIGURE 3: IMPACT OF INCREASING NUMBER OF FIRMS



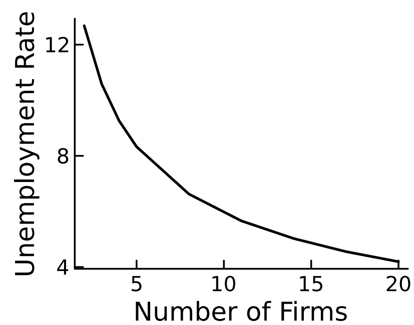
(A) WAGES



(B) FIRM PROFITS



(C) VACANCIES



(D) UNEMPLOYMENT

percent increase in the average wage in a labor market. This substantial gain in wages is paired with a substantial drop in each firm's profit. This is because each individual firm's labor market power decreases as the number of firms increases. The decrease in profits leads to firms cutting back on the number of vacancies they post. However, the decrease in an individual firm's vacancies is made up for by the growth in the number of firms. So, total vacancies increases as the number of firms increases. This leads to a reduction in the unemployment rate as the number of firms increases.

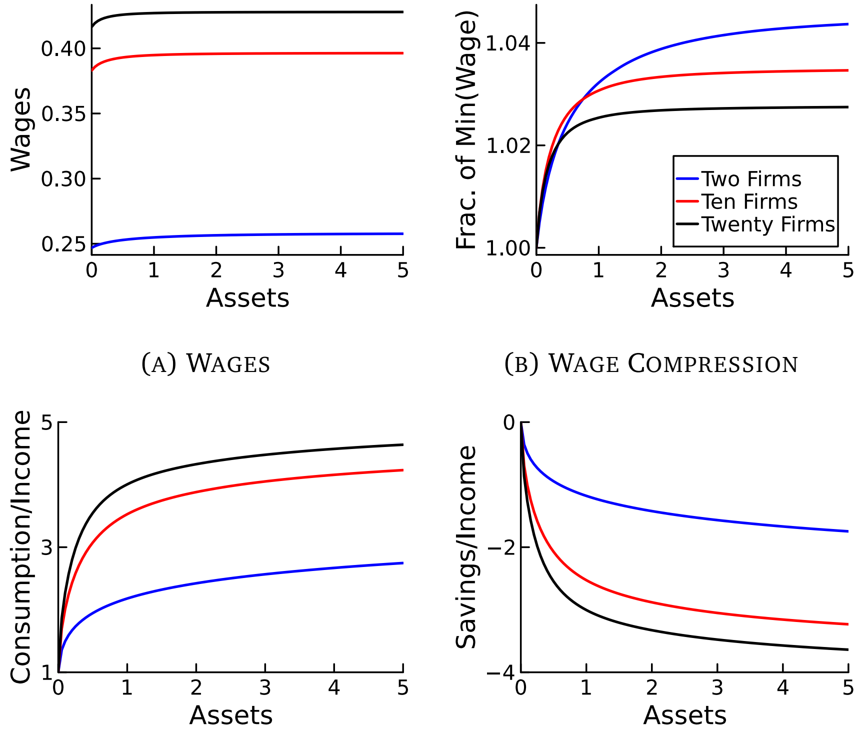
There heterogeneity in the impact of labor-market concentration. Figure 4 examines outcomes for households with different asset levels with two, ten, and twenty firms in the market. Panel 4a shows the wage offer an unemployed household with assets a receives after a match. As concentration decreases, the wage offer function shifts upwards. The wage function is increasing in assets meaning households at the borrowing constraint are receiving lower offers than households with more assets. This is because households with more assets can more readily afford to reject low-wage offers and wait for another job offer. The degree of the wage haircut depends on the number of firms in the market. Panel 4b displays the wage offer function as a fraction of the wage offer given to the households at the lower bound. The haircut faced by borrowing constrained households in a market with two firms is more than double compared to a household in a market with twenty firms. These curves flatten out as assets increase. however, it flattens more slowly in the case of two firms when compared with twenty. This is because the probability of an unemployed household at a specific asset level becoming borrowing constrained is inversely related to market concentration.

Additionally, households in more concentrated labor markets are less willing to consume their savings in unemployment. Panel 4c displays the the fraction of income an unemployed household is consuming and Panel 4d displays the fraction of income an unemployed households saves. It is important to note, that, at a given level of asset, an unemployed household's income is the same across all three lines. The consumption of a unemployed household in a labor market with twenty firms is, on average, more than double the consumption of a household in a labor market with two firms. This is because households in less concentrated markets are burning their savings more quickly than those in more concentrated markets. Household are willing to consume more because (1) the expected length of unemployment spells decreases as the number of firms increases and (2) the wage haircut for having less assets is lower in less concentrated markets.

3.3 The Role of Social Insurance

In the model, an unemployed household receives a guaranteed payment b for the entirety of their unemployment spell. Figure 5 examines how wages would change under different levels of this benefit. As the unemployment benefit increases, the wage offer curve shifts up. The increased benefit increases the value of being unemployed which puts workers in better bargaining positions with potential employers. Additionally, the low asset workers are taking a smaller wage haircut as the unemployment benefit increases. This is because the benefit serves as a substitute for the household's savings as a source of insurance. This is especially important for asset poor households who have limited insurance ability. Thus, social insurance can both

FIGURE 4: THE HETEROGENEOUS IMPACT OF CONCENTRATION ON UNEMPLOYED HOUSEHOLDS



(A) WAGES

(B) WAGE COMPRESSION

(C) CONSUMPTION

(D) SAVING

reduce firm's average and heterogeneous impact of monopsony power.

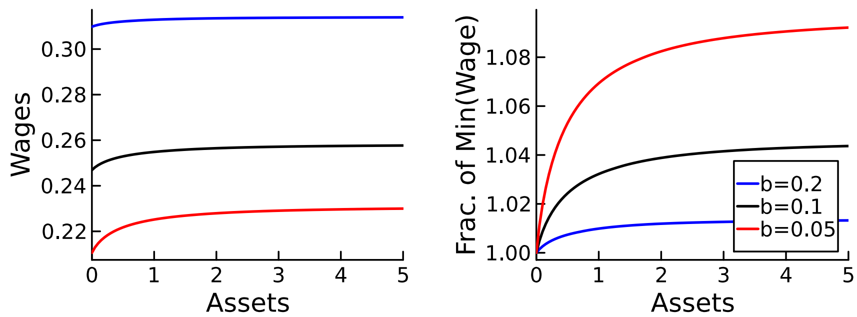
4 QUANTIFYING THE THEORY

We calibrate the model to quantitatively study the impact of concentration on inequality and conduct policy counterfactuals. In Section 4.1, we outline our calibration strategy for determining the parameters. In Section 4.2, we validate the model by comparing model-generated moments at chosen parameters to their empirical counterparts.

4.1 Calibration

We match long-run averages of empirical moments for the US economy over the period 1990–2022 to the moments implied by the stationary equilibrium of our model. We assume the economy consists of a set of labor markets, each defined as a combination of Metropolitan Sta-

FIGURE 5: CHANGING THE LEVEL OF UNEMPLOYMENT BENEFIT



(A) WAGES (B) WAGE COMPRESSION

Parameter	Description	Value	Target/Source
	Externally Calibrated		
γ	Coefficient of relative risk aversion	2	
ρ	Discount rate	0.05	
r	Interest rate	0.03	
α	Elasticity of the matching function	0.5	
	Internally Calibrated		
A	Efficiency of the matching function		UE rate
δ	Job destruction rate		Unemployment rate
b	Unemployment flow value		$0.45 \times$ wages
$\{z_j\}_{j=1}^{J}$	Levels of the productivity distribution		Wage distribution
$f_m(z)$	Mass of the productivity distribution	–	BDS
β	Worker's bargaining power		$Corr(w,a)$ | firm size
σ	Vacancy posting cost parameter		

tistical Areas (MSAs) and 2-digit NAICS sectors.[3] The model is solved and simulated separately for each labor market. Whenever the corresponding empirical moment can be measured at the market level, we select market-specific parameters and index markets by m. A unit of time is defined as one month.

Preferences. The preferences over consumption are represented by the Constant Relative Risk

[3]This choice is driven by data availability.

Aversion (CRRA) utility function,

$$u(c) = \begin{cases} \dfrac{c^{1-\gamma}-1}{1-\gamma}, & \gamma > 0 \\ \log(c), & \gamma = 1, \end{cases}$$

where γ is the coefficient of relative risk aversion. In the baseline calibration, we set $\gamma = 2$. The workers' discount rate is set to 5 percent, and the risk-free interest rate is set to 3 percent.

Matching technology. The matching function takes the Cobb-Douglas form and is given by $m(u_m, v_m) = Au_m^{\alpha} v_m^{1-\alpha}$, where A represents the matching efficiency, and α denotes the elasticity of matches with respect to the stock of unemployed workers. The match elasticity is set to the standard value of 0.5. The matching efficiency parameter, A, is pinned down by the aggregate monthly UE rate of 0.45 (Shimer, 2005).

Separation rate (δ). We utilize the aggregate UE rate of 0.45 with the steady-state unemployment rate of 5.7 percent from Shimer (2005) to pindown δ, which is assumed to be the same across all markets.

Flow value of unemployment (b). We calibrate b to equal 0.45 replacement rate of wages for individual workers. (Replacement rates available here.)

Firm productivity distribution. In our model, a firm's productivity is closely tied to its size. Since productivity is unobservable, we utilize data available by firm size to proxy for the firm-productivity distribution. Due to the lack of detailed measures of firm size in publically available data, we assume firm productivity takes five values corresponding to the five firm size bins consistently observable in the SIPP and BDS. This allows us to quantify the relationship between wages and assets within each firm bin using SIPP data and get the share of firms in each bin from the BDS. [4] While the shares for five productivity types are directly observable from the BDS, the five productivity values are pinned down by the distribution of wages, conditional on assets within each bin.

Worker's bargaining power (β). A worker's wages in the model depend on their outside

[4]Note that technically, the smallest firm size bin in SIPP is "less than 20", but it is "less than 26" in the BDS data. Alternatively, we can do 4 bins instead of 5.

option, which depends on their assets. Conditional on firm type, a higher β would lead to a lower correlation between assets and wages for identical workers. Essentially, if workers have higher bargaining power, they are able to get a higher share of the firm surplus and the relationship between their outside option and wages is weakened. We target the correlation between assets and wages to pin down β.

Vacancy posting cost. Flow cost of posting a vacancy is given by the iso-elastic function $c(v_i) = v_i^{1+\sigma}/(1+\sigma)$. The cost parameter σ determines the distribution of vacancies across firms. In particular, keeping else constant, a higher σ means vacancies are more concentrated at high-productivity firms. Once we add capital to the model, we also target the labor share of income.

4.2 Validation

5 Results and Policy Counterfactuals

6 Conclusion

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A Model Appendix

A.1 Reservation wage/productivity

Reservation wage for firm i for asset a worker $w_i^r(a)$ is characterized by:

$$W(a, w_i^r(a)) = \tilde{U}_i(a)$$

If in equilibrium, $z_i > z_j$ implies $v_i > v_j$ and $w_i(a) > w_j(a)$ for all a, then $\tilde{U}_i(a) < \tilde{U}_j(a)$. Hence, we would have that $w_i^r(a) < w_j^r(a)$. High-productivity firms are more sought after. This implies that there is a threshold $z_r(a)$, such that the worker accepts all offers from firms with $z > z_r(a)$ in the equilibrium. If firms ordered by productivity $z_1 < z_2 < \ldots < z_M$ then denote $i_r(a) = \arg\min_i \{z_i | z_i > z_r(a)\}$. In this case we can write the value functions as:

$$\rho U(a) = \max_{c \geq 0} \; u(c) + \sum_{i=i_r(a)}^{M} \lambda_i \left[W_i(a) - U(a)\right]$$
$$\text{s.t.} \quad \dot{a} = ra + b - c$$

Also, intuitively, lower asset workers should have lower $i_r(a)$?

A.2 Optimal policy functions

A worker with asset level a and flow income ω can be in one of the following $2M + 1$ states,

$$\chi \in \{u_0, \tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_M, e_1, e_2, \ldots, e_M\}.$$

Here, u_0 represents the state of being unemployed and not excluded from any firm, $\tilde{u}_i$ represents the state of being unemployed and excluded from firm i, and e_i represents the state of being employed at firm i.

Denote the optimal consumption and saving policy functions by $c(a, \omega, \chi)$ and $s(a, \omega, \chi)$, which are characterized by the following equations:

$$u'(c(a, b, u_0)) = \partial_a U(a)$$
$$u'(c(a, b, \tilde{u}_i)) = \partial_a \tilde{U}_i(a) \quad \text{for } i = 1, 2, \ldots, M$$
$$u'(c(a, w, e_i)) = \partial_a W(a, w) \quad \text{for } i = 1, 2, \ldots, M$$

Note that $\omega = b$ in all of the unemployed states. Also, the consumption policy function in all employed states is identical. Finally, the optimal job-offer acceptance function, $I_i(a)$, is defined as follows:

$$I_i(a) = \begin{cases} 1, & W_i(a) \geq \tilde{U}_i(a) \\ 0, & \text{otherwise} \end{cases}$$

A.3 Steady-state distribution

Let $g(a, \omega, \chi)$ denote the steady-state joint distribution over assets, flow income, and labor market states. Let $g^u_j(a)$ represent the proportion of unemployed workers available to firm j (i.e., not in firm j's excluded set), defined as $g^u_j(a) = g(a, b, u_0) + \sum_{i \neq j} g(a, b, u_i)$. Also, note that the steady-state unemployment rate can be written as:

$$u = \sum_{i=0}^{M} \int g(a, b, u_i) \, \mathrm{d}a$$

The following Kolmogorov Forward (or Fokker-Planck) equations characterize the steady-state distribution:

- Unemployment without exclusion:

$$-\frac{d}{da}\left[s(a, b, u_0)g(a, b, u_0)\right] - g(a, b, u_0)\sum_{i=1}^{M}\lambda_i + \delta\sum_{i=1}^{M}\int g(a, w, e_i) \, \mathrm{d}w = 0$$

  For $\omega \neq b$, $g(a, \omega, u_0) = 0$.

- Unemployment with exclusion, for $j = 1, 2, \ldots, M$,

$$-\frac{d}{da}\left[s(a, b, \tilde{u}_j)g(a, b, \tilde{u}_j)\right] - g(a, b, \tilde{u}_j)\sum_{i \neq j}\lambda_i + \lambda_j[1 - I_j(a, b)]\left(g(a, b, u_0) + \sum_{i \neq j} g(a, b, \tilde{u}_i)\right) = 0$$

  For $\omega \neq b$, $g(a, \omega, u_j) = 0$ for $j = 1, 2, \ldots, M$.

- Employment, for $j = 1, 2, \ldots, M$,

$$-\frac{d}{da}\left[s(a, \omega, e_j)g(a, \omega, e_j)\right] - \delta g(a, \omega, e_j) + \lambda_j I_j(a, b)\left(g(a, b, u_0) + \sum_{i \neq j} g(a, b, u_i)\right) = 0$$

A.4 Comparison to Free-Entry Condition

If firms small enough, reasonable to assume firm solves $\max_{v_i} \Pi_i(v_i|V_i)$. This corresponds to free entry condition:

$$\int \frac{z_i - w_i(a)}{r+\delta} \cdot \tilde{g}_i^u(a)\,\mathrm{d}a = \frac{c(v_i)}{q(u,v)}$$

A.5 Wage Determination

$$\rho W(a,w) = u(c_e(a,w)) + \partial_a W(a,w)[ra + w - c_e(a,w)] + \delta[U(a) - W(a,w)]$$

$$W(a,w) = \frac{u(c_e(a,w)) + u'(c_e(a,w))[ra + w - c_e(a,w)] + \delta U(a)}{\rho+\delta}$$

By envelope theorem (is this correct?),

$$\partial_w W(a,w) = \frac{u'(c_e(a,w))}{\rho+\delta}$$

$$w_i(a) \in \arg\max_w \Big(W(a,w) - \tilde{U}_i(a)\Big)^{\beta}\Big(J_i(w) - V_i\Big)^{1-\beta}$$

FOC:

$$\beta\Big(J_i(w) - V_i\Big)\partial_w W(a,w) + (1-\beta)\Big(W(a,w) - \tilde{U}_i(a)\Big)\partial_w J_i(w) = 0$$

For firm's optimization, derivative of w with respect to v_i will involve taking the derivative of $W(a,w)$ with respect to w. While taking the later derivative, firm doesn't take $c_e(a,w)$ as given.

A.6 Aggregation

Note that the assumed matching technology implies that the contact rate $\lambda_m = A(v_m/u_m)^{1-\alpha}$. According to the model the UE transition rate is given by $\lambda_m \times P_m$, where P_m is the probability that a match is formed conditional on meeting. The aggregate UE rate then would be:

$$A \cdot \frac{\sum_m (v_m/u_m)^{1-\alpha} \times P_m \times u_m}{u}$$

So, matching efficiency A would be pinned by the aggregate UE rate.

The aggregate steady-state unemployment rate would be:

$$u = \sum_m \frac{\lambda_m P_m}{\lambda_m P_m + \delta} \pi_m^L$$

This will help pin δ.

B Extensions

B.1 Unemployment flow depends on past wages

Value of being employed with assets a and wages w:

$$\begin{aligned} \rho W(a,w) = \max_c \; & u(c) + \partial_a W(a,w)[ra + w - c] \\ & + \delta[U(a,b(w)) - W(a,w)] \end{aligned} \tag{4}$$

Value of being unemployed with assets a and flow income b:

$$\begin{aligned} \rho U(a,b) = \max_c \; & u(c) + \partial_a U(a,b)[ra + b - c] \\ & + \sum_{i=1}^{M} \lambda_i \left(\max\{W(a,w_i(a,b)), \tilde{U}_i(a,b)\} - U(a,b) \right) \end{aligned} \tag{5}$$

Here, $w_i(a,b)$ denotes bargained wage at firm i for an unemployed worker with assets a and flow income b.

Value of being unemployed and excluded from firm j with assets a and flow income b:

$$\begin{aligned} \rho \tilde{U}_j(a,b) = \max_c \; & u(c) + \partial_a \tilde{U}_j(a,b)[ra + b - c] \\ & + \sum_{i \neq j} \lambda_i \left(\max\{W(a,w_i(a,b)), \tilde{U}_i(a,b)\} - \tilde{U}_j(a,b) \right) \end{aligned} \tag{6}$$

Let $g(a,\omega,\chi)$ denote the steady-state distribution over assets, flow income ω and labor market state χ which can take one of the following $2M+1$ values:

$$\chi \in \{u_0, \tilde{u}_1, \ldots, \tilde{u}_M, e_1, \ldots, e_M\}$$

$s(a,\omega,\chi)$ the denotes the optimal saving policy function. Let $I_i(a,b)$ denote the offer acceptance policy function.

Kolmogorov Forward equations:

Unemployment without exclusion:

$$-\frac{d}{da}\left[s(a,\omega,u_0)g(a,\omega,u_0)\right]-g(a,\omega,u_0)\sum_{i=1}^{M}\lambda_i+\delta\sum_{i=1}^{M}\int_{w:\omega=b(w)}g(a,w,e_i)\,\mathrm{d}w=0$$

Unemployment with exclusion, for $j=1,2,\ldots,M$,

$$-\frac{d}{da}\left[s(a,\omega,\tilde{u}_j)g(a,\omega,\tilde{u}_j)\right]-g(a,\omega,\tilde{u}_j)\sum_{i\neq j}\lambda_i+\lambda_j[1-I_j(a,b)]\left(g(a,\omega,u_0)+\sum_{i\neq j}g(a,\omega,\tilde{u}_i)\right)=0$$

Employment, for $j=1,2,\ldots,M$,

$$-\frac{d}{da}\left[s(a,\omega,e_j)g(a,\omega,e_j)\right]-\delta g(a,\omega,e_j)+\lambda_j\int_{b:w_j(a,b)=\omega}I_j(a,b)\left(g(a,b,u_0)+\sum_{i\neq j}g(a,b,u_i)\right)\mathrm{d}b=0$$

B.2 Worker heterogeneity

B.3 On-the-job search

B.4 Capital and equity