Innovation and Competition Policy

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ABSTRACT

It has become increasingly apparent to policymakers that optimal antitrust policy requires looking beyond traditional static analyses and considering the dynamic effects of policy. Such analysis is challenging as existing studies of dynamic competition policy are limited. This paper attempts to bridge this knowledge gap by developing a novel structural growth model containing the major motivation of mergers and acquisitions (M&A) activity. To enable estimation of the model, frontier modeling techniques in artificial intelligence via natural language processing (NLP/AI) are employed to classify whether parties to an M&A transaction are currently operating in similar markets or whether acquirers are using M&A as an entry mechanism into to new markets. Examining the overall impact of M&A on growth reveals a double-edged sword: policies that either completely shut down M&A or allow unrestricted M&A both experience significantly lower growth rates than the baseline estimate. This motivates an optimal antitrust policy – one that provides actionable suggestions to policy makers.

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1 Introduction

In 2004, the world of online mapping and navigation was dominated by a company named MapQuest. At the same time, a small tech company called Where 2 Technologies had also developed a competitor product called Expedition — one that time would prove to be vastly superior to the existing products. Where 2 secured an initial funding deal from a venture capital firm but any hope of a deal is lost as Yahoo! entered the market the next day. The consensus from venture firms was that it would require too much investment and risk to enable Where 2 to compete with an internet giant like Yahoo. However, one of those VC funds connected Where 2 with another client who was interested in bringing Where 2's product to the market and out-compete Yahoo. That client was Google and Expedition was re-branded as Google Maps. It is fair to say that, without having been acquired, the return on investment for Where 2 Technologies' product would have been scarce.

This is not just an anecdotal story of one tech startup. Mergers and Acquisitions (M&A) has become the primary channel for startup companies to receive a return on their investments. Figure 1 displays methods of VC firm exits, the point where venture capital firms exit their investments and receive returns on their investment. It shows that from 2010 to 2019, on average, 78% of VC firms exited via acquisitions compared to 8% which were exits through public offerings. Looking back to the 1990s, that pattern is reversed. From the acquirers' perspective, M&A is a large channel of investment. Data on M&A transactions, which will be detailed in Section 3.1, shows that M&A transaction values, on average, exceed $300 billion a year. However, M&A is a channel that is rarely considered in the endogenous growth literature. By excluding M&A activity, the literature is missing a significant incentive for investment and a major mechanism for firm growth.

In addition to being a source of significant investment incentives, M&A is also one of the primary sources for firms to consolidate markets and gain monopoly power.

The first contribution of this paper is to develop a dynamic, endogenous growth model which captures the majority of stories of M&A and their relationships to economic growth. This model is able to simultaneously capture the q-theoretic motivations for M&A proposed by Jovanovic and Rousseau (2002) as well as product complimentary motivation studies by Rhode-Kropf and Robinson (2008). Incorporating oligopolistic competition, the model also captures the motives for and consequences of market consolidation. The inclusion of numerous potential motives and consequences for M&A

allows the model to provide a rich laboratory for experimentation on U.S. antitrust policy.

While the model's richness provides significant power in studying the aggregate effects of mergers and acquisitions, it presents several measurement challenges. The second contribution of this paper is to provide solutions for two of the most significant challenges. One challenge posed by the model is to determine whether an M&A transaction is between firms that are currently operating in the same market or whether the acquiring company is using the transaction to enter the market. An accurate estimate is critical since these two cases have different market consolidation consequences. Transaction data does not directly provide an indication of this relationship between the transacting parties. However, it does provide a textual description of the acquired firm. Using state of the art natural language processing combined with a broad textual description of the acquiring firm, in the form of corporate annual reports, to estimate the relation between the parties. The results show that 61% of MA transactional value is for transactions that occur between firms in the same market. The second measurement challenge is providing an accurate depiction of US antitrust policy. Quantitative data on antitrust decision making is extremely limited making strictly empirical estimates of the antitrust policy function nearly impossible. This paper proposes a novel approach to this problem by utilizing a simple discrete choice model to bridge the gaps in data. The resulting antitrust policy function provides an increased degree of accuracy to this papers results as well as providing an estimate applicable beyond this project.

The paper's third contribution is to employ the estimated model to illustrate the impact of M&A on growth. The key result is to show that M&A is a double-edged sword: where a complete ban on MA results in a **xxx**% decrease in growth and a **xxx**% decrease in consumption equivalence welfare. For the case where there is no MA, the results show that the lack of an outside option causes a significant decrease in expansion rates of low and medium productivity firms.

It also decreases in all innovation rates as the reduced entry margin reduces the overall level of research.

On the other hand, when there is no restriction on M&A, there is a nearly increase in the number of monopolies.

Literature Review

Despite the significant concern surrounding the dynamic effects of antitrust policy, the literature is incredibly sparse and rarely considered the growth angle. The closest prior paper to this one in goal and approach is Cavenaile, Celik and Tian (2021b). This paper used the Cavenaile et al. (2021b) model as a base and then made several key advancements in order to increase its applicability. In the Cavenaile et al. (2021b) paper, M&A opportunities arrive exogenously at a common rate for all pairs of firms in a market and the pair of potential merger partners then negotiate bilaterally. Their decision to have a rate be exogenous results in limited ability of firms to respond to changes in antitrust policy. One advancement of the current paper is to replace the exogenous opportunities with endogenous search. The impact of an exogenous versus endogenous rate is most obvious when considering a situation where antitrust policy is shut down, i.e. all M&A transactions occur freely. In this situation, Cavenaile et al. (2021b)'s model demonstrates that there is only a -0.88% change in the growth rate and a minor impact on market concentration. When running the identical experiment, the current paper's model[1] results stand in stark contrast with a **xxx**% change in growth and a **xxx**% increase in the number of monopolies, which more realistically reflects both historical changes.

Another additional significant improvement that this paper makes is to estimate the true U.S. antitrust policy function. Their paper makes the assumption that all transactions above the screening threshold[2] are obstructed with equal probability — an assumption that does not hold true in the empirical data. The consequence of their assumption is to underestimate antitrust enforcement for transactions that result in a high degree of market consolidation since transactions with extremely high HHIs would have the same probability of being blocked as a transaction that is just over the screening threshold but minimally impacts market concentration. This choice would both bias their estimation (the parameters values are chosen to match the distribution of markups of the United States while allowing a higher frequency of large increase in markup transactions) as well as their result that shutting down antitrust enforcement has modest welfare consequences.

Fons-Rosen, Roldan-Blanco and Schmitz (2021)

[1]Located in section **??**

[2]For the latest published guidelines, the screening threshold is a HHI of more than 0.15 and a change in HHI of more than 0.01.

Additional sources to incorporate in literature review:

Model of growth and markup: Peters (2020); Cavenaile, Celik and Tian (2021a); Cavenaile et al. (2021b); ; and Pearce and Wu (2022).

M&A and Innovation: Phillips and Zhdanov (2013), Akcigit, Celik and Greenwood (2016)

Empirical M&A: Higgins and Rodriguez (2006), Hoberg and Phillips (2010), Wang, Wu and Lai (2022), Gerard and Phillips (2021) ‘ + .0

2 Model

Todo: Section introduction.
This section describes the environment and describes equilibrium. Careful derivations of results are reserved for Appendix A.

2.1 Environment

Household: There is a single infinitely-lived representative household who discounts the future at rate ρ. The household is endowed with one unit of labor which it supplies to the labor market at wage rate w_t. The household owns all the assets in the economy which evolves according to:

$$\dot{A}_t = r_t A_t + w_t - P_t C_t.$$

where A_t is the household's assets with rate of return r_t, and where C_t is the consumption good with price P_t. The household determines its consumption-savings plan to maximize its discounted lifetime utility of:

$$U = \int_0^\infty e^{-\rho t} \log C_t \, \mathrm{d}t.$$

Firms: The economy is populated by a measure m of multi-product firms indexed by f. Firms can be multi-product. Across all of a firm's product markets, it has a common, time-invariant productivity z_f drawn from a distribution Z. The set of product markets in which a firm f competes at time t is denoted by $I_{f,t}$. Within each of these product markets $i \in I_{f,t}$, the firm produces a single good with quality $q_{f,i,t}$.

Product Markets: The economy contains a unit measure of differentiated product markets indexed by $i \in [0,1]$. For each product market i, there is a set of firms $F_{i,t}$ and a competitive fringe, each of whom produce differentiated varieties of products. These product varieties each have their own qualities denoted $q_{f,i,t}$ for a firm $f \in F_{i,t}$, and $q_{c,i,t}$ for a fringe firm. Within a product market i, the firms' goods are aggregated together by a competitive firm employing technology:

$$y_{i,t}^{\frac{\theta-1}{\theta}} = \sum_{f \in F_{i,t}} q_{f,i,t}^{\frac{1}{\theta}} y_{f,i,t}^{\frac{\theta-1}{\theta}} + q_{c,i,t}^{\frac{1}{\theta}} y_{c,i,t}^{\frac{\theta-1}{\theta}}$$

where $y_{i,t}$ is the total quantity of aggregated goods, $y_{f,i,t}$ is a firm f's output, and $y_{c,i,t}$ is the competitive fringe's output.

While the competitive fringe are price takers, the firms strategically compete with each other in Cournot competition. This structure enables the firms to set an endogenous markup based on market conditions including the relative quality of their good. The red provides incentive for a firm to improve product quality; the higher their relative quality, the higher their profit. However, it is also a source of misallocation: the higher a firm's market share, the higher the markup they are able to charge.

Final Goods: All of the product markets' goods $y_{i,t}$ are aggregated into a final good Y_t by a competitive final goods firm. This final goods firm employs a unitary elasticity of substitution aggregator function[3] specified by:

$$\log Y_t = \int_0^1 \log y_{i,t}, \mathrm{d}i.$$

Production: For a firm f to produce their output $y_{f,i,t}$ in market $i \in I_{f,t}$, they combine their productivity $z_{f,t}$ with a labor input $h_{f,i,t}$, hired at a wage rate w_t, according to

$$y_{f,i,t} = z_{f,t} h_{f,i,t}.$$

The competitive fringe employs an identical technology where their productivity is identically one.

Research and Development: The firms conduct research and development to achieve two goals: (1) improve the quality of the products they currently produce and (2) expand

[3]The assumption that the aggregator has unitary elasticity is not critical when computing the solution to the model — it can be easily replaced with a flexible constant elasticity of substitution aggregator. However, the unitary case provides an increased degree of clarity in welfare analysis and is consistent with the previous literature.

into new markets by creating new products. Each good's quality sits on a quality ladder with step size λ. Each time a firm improves its product, the good's quality takes a single step up the ladder; its quality goes from $q_{f,i,t}$ to $\lambda q_{f,i,t}$. Improvements arrive to the firm exogenously according to a Poisson arrival process at an endogenous rate $\mathcal{I}_{f,i,t}$. The cost of achieving that rate is $c_{\mathcal{I}}\left(\mathcal{I}_{f,i,t}\right)Y_t$ units of goods. The improvements are market-specific and the firm can simultaneously attempt to improve all its products. If a firm's product quality in a market falls more than the quality gap threshold of $\bar{n}$ steps behind the highest quality product, the product becomes obsolete and the firm can no longer produce in that market. This mechanism captures the creative destruction channel of growth.

Recall that firms can have a single product in each market. Therefore, expansion to new markets is achieved by generating new products, which occurs in a similar manner to improvements. New products arrive to the firm according to a Poisson arrival process at an endogenous rate $\mathcal{X}_{f,t}$. The cost of achieving that rate is $c_{\mathcal{X}}\left(\mathcal{X}_{f,t}\right)Y_t$ units of goods. The research effort is undirected in the sense that the firm cannot target which market to enter. When they develop a new product, the firm receives a uniform random draw of which market it enters. Upon entering the new market, the quality of the new good is a step above the maximum quality currently existing in that market. The new product quality q' is

$$q'_{i,t} = \lambda \max\left\{q_{f,i,t} : f \in F_{i,t}\right\}.$$

This is intended to capture new products being superior versions of existing products. In each product market, there is a maximum size cap of $\bar{F}$ firms. If the market which got drawn already has $\bar{F}$ firms prior to the new firm entering, then the firm with the least profitable product, in terms of sales minus wages paid to production workers, in that market exits as another victim of creative destruction[4].

The competitive fringe does not perform research and development. Instead, their good's quality is a function of the market leader's quality. Specifically,

$$q_{c,i,t} = \psi \max\left\{q_{f,i,t} : f \in F_{i,t}\right\}.$$

[4]A common, alternative assumption is that the market is closed to new entry after reaching the size cap. However, in this framework, this results in an unrealistic result where the firm with the highest quality will cut research if one of its competitors is $\bar{F}$ steps behind. Recall that when firms leave a market if their product quality is too far from the leading firm, and that new market entrants emerge with a product with superior quality to the current market leader. As a result, current market leaders can ensure that they are not overtaken by new entrants by stopping innovation in order to not force additional firms out.

That means innovations by the market leaders results in a positive spillover by also increasing good quality for the competitive fringe.

Mergers and Acquisitions: The second source of firm growth is through mergers and acquisitions. Firms can perform M&A both within markets where they currently operate as well as using M&A as a way to enter into new markets. In model, M&A transactions are for a single product. While M&A transactions are traditionally thought of as one company buying the entirety of another, one company buying only a portion of another is frequently observed in data[5].

Beginning with transactions where the buyer is not currently active in the market (the entry margin), there is a decentralized M&A market for potential buyers and sellers. Opportunities for a buyer or a seller to enter the market arrive exogenously at an endogenous rate $\eta_{f,i,t}$ for firm f in attempting to sell its good in market i

Within a market, the

Both types of transactions are subject to antitrust regulation prior to being allowed to commence. The antitrust regulator is assumed to be able to accurately predict the post-merger HerfindahlâHirschman index (HHI) as well as the change in HHI. Given those estimates, the antitrust regulator will choose to block a transaction with probability $B(hhi, \Delta hhi)$ where hhi is the post-merger HHI and Δhhi is the change as result of the transaction. If the regulator blocks a transaction, there is nothing transferred between the parties. Their only loss is the wasted effort of attempting the transaction. Additionally, it is assumed that once the regulator blocks the transaction, the parties can not reattempt a modified version of the transaction.

Firm's Value Function: The firm f's state is their productivity z_f and the state of the the markets which they compete s.

Each division's state is the firm's productivity $z_{f,t}$, the firm's product's quality $q_{f,i,t}$, and the similar states of all of the firm's competitors $o_{f,i,t} = \left\{ \left(z_{\tilde{f},t}, q_{\tilde{f},i,t} \right) \right\}_{\tilde{f} \neq f}$. In order to improve clarity in the following value function, the full state will be omitted and only changes to the state, indicated by the symbol $\rightarrow$, will be included. Each division's value is specified by **todo** along with boundary conditions such

[5]For example, Google purchasing only Motorola's cell phone division.

2.2 Balanced Growth Path

The equilibrium concept for this environment is a balanced growth path:

Definition 1. *A* ***balanced growth path*** *is*

Determining the price schedules $p_q(z', s|z)$ and $p_q(z|z', s)$ is the only less standard part of solving for the equilibrium. Given that the two prices schedules have been endowed with very little structure in the model, it seems like a nearly impossible challenge. However, the theory of optimal transport problems[6] provides a tractable solution. The surplus for a productivity z firm acquiring a product whose state is $s = (q, \{(\tilde{z}, \tilde{q})\})$ from a productivity z' firm is

$$X(z, z', s) = D\left(z, q, \{(\tilde{z}, \tilde{q})\}\right) - D\left(z', q, \{(\tilde{z}, \tilde{q})\}\right).$$

Let $\pi(z)$ be the measure of buyers of productivity in the market and let $\pi\left(z', s\right)$ be the measure of products in the M&A market available for sale. Consider the market planner's problem

$$\max_{\pi(z,z',s)} \sum_{(z,z',s)} \pi(z, z', s) X(z, z', s)$$
$$s.t. \sum_{z} \pi(z, z', s) \leq \pi\left(z', s\right) \tag{1}$$
$$\sum_{(z',s)} \pi(z, z', s) \leq \pi(z) \tag{2}$$
$$\pi\left(z', s\right), \pi(z) \geq 0$$

where $\pi(z, z', s)$ is the measure of products of state s transferred between productivity z and z' firms[7]. Optimal transport theory shows that the allocations from this planner problem correspond to the equilibrium allocation. It also shows that the dual value $\nu(z', s)$ of (1) will correspond to the market value of the seller's product and $\nu(z)$ of (2) will correspond to the buyer's surplus from the transaction. Using this information the equilibrium price schedules are

$$p_q(z', s|z) = X(z, z', s) - \nu(z', s)$$
$$p_q(z|z', s) = X(z, z', s) - \nu(z).$$

[6]The theory of optimal transport problems is beyond the scope of this paper. Galichon (2016) provides an excellent introduction.

[7]Under the assumption of this problem, everyone must trade and there is no opting out once they reach the market. However, attempting to enter the market is an endogenous choice of the firms which means that firm which will not want to trade will not enter the market

2.3 Numerical Solution

The model does not admit an analytical solution and must be solved numerically. Throughout estimation and for all of the result the model was discretized such that every market could have a maximum of five firms (excluding the competitive fringe), a maximum of six quality steps between competitors, and every firm can take on one of three distinct productivity states. This results in 32,131 potential markets and 133,980 unique firm state for which solving a value function is required. While this results in a large system, it contains several nice numerical properties that enables relatively rapid solutions — especially if utilizing parallel processing via a GPU.

3 M&A Data

The model described in the previous subsection made a distinction for M&A activity between competitors within a market and as a mechanism for firms to enter into new markets. While this does allow the model to analyze a wide verity of M&A situations, it also creates a difficult measurement challenge of how to distinguish between the two types of transactions. Approaches that compare industry codes, such as NAICS or SIC codes, are insufficient since neither are specific enough to determine if two firms are in a similar market, nor can they capture the multi-product nature of modern firms. This section develops an alternative approach that applies state of the art natural language processing (NLP) techniques to compare textual descriptions of both target and acquiring companies.

3.1 M&A Transaction Data

The most comprehensive data set on M&A transactions in the United States is SDC Platinum data set by Refinitiv. After 1992, it covers all corporate transactions involving at least 5% of the equity of a company. For each transaction, the data set contains financial variables on the acquired firm prior to the transaction, information on the terms of the deal, and, if they are a public company, identifying information allowing linking to other data sets including Compustat and SEC filings. From these transactions, select transactions where

1. The acquiring company is a US publicly traded corporation,

2. The transaction resulted in acquiring firm acquiring a majority stake in the company,

3. The parties were unrelated,

4. The transaction was completed.

When the acquiring listed in a transaction record is the subsidiary of another corporation, the ultimate parent of the group is set as the acquirer. Discussing the goals of the requirements, The first requirement is critical for this approach since, as discussed in the next section, there exists high quality textual descriptions of US publicly traded corporations.The second requirement's goal is to ensure control was transferred to the acquiring firm and to exclude the purchases of minority positions. The final two requirements attempt to exclude actions such as corporate reorganizations that appears like M&A transactions for legal reasons.

After having selected the target transactions, the next step is to link this transactions with the corporate annual report of the acquirer's company. US publicly traded corporations are required annually to provided the Securities and Exchange Commission a detailed report of their business and operations along with their financial statements. These annual reports are frequently described as 10-Ks. Electronic copies of these documents are freely accessible from the SEC's Electronic Data Gathering, Analysis, and Retrieval system (EDGAR) which, staring in 1997, is a nearly complete record of all corporate filings with the SEC including the 10-Ks. The transactions from the SDC platinum data set are matched with the 10-K for the acquiring corporation the year prior to the transfer. The year prior is chosen to in order to obtain an accurate depiction of the acquiring business prior to the transaction as well as to minimize the number of documents which directly refer to the M&A transaction[8].

Given the availability restrictions of Edgar data, the data is restricted to transactions from 1997-2019. Not every transaction claiming to be acquired by a public corporation in a specific year is able to be linked. This seems to likely to either to have been mistakenly labeled as having been purchased by a public corporation or there was clarical error when recording the acquirer's identifying information. Table **??** shows the summary statistics for

[8]Since the approach undertake will be to compare text similarity, having references to the transaction in the comparison text will add noise

From SDC Platinum, I subset for transactions where the acquiring company is a U.S. publicly traded corporation. This consisted of over 38k transactions with a total market value of $7.8 trillion between 1997 and 2019.

3.2 Comparing Companies Using Natural Language Processing

The table below shows a comparison example of text description from ADE Corporation's acquisition of Phase Shift Technology in 1999.

Acquiring company description	**Target company description**
ADE Corporation	**Phase Shift Technology**
The Company's metrology and inspection products use its proprietary non-contact capacitive, magnetic and optical technologies to measure the dimensional, magnetic and surface characteristics of semiconductor wafers and devices and computer hard disks.	Manufacture high-performance, non-contact surface metrology equipment using advanced optical interferometric technology; its products are used in data storage for production control, failure analysis, and advanced research of surface shape and surface microstructure.

3.3 Natural Language Processing

In order to classify transactions, I implemented a variant of Bidirectional Transformers for Language Understanding (BERT) to search for the company description from the AUP's annual reports. This approach is an improvement over past macroeconomic attempts to compare textual data, such as Hoberg and Phillips (2010) which used a vector space model on product descriptions from SEC 10-K filings to compute similarity between firms.

Conceptually, BERT is a language representation model developed by Google that learns contextual relations between words (or sub-words) in a text both left-to-right and right-to-left ("bidirectional"). The bidirectional pre-training results in significantly improved accuracy over prior language models that were only pre-trained on reading sentences left-to-right. Further, BERT achieves state-of-the-art performance for a large suite of both sentence-level and token-level tasks.

After applying BERT to the merged SDC Platinum and SEC company filings, I receive TRUE/FALSE output for each transaction for whether or not the acquiring company is in the same market as the target company. Across all transactions, M&A activity by acquirers that already existed in the target's market comprised 61% of all M&A value. For acquisitions in the tech sector and pharmaceutical sector, it was 42% and 84%, respectively.

4 Estimation

4.1 Capturing US Antitrust Policy

Accurately modeling antitrust policy is crucial for both estimating the model and providing realistic policy analysis. In the United States, the primary form of antitrust policy is the premerger notification program where the parties to a M&A transaction, whose value exceeds a legal threshold[9], must seek approval from the antitrust regulator[10] for the transaction prior to it commencing. This process has three basic steps:

1. The transacting parties submit information to the regulator about their proposed transaction.

2. The antitrust regulator reviews the submission and can either approve the transaction or request additional information related to anticompetitive concerns. The request for additional information is called "entering second review."

3. The regulator assesses the second review material and makes a determination on whether it will attempt to legally block the transaction or allow it to transact.

A significant challenge when studying antitrust policy is a lack of transparent data surrounding the decision-making process. The United States antitrust authorities release two quantitative resources which are both limited in scope. The first resource is the *Hart-Scott-Rodino Annual Report* (HSR report) in which the Federal Trade Commission and the Department of Justice Antitrust Division are required to report information related to that year's actions for the premerger notification program. Quantitatively, it contains:

- the number of transactions reported to the antitrust regulator

[9] The threshold is defined by Hart-Scott-Rodino Antitrust Improvements Act and changes annually. For transactions occurring in 2022, the threshold $101 million.

[10] In the United State, antitrust regulation is the dual duties of the Federal Trade Commission and the Department of Justice Antitrust Division. While there are differences between the two agencies, those differences are beyond the scope of this paper. In general, the two will be referred together as the antitrust regulator.

- the number of requests for second reviews
- the number of transactions that the regulator attempted to block[11]

The report also qualitatively describes the reasoning behind selected actions — frequently discussing concerns about market concentration — but lacks information about the measures of market concentration, such as HHI, in both those selected transaction and th0e overall sample of transactions. However, the second source, *Horizontal Merger Investigation Data, Fiscal Years 1996-2011*[12], provides enough of a glimpse to push forward. The report provides summary statistics on every second request investigation, between 1996 and 2011, where the antitrust regulators reason to perform the investigation was concerns about horizontal theories of competitiveness concerns[13]. Crucially, Table 3.1 of the report (reproduced as Table 3 in the appendix of this paper) provides, for bins of HHI and change change in HHI, counts on the number of transactions which were and were not blocked. The remainder of this section will discuss how to turn this data into an antitrust policy function.

Recall from the model that antitrust policy is characterised as a Bernoulli random variable $B(hhi, \Delta hhi)$ where $B(hhi, \Delta hhi) = 1$ means the regulator blocks the merger and $B(hhi, \Delta hhi) = 0$ means the regulator allows the transaction. Let P_B be the probability mass function of B. The mass function, conditional on a transactions $(hhi, \Delta hhi)$, can be written as

$$P_B\left(B|hhi, \Delta hhi\right) = P_B\left(B|hhi, \Delta hhi, S=1\right) P_S\left(S=1|hhi, \Delta hhi\right) \tag{3}$$

where $P_B\left(B|hhi, \Delta hhi, S=1\right)$ is the probability that a transaction is blocked conditional on having received a second review[14] and $P_S\left(S=1|hhi, \Delta hhi\right)$ is the probability of a transaction receiving a second review conditional on having characteristics $(hhi, \Delta hhi)$. These objects will be considered separately.

Consider the problem of the antitrust regulator, after having entered second review, determining if they want to challenge a transaction with characteristics $(hhi, \Delta hhi)$. The simplest way to analyze this problem is to set it up as a discrete choice problem where the value of denying the transaction is given by

$$G\left(hhi, \Delta hhi\right) + \varepsilon$$

[11] A table of these counts for the sample years is included in the appendix as Table **??**

[12] Commission (2013)

[13] **To do**

[14] A transaction can not be blocked without first getting a second review. Thus $P_B\left(B|hhi, \Delta hhi, S=0\right) = 0$.

and the value of approving the transaction is zero. The G function can be thought specifying the regulator's goal to prevent concentration while ε accounts for non-concentration factors[15] influencing the regulator's decision. Table 3.1 of the *Horizontal Merger Investigation Data* report can be thought of as the outcome of this discrete choice problem. Making the assumption that ε is logistically distributed, a logistics regression on Table 3.1 characterises the solution to the government's discrete choice problem. Prior to performing that regression, a linear functional form of G must be chosen. Given there is limited research into the form of G, a fourth degree polynomial

$$G\left(hhi, \Delta hhi\right) = \sum_{n=0}^{4} \sum_{k=0}^{n} \beta_{n,k} \left(hhi\right)^{n-k} \left(\Delta hhi\right)^{k}$$

to allow for maximum flexibility in representing the solution. The result of the regression, the representation of the regulators choice function $P_B\left(B|hhi, \Delta hhi, S=1\right)$, is displayed in Figure **??**[16].

Moving on to the probability of having a firm enter secondary review, estimation is much more difficult. There is no comparable version of Table 3.1 for entering secondary review and no other data source allowing for a similar estimation exercise. In fact, the only available data on the choice of sending a transaction to secondary review is the fraction of transactions, on average 3.1% over 1997 to 2019, that received secondary review[17]. Given that, the approach will be to exploit the discrete choice problem from the previous exercise and combine it with the aggregate counts. Suppose that the antitrust regulator is capacity constrained and can not send more than 3.1% of transactions to secondary review. Also, suppose that they can observe every merger every merger they will need to pick between to send to secondary review. The question is who would they pick? A question that be answered by connected back to the discrete choice problem.

Let $T = \{(hhi_j, \Delta hhi_j)\}$ be the set of all transactions submitted to the regulator for review. The regulators problem is to choose at most 3.1% of them to review. It will select a set of transactions $\tilde{T}$ in order to maximize the total expected value of its second stage problem

$$\int_{\tilde{T}} \mathbb{E}_{\varepsilon}\left[\max\left\{G\left(hhi, \Delta hhi\right) + \varepsilon, 0\right\}\right] \, \mathrm{d}f_T\left(hhi, \Delta hhi\right)$$

[15] Factors such as **TODO**

[16] The coefficients can be found in Appendix **??**

[17] These counts are also sliced by major industry and transaction size, either of which is useful for this project.

where f_T is the measure of transactions. While this problem appears challenging at first, the solution can be characterized as finding a threshold R such that transactions where $P_B(B|hhi, \Delta hhi) \geq R$ are selected for secondary review and the total number of transactions reviewed is subject to the capacity constraint. While being able to observe both the capacity constraint and $P_B(B|hhi, \Delta hhi)$, the threshold condition is not able to be directly estimated since their is no information on the distribution of M&A transactions f_T. However, it can be estimated indirectly through the model as detailed in the next subsection.

4.2 Estimation

In addition to the threshold condition R, the model has 17 parameters (A_a, $A_{\mathcal{I}}$, A_w, $A_{\mathcal{X}}$, α, β, γ_a, $\gamma_{\mathcal{I}}$, γ_w, $\gamma_{\mathcal{X}}$, λ, m, ψ, ρ, θ, ω, ζ) as well as the productivity distribution for z. With the exception of the discount rate ρ, the fringe's goods quality ψ, and α, the remaining parameters are estimated. For the discount rate ρ, the model assumes a 6% interest rate. Since the economy is growing at 2.4% in the baseline, that implies the discount rate is $\rho = 0.036$. For the fringe's quality ψ, the assumption is that the fringe's good is always lower quality when compared to a strategic firm. The easiest way to implement this assumption is to set ψ such that it is one step further down the quality ladder than the maximum allowed gap of $\bar{n}$. That parameter α is intended to enable the model to the like-buys-like story of Rhode-Kropf and Robinson (2008) and is thus set at half. Also, in order to correspond with the M&A data set detailed in Section 3.1, data is restricted to 1997-2019.

Prior to discussing the estimation moments, it's important to establish the units of measurement in the model. Moments will either be aggregates or firm-level statistics. While within a firm, the divisions' and headquarters' problems were each solved independently, they are still part of a single unit of measurement: the firms. During simulations, the firms need to keep track of their divisions and add up their divisions' variables into firm-level ones. This is critical when utilizing identification moments such average profits. If the division values are dollar values from real world financial statements such as sales or profits, then the dollars are summed into a single firm-level value. For other values like mark up or market share, they are sales-weighted average between all the firm's divisions.

What follows is a brief description of the identifying data moments utilized to discipline the model.The moments' values, as well as the values of the estimated parameters, can be found in Table 1. A more detailed description of the identifying moments as well as how to construct model objects to match the data can be found in Appendix B.

To capture the US's growth dynamics, the model is estimated to match the US growth rate as well as the ratio of the National Income and Product Accounts estimate of investment intellectual product product (less artistic originals)[18] Given the paper's interest, it is also critical to contain an accurate depiction of competition and research effort. For all parameter sets, this model will produce a inverted U relationship between the level of competition and R&D spending. The goal is to match the inverted U shape in data. Following Cavenaile et al. (2021a), the model is estimated to match a quadratic regression of relative sales on the log of RD spending[19]. The model is also estimated the sales weighted average and standard deviation of markup reported in .

Moving on the M&A process, the model set to match the ratio of the sum of M&A equity values[20] to GDP as well as the within sector MA ratio established in Section 3. To match the pattern of trade, the model matches the mean and standard deviation of M&A premium as observed in the SDC Data.

Finally the productivity values z.The goal is to match the skewness of the US sales distribution as well as match an empirical pattern about M&A. This is, most acquiring firms set in the top of the sales distribution while while the acquired firm come uniformly from all but the top bins. To acheive those goals, the three productivity points are set such that the first is meant to represent the bottom 50% of forms, the next productivity meant to represent the 50%-90%, and the third productivity meant to capture the top ten percent of firms. The middle productivity is normalized to one and then the other twos are set to match the fraction of compustat sales which fall in their bins over th sample period.

[18]Traditionally, RD spending from the NSF has been used instead. However, this does not include non-scientific R&D. The NIPA estimate includes this scientific R&D as well as software purchase. They are intended to proxy for the non-scientific spending.

[19]In Cavenaile et al. (2021a), they used patent counts and improvement arrival rates. Given the multi-product nature of this model, RD spending adds up to a firm level variable more easily than arrival rates.

[20]There are several measures of M&A transactions values. Equity value is the total transaction value net of debt. Since the paper abstracts from financial issues, it is the appropriate measure.

5 Growth and M&A

5.1 Illustrative Example

To begin elucidating the impact of M&A on economic growth, it is best to consider a simple illustrative example that compares a baseline economy with an economy where the antitrust authority imposes a blanket ban on M&A activity.

Consider a duopoly market where one firm's productivity is equal to the highest productivity in the model and the other firm's productivity is equal to the lowest. These firms represent the extreme ends of productivity and allow us to examine the corresponding impact of productivity on two variables: (1) gross profit and (2) revenue minus wages paid to production labor. For each firm, Figure 4 displays the baseline case for the level and the change for gross profit, and revenue minus wages paid to production labor. The figure clearly shows that for the low-productivity firm, the marginal gross profit gain from moving a step up the quality ladder is extremely small. If gross profits from production were the only possible return on the R&D investment, then the low-productivity firm would have very little incentive to perform any R&D. Panel A of Figure 5 illustrates this lack of incentive which contrasts the firm's research intensity between the baseline economy and the fully-restricted economy with no M&A activity permitted[21] activity. Panel A reveals that under a complete M&A ban, the low-productivity firm, **todo**, performs xxx% less innovation R&D than in the baseline case.

This reduction in R&D spending is not just limited to the low-productivity firm. Turning to the research intensity for the high-productivity firm, Panel B of Figure 5 shows that the high-productivity firm performs xxx% less innovation R&D in this market compared to the baseline. Unlike the low-productivity firm in the baseline case, the high-productivity firm is not performing R&D in hopes of a future M&A transaction. In fact, the firm is attempting to reduce the likelihood that an M&A transaction occurs within their market since new firms that enter the market via acquisition have a productivity that supersedes the current high-productivity firm. The arrival rate for an outside buyer for a low-productivity firm is revealed in Figure **??** which shows that the acquisition rate increases rapidly as the low-productivity firm's quality increases. This places pressure on the high-productivity firm perform R&D even as they are dominating their competitors.

[21] While general equilibrium effects change the profit functions in Figure 4, the changes are minor between the baseline and the case of fully restricted M&A activity. The result remains that the low-productivity firm has minimal profit growth.

5.2 The Impact of M&A Across The Firm Distribution

The previous example was just the story of one market. The model's high degree of heterogeneity allows it to simultaneously capture a wide verity of incentives and consequences of M&A at the micro level. One of the clearest ways to appreciate these micro effects is by using the model to calculate micro-elasticities. There are many candidate elasticities that could be considered, but the one that fits most clearly in the with the overall topic of competition policy is the regulator's blocking elasticity. Recall the blocking probability of an aribtrary M&A transaction is $B(hhi, \Delta hhi)$. The elastiticity is calculated by taking small additive perturbation of that probability — as if every trasanction in the economy became slightly more likely to be blocked.

Beginning with expansion R&D elasticity $\varepsilon_B^{\mathcal{X}}$, the average elasticity is xxx. Breaking this down by a firm's productivity type, the elasticities are:

Productivity z	Low	Medium	High
Elasticity $\varepsilon_B^{\mathcal{X}}$	xxx	xxx	xxx

Todo:Commentary

Moving to the improvement elasticity, Figure 6 shows the distribution across all divisions of the improvement elasticity $\varepsilon_B^{\mathcal{I}}$. todo:description The graph shows that xxx% of divisions with positive elastiticities and the remaining have negative elatiticties. Table **??** provides summary statistics describing this two groups. We see Comments

5.3 Aggregate Growth

These micro-effects aggregate In the balanced-growth path, the growth rate is defined by

$$g = \sum_s \mathcal{I}_\ell(s) \log(1+\lambda)\, \mu_m(s) + \sum_z \mathcal{X}(z) \log(1+\lambda)$$
$$+ \sum_s$$

par

5.4 M&A is a Double Edged Sword

The previous sections have shown how reduced M&A activity can significantly dampen growth. A reasonable hypothesis is that loosening antitrust policy may therefore increase the growth rate. A version of this hypothesis is an experiment where there are no antitrust restrictions on M&A such that any firm can trade with any firm regardless of market concentration. The result from this experiment is shown in Table 2 alongside the results from the estimated baseline and the fully restricted M&A policy that was previously considered. Similar to the fully restricted M&A case, unrestricted M&A activity also causes a drop in the growth when compared 43to the baseline estimate — in this case, a drop of xxx. While the drop in growth in the fully restricted MA experiment was also caused by limited outside options for firms to sell their divisions, the drop here is due to too much M&A resulting in a significant number of monopolies. Table 2 reveals that the number of markets in monopolies increases 37x, from around one percent of all firms to thirty-seven percent of all firms. The lack of competition severely reduces the motive for a firm to perform R&D. These two extremes provide motivation for the importance of the optimal antitrust policy.

6 Competition Policy

Needing to consider dynamic issues as well as their traditional static, the antitrust policymaker faces a difficult problem. The simplest way to observe the balance they need to strike is to consider household welfare. In the balanced growth path, the welfare function is

$$U = \frac{1}{\rho}\log\left(C_0\right) + \frac{1}{\rho^2}g.$$

Antitrust policies are normally concerned with reducing the level of static misallocation. Such a reduction would normally increase the base level of consumption C_0. Suppose a change in antitrust policy resulted in a 1% increase in C_0. The welfare function say that the policy will be welfare improving only if any resulting loss in the growth rate is less than 0.3pp.

6.1 Meeting a Purely Static Objective

6.2 The Impact of Changing Policy

6.3 Optimal Policy

The final item this paper will examine is two versions of the optimal antitrust problem: one focused on an HHI based rule and the other on

7 Conclusion

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Tables

Table 1: Estimation Moments and Estimated Parameters

Parameter	Value	Moment	Data	Model
A_a		M&A-GDP Ratio	5.00%	
$A_{\mathcal{I}}$		R&D-GDP Ratio	3.48%	
A_w		Within Market M&A Ratio	61.03%	
$A_{\mathcal{X}}$		Markup Standard Deviation	34.60%	
β		Avg. Merger Premium	41.56%	
(γ_A, γ_W)		$\beta\,(R\&DSpending, RelativeSales)$	1.33	
$(\gamma_{\mathcal{I}}, \gamma_{\mathcal{X}})$		$\beta^2\,(R\&DSpending, RelativeSales)$	-1.169	
λ		GDP Growth Rate	2.41%	
m		Labor Share	0.65	
θ		Avg. Markup	34.98%	
ω		Killer Acquisition Rate	5.3%	
ζ		Merger Premium Standard Deviation	0.4491	
z_1		Bottom 50 Share of Sales	1.4%	
z_3		Top 10 Share of Sales	80.73%	

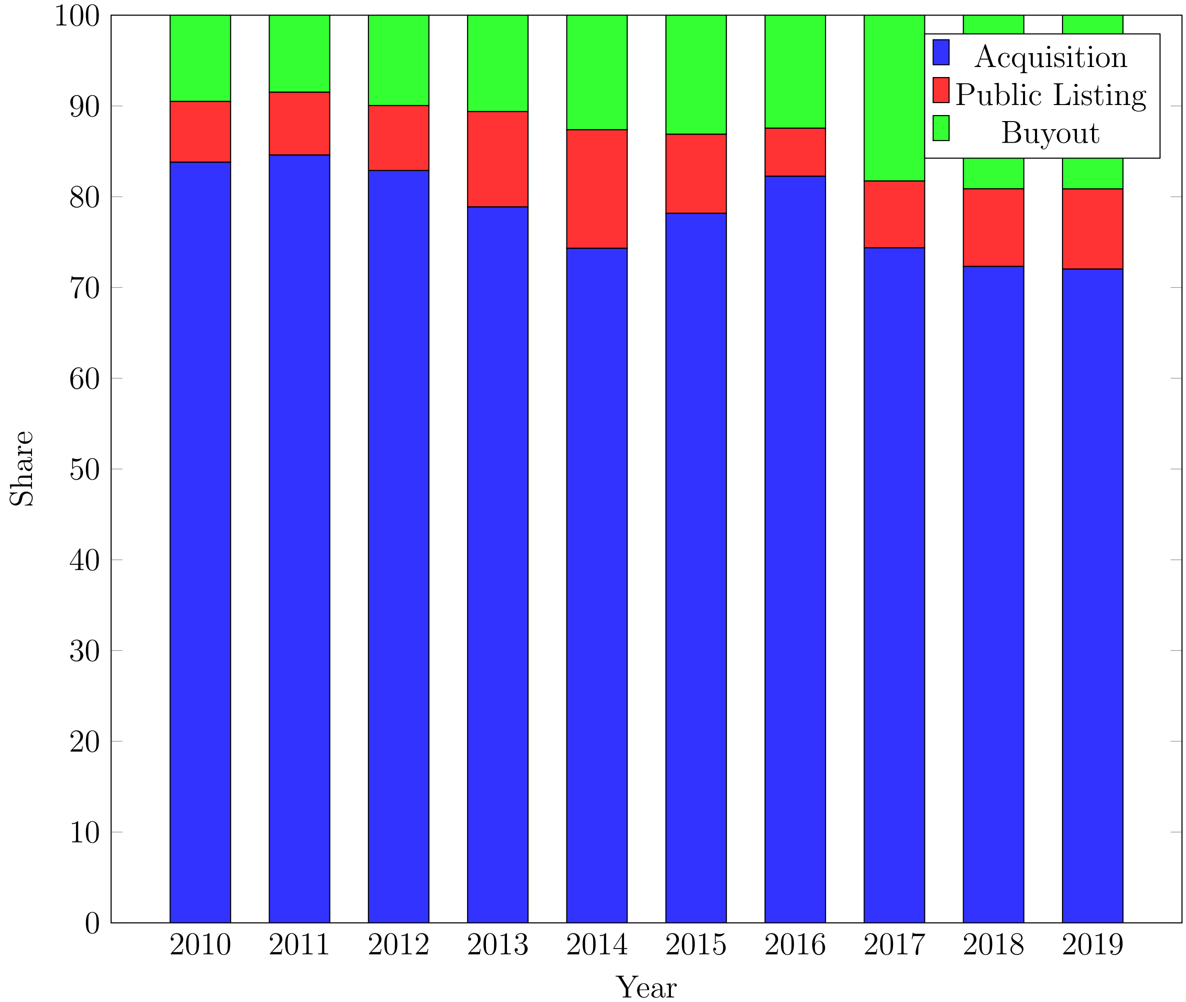
Note: Estimates of $\beta\,(R\&DSpending, RelativeSales)$ and $\beta^2\,(R\&DSpending, RelativeSales)$ are from Cavenaile et al. (2021a)

Table 2: The Estimated, No M&A , and No Blocking

	Estimated	No M&A	No Blocking
Growth Rate	1.95	1.83	1.6
R&D Intensity	3	2.8	2.3
Avg. Markup			
Sd. Markup			
1	1	0.01	37
2	4	0.8	22
3+	95	99	41

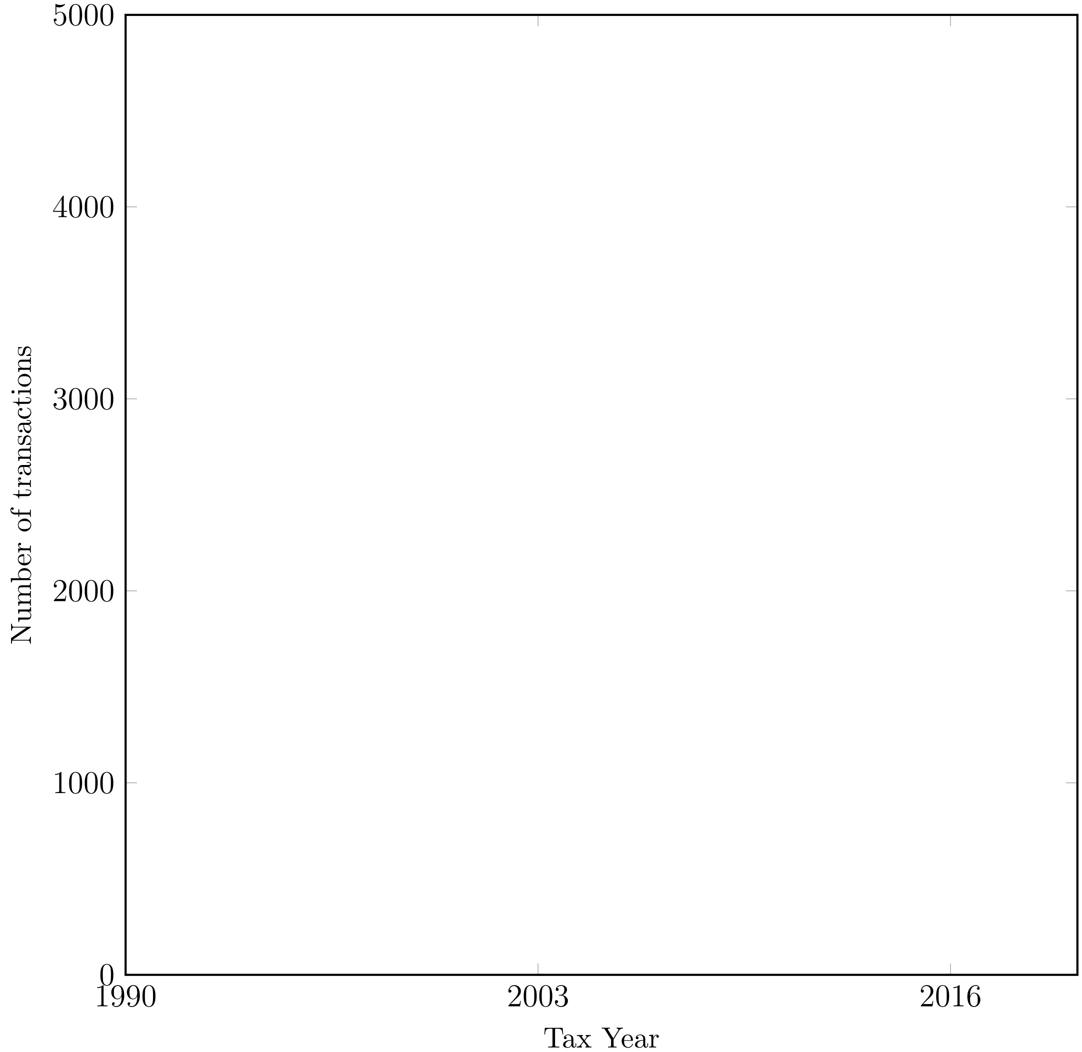
Figures

Figure 1: Venture Capital Exits By Type of Exit



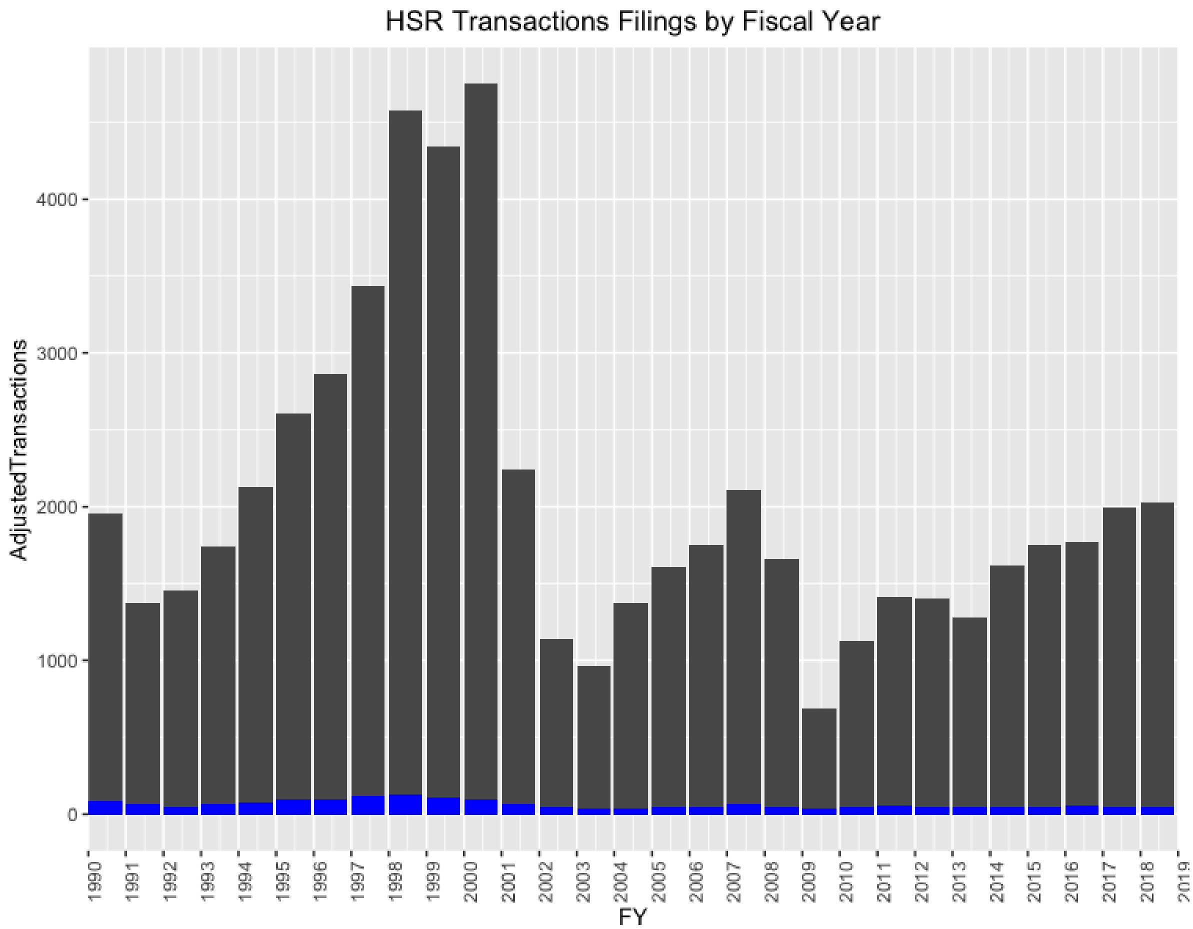
Note: This data was retrieved from the Q3 2022 Pitchbook-NVCA Venture Monitor.

Figure 2: Estimated Antitrust Blocking Probability



Note:

Figure 3: Historical Antitrust Action Over Time



Note: **TODO: Explain that I am using adjusted transactions instead of total transactions reported and why**

Figure 4: Illustrative Example, Gross Profits

Note: A firm's product quality sits on a quality ladder. The quality lead is the number of steps the low-productivity firm's product is above (or, if negative, below) the low-productivity firm.

Figure 5: Illustrative Example, R&D Effort: Innovation ($\mathcal{I}_{f,i,t}$)

Note: A firm's product quality sits on a quality ladder. The quality lead is the number of steps the low-productivity firm's product is above (or, if negative, below) the low-productivity firm.

Figure 6: Elasticity Distribution

Note:

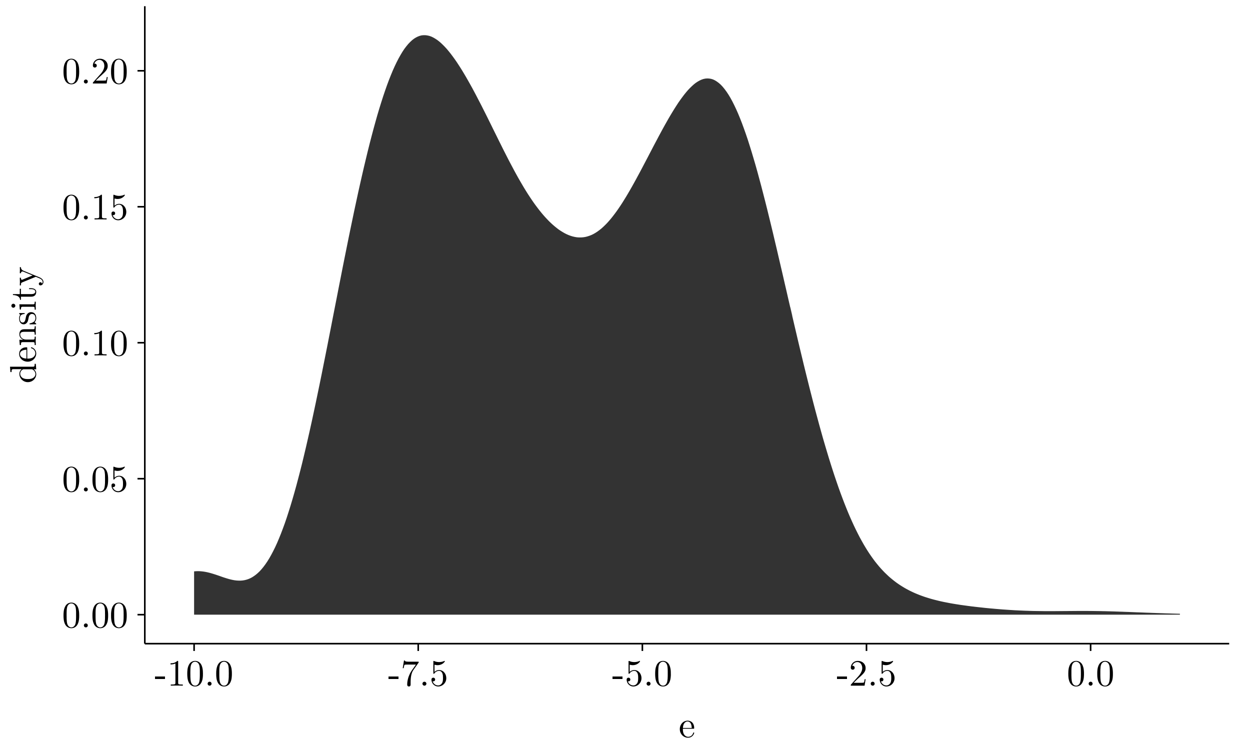
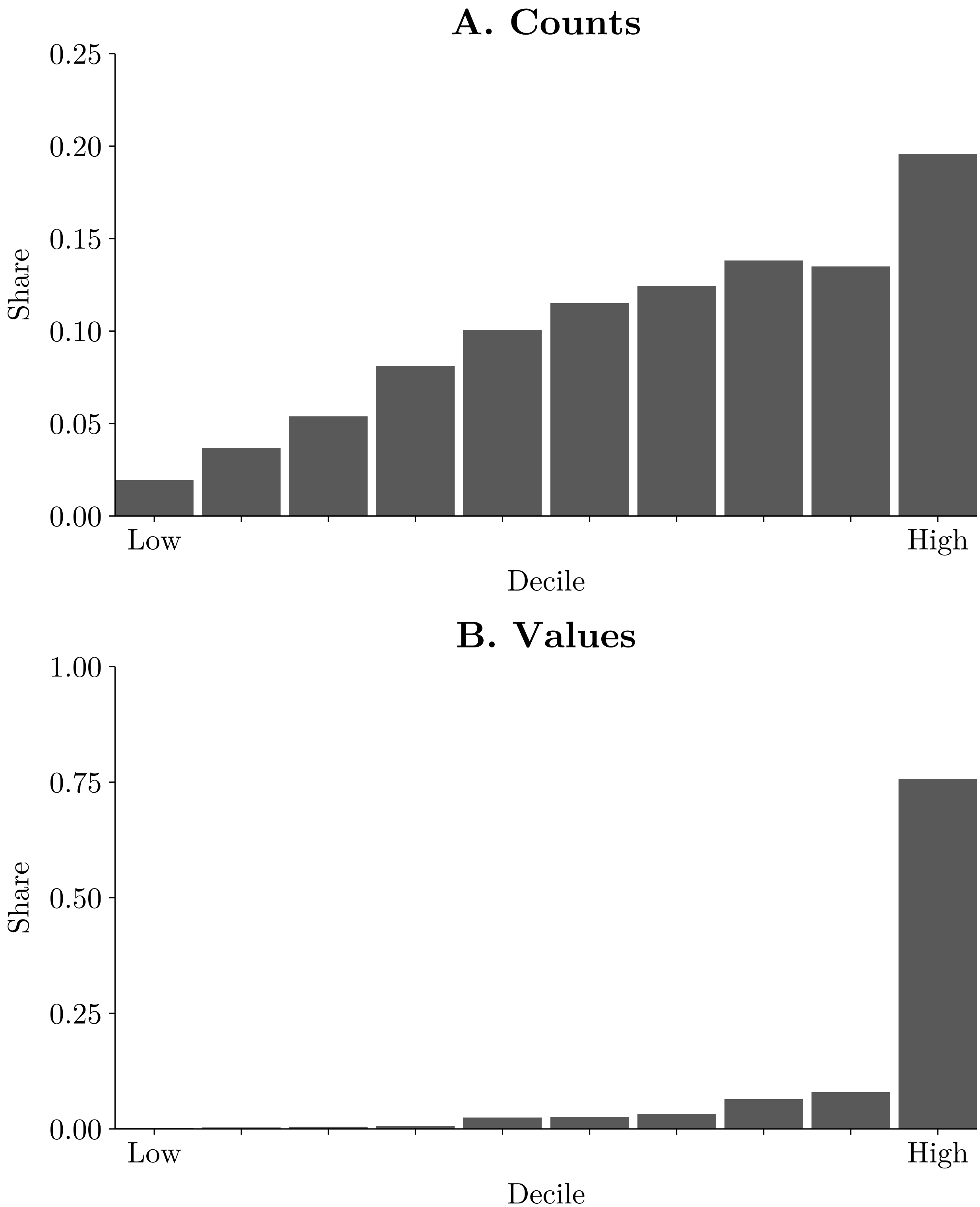


Figure 7: Share of Acquirers and Transaction Value



Note:

A Derivations

Household: To compute welfare, note that, on the balanced growth path, consumption C_t grows at rate g. So,

$$C_t = C_0 e^{gt}.$$

Applying this observation to the household's utility function, we observe

$$\begin{aligned} U &= \int_0^\infty e^{-\rho t} \log(C_t) \, \mathrm{d}t \\ &= \int_0^\infty e^{-\rho t} \log(C_0 e^{gt}) \, \mathrm{d}t \\ &= \log(C_0) \int_0^\infty e^{-\rho t} \, \mathrm{d}t + g \int_0^\infty t e^{-\rho t} \, \mathrm{d}t \\ &= \frac{1}{\rho} \log(C_0) + \frac{1}{\rho^2} g. \end{aligned}$$

This is the welfare shown in (**??**).

Demand: To start, we will derive the demand system. The final goods firm, taking prices as given, solves

$$\begin{aligned} \max_{Y_t, y_{i,t}} \; & P_t Y_t - \int_0^1 p_{i,t} y_{i,t}, \mathrm{d}i \\ s.t. \; & \log Y_t = \int_0^1 \log y_{i,t} \, \mathrm{d}i \\ & Y_t, y_{i,t} \geq 0. \end{aligned}$$

The first-order condition imply

$$p_{i,t} y_{i,t} = P_t Y_t. \tag{4}$$

Next, we need to move on to the product market aggregator. Taking prices as given, they solve

$$\begin{aligned} \max_{y_{i,t}, y_{f,i,t}, y_{c,i,t}} \; & p_{i,t} y_{i,t} - \sum_{f \in F_{i,t}} p_{f,i,t} y_{f,i,t} - p_{c,i,t} y_{c,i,t} \\ s.t. \; & y_{i,t}^{\frac{\theta-1}{\theta}} = \sum_{f \in F_{i,t}} q_{f,i,t} y_{f,i,t}^{\frac{\theta-1}{\theta}} + q_{c,i,t} y_{c,i,t}^{\frac{\theta-1}{\theta}} \\ & y_{i,t}, y_{f,i,t}, y_{c,i,t} \geq 0. \end{aligned}$$

The first-order conditions result in

$$
\begin{aligned}
y_{f,i,t} &= q_{f,i,t}^{\theta} p_{f,i,t}^{-\theta} p_{i,t}^{\theta} y_{i,t} \\
y_{c,i,t} &= q_{c,i,t}^{\theta} p_{c,i,t}^{-\theta} p_{i,t}^{\theta} y_{i,t}.
\end{aligned}
$$

Since the aggregation firm are competitive, the product market's price index $p_{i,t}$ is given by

$$
p_{i,t} = \left(\sum_{f \in F_{i,t}} q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta} p_{c,i,t}^{1-\theta} \right)^{\frac{1}{1-\theta}}.
$$

Combining this price index, the first-order conditions, and (4), we get the result

$$
\begin{aligned}
y_{f,i,t} &= \frac{q_{f,i,t}^{\theta} p_{f,i,t}^{-\theta}}{\sum_{f \in F_{i,t}} q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta} p_{c,i,t}^{1-\theta}} P_t Y_t \\
y_{c,i,t} &= \frac{q_{c,i,t}^{\theta} p_{c,i,t}^{-\theta}}{\sum_{f \in F_{i,t}} q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta} p_{c,i,t}^{1-\theta}} P_t Y_t
\end{aligned}
$$

which is the same as (**??**).

Static Bertrand Game: Each superstar firm competes against the other superstars and the fringe in a static Bertrand game. Taking the prices of the other firms as given, firm f in market i solves

$$
\begin{aligned}
\max_{p_{f,i,t}, y_{f,i,t}, h_{f,i,t}} \; & p_{f,i,t} y_{f,i,t} - w_t h_{f,i,t} \\
s.t. \; & y_{f,i,t} = z_{f,t} h_{f,i,t} \\
& y_{f,i,t} = \frac{q_{f,i,t}^{\theta} p_{f,i,t}^{-\theta}}{\sum_{f \in F_{i,t}} q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta} p_{c,i,t}^{1-\theta}} P_t Y_t.
\end{aligned}
$$

This is equivalent to solving

$$
\max_{p_{f,i,t}} \left(p_{f,i,t} - \frac{w_t}{z_{f,t}} \right) q_{f,i,t}^{\theta} p_{f,i,t}^{-\theta} \left(\sum_{f \in F_{i,t}} q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta} p_{c,i,t}^{1-\theta} \right)^{-1} P_t Y_t
$$

The first-order condition is

$$
0 = p_{f,i,t} - \theta \left(p_{f,i,t} - \frac{w_t}{z_{f,t}} \right) - (1-\theta) \left(p_{f,i,t} - \frac{w_t}{z_{f,t}} \right) \frac{q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta}}{\sum_{f \in F_{i,t}} q_{f,i,t}^{\theta} p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta} p_{c,i,t}^{1-\theta}}.
$$

We can rewrite this as

$$
\begin{aligned}
p_{f,i,t} &= \frac{\varepsilon_{f,i,t}}{\varepsilon_{f,i,t}-1}\frac{w_t}{z_{f,t}} \\
\varepsilon_{f,i,t} &= \theta + (1-\theta)s_{f,i,t} \\
s_{f,i,t} &= \frac{q_{f,i,t}^{\theta}p_{f,i,t}^{1-\theta}}{\sum_{f\in F_{i,t}} q_{f,i,t}^{\theta}p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta}p_{c,i,t}^{1-\theta}}
\end{aligned}
$$

which is (**??**). Something worth noting is that $s_{f,i,t}$ is firm f's share of market i. From the demand function, we get

$$
\begin{aligned}
\frac{p_{f,i,t}y_{f,i,t}}{\sum_{f\in F_{i,t}} p_{f,i,t}y_{f,i,t} + p_{c,i,t}y_{c,i,t}} &= \frac{p_{f,i,t}y_{f,i,t}}{p_{i,t}y_{i,t}} \\
&= \frac{p_{f,i,t}y_{f,i,t}}{P_tY_t} \\
&= \frac{1}{P_tY_t}\left(p_{f,i,t}\frac{q_{f,i,t}^{\theta}p_{f,i,t}^{-\theta}}{\sum_{f\in F_{i,t}} q_{f,i,t}^{\theta}p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta}p_{c,i,t}^{1-\theta}}P_tY_t\right) \\
&= \frac{q_{f,i,t}^{\theta}p_{f,i,t}^{1-\theta}}{\sum_{f\in F_{i,t}} q_{f,i,t}^{\theta}p_{f,i,t}^{1-\theta} + q_{c,i,t}^{\theta}p_{c,i,t}^{1-\theta}} \\
&= s_{f,i,t}.
\end{aligned}
$$

Given the solution to this system, we can pin down output and profits. Since we have already computed market share, we know

$$p_{f,i,t}y_{f,i,t} = s_{f,i,t}P_tY_t.$$

That means output is

$$y_{f,i,t} = \frac{s_{f,i,t}}{p_{f,i,t}}P_tY_t.$$

Profits are given by

$$
\begin{aligned}
\pi_{f,i,t} &= \left(p_{f,i,t} - \frac{w_t}{z_{f,t}}\right)y_{f,i,t} \\
&= \frac{1}{\varepsilon_{f,i,t}-1}\frac{w_t}{z_{f,t}}\frac{s_{f,i,t}}{p_{f,i,t}}P_tY_t \\
&= \frac{s_{f,i,t}}{\varepsilon_{f,i,t}}P_tY_t
\end{aligned}
$$

giving the result shown in (**??**).

Lemma 1. *An increase in a firms quality $q_{f,i,t}$ or productivity $z_{f,t}$ results an increase in markups $m_{f,i,t}$ and a decrease in product level output $y_{i,t}$. The entry of a new superstar firm, regardless of its quality and productivity, does the opposite.*

Proof. **TODO** □

Growth: Starting with the definition of log output, we see

$$
\begin{aligned}
\log\left(Y_t\right) =& \frac{\theta}{\theta-1}\int_\Omega \log\left(\sum_{f\in F_t(s)} q_{f,t}(s) y_{f,t}(s)^{\frac{\theta-1}{\theta}} + q_{c,t}(s) y_{c,t}(s)^{\frac{\theta-1}{\theta}}\right) \mathrm{d}\mu_t(s) \\
=& \frac{\theta}{\theta-1}\int_\Omega \log\left(q_{c,t}(s) y_{c,t}(s)^{\frac{\theta-1}{\theta}}\right) + \log\left(\sum_{f\in F_t(s)} \frac{q_{f,t}(s)}{q_{c,t}(s)} \left(\frac{y_{f,t}(s)}{y_{c,t}(s)}\right)^{\frac{\theta-1}{\theta}} + 1\right) \mathrm{d}\mu_t(s) \\
=& \frac{\theta}{\theta-1}\int_\Omega \log\left(q_{c,t}(s) y_{c,t}(s)^{\frac{\theta-1}{\theta}}\right) + \log\left(\sum_{f\in F_t(s)} \left(\frac{q_{f,t}(s)}{q_{c,t}(s)}\right)^{\theta} \left(\frac{p_{f,t}(s)}{p_{c,t}(s)}\right)^{1-\theta} + 1\right) \mathrm{d}\mu_t(s).
\end{aligned}
$$

The competitive firm's output is

$$
y_{c,t}(s) = \left(\sum_{f\in F_t(s)} \left(\frac{q_{f,t}(s)}{q_{c,t}(s)}\right)^{\theta} \left(\frac{p_{f,t}(s)}{p_{c,t}(s)}\right)^{1-\theta} + 1\right)^{-1} \frac{Y_t}{w_t}.
$$

Thus log output is

$$
\log\left(Y_t\right) = \frac{\theta}{\theta-1}\int_\Omega \log\left(q_{c,t}(s)\right) + \frac{1}{\theta}\log\left(\sum_{f\in F_t(s)} \left(\frac{q_{f,t}(s)}{q_{c,t}(s)}\right)^{\theta} \left(\frac{p_{f,t}(s)}{p_{c,t}(s)}\right)^{1-\theta} + 1\right) \mathrm{d}\mu_t(s) + \log\left(\frac{Y_t}{w_t}\right).
$$

Letting

$$
h_t(s) = \frac{1}{\theta}\log\left(\sum_{f\in F_t(s)} \left(\frac{q_{f,t}(s)}{q_{c,t}(s)}\right)^{\theta} \left(\frac{p_{f,t}(s)}{p_{c,t}(s)}\right)^{1-\theta} + 1\right),
$$

we see that

$$
\begin{aligned}
\log\left(Y_{t+\Delta}\right) - \log\left(Y_t\right) = & \frac{\theta}{\theta-1}\int_\Omega \left(\log\left(q_{c,t+\Delta}(s')\right) - \log\left(q_{c,t}(s)\right)\right) \pi_{t,t+\Delta}(s'|s)\, \mathrm{d}\mu_t(s) \\
& + \frac{1}{\theta-1}\int_\Omega \left(h_{t+\Delta}(s') - h_t(s)\right) \pi_{t,t+\Delta}(s'|s)\, \mathrm{d}\mu_t(s) \\
& + \log\left(\frac{Y_{t+\Delta}}{w_{t+\Delta}}\right) - \log\left(\frac{Y_t}{w_t}\right).
\end{aligned}
$$

On the balanced growth path, we know that $\frac{Y_t}{w_t}$ is constant over time. Also, since the terms of h depend only on relative shares which remain the same on the balanced growth path, we know

$$
\int_\Omega \left(h_{t+\Delta}(s') - h_t(s)\right) \pi_{t,t+\Delta}(s'|s)\, \mathrm{d}\mu_t(s) = 0.
$$

So, on the balanced growth path

$$\log\left(Y_{t+\Delta}\right)-\log\left(Y_t\right)=\frac{\theta}{\theta-1}\int_{\Omega}\left(\log\left(q_{c,t+\Delta}(s')\right)-\log\left(q_{c,t}(s)\right)\right)\pi_{t,t+\Delta}(s'|s)\,\mathrm{d}\mu(s).$$

Remember that $q_{c,t}(s)=\psi q_{\ell,t}(s)$ where $q_{\ell,t}(s)=\max\left\{q_{f,t}(s)\right\}$. So

$$\log\left(Y_{t+\Delta}\right)-\log\left(Y_t\right)=\frac{\theta}{\theta-1}\int_{\Omega}\left(\log\left(q_{\ell,t+\Delta}(s')\right)-\log\left(q_{\ell,t}(s)\right)\right)\pi_{t,t+\Delta}(s'|s)\,\mathrm{d}\mu(s).$$

which means we can compute the difference if we know the probability that the leading firm's quality evolves. Let $L_t(s)$ be the set of firms with the leading quality (their can be multiple). The quality can evolve in one of four ways:

1. One of the firms in $L_t(s)$ innovates. This occurs at rate $\mathcal{I}_{f,t}(s)$ for $f\in L_t(s)$.
2. A new firm with productivity z enters into the market. This occurs at rate $\mathcal{X}_t(z)$.
3. A within MA transaction occurs where **todo**
4. **todo**

So, we see **TODO**

Thus, in the balanced growth path, the growth rate is

$$g=\frac{\theta}{\theta-1}\int_{\Omega}\left(\sum_{f\in L(s)}\mathcal{I}_f(s)+\sum_{z\in Z}\mathcal{X}(z)\right)\log(\lambda)+$$

B Data Details

Year Acquisition Public Buyout 2010 84 7 10 2011 85 7 8 2012 83 7 10 2013 79 11 11 2014 74 13 13 2015 78 9 13 2016 82 5 12 2017 74 7 18 2018 72 9 19 2019 72 9 19

2010 626 50 71 2011 648 53 65 2012 741 64 89 2013 743 99 100 2014 842 148 143 2015 870 97 146 2016 839 54 127 2017 798 79 196 2018 896 106 237 2019 922 113 245

2010 0.83802 0.06693 0.09505 2011 0.84595 0.06919 0.08486 2012 0.82886 0.07159 0.09955 2013 0.78875 0.10510 0.10616 2014 0.74316 0.13063 0.12621 2015 0.78167 0.08715 0.13118 2016 0.82255 0.05294 0.12451 2017 0.74371 0.07363 0.18267 2018 0.72316 0.08555 0.19128 2019 0.72031 0.08828 0.19141

C Extra Figures

Table 3: FTC Enforcement Actions By Post-Merger Concentration

A. Total Investigations

B. Blocked Transactions

Note: **TODO**