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```
%%Math 240 Matlab Project 3
% Spring 2020
%
% Section [0342]
%
% Author: [May Kyaw]
% Problem 1
```

(a)

```
clear
clc
close all
v1 = [-6 4 -9 4]'
v2 = [8 -3 7 -3]'
v3 = [-9 5 -8 3]'
x = [4 7 -8 3]'
A = [v1 v2 v3 x]
% x is in H = Span{v1,v2,v3}
%show that the vector equation  $x_1v_1+x_2v_2+x_3v_3=x$  has a solution
%The augmented matrix [v1 v2 v3 x] in rref
disp('A = ');
disp(A);
fprintf('Row Reduced form : \n');
disp(rref(A));
%The first three columns show that B is a basis for H
% Since this system has a solution, x is in H
%The solution allows us to find the B-coordinate vector for x: since
% $x=x_1v_1+x_2v_2+x_3v_3=3v_1+5v_2+2v_3$ 
% $x\beta=[3 \ 5 \ 2]$ 
fprintf('xβ=[3 5 2] \n');
```

v1 =

-6  
4  
-9  
4

v2 =

8  
-3  
7  
-3

v3 =

-9  
5

---

```

-8
3

x =

4
7
-8
3

A =

-6      8      -9      4
4      -3      5      7
-9      7      -8     -8
4      -3      3      3

A =

-6      8      -9      4
4      -3      5      7
-9      7      -8     -8
4      -3      3      3

Row Reduced form :

1      0      0      3
0      1      0      5
0      0      1      2
0      0      0      0

```

```
x\beta=[3 5 2]
```

Problem 2 (a)

```

format short
t = [0 0.1 0.2 0.3];
f = @(t) [1 cos(t) cos(t)^2 cos(t)^3];
%when plugging in the t it gives you the four linear equations
%for the four unknown equations

%Defining what the coefficient matrix A is for the linear system
A = zeros(4);
for i=1:4
A(i,:)=f(t(i));
end
fprintf('\nA = \n');
disp(A);

```

```

A =

1.0000    1.0000    1.0000    1.0000
1.0000    0.9950    0.9900    0.9851
1.0000    0.9801    0.9605    0.9414

```

---

1.0000	0.9553	0.9127	0.8719
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(b)

```
%Prints out rref(A)
fprintf('Row Reduced form : \n');
disp(rref(A));
%Prints out det(A)
fprintf('det(A) = ');
disp(det(A));
```

```
Row Reduced form :
    1    0    0    0
    0    1    0    0
    0    0    1    0
    0    0    0    1
```

```
det(A) =    6.5176e-11
```

(c) From the example we are able to see that  $\det(A)$  is pretty close to zero

%but is not zero. That  $\text{rref}(A)$  contains all of its diagonal 1 as it is  
a  
%full rank matrix so because of all of these reasons  $A$  is invertible.

(d)

```
t = [0 0.2 0.5 1];
f = @(t) [1 cos(t) cos(t)^2 cos(t)^3];
%Plugging in the new values of t into the equation

%Defining what the coefficient matrix A is for the linear system
A = zeros(4);
for i=1:4
    A(i,:)=f(t(i));
end
fprintf('\nA = \n');
disp(A);

%Prints out rref(A)
fprintf('Row Reduced form : \n');
disp(rref(A));
%Prints out det(A)
fprintf('det(A) = ');
disp(det(A));
```

```
A =
    1.0000    1.0000    1.0000    1.0000
    1.0000    0.9801    0.9605    0.9414
    1.0000    0.8776    0.7702    0.6759
    1.0000    0.5403    0.2919    0.1577
```

---

```

Row Reduced form :
    1    0    0    0
    0    1    0    0
    0    0    1    0
    0    0    0    1

```

```

det(A) =      1.7052e-05

```

(e)

```

%We know that 1=1(sin^2t)+1(cos^2t) vt
%then 1 is expressed as {(sin^2t),(cos^2t)} and because the set of the
%function is {1,(sin^2t),(cos^2t)} it is linearly dependent
%also since
%sin^2t+cos^2t=1
%1-sin^2t-cos^2t=0
%{1,(sin^2t),(cos^2t)} would be a linearly dependent function

```

Problem 3 (a)

```

%1=1
%cost=cost
%cos2t= -1+2cos^2t
%cos3t= -3cost+4cos^3t
%cos4t= 1-8cos^2t+8cos^4t
%cos5t= 5cost-20cos^3t+16cos^6t
%cos6t= -1+18cos^2t-48cos^4t+32cos^6t
A=[1 0 -1 0 1 0 -1;0 1 0 -3 0 5 0;0 0 2 0 -8 0 18;0 0 0 4 0 -20 0;0 0
  0 8 0 -48;0 0 0 0 0 16 0;0 0 0 0 0 0 32]
fprintf('Row Reduced form : \n');
disp(rref(A));
fprintf('Rank of A\n')
rank(A)
%So C would be considered to be linearly independent
fprintf('C is linearly independent\n')

```

A =

1	0	-1	0	1	0	-1
0	1	0	-3	0	5	0
0	0	2	0	-8	0	18
0	0	0	4	0	-20	0
0	0	0	0	8	0	-48
0	0	0	0	0	16	0
0	0	0	0	0	0	32

```

Row Reduced form :
    1    0    0    0    0    0    0
    0    1    0    0    0    0    0
    0    0    1    0    0    0    0
    0    0    0    1    0    0    0
    0    0    0    0    1    0    0
    0    0    0    0    0    1    0

```

---

0 0 0 0 0 0 1

Rank of A

ans =

7

C is linearly independent

(b)

%Since  $\dim H = 7$  because B is a basis of H.  
%C is a linearly independent set  
%the vectors in C lie in H because of the trigonometric identities.  
%So because of the Basis Theorem, C is basis for H.

Problem 4 (a)

```
format rat
A=[-2 2 1 8 2;1 -10 22 11 11;1 -4 7 1 3;-2 -4 16 18 10]
fprintf('Rank of A\n')
rank(A)
```

A =

-2	2	1	8	2
1	-10	22	11	11
1	-4	7	1	3
-2	-4	16	18	10

Rank of A

ans =

2

(b)

%A is a  $m \times n$  matrix where the  $m=4$  and the  $n=5$   
%When using the rank nullity theorem  
%rank(A)+dim(Nul A)=n  
%dim(Nul A)=4 - rank(A)=5-2=3  
%dim(Col A)= rank(A) = 2  
%dim(Row A)= rank(A) = 2

```
fprintf('dim(Nul A): %d\n', size(A,2) - rank(A))
fprintf('dim(Col A): %d\n', rank(A))
fprintf('dim(Row A): %d\n', rank(A))
```

---

```
dim(Nul A): 3
dim(Col A): 2
dim(Row A): 2
```

(c)

```
fprintf('\nRREF of A\n')
rref(A)

%(i) Null(A)
%A=[1 0 -3 -17/3 -7/3;0 1 -5/2 -5/3 -4/3]
%B=[c1;c2;c3;c4;c5]
%A*B= [0;0;0;0]
%c1=(17s/3)+(3t)+(7u/3)
%c2=(5s/3)+(5t/2)+(4u/3)
%c3=t
%c4=s
%c5=u
%Null(A)={3,5/2,1,0,0}, {17/3,5/3,0,1,0},{7/3,4/3,0,0,1}
fprintf('\n Null(A)={3,5/2,1,0,0}, {17/3,5/3,0,1,0},{7/3,4/3,0,0,1}
\n')
%(ii) Col(A)
fprintf('\n Col(A) \n')
A=[-2 2 1 8 2;1 -10 22 11 11]
%(iii) Row(A)
fprintf('\n Row(A) \n')
A=[-2 1 1 -2;2 -10 -4 -4]
```

*RREF of A*

*ans =*

1	0	-3	-17/3	-7/3
0	1	-5/2	-5/3	-4/3
0	0	0	0	0
0	0	0	0	0

$\text{Null}(A)=\{3,5/2,1,0,0\}, \{17/3,5/3,0,1,0\},\{7/3,4/3,0,0,1\}$

$\text{Col}(A)$

*A =*

-2	2	1	8	2
1	-10	22	11	11

---

$\text{Row}(A)$

$A =$

$$\begin{array}{cccc} -2 & 1 & 1 & -2 \\ 2 & -10 & -4 & -4 \end{array}$$

Problem 5 (a)

$$\begin{aligned} v1 &= [4 \ 3 \ 7 \ 3]' \\ v2 &= [2 \ 1 \ 1 \ -1]' \\ v3 &= [1 \ 1 \ 3 \ 2]' \\ v4 &= [-3 \ 5 \ 8 \ 1]' \\ v5 &= [5 \ 3 \ 5 \ 0]' \end{aligned}$$

$v1 =$

$$\begin{array}{c} 4 \\ 3 \\ 7 \\ 3 \end{array}$$

$v2 =$

$$\begin{array}{c} 2 \\ 1 \\ 1 \\ -1 \end{array}$$

$v3 =$

$$\begin{array}{c} 1 \\ 1 \\ 3 \\ 2 \end{array}$$

$v4 =$

$$\begin{array}{c} -3 \\ 5 \\ 8 \\ 1 \end{array}$$

$v5 =$

$$\begin{array}{c} 5 \\ 3 \end{array}$$

---

5  
0

(b)

```
A = [v1 v2 v3 v4 v5];  
disp('A = ');  
disp(A);  
fprintf('\nRREF of A\n')  
rref(A)  
C = colspace(sym(A));  
disp('Basis = ');  
disp(double(C));
```

```
A =  
      4      2      1      -3      5  
      3      1      1      5      3  
      7      1      3      8      5  
      3     -1      2      1      0
```

*RREF of A*

```
ans =  
      1      0      1/2      0      1/2  
      0      1     -1/2      0      3/2  
      0      0      0      1      0  
      0      0      0      0      0
```

```
Basis =  
      1      0      0  
      0      1      0  
      0      0      1  
    -3/23   -50/23   33/23
```

(c)

```
%This displays that [v1 v2 v3 v4 v5] is not linearly independent set  
%Since we only have three non zero pivot. So dimension of W = 3.  
%And we can choose last three column of A for basis for W as follow.  
  \{t^2, -3+2t, -2+t\}  
%Because these are linearly independent.
```

(d)



---

```
%W does not equal P3
%dimW=3 and dimP3=4
%3<4
%so W does not equal P3
```

Problem 6 (a)

```
M = 4;
A = zeros(2, 3, M);
A(:, :, 1) = [2 -3 4; -2 1 2];
A(:, :, 2) = [0 -4 0; -3 2 1];
A(:, :, 3) = [2 0 4; 5 -2 -1];
A(:, :, 4) = [1 -12 2; 3 2 -2];

N = 6;
E = zeros(2, 3, N);
E(:, :, 1) = [1 0 0; 0 0 0]; E(:, :, 2) = [0 1 0; 0 0 0];
E(:, :, 3) = [0 0 1; 0 0 0]; E(:, :, 4) = [0 0 0; 1 0 0];
E(:, :, 5) = [0 0 0; 0 1 0]; E(:, :, 6) = [0 0 0; 0 0 1];
```

(a)

```
v = zeros(2*3, M);
for i = 1:1:M
    T = A(:, :, i); T = T'; T = T(:);
    v(:, i) = T;
end
disp('v1 ='), disp(v(:,1))
disp('v2 ='), disp(v(:,2))
disp('v3 ='), disp(v(:,3))
disp('v4 ='), disp(v(:,4))
```

v1 =

```
2
-3
4
-2
1
2
```

v2 =

```
0
-4
0
-3
2
1
```

v3 =

```
2
0
4
5
-2
-1
```

---

v4 =

1  
-12  
2  
3  
2  
-2

(b)

```
rref_v = rref(v);  
disp('RREF of v ='), disp(rref_v)  
fprintf('Column 4 (v4) can be written as linear combination of other  
three columns.\n')  
fprintf('v4 = -2*v1 + (9/2)*v2 + (5/2)*v3 so the vectors v1, v2, v3  
and v4 are linearly dependent.\n')
```

RREF of v =

1	0	0	-2
0	1	0	9/2
0	0	1	5/2
0	0	0	0
0	0	0	0
0	0	0	0

Column 4 (v4) can be written as linear combination of other three columns.

v4 = -2\*v1 + (9/2)\*v2 + (5/2)\*v3 so the vectors v1, v2, v3 and v4 are linearly dependent.

(c)

```
fprintf('Thus, A4 = -2*A1 + (9/2)*A2 + (5/2)*A3, same as the  
relationship between coordinate vectors.\n')  
disp('A4 ='), disp(A(:, :, 4))  
T = -2*A(:, :, 1) + 9/2*A(:, :, 2) + 5/2*A(:, :, 3);  
disp('-2*A1 + (9/2)*A2 + (5/2)*A3 ='), disp(T)
```

Thus, A4 = -2\*A1 + (9/2)\*A2 + (5/2)\*A3, same as the relationship between coordinate vectors.

A4 =

1	-12	2
3	2	-2

-2\*A1 + (9/2)\*A2 + (5/2)\*A3 =

1	-12	2
3	2	-2

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