1. - Couldn't find the Gaddis HW pages, will try later

2. ```
   - Compute $\arg\max \prod_i \vec{v}$ where $\vec{v} = \left[ v_1, v_2,
   \dots, v_i\right]$ such that $v_i \in \mathbb{N}$ and $\sum_i \vec{v} = n$
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   Proof by induction:
   Suppose we have $\vec{v} = [v_1, v_2, \dots, v_l]$ satisfying $\sum_i \vec{v} = n$
   One optimization that we can make is that for any $\vec{v}$ with elements $v_i \neq v_j$, we can construct a vector $\vec{\lambda}$, where $\lambda_i = \lambda_j = \frac{v_i + v_j}{2}$, and $\lambda_{a \neq i, a \neq j} = v_a$. We can then argue that $\prod_i \vec{\lambda} > \prod_i \vec{v}$
   Proof
   Let $p = \prod_i \vec{v}$. We can say that $\prod_i \vec{\lambda} = \frac{p}{v_i v_j} \cdot \lambda_i \lambda_j$. Therefore, $\frac{\prod_i \vec{\lambda}}{p} = \frac{\frac{p}{v_i v_j} \cdot \lambda_i \lambda_j}{p} = \frac{\lambda_i \lambda_j}{v_i v_j}$
   To prove that $\prod_i \vec{\lambda} > \prod_i \vec{v}$, we can prove that $\frac{\lambda_i \lambda_j}{v_i v_j} > 1$
   $\lambda_i \lambda_j = \left(\frac{v_1 + v_2}{2}\right)^2$

   $$\frac{\lambda_i \lambda_j}{v_i v_j} > 1$$

   $$\frac{\left(\frac{v_i + v_j}{2}\right)^2}{v_i v_j} > 1$$

   $$\frac{(v_i + v_j)^2}{4 v_i v_j} > 1$$

   $$(v_i + v_j)^2 > 4 v_i v_j$$

   $$v_i^2 + 2 v_i v_j + v_j^2 > 4 v_i v_j$$

   $$v_i^2 - 2 v_i v_j + v_j^2 > 0$$

   $$(v_i - v_j)^2 > 0$$

   Since $v_i \neq v_j$, $v_i - v_j$ cannot be 0, therefore its square will always be greater than 0.
   This proves that the optimal list (allowing $v_i \in \mathbb{Q}, v_i \geq 1$) has all elements equal.

Solving for v_i

Now that we know all entries of $\vec{v}$ are equal, we can solve for the value of any entry, which we will denote v_i.
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$$\sum_i \vec{v} = n$$

$$\sum_i^l v_i = n$$

$$l \cdot v_i = n$$

$$v_i = \frac{n}{l}$$

The next thing to determine is the optimal length. Since we know that all elements of the vector all the same, we now have a simple expression for the term we want to maximize

$$\prod_i \vec{v} = \prod_i v_i = v_i^l = \left(\frac{n}{l}\right)^l$$

Where $1 \leq l \leq n$
To optimize this term, we can use simple calculus. Let $f(x) = \left(\frac{n}{x}\right)^x$. To find the extrema, we just find the zeroes of $f'(x)$

$$f'(x) = \frac{d}{dx}\left(\frac{n}{x}\right)^x$$

$$= \frac{d}{dx}\left(e^{x \ln \frac{n}{x}}\right)$$

$$= e^{x \ln \frac{n}{x}} \cdot (\ln(\frac{n}{x}) - 1)$$

A power term can never be zero, so for this to be zero, $\ln\left(\frac{n}{x}\right) - 1 = 0$
This means that $\ln\left(\frac{n}{x}\right) = 1$, so $\frac{n}{x} = e$, so $x = \frac{n}{e}$
l must be an integer, so we can say that $l = \mathrm{round}(\frac{n}{e})$
This gives us our final (unrounded) vector

$$\vec{v} = [v_1, v_2, \ldots, v_l]$$

such that $l = \mathrm{round}(\frac{n}{e})$ and for $i \in \mathbb{N}$ and $1 \leq i \leq l$, $v_i = \frac{n}{l}$

Rigorous definition of $\mathrm{round}(x)$:

$$\mathrm{round}(x) : \mathbb{R} \to \mathbb{Z} = \begin{cases} \lfloor x \rfloor & \text{if } x - \lfloor x \rfloor < \frac{1}{2} \\ \lceil x \rceil & \text{if } x - \lfloor x \rfloor > \frac{1}{2} \end{cases}$$

$\mathrm{round}(x)$ is not defined for $x = w \in \mathbb{Z} + \frac{1}{2}$, but this is not a problem for our use case.
Proof by contradiction:
Suppose we did have an undefined value for $\mathrm{round}(x)$ in our math. The only place we use $\mathrm{round}(x)$is in $\mathrm{round}(\frac{n}{e})$, so that would imply that $\frac{n}{e} = w \in \mathbb{Z} + \frac{1}{2}$
$\frac{n}{e} = \frac{2w+1}{2}$
$n = e\frac{2w+1}{2}$

$e = \frac{2n}{2w+1}$

This would imply that $e \in \mathbb{Q}$, since it equals the ratio of 2 whole numbers. However, e is known to be transcendental, and therefore we have a contradiction. Therefore, we will never run into the edge case for $\mathrm{round}(x)$; it will always be defined for our inputs.

The above equations always give the optimal result, but they do it for elements in the rational numbers, not the integers. I thought I had a system for rounding that always produced the optimal output (shown below), but I found some counterexamples, and I can't figure out how to move forward. I apologize for taking so long on this homework (and not even completing it fully), while also not answering this question satisfactorily. For a computer algorithm, one could start with the guess generated by the above equations, and just test a few different combinations of lengths and entries near the given values. I don't think I'm quite skilled enough at math to figure out a closed form solution for solving this with natural number entries.

To round these terms, we can write $v_i = w + \frac{a}{b}$, where $w \in \mathbb{N}$. This is valid since we know that v_i is a rational number, being a ratio of two integers. We can then say that for a vector $\vec{\lambda}$ with rounded terms, $\lambda_i = w$ when $(i - 1 \mod b) \leq b - a$, and $w + 1$ otherwise. For example, the vector $\vec{v} = [3\frac{1}{3}, 3\frac{1}{3}, 3\frac{1}{3}]$ would round to $[3, 3, 4]$, and the vector $\vec{v} = [3\frac{2}{3}, 3\frac{2}{3}, 3\frac{2}{3}]$ would round to $[3, 4, 4]$. We can also explicitly solve for the elements. The two unique values in the vector, λ_a and λ_b, can solved for easily.

$$\lambda_a = \lfloor v_i \rfloor, \lambda_b = \lceil v_i \rceil$$

We also need to solve for the number of each term, in order to calculate $\prod_i \vec{\lambda}$.
Let $(\lambda_a)^\alpha \cdot (\lambda_b)^\beta = \prod_i \vec{\lambda}$, and $\alpha + \beta = l$
First, notice that because $v_i = w + \frac{a}{b} = \frac{n}{l}$, $l = \frac{n}{w + \frac{a}{b}} = \frac{n}{\frac{wb+a}{b}} = b\frac{n}{wb+a}$, which means that $l \mod b = 0$. This means that stepping though all values of i, from 0 to l, always completes $\frac{l}{b}$ cycles. During each cycle, the number of i values that get mapped to λ_a is the number of $1 \leq i \leq b$ values that satisfy $i - 1 \leq b - a$, which just equals $b - a$. Therefore, we have $\frac{l}{b}$ cycles, each with $b - a$ values that map to λ_a, so

$$\alpha = \frac{l}{b}(b - a)$$

We can now easily solve for β

$$\alpha + \beta = l$$

$$\beta = l - \alpha$$

$$\beta = l - \frac{l}{b}(b - a)$$

$$\beta = l - \frac{l(b - a)}{b}$$

$$\beta = \frac{lb - l(b-a)}{b}$$

$$\beta = \frac{lb - lb + la}{b}$$

$$\beta = \frac{la}{b}$$

Therefore, we find that

$$\prod_i \vec{\lambda} = \lfloor v_i \rfloor^{\frac{l}{b}(b-a)} \cdot \lceil v_i \rceil^{\frac{la}{b}}$$