1

$$f_\xi(x,y) = \frac{e^{-2|y|}}{\pi(1+x^2)} \qquad \begin{array}{l} x \in [-\infty; +\infty] \\ y \in [-\infty; +\infty] \end{array}$$

$$\frac{4}{\pi}\int_0^\infty\int_0^\infty \frac{e^{-2y}}{1+x^2}\,dx\,dy = \int_0^\infty e^{-2y}\,\mathrm{arctang}(x)\,dy =$$

$$= \int_0^\infty e^{-2y}\left(\frac{\pi}{2}-0\right)dy = -\frac{\overset{2}{\cancel{4}}\cdot\cancel{\pi}}{\cancel{\pi}\cdot\cancel{2}}\,\frac{e^{-2y}}{2}\Big|_0^\infty = 0+1 = 1$$

Является

②

a)

η	-1	0	1
P	$\frac{11}{24}$	$\frac{1}{4}$	$\frac{7}{24}$

ξ	-1	1
P	$\frac{1}{2}$	$\frac{1}{2}$

b) 1) $E(\xi,\eta) = \begin{pmatrix}-1\\-1\end{pmatrix}\cdot\frac{1}{8} + \begin{pmatrix}-1\\1\end{pmatrix}\cdot\frac{1}{3} + \begin{pmatrix}0\\-1\end{pmatrix}\cdot\frac{1}{12} + \begin{pmatrix}0\\1\end{pmatrix}\cdot\frac{1}{6} + \begin{pmatrix}1\\-1\end{pmatrix}\cdot\frac{7}{24} =$

$= \begin{pmatrix}-\frac{1}{8}\\-\frac{1}{8}\end{pmatrix} + \begin{pmatrix}-\frac{1}{3}\\\frac{1}{3}\end{pmatrix} + \begin{pmatrix}0\\-\frac{1}{12}\end{pmatrix} + \begin{pmatrix}0\\\frac{1}{6}\end{pmatrix} + \begin{pmatrix}\frac{7}{24}\\-\frac{7}{24}\end{pmatrix} = \begin{pmatrix}-\frac{1}{6}\\0\end{pmatrix}$

2) $cov(\xi;\eta) = (-1+\frac{1}{6})\cdot(-1-0)\cdot\frac{1}{8} + (-1+\frac{1}{6})\cdot(1-0)\cdot\frac{1}{3} + (0+\frac{1}{6})\cdot(-1-0)\cdot\frac{1}{12} +$

$+ (0+\frac{1}{6})\cdot(1-0)\cdot\frac{1}{6} + (1+\frac{1}{6})\cdot(-1-0)\cdot\frac{7}{24} = -\frac{5}{6}(-1)\cdot\frac{1}{8} + (-\frac{5}{6}\cdot 1\cdot\frac{1}{3}) +$

$+ \frac{1}{6}\cdot(-1)\cdot\frac{1}{12} + \frac{1}{6}\cdot 1\cdot\frac{1}{6} + \frac{1}{6}(-1)\cdot\frac{7}{24} = \frac{5}{6\cdot 8} + (-\frac{5}{6\cdot 3}) + (-\frac{1}{6\cdot 12}) +$

$+ \frac{1}{6}\cdot\frac{1}{6} + (-\frac{7}{6\cdot 24}) = \frac{5}{48} - \frac{5}{18} - \frac{1}{72} + \frac{1}{36} - \frac{7}{6\cdot 24} =$

$= \frac{1}{6}\left(\frac{5}{8} - \frac{5}{3} - \frac{1}{12} + \frac{1}{6} - \frac{7}{24}\right) = \frac{1}{6}\left(\frac{15-40-2+4-7}{24}\right) = -\frac{1}{6}\cdot\frac{30}{24} = -\frac{5}{24}$

$cov(\xi,\xi) = D(\xi) = (-1-0)^2\cdot\frac{1}{8} + (-1-0)^2\cdot\frac{1}{3} + (-1-0)^2\cdot\frac{1}{12} + (1-0)^2\cdot\frac{1}{6} +$

$+ (-1-0)^2\cdot\frac{7}{24} = \frac{1}{8} + \frac{1}{3} + \frac{1}{12} + \frac{1}{6} + \frac{7}{24} = \frac{3+8+2+4+7}{24} = \frac{24}{24} = 1$

$cov(\eta,\eta) = (-1+\frac{1}{6})^2\cdot\frac{1}{8} + (-1+\frac{1}{6})^2\cdot\frac{1}{3} + (0+\frac{1}{6})^2\cdot\frac{1}{12} + (0+\frac{1}{6})^2\cdot\frac{1}{6} +$

$+ (1+\frac{1}{6})^2\cdot\frac{7}{24} = (-\frac{5}{6})^2\cdot\frac{1}{8} + (-\frac{5}{6})^2\cdot\frac{1}{3} + (\frac{1}{6})^2\cdot\frac{1}{12} + (\frac{1}{6})^2\cdot\frac{1}{6} + (\frac{7}{6})^2\cdot\frac{7}{24} =$

$= \frac{25}{36\cdot 8} + \frac{25}{36\cdot 3} + \frac{1}{36\cdot 12} + \frac{1}{36\cdot 6} + \frac{49\cdot 7}{36\cdot 24} = \frac{1}{36}\left(\frac{25}{8} + \frac{25}{3} + \frac{1}{12} + \frac{1}{6} + \frac{49\cdot 7}{24}\right) =$

$= \frac{1}{36}\left(\frac{25\cdot 3 + 25\cdot 8 + 2 + 4 + 49\cdot 7}{24}\right) = \frac{26}{36}$

$$Cov = \begin{pmatrix} \frac{26}{36} & \frac{5}{24} \\ \frac{5}{24} & 1 \end{pmatrix}$$

$$Cor = \begin{pmatrix} 1 & \frac{5}{4\sqrt{26}} \\ \frac{5}{4\sqrt{26}} & 1 \end{pmatrix}$$

c) $\frac{5}{4 \cdot \sqrt{26}} \approx 0{,}245$ - Величины относительно независимы и слабо коррелируют

③

a)

$\xi_1 \backslash \xi_2$	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$\} \varphi_1$

$\xi_1 \backslash \xi_2$	1	2	3	4
1	1	1	1	1
2	1	1	0	1
3	1	0	1	0
4	1	1	0	1

$\} \varphi_2$

b)

$\varphi_2 \backslash \varphi_1$	2	3	4	5	6	7	8
0	0	0	0	$\frac{2}{16}$	0	$\frac{2}{16}$	0
1	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	0	$\frac{1}{16}$

c) 1) $E(\varphi_1, \varphi_2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \frac{1}{16} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \frac{2}{16} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} \cdot \frac{3}{16} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} \cdot \frac{2}{16} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot \frac{2}{16} +$

$+ \begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \frac{3}{16} + \begin{pmatrix} 7 \\ 0 \end{pmatrix} \cdot \frac{2}{16} + \begin{pmatrix} 8 \\ 1 \end{pmatrix} \cdot \frac{1}{16} = \begin{pmatrix} \frac{2}{16} \\ \frac{1}{16} \end{pmatrix} + \begin{pmatrix} \frac{6}{16} \\ \frac{2}{16} \end{pmatrix} + \begin{pmatrix} \frac{12}{16} \\ \frac{3}{16} \end{pmatrix} + \begin{pmatrix} \frac{10}{16} \\ 0 \end{pmatrix} +$

$+ \begin{pmatrix} \frac{10}{16} \\ \frac{2}{16} \end{pmatrix} + \begin{pmatrix} \frac{18}{16} \\ \frac{3}{16} \end{pmatrix} + \begin{pmatrix} \frac{14}{16} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{8}{16} \\ \frac{1}{16} \end{pmatrix} = \begin{pmatrix} 5 \\ \frac{3}{4} \end{pmatrix}$

2) $cov(\varphi_1; \varphi_2) = (2-5)\cdot(1-\frac{3}{4})\cdot\frac{1}{16} + (3-5)\cdot(1-\frac{3}{4})\cdot\frac{2}{16} +$

$+ (4-5)\cdot(1-\frac{3}{4})\cdot\frac{3}{16} + (5-5)(0-\frac{3}{4})\cdot\frac{2}{16} + (5-5)(1-\frac{3}{4})\cdot\frac{2}{16} +$

$+ (6-5)(1-\frac{3}{4})\cdot\frac{3}{16} + (7-5)(0-\frac{3}{4})\cdot\frac{2}{16} + (8-5)(1-\frac{3}{4})\cdot\frac{1}{16} =$

$= \cancel{-\frac{3}{4\cdot16}} + (-\frac{1}{16}) \cancel{-\frac{3}{4\cdot16}} + \cancel{\frac{3}{4\cdot16}} - \frac{3}{16} + \cancel{\frac{3}{4\cdot16}} = -\frac{1}{16} - \frac{3}{16} = -\frac{4}{16} = -\frac{1}{4}$

$cov(\varphi_1; \varphi_1) = (2-5)^2\cdot\frac{1}{16} + (3-5)^2\cdot\frac{2}{16} + (4-5)^2\cdot\frac{3}{16} + (6-5)^2\cdot\frac{3}{16} +$

$+ (7-5)^2\cdot\frac{2}{16} + (8-5)^2\cdot\frac{1}{16} = \frac{9}{16} + \frac{4\cdot2}{16} + \frac{3}{16} + \frac{3}{16} + \frac{4}{16} + \frac{9}{16} =$

$= \frac{36}{16} = \frac{9}{4}$

$cov(\varphi_2; \varphi_2) = (1-\frac{3}{4})^2\cdot\frac{1}{16} + (1-\frac{3}{4})^2\cdot\frac{2}{16} + (1-\frac{3}{4})^2\cdot\frac{3}{16} + (0-\frac{3}{4})^2\cdot\frac{2}{16} +$

$+ (1-\frac{3}{4})^2\cdot\frac{2}{16} + (1-\frac{3}{4})^2\cdot\frac{3}{4} + (0-\frac{3}{4})^2\cdot\frac{2}{16} + (1-\frac{3}{4})^2\cdot\frac{1}{16} =$

$= \frac{1}{16}\cdot\frac{1}{16} + \frac{1}{16}\cdot\frac{2}{16} + \frac{1}{16}\cdot\frac{3}{16} + \frac{9}{16}\cdot\frac{2}{16} + \frac{1}{16}\cdot\frac{2}{16} + \frac{1}{16}\cdot\frac{3}{4} + \frac{9}{16}\cdot\frac{2}{16} +$

$+ \frac{1}{16}\cdot\frac{1}{16} = \frac{1+2+3+18+2+3+18+\cancel{8}1}{16^2} = \frac{48}{16^2}$

$$Cov\ matrix = \begin{pmatrix} \frac{9}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{48}{16^2} \end{pmatrix}$$

3) $$Cor\ matrix = \begin{pmatrix} 1 & \frac{-2}{3\sqrt{3}} \\ \frac{-2}{3\sqrt{3}} & 1 \end{pmatrix}$$

(4) $f_x = \frac{1}{2\pi}$

$$E\eta_1 = \int_{-\pi}^{\pi} \frac{1}{2\pi} \cos\xi \, d\xi = 0$$

$$E\eta_2 = \int_{-\pi}^{\pi} \frac{1}{2\pi} \sin\xi \, d\xi = 0$$

$$D\eta_1 = \int_{-\pi}^{\pi} \cos^2\xi \cdot \frac{1}{2\pi} \, d\xi = \frac{1}{2\pi} \cdot \cos\xi \sin\xi + \int_{-\pi}^{\pi} \sin^2\xi \, dx =$$

$$\int 1 - \int \cos^2\xi$$

$$= \frac{1}{2\pi} \cdot \frac{1}{2} \left(x + \cos\xi \cdot \sin\xi\right) \Big|_{-\pi}^{\pi} = \frac{-1 \cdot 0 + \pi}{4\pi} + \frac{-1 \cdot 0 - \pi}{4\pi} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$D\eta_2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin^2\xi \, d\xi = \frac{1}{2\pi} \cdot \frac{1}{2} \int_{-\pi}^{\pi} 1 - \cos 2\xi \, d\xi = \frac{1}{4\pi} \left(\xi - \frac{\sin 2\xi}{2}\right) \Big|_{-\pi}^{\pi} =$$

$$= \frac{\pi}{4\pi} - \frac{0}{2} - \frac{(-\pi)}{4\pi} - \frac{0}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$cov(\eta_1; \eta_2) = \int_{-\pi}^{\pi} \cos\xi \cdot \sin\xi \cdot \frac{1}{2\pi} \, d\xi = \frac{1}{2\pi} \frac{\sin^2\xi}{2} \Big|_{-\pi}^{\pi} = 0$$

$$cov\_matrix(\eta_1; \eta_2) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

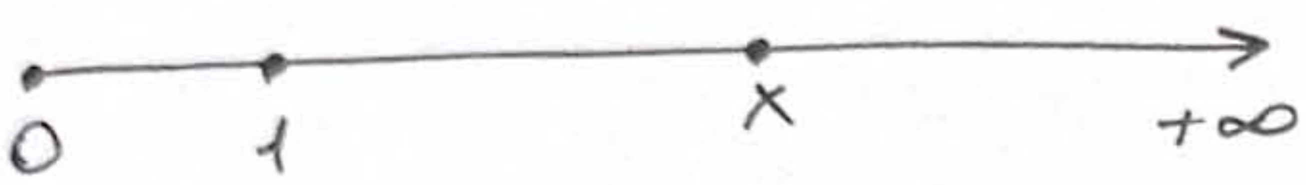
c) Корреляции нет, зависимость есть $(\eta_1^2 + \eta_2^2 = 1)$

$$cor\_matrix(\eta_1; \eta_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(5)

$\xi \sim Exp_2$ $\qquad\qquad\qquad\qquad$ $\eta \sim U_{0,1}$

$$p(x) = \begin{cases} 0, & x < 0 \\ 2e^{-2x}, & x \geq 0 \end{cases} \qquad\qquad p(x) = \begin{cases} 0, & x \notin [0,1] \\ 1, & x \in [0,1] \end{cases}$$



$$g(x) = 2e^{-2x}$$

$$g * p(x) = \int_0^x p(u) \cdot g(x-u)\, du$$

1) $x > 1$; 2) $x \leq 1$

1) $x > 1$

$$\int_0^1 1 \cdot g(x-u)\, du = \int_0^1 2e^{-2(x-u)}\, du = 2 \cdot e^{-2x} \cdot \int_0^1 e^{2u}\, du = 2 \cdot e^{-2x} \cdot \frac{e^{2u}}{2}\Big|_0^1 =$$

$$= e^{-2x}(e^2 - 1)$$

2) $x \leq 1$

$$\int_0^x 1 \cdot g(x-u) = \int_0^x 2 \cdot e^{-2(x-u)}\, du = e^{-2x} \cdot e^{2u}\Big|_0^x = e^{-2x} \cdot (e^{2x} - 1) =$$

$$= 1 - e^{-2x}$$

$$f_{\xi+\eta}(x) = \begin{cases} e^{-2x}(e^2 - 1), & x > 1 \\ 1 - e^{-2x}, & x \leq 1 \end{cases}$$