Q2) Adiabatic shocks: numerical exercise

2.1) From Class: $\frac{\rho_1}{\rho_2} = \frac{\gamma-1}{\gamma+1} + \frac{2}{M_1^2(1+\gamma)}$

From plots: $\rho_1 \sim 1\ kg/m^3$, $\rho_2 \sim 0.5\ kg/m^3$, $M_1 \sim 0.65$

$$\Rightarrow \frac{1}{0.5} \overset{?}{=} \frac{5/3-1}{5/3+1} + \frac{2}{(0.65)^2(1+5/3)}$$

$$2 \overset{?}{=} \frac{2/3}{8/3} + \frac{2}{(0.65)^2(8/3)}$$

$$2 \approx 2.03$$

Yes, this agrees with the shock jump conditions derived in class.

2.2) With a $dx = 0.4$ and $dt = 0.04$, we have a shock with a width ~ 5 grid points.

The value of dx sets this width. A higher value of dx results in a larger width. This is a result of having too much spatial discretization error which results in what is called artificial viscosity. This ends up smearing the numerically resolved shock. As we decrease dx, we can see that the shock front is sharper as a result; Approaching a width independent of dx and dt, i.e. a numerically converged solution. (see plots)