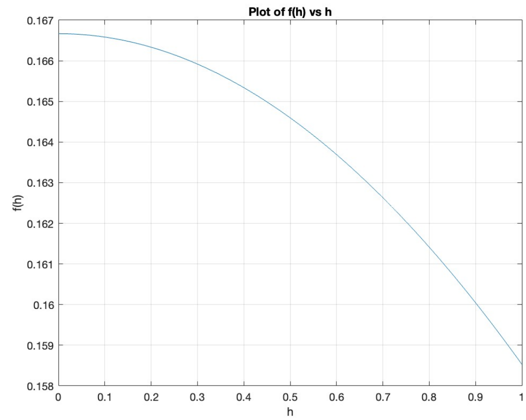
Computing *Assignment 1*

Q(a). The given funciton is $F(h) = \frac{(h-sin(h))}{h^3}$. I created a plot of $F(h)$ using range of values of h. The interval that I chose to plot my graph was $h \in [0, 1]$ with step size '0.0001'.
I was able to generate the graph given below from which it is clear that $F(h) \to \frac{1}{6}$



If the selected h values are too small, it may become a source of error as MATLAB is only able to store values upto certain significant digits, so incase the chosen value is too small, numerical precision limitations may affect the accuracy of the result. $\frac{0}{0}$ is causing our result to be undefined.

Q(b) As the given limit is in the form of $\frac{0}{0}$, we can utilize the L'hopitals rule in order to evaluate the limit analytically as $h \to 0$ as follows:

$$\lim_{h\to 0} \frac{(h - sin(h))}{h^3} = \lim_{h \to 0} \frac{(1 - cos(h))}{3h^2} \qquad \text{...differentiating num. \& den. w.r.t d(h)}$$

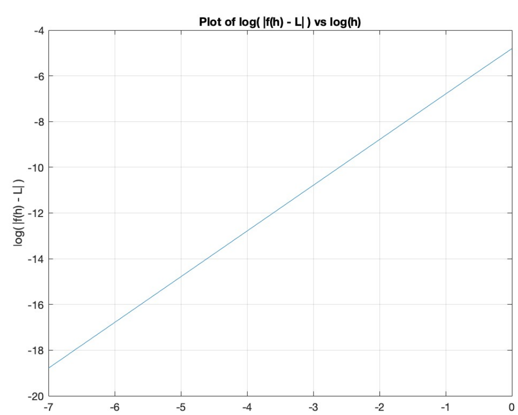
$$\lim_{h\to 0} \frac{sin(h)}{6h} = \lim_{h\to 0} \frac{cos(h)}{6} = \frac{1}{6} \lim_{h\to 0} cos(h) = \frac{1}{6}$$

$$\lim_{h\to 0} F(h) = \frac{1}{6}$$

Moving forward, we want to find the largest value of p for which $F(h) = L+O(h^p)$. I will do so using the plot of $\log(|F(h) - L|)$ v/s $\log(h)$. We know that for a straight line, the equation is $y = mx + b$. Hence, the slope of the $\log(|F(h) - L|)$ v/s $\log(h)$ graph should give us the approximate value of p as:

$$\log|F(h) - L| = m \log h + b \Rightarrow \log(|F(h) - L|) - \log h^m = b \Rightarrow \log \frac{|F(h) - L|}{h^m} = b$$

$$\Rightarrow 10^b = \frac{|F(h) - L|}{h^m} \Rightarrow h^m(c) = |F(h) - L|$$



I computed the slope of the graph to be 1.99, using MATLAB. Hence, the largest value of p for which the equation $F(h) = L + O(h^p)$ holds ≈ 2