Computing Assignment 3

A. The values of the three iterative approaches were determined up to a maximum of 75 iterations in order to ascertain whether they converge. The linked approaches were shown to be convergent if the error $|p_n - p| < 10^{-6}$ was confirmed to be true. The outcomes were as follows:

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Method 1: Approximation = 1.49130148, Iterations = 5, Converged = 1
Method 2: Approximation = 1.60957651, Iterations = 75, Converged = 0
Method 3: Approximation = 1.49130061, Iterations = 66, Converged = 1
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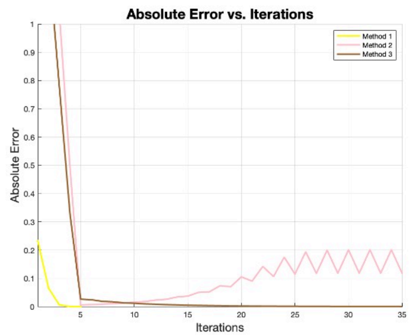
The following observation is drawn based on the received statistics-

- Method1 & Method3 converge within 75 iterations
- Method2 doesn't converge for 75 iterations
- Method1 converges faster than Method3 (ROC1 > ROC3)

B. A plot of absolute error vs. iterations is presented below to further solidify our conclusions in partA.



The following conclusions can be drawn from the graph

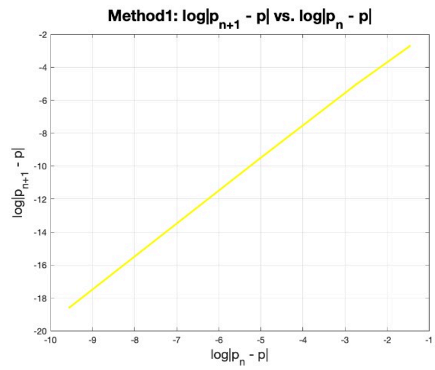
- Method2 diverges
- Method1 converges faster than Method3

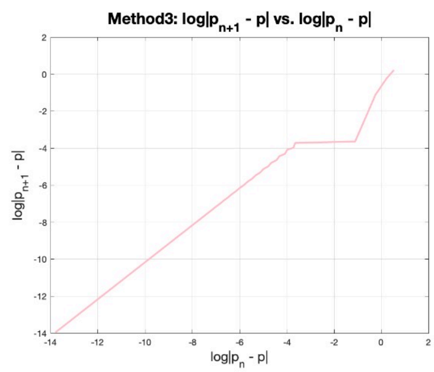
C. Take into consideration the following to calculate the asymptotic error constant (λ) and order of convergence (α). Suppose $\{p_n\}_{n=0}^{\infty}$ is a sequence that converges to p with $p_n \neq p$ for all n. If positive constants λ & α exist with $\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|} = \lambda$, then sequence $\{p_n\}_{n=0}^{\infty}$ converges to p of order α with asymptotic error constant λ.

Taking $e_{n+1} = |p_{n+1} - p|$ and $e_n = |p_n - p|$, we get $\frac{e_{n+1}}{e_n^{\alpha}} = \lambda$. Using log properties to simplify, we obtain: $\lambda \log e_{n+1} = \log \lambda + \alpha \log e_n$, which is basically a linear equation in the form of $y = mx + c$. In order to investigate this further, we display a log-log graph to find the corresponding order (α) & (λ).

The following conclusions can be drawn from the graph

1. Method1: $\alpha = 1.967575 \approx 2$ & $\lambda = e^{0.2565872} \approx 1.29$ i.e. Quadratically convergent
2. Method3: $\alpha = 0.9685492 \approx 1$ & $\lambda = e^{-0.4800419} \approx 0.61$ which is less than 1 and thus i.e. Linearly Convergent. see Def. 2.7 of text





Good job! 10