Introduction :

AC is acronym of alternating current. The Magnitude and direction of alternating current changes. In DC there are no such changes takes place.

Defination :

Any alternating quantity (voltage, current, Power) is defined as the one which changes its quantity as well as direction (polarity) with respect to time.

The alternating quantity can have other shapes such as triangular wave, square wave or a trapezoidal waveform. However we use sine waveforms because it has following advantages:

1. It produces less disturbance in an Electrical circuit
2. It is Easier and Economical to produce a sine waveform.
3. It can produce Rotating Magnetic field.
4. It can easily represented in phasor form and Easier for Mathematical Calculations.

All our applainces such as TV, refrigerator washing Machines, air conditioner, fans etc. operate on the alternating voltage (ac voltage).

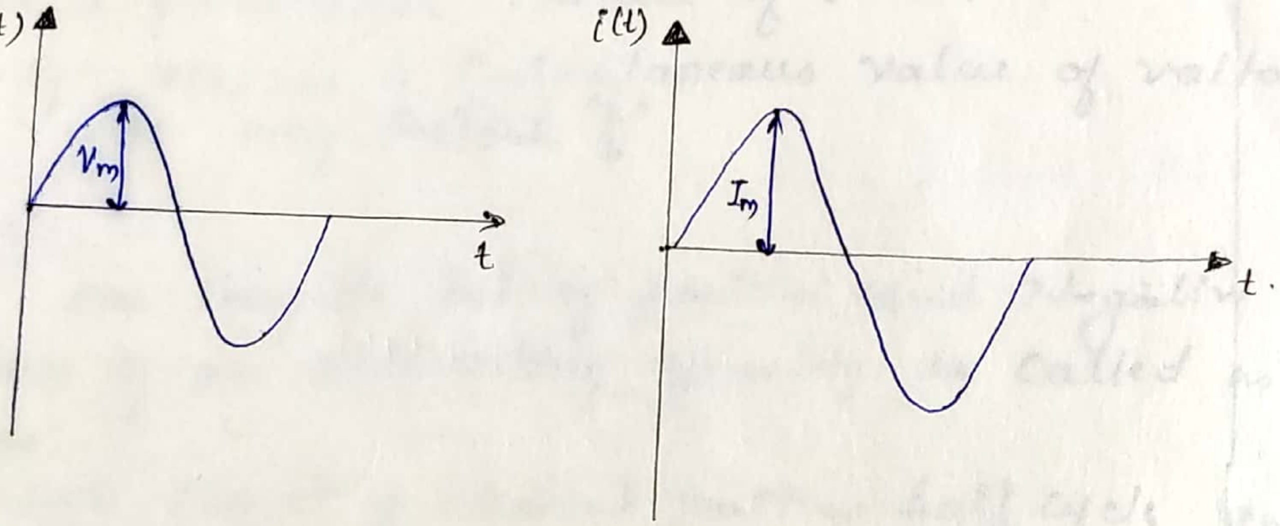
Some Important Definition in AC Terminology

1) **Waveform:**

A waveform is a graph of magnitude of a quantity with respect to time.

Time is plotted on x-axis and on y-axis the plotted quantities will be voltage, current or power.

Waveforms of Sinusoidal AC



Graphical representation

Mathematical Representation for both Waveforms as

$$v(t) = V_m \sin\theta = V_m \sin\omega t = V_m \sin(2\pi f t.)$$

$$i(t) = I_m \sin\theta = I_m \sin\omega t = I_m \sin(2\pi f)t.$$

Where,

$v(t) \Rightarrow$ Instantaneous Voltage.

$i(t) \rightarrow$ Instantaneous current.

$f \rightarrow$ Frequency in Hz. ($f = 1/T$).

$V_m \rightarrow$ Peak value (Maximum value) of voltage.

$I_m \rightarrow$ peak value (Maximum value) of current

2) Amplitude

The Maximum or peak value of an alternating quantity is called as its amplitude. The Amplitude is denoted by V_m for voltage & I_m for current.

3) Instantaneous value

The instantaneous value of an ac quantity is defined as value of the quantity at a particular instant of time.

for Eg: $v(t)$ is a instantaneous value of voltage at any instant 't'

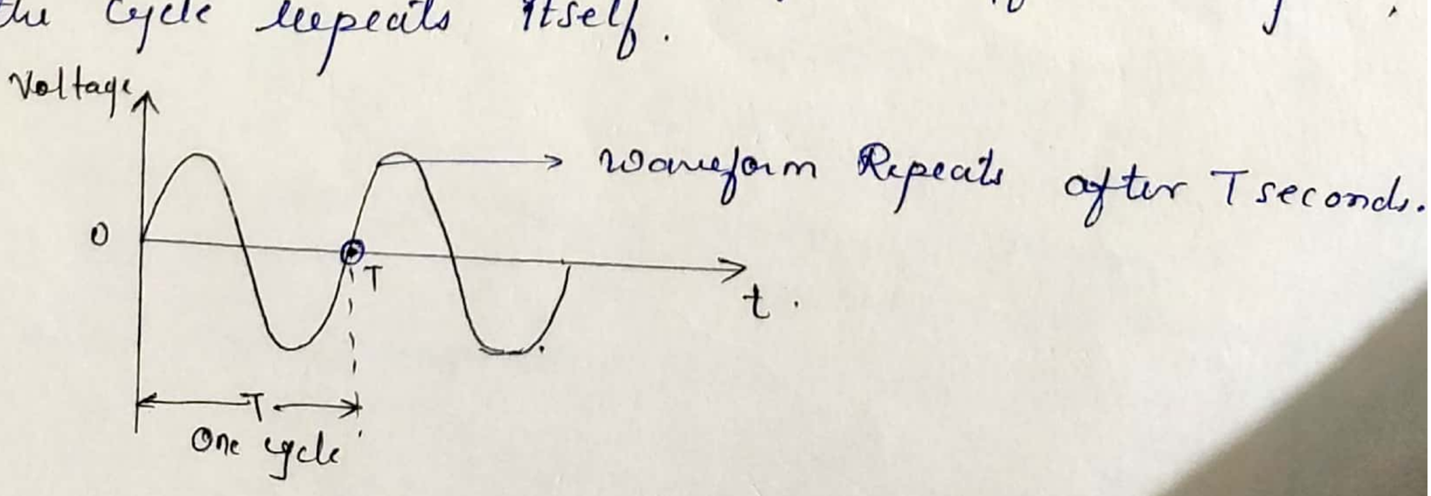
) Cycle

One Complete set of positive and Negative values of an alternating quantity is called as cycle.

One cycle consist of identical positive half cycle and Negative half cycle.

) Time period

The Time period is defined as the time taken in seconds by the Waveform of an a.c quantity to complete one cycle. After Every T, the cycle repeats itself.



Frequency

Frequency is defined as number of cycles completed by an alternating quantity in one second. It is denoted by 'F'

Unit is cycles / seconds or Hertz. (H_z)

$$\text{Frequency } (F) = \frac{\text{cycles}}{\text{seconds}}$$

$$= \frac{1}{\text{seconds/cycle}} = \frac{1}{T}$$

$$\boxed{f = \frac{1}{T}} \; H_z$$

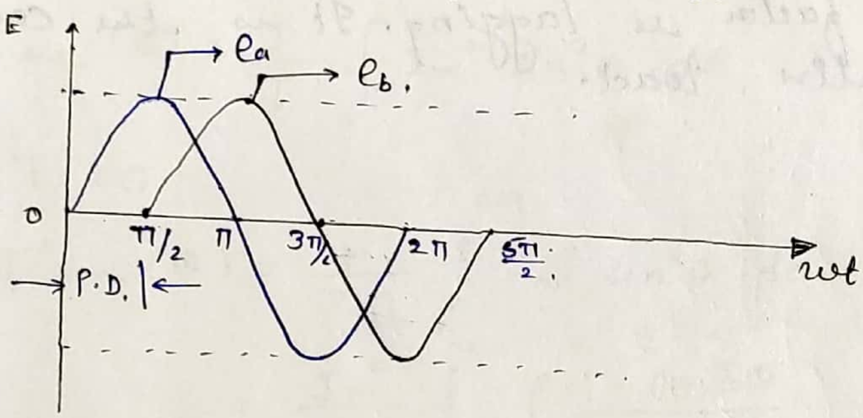
Therefore, with increase in time period, the frequency decreases and vice versa.

(7) **Phase**

It is a part of time period after the alternating quantity passed through zero position.

(8) **Phase difference**

If the two alternating quantity do not reach their zero value in the same direction simultaneously, they have phase difference. The difference in phase angle between two quantities of same reference axis is known as phase difference.



e_a & e_b has a phase difference of $\pi/2$.

(9) **Angular velocity: (ω)**

The angular velocity is the rate of change of angle (ωt) with respect to time.

$$\therefore \quad \omega = \frac{d\theta}{dt}.$$

Where, $d\theta$ is the change in angle in time dt

If $dt = T$; i.e. time period (one cycle).

then ~~cor~~ corresponding change in θ. is 2π radian

$$d\theta = 2\pi.$$

$$\therefore \quad \omega = \frac{2\pi}{T.} \qquad \text{As} \quad T = 1/f.$$

$$\therefore \quad \boxed{\omega = 2\pi f}$$

(10) Power factor.

It is the cosine of angle between volt & current of same circuit.

$$P.f = \cos\phi = \frac{R}{Z}.$$

(11) Leading Power factor

When current leads. the applied voltage power factor is leading. It is in case of Capacitive load.

(12) Lagging Power factor

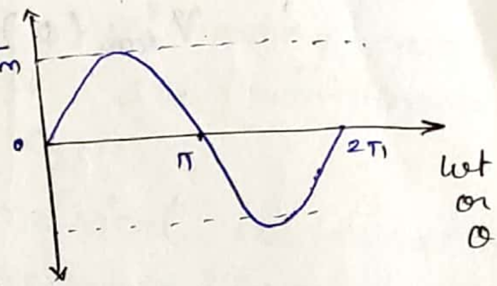
when current lags the applied volt Power factor is lagging. It is the case of Inductive load.

Root Mean Square of current

It is the value of DC current when flowing through a given circuit for a given time produces the same amount of heat as produced by alternating current when flowing through a same circuit for the same time.

RMS value can be calculated as.

$$I^2_{rms}(\theta) = \frac{1}{T}\int_0^T I^2(\theta).d\theta.$$



where T is the time period,

As $I(\theta) = I_m \sin\theta.$

$$\therefore I^2_{rms}(\theta) = \frac{1}{T}\int_0^T I_m^2 \sin^2\theta . d\theta.$$

As $T = 2\pi$.

$$\therefore I^2_{rms}(\theta) = \frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta . d\theta.$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right).d\theta.$$

$$I^2_{rms}(\theta) = \frac{I_m^2}{2\pi}\times\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}.$$

$$= \frac{I_m^2}{2\pi}\times\frac{1}{2}\left[\cancel{(2\pi-0)-(0-}\ \left(2\pi - \frac{\sin 4\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2\pi}\times\frac{1}{2}\left[2\pi - 0 - 0 - 0\right]$$

$$= \frac{I_m^2}{\cancel{2\pi}}\times\frac{1}{2}\times\cancel{2\pi}$$

$$I^2_{rms}(\theta) = \frac{I_m^2}{(\sqrt{2})^2} \Rightarrow I_{rms}(\theta) = \frac{I_m}{\sqrt{2}}$$

$$\therefore \boxed{I_{rms}(\theta) = \frac{I_m}{\sqrt{2}} = 0.707\, I_m}$$

Root mean square value of voltage :

The Mean of square of Instantaneous value of voltage over a complete cycle is given as.

$$V_{rms}^2(\theta) = \frac{1}{T}\int_0^T v^2(\theta).d\theta.$$

As $T = \frac{1}{2\pi}$ and $v(\theta) = V_m \sin \omega t$.

$$\therefore V_{rms}^2(\theta) = \frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta .d\theta.$$

$$= \frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right).d\theta.$$

$$= \frac{V_m^2}{2\pi} \times \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

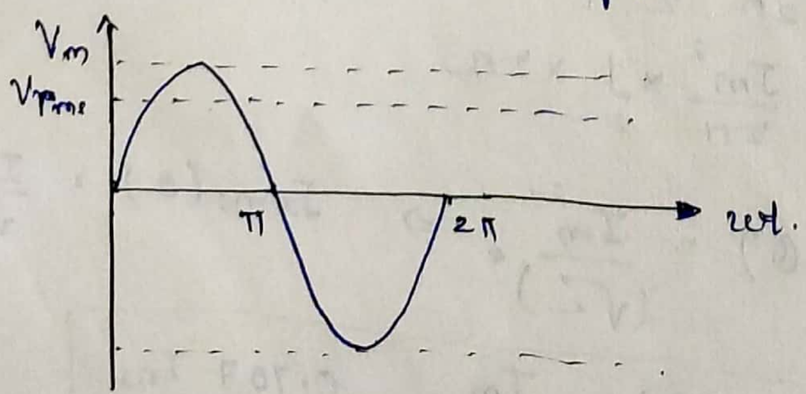
$$= \frac{V_m^2}{2\pi} \times \frac{1}{2}\left[2\pi - \frac{\sin 4\pi}{2} - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{V_m^2}{2\pi} \times \frac{1}{2}\left[2\pi + 0 - 0 - 0\right]$$

$$V_{rms}^2(\theta) = \frac{V_m^2}{2}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m.}$$

Or. RMS value $= \frac{\text{Max. value}}{\sqrt{2}}$



Mean or Average value:

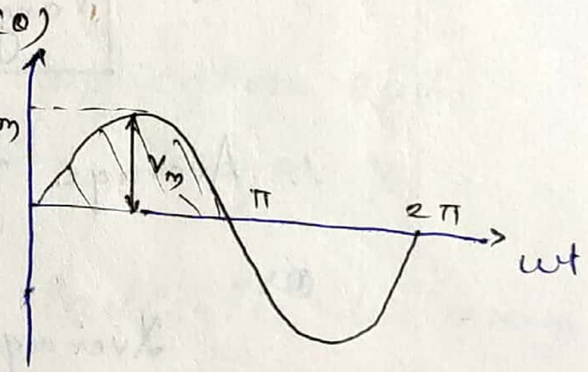
Average value of current:

The average of value of alternating current is the D.C. current which transfer across any circuit, the same charge as if transferred by that alternating current during same time.

(The average value of an alternating quantity is equal to the average of all, the instantaneous values over a period of half cycle.)
The average value of a symmetrical alternating waveform is determined from just one half cycle because for full cycle, is zero.

The average value can be calculated as.

$$I_{av} = I_{dc} = \frac{1}{\pi}\int_0^{\pi} i(\theta).d\theta.$$



As $i(\theta) = I_m \sin\omega t$
$= I_m \sin\theta$.

$$\therefore I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta.d\theta.$$

$$= \frac{1}{\pi} \times I_m \int_0^{\pi} \sin\theta.d\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[-\cos\pi - (-\cos 0)\right]$$

$$= \frac{I_m}{\pi}\left[-(-1) - (-1)\right] = \frac{I_m}{\pi}(1+1)$$

$$= \frac{2I_m}{\pi}$$

$$\therefore \boxed{I_{avg} = 0.637\, I_m}$$

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Average value of Voltage

The average value of voltage can be Calculated as.

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v(\theta).d\theta.$$

But $v(\theta) = V_m \sin\theta$.

$$\therefore\ V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta . d\theta.$$

$$\therefore\ V_{av} = \frac{V_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi - (-\cos 0)\right]$$

$$= \frac{V_m}{\pi}(1+1) = \frac{2V_m}{\pi}.$$

$$\therefore\ \boxed{V_{avg} = 0.637 V_m}.$$

$\therefore$ Average value = 0.637 × Max. value.

Or.

$$\text{Average value} = \frac{2 \times \text{Maximum value}}{\pi}.$$

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* Practical Significance of RMS value

1. The specified value of a.c. supply voltage is 230 V, this is the RMS value of a.c supply voltage, so generally RMS value are specified.
2. The intensity of Bulbs, heat Produced by the iron, water heater, boiler etc is Proportional to the square of RMS value of a.c voltage or current
3. The a.c. voltmeter and ammeter indicate RMS value of a.c. voltage or current being measured.

* Practical significance of Average value

1. Average value is required in finding the charge transferred in Capacitor circuits and Electro-Chemical circuit.
2. It is also required in the applications such as dc motor control, battery chargers, rectifying circuits etc.
3. The dc ammeter & voltmeter indicates the average value
4.

Form factor

The form factor of an alternating quantity is defined as ratio of RMS value to average value.

$$\therefore \text{form factor} = K_f = \frac{\text{RMS value}}{\text{Avg. value}} = \frac{0.707\ \text{Max}}{0.637\ \text{max.}}$$

$$\therefore \underline{K_f = \text{form factor} = 1.11.}$$

$K_f = 1.11$ is valid only for sinusoidal (sine or cosine) ac quantities. For other shapes of ac quantities the form factor is different from 1.11.

Peak factor / Crest factor / Amplitude factor

The Peak factor of an alternating quantity is defined as the Ratio of Max. value to RMS value.

$$\therefore \text{Peak factor} = k_p = \frac{\text{Max value}}{\text{RMS Value}}$$

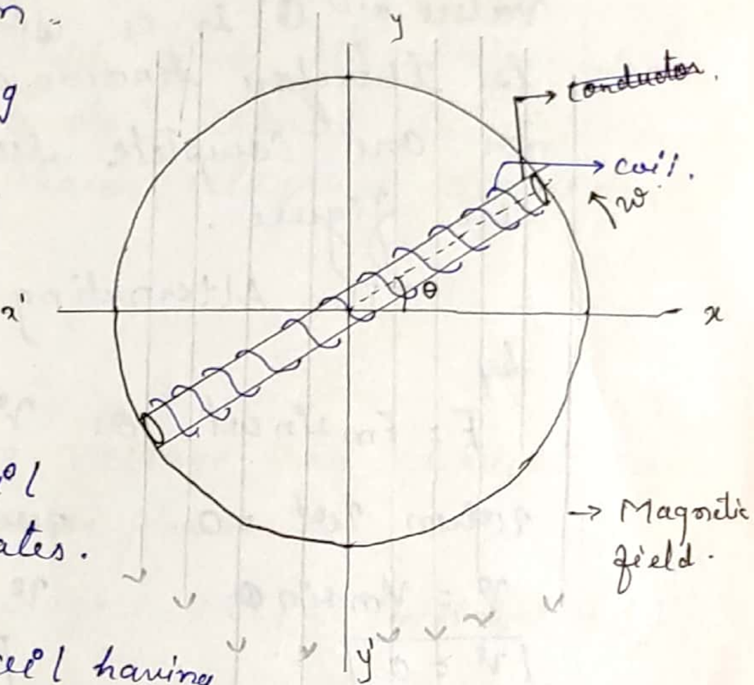
$$= \frac{\text{Max. value}}{(\text{Max. value}/\sqrt{2})}$$

$$\boxed{k_p = \sqrt{2}}$$

$$\therefore \text{Peak factor} = k_p = \sqrt{2} = 1.414$$

Generation of alternating voltage & current

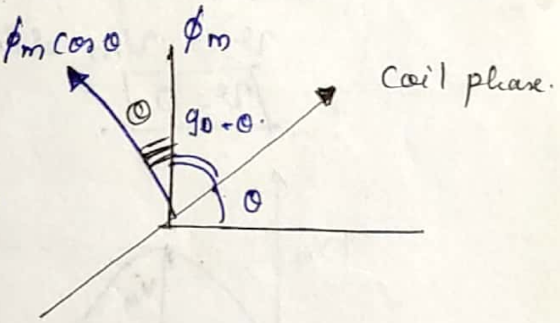
By faraday's law of Electromagnetic Induction.
when ever flux linking with the coil changes, an emf is induced in it.
The Magnitude of EMF generated in the coil depends upon strength of Magnetic field & speed ~~of~~ at which coil or Magnetic field rotates.



Let consider that coil having 'N'. no. of turns is Rotating in an uniform magnetic field at an angular velocity of ω rad/sec.

It starts with intial position of x-x' and rotated by an angle $\theta°$ in time 't' sec in anticlockwise direction.

$\theta = \omega t$.



At this instant, flux are perpendicular to the coil $\phi_m \cos\omega t$.

where, ϕ_m is maximum possible flux.

By faraday's law.

$$E = -N\frac{d\phi}{dt}.$$

$$= -N\frac{d}{dt}(\phi_m \cos\omega t).$$

$$= -N\phi_m \frac{d}{dt}\cos\omega t = +\omega N\phi_m \sin\omega t$$

$$\boxed{E = E_m \sin\omega t}$$

where, $E_m = N\phi_m\omega$

Or, $v = V_m \sin \omega t$

Hence voltage produced will depend on the value of $\sin \omega t$ and curve obtained for diff value of 'θ' is a sine curve. The Emf induced is therefore known as Sinusoidal EMF Waveform for one complete rotation of coil is as shown in figure.

The Alternating EMF can be represented by.

$E = E_m \sin \omega t$ or $v = V_m \sin \omega t$.

when $\omega t = 0$.

$v = V_m \sin 0$

$\boxed{v = 0}$.

when $\omega t = 90°$.

$v = V_m \sin 90°$

$\boxed{v = V_m}$.

when $\omega t = 180°$.

$v = V_m \sin 180°$

$\boxed{v = 0}$

when $\omega t = 270°$.

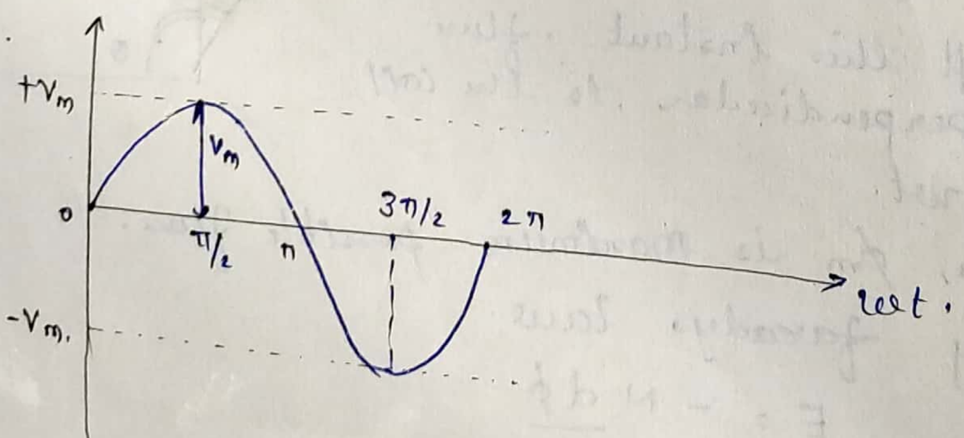
$v = V_m \sin 270$.

$\boxed{v = -V_m}$

when $\omega t = 360°$.

$v = V_m \sin 360°$.

$\boxed{v = 0}$.



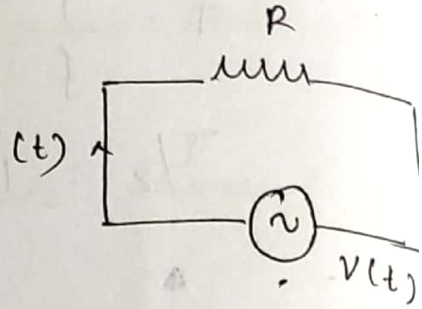
Similarly alternating current represented by

$i = I_m \sin \omega t$.

$i = I_m \sin 2\pi f t$

Basic A.C circuit

① A.C. through Pure Resistance:



$v(t) = V_m \sin\omega t$

Consider an A.C. circuit containing a non-inductive Resistance of R Ω connected across a sinusoidal voltage given by:

$$v(t) = V_m \sin \omega t \quad — (1).$$

When the current flows through Pure Resistance R changes, no back emf. setsup.

$$v = iR.$$

$$v(t) = i(t) \cdot R.$$

$$i(t) = \frac{v(t)}{R} = \frac{V_m \sin \omega t}{R}.$$

As $\frac{V_m}{R} = I_m$.

$$\therefore \quad i(t) = I_m \sin \omega t \quad —(2).$$

From Eqn ① & ② it can be concluded that in case of Resistances the current and voltage are in same phase.

Phasor diagram:

Power :

Instantaneous Power

$$P = v(t)\, i(t)$$

$$P = V_m I_m \sin^2 \omega t$$

$$P = V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

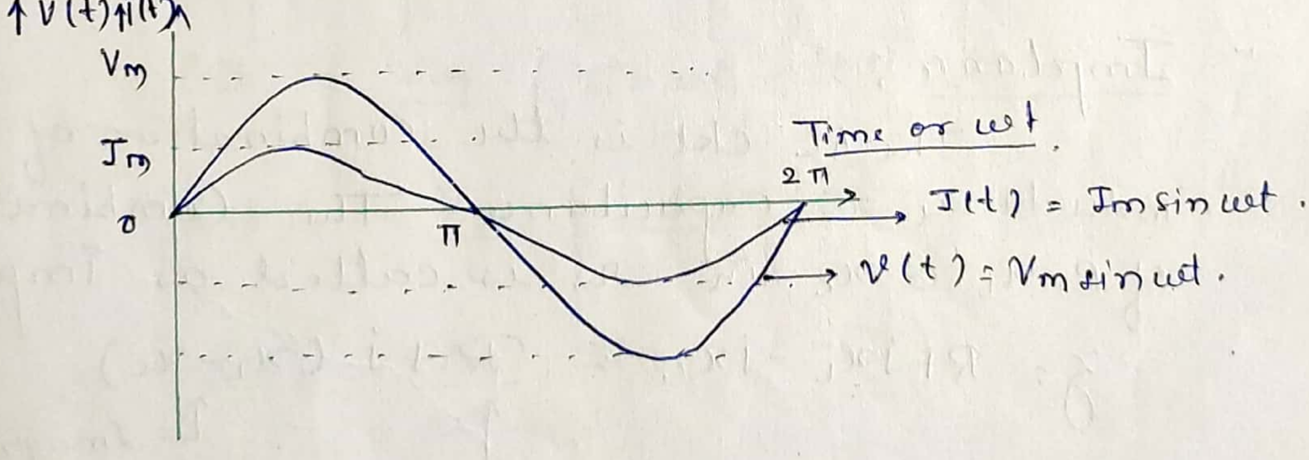
→ for a complete cycle the average power value of $\frac{V_m I_m}{2} \cos 2\omega t$ is zero

$$\therefore \quad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad P_{av} = V_{rms} \cdot I_{rms}$$

$\therefore$ $P_{av} = V_{rms} \cdot I_{rms}$ is the Power Eq^n in case of pure Resistance.

Waveform :



Reactance and Impedence

In d.c. circuit, we have defined R as opposition to the flow of current. Similarly for ac circuit, we defined two terms namely reactance 'X' and impedence 'Z'

Reactance

Reactance can be of two types.

1. Inductive Reactance (X_L)
2. Capacitive Reactance (X_C).

1. Inductive Reactance

$$X_L = 2\pi f L = \omega L \ (\Omega).$$

2. Capacitive Reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \ (\Omega).$$

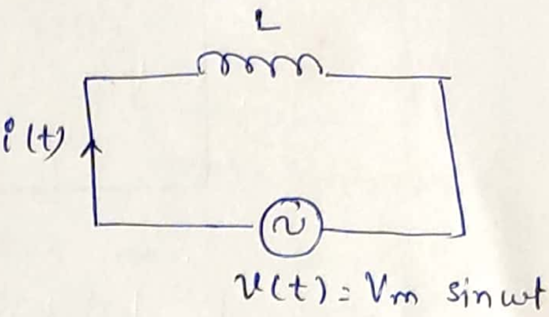
Impedance :

The AC ckt is the combination of Re inductance & capacitance. The combination of R, X_L & X_C in Ω is called as impedence

$$Z = R + jX_L - jX_C = \underbrace{R}_{\text{Real part}} + j\underbrace{(X_L - X_C)}_{\text{Imaginary part}}.$$

(2) A.C. through Pure Inductance

Whenever an alternating voltage is applied to a purely inductive coil, a back EMF is produced, due to self inductance of coil. The Back EMF opposes the rise & fall of current through the coil.



Let the self ~~EMF~~ inductance of coil and voltage induced in it given by.

$$V_L = v(t) = L \frac{di(t)}{dt}$$

As,

$$v(t) = V_m \sin wt \quad \text{— (1)}$$

$$V_m \sin wt = L \frac{di(t)}{dt}$$

Integrating above Eqn.

$$i(t) = \frac{V_m}{L} \int \sin wt \cdot dt$$

$$i(t) = \frac{V_m}{L} \left[- \frac{\cos wt}{w} \right]$$

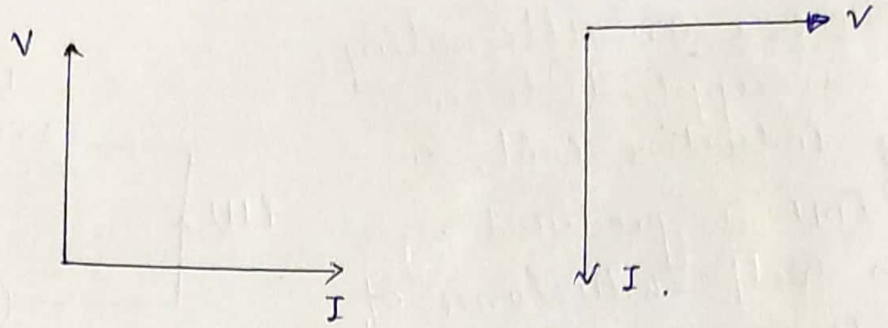
$$i(t) = \frac{V_m}{wL} \sin \left(wt - \frac{\pi}{2} \right)$$

$$i(t) = \frac{V_m}{x_L} \sin \left(wt - \frac{\pi}{2} \right)$$

$$i(t) = I_m \sin \left(wt - \frac{\pi}{2} \right) \quad \text{— (2)}$$

From Expression of $v(t)$ and $i(t)$, it can be concluded that, the current through pure inductor lags to the applied voltage by an angle $\pi/$

Phasor diagram / vector diagram



Power :

Instantaneous Power.

$$P = v \times I.$$

$$P = V_m \sin wt \times I_m \sin\left(wt - \frac{\pi}{2}\right)$$

$$\sin\left(wt - \frac{\pi}{2}\right) = -\cos wt.$$

$$P = -V_m I_m \cos wt \sin wt.$$

But $\sin wt \cos wt = \dfrac{\sin 2wt}{2}$

$$\therefore P = -\frac{V_m I_m}{2} \sin 2wt$$

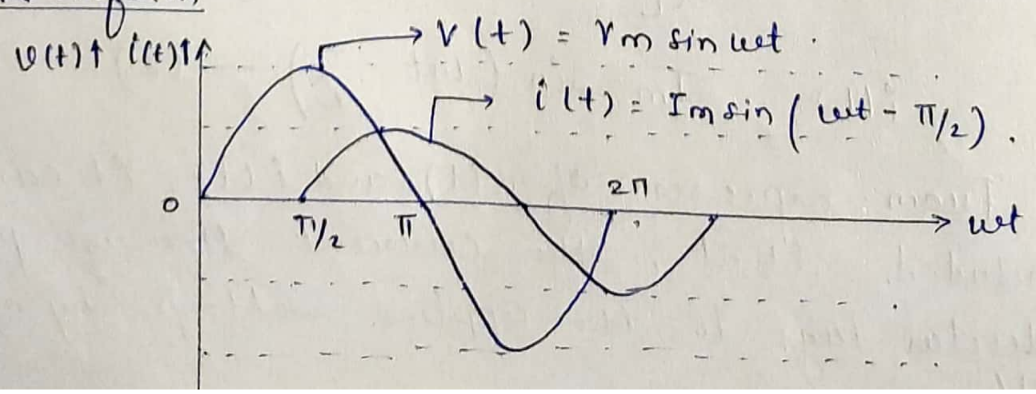
⇒ **Average power**

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2wt \cdot dwt.$$

$$\boxed{P_{av} = 0}.$$

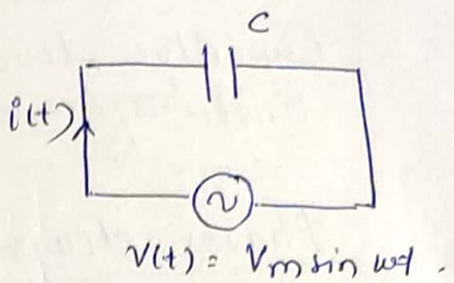
The average value over a complete cycle is 0

Waveform



③ AC through Pure Capacitor

when an alternating voltage is applied to the plates of capacitor is given by



$v(t) = V_m \sin wt$ —(1).

The capacitor is charge in one direction & then opposes its direction for other half cycle.

Let v = P.d developed between plates at any instant.

q = Charge on plates at that instant.

then

$q = Cv$.

and as $v(t) = V_m \sin wt$.

$\therefore \quad q = C \times V_m \sin wt$. —(1).

Current is Rate of flow of charge.

$$\frac{dq}{dt} = i(t).$$

$\therefore$ Differentiate Eqn ①.

$$\therefore \quad \frac{dq}{dt} = C V_m \frac{d}{dt} \sin wt.$$

$$\therefore \quad i(t) = C V_m \times \cos wt \times w.$$

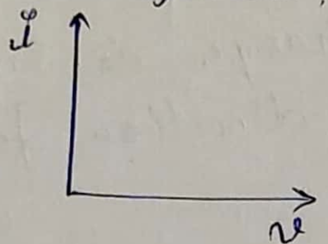
$$= \frac{V_m}{1/wC} \cos wt.$$

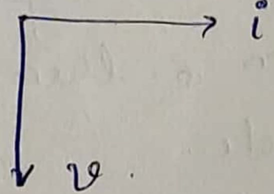
$$\therefore \quad i(t) = \frac{V_m}{X_c} \sin\left(wt + \frac{\pi}{2}\right).$$

$$\therefore \quad \boxed{i(t) = \frac{V_m}{X_c} \sin\left(wt + \frac{\pi}{2}\right)} \quad —(2).$$

from the Expression of $v(t)$ & $i(t)$ it
be Concluded that the current through pu
Capacitor leads to the applied voltage by an
angle '$\pi/2$'

Phasor diagram



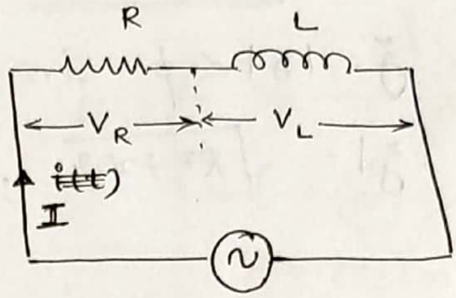


Series A-C Circuit :

Series R-L circuit.

The series R-L circuit is shown which are Connected across a voltage source.

$\underline{v = V_m \sin \omega t}$



$v(t) = V_m \sin \omega t$.

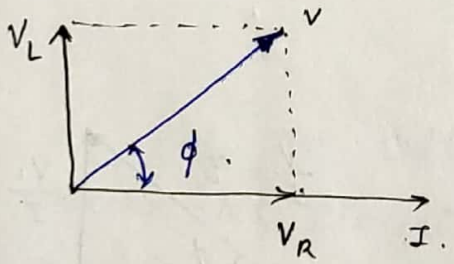
The current flowing through R & L is same as they are Connected in series. But voltage across R & L is.

$V_R = R \cdot I$.

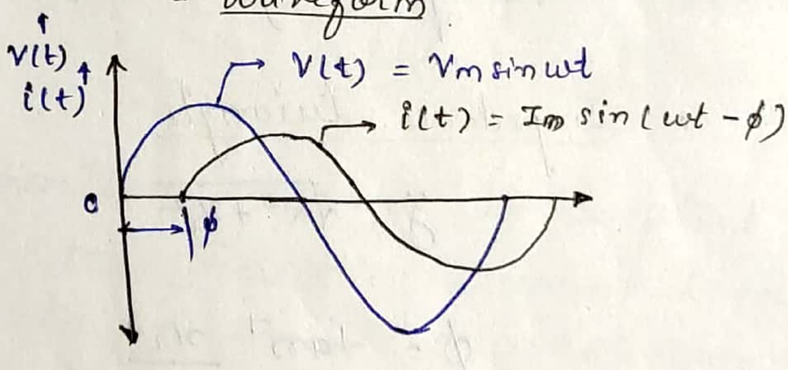
$V_L = I \cdot X_L$

The applied voltage 'v' is the vector sum of V_R & V_L

- Phasor diagram



- Waveform



If an alternating voltage is applied to series R-L circuit then Resulting current lags to applied voltage by an angle ϕ as shown.

$\therefore$ $\underline{v = V_m \sin \omega t = V \angle 0}$ is applied to a series R-L circuit the Eqn of current is.

$$\underline{i = I_m \sin(\omega t - \phi) = I \angle -\phi \text{ Amp.}}$$

- Impedence of R-L circuit

$$\underline{Z = R + jX_L} \text{ in Rectangular}$$

$$Z = |Z| \angle \phi \text{ in polar.}$$

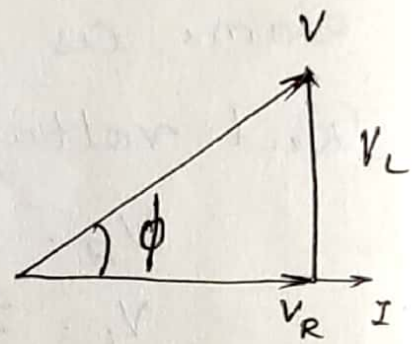
$$|Z| = \sqrt{R^2 + X_L^2} \quad ; \quad \phi = \tan^{-1} \frac{X_L}{R}.$$

- <u>Voltage triangle</u>

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$



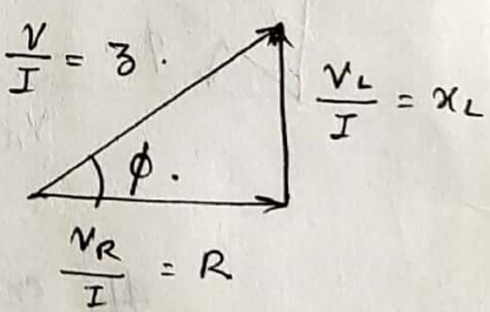
$$I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z}.$$

The quantity $\sqrt{R^2 + X_L^2}$ is known as impedence

- <u>Impedence triangle</u>

$$Z = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1} \frac{X_L}{R}$$



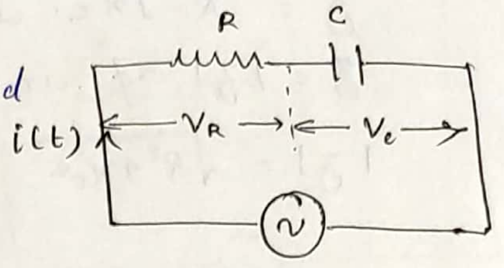
* Power factor $= \cos\phi = \frac{R}{Z}.$

* lagging in Nature

Series RC circuit

The series RC circuit is shown which are connected across a voltage source.

$v(t) = V_m \sin \omega t$

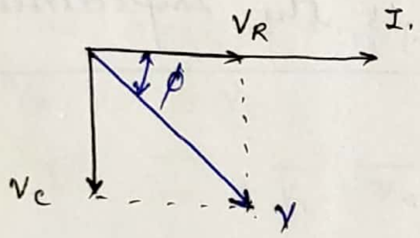


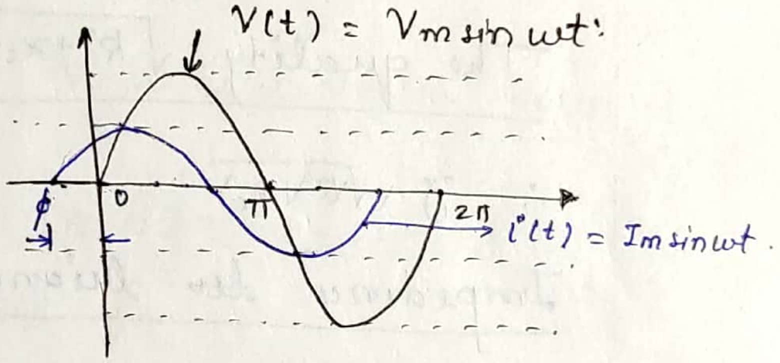
The current flowing through RC series ckt is same. But, voltage across R & C is

$V_R = I.R.$

$V_c = I.X_c.$

The applied voltage 'v' is the vector sum of V_R & V_L





If an alternating voltage is applied to series RC circuit the Resulting current leads to applied voltage by an angle ϕ as shown.

$\therefore$ $\underline{v = V_m \sin \omega t} = V \angle 0$ is applied to series RC circuit then the Eq^n of current is.

$i = I_m \sin(\omega t + \phi) = I \angle \phi$ Amp.

- <u>Impedance of RC circuit</u>

$Z = R - jX_C$ in Rectangular

$Z = |Z| \angle \phi$ in polar.

$|Z| = \sqrt{R^2 + X_C^2}$ $\phi = \tan^{-1}\left(+\frac{X_C}{R}\right)$.

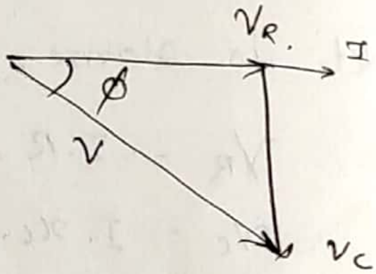
<u>Voltage Triangle</u>

$V = \sqrt{V_R^2 + V_C^2}$

$= \sqrt{(IR)^2 + (IX_C)^2}$

$V = I\sqrt{R^2 + X_C^2}$

$\therefore \frac{V}{I} = \sqrt{R^2 + X_C^2} = Z$.



The quality $\sqrt{R^2 + X_C^2}$ is the Impedance.

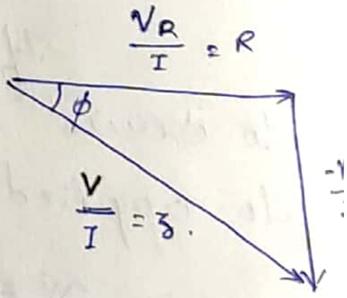
$\therefore Z = \sqrt{R^2 + X_C^2}$.

<u>Impedence triangle</u>

$\therefore Z = \sqrt{R^2 + X_C^2}$.

$= R - jX_C$.

$\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$.

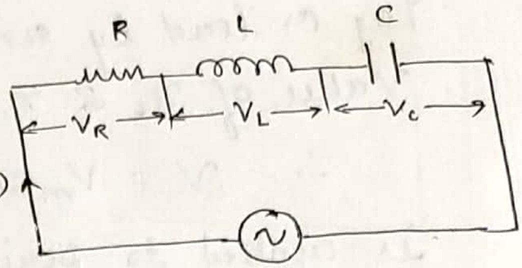


* Power factor $= \cos\phi = \frac{R}{Z}$
* leading in Nature

Series RLC circuit

The series R-L-C ckt is shown. which are connected across a voltage source.

$v(t) = V_m \sin\omega t$



$v(t) = V_m \sin\omega t$.

The current through RLC series ckt is same, But voltage across R, L & C are,

$V_R = I.R \Rightarrow$ Voltage across 'R'.

$V_L = I.x_L \Rightarrow$ voltage across 'L'.

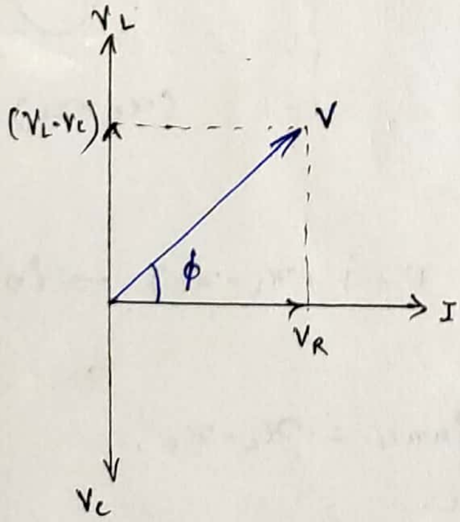
$V_C = I x_c \Rightarrow$ voltage across 'C'

The applied voltage 'v' is the vector sum of V_R, V_L & V_C,

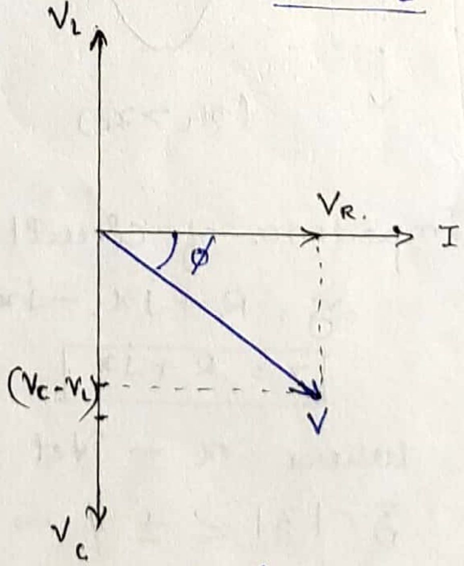
$$V = \overline{V_R} + \overline{V_L} + \overline{V_C} = IR + jIx_L - jIx_c$$

$$= I(R + jx_L - jx_c)$$

when, $x_L > x_c$.
$V_L > V_c$



when, $x_c > x_L$
$V_c > V_L$



I lags the applied voltage 'v' by an angle 'ϕ'

I leads to applied voltage 'v' by an angle ϕ.

If an alternating voltage is applied to series R-L-C circuit then resulting current will lag or lead by an angle ϕ will depend on the value of x_L & x_c.

$\therefore$ $v = V_m \sin \omega t = v \angle 0$

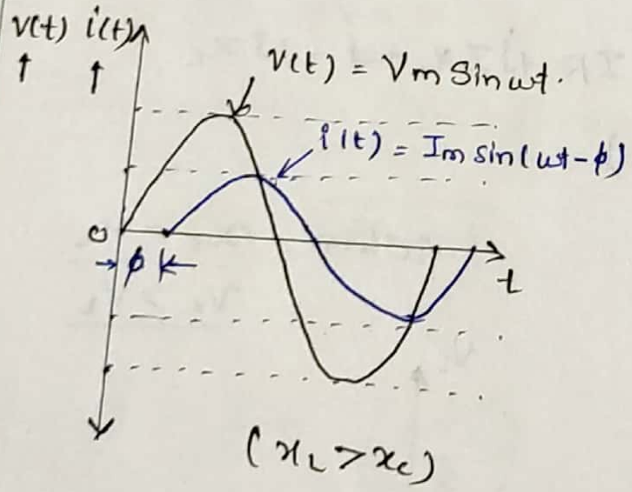
is applied to series RLC ckt then the eqn of current is

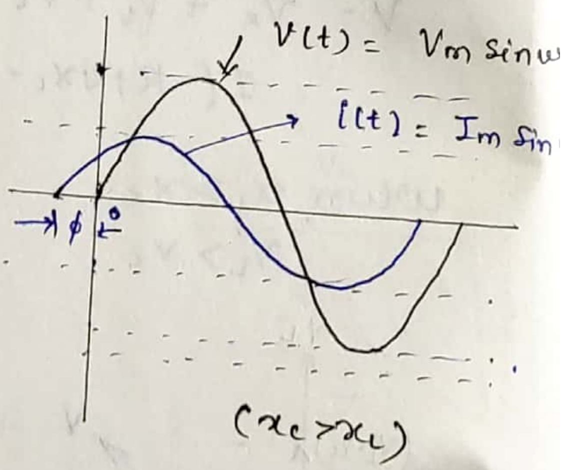
$$i = I_m \sin(\omega t \pm \phi).$$

The +ve sign indicate that current lead to the applied voltage by an angle ϕ i.e ($x_c >$

The -ve sign indicate that current lags to the applied voltage by an angle ϕ i.e. ($x_L > x_c$).

Waveform.





Impedence of circuit

$Z = R + jx_L - jx_c = R + j(x_L - x_c) \rightarrow$ in Rectang

$\boxed{Z = R + jx}$

Where $x \rightarrow$ Net Reactance $= x_L - x_c$.

$Z = |Z| \angle \pm \phi \rightarrow$ polar.

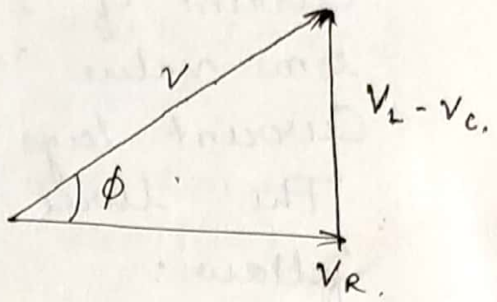
where $|Z| = \sqrt{R^2 + (x_L - x_c)^2}$ and $\phi = \tan^{-1}\left(\frac{x_L - x_c}{R}\right)$.

<u>Voltage Triangle</u> $(x_L > x_C)$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (Ix_L - Ix_C)^2}$$

$$= I\sqrt{R^2 + (x_L - x_C)^2}$$



$$\frac{V}{I} = \sqrt{R^2 + (x_L - x_C)^2}$$

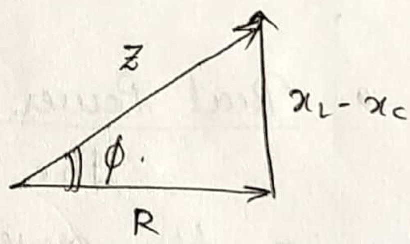
$$I = \frac{V}{\sqrt{R^2 + (x_L - x_C)^2}} = \frac{V}{Z}$$

The quantity $\sqrt{R^2 + (x_L - x_C)^2}$ is known as Impedence.

- <u>Impedence triangle</u>. $(x_L > x_C)$

$$Z = \sqrt{R^2 + (x_L - x_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{x_L - x_C}{R}\right).$$



Active, Reactive & Apparent Power :

Let a series RL circuit draws a current of 'I' when an alternating voltage of rms value 'V' is applied to it. Suppose the current lags behind the applied voltage by ϕ. The three Powers drawn by ckt are as follows:

Apparent Power : (S)

It is the Product of rms value of voltage & current
It is denoted by 'S'
It's unit is VA or KVA.

$$\text{Apparent Power} = S = V \times I \quad \text{volt Ampere}$$
$$= I^2 Z \quad VA.$$

Real Power or True Power or Active Power : (P)

The True Power or real Power is defined as the average power taken by or consumed by a given circuit or Power dissipated by Resistor.
It is denoted by 'P'
Its unit is watt.

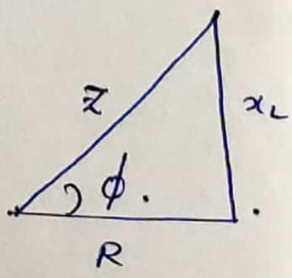
$$\text{Active Power} = P = I^2 R = I^2 (Z\cos\phi) = V \cdot I \cos\phi.$$

As $\cos\phi = \frac{R}{Z}$

$\therefore R = Z\cos\phi$

$\therefore P = I^2 Z \cos\phi.$

$= I(I \cdot Z)\cos\phi.$



$\underline{P = V \cdot I \cos\phi}$ or

$P = I^2 R.$

Reactive Power :

It is Power developed in Reactive Part. It is defined as the Product of V & I and Sine of angle between V & I i.e. ϕ.

Its unit is KVAR or VAR (volt ampere reactive).

Reactive Power : $Q = I^2 \cancel{Z} \cdot x_L$

$= I^2 Z \sin\phi$.

$\therefore$ from Impedence triangle,

$x_L = Z \sin\phi$.

$\therefore Q = I^2 Z \sin\phi$.

$Q = I(I \cdot Z) \cdot \sin\phi$.

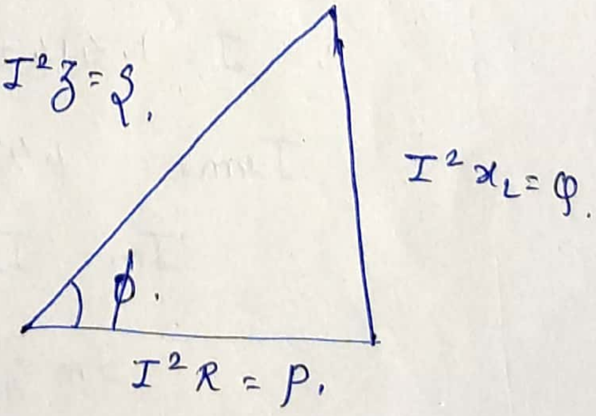
$\therefore$ $\underline{Q = VI \sin\phi \text{ VAR}}$.

Or.

$\underline{Q = I^2 x_L \text{ VAR}}$

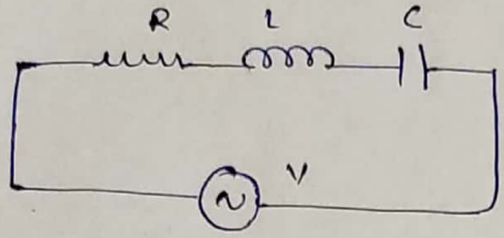
- from power triangle

$S = \sqrt{P^2 + Q^2}$



$\Rightarrow \cos\phi = \dfrac{P}{S}$

Series Resonance.



A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance the circuit behaves as a pure resistance & the net reactance is zero.

$$\bar{Z} = R + jx_L - jx_C$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right).$$

⇒ At resonance ; Z must be resistive :
Therefore the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$; $f_0 \to$ resonant frequency.

⇒ Power factor.

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (x_L - x_C)^2}}$$

$$\cos\phi = \frac{R}{R}.$$

$$P.f = \frac{R}{R} = 1.$$

⇒ Current : Since Impedance is minimum, the current is maximum at resonance. Thus circuit accepts more current and as such, R-L-C under resonance is called as acceptor circuit ;

$$I_0 = \frac{V}{Z} = \frac{V}{R}.$$

⇒ . Voltage :

At resonance.

$$x_L = x_C.$$

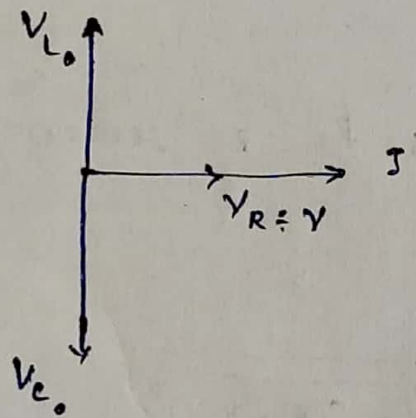
$$x_L \times I_0 = x_C \times I_0.$$

$$V_{L_0} = V_{C_0}.$$

Potential difference across inductance Equal to Potential difference across Capacitance being Equal & opposite ; Cancel Each other.

→ It is called as voltage Magnification Circuit.

<u>Phasor diagram</u>.



Reactance & Impedance :

In dc circuit resistance is the opposition to flow of current but in ac circuit resistance and reactance and Impedance are the opposition to flow of current.

① Reactance :

There are two types of Reactance.
① Inductive Reactance.
② Capacitive Reactance

ⓐ Inductive Reactance:

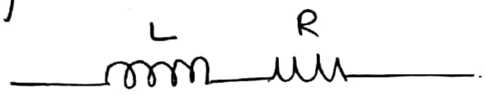
$$x_L = \cancel{2\pi\omega L} \; 2\pi f L = \omega L (\Omega)$$

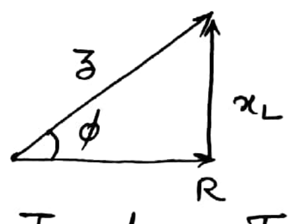
ⓑ Capacitive Reactance :

$$x_c = \frac{1}{2\pi f c} = \frac{1}{\omega c} \Omega$$

② Impedance :

The ac circuit is combination of R, L, and C. The value of whole branch in ohm is called as Impedance

①


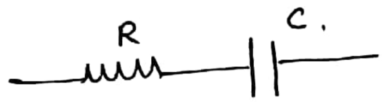


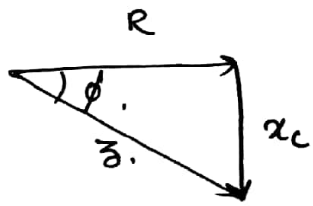
Impedance Triangle.

$$Z = R + j x_L$$

$$|Z| = \sqrt{R^2 + x_L^2}$$

$$\cos\phi = \frac{R}{Z}.$$

②




$$Z = R - j x_c$$

$$|Z| = \sqrt{R^2 + x_c^2}$$

$$\cos\phi = \frac{R}{z}.$$

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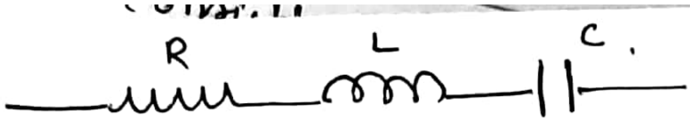
(3) 

$$Z = R + jx_L - jx_c$$

$$Z = R + j(x_L - x_c).$$

$$Z = R + jX.$$

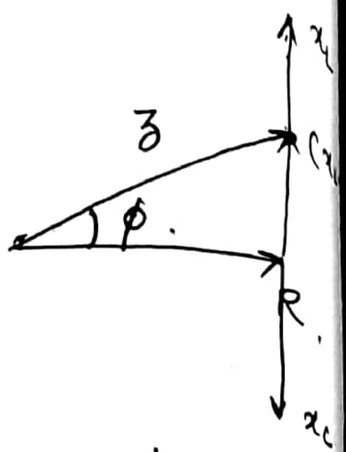
$X = x_L - x_c \rightarrow$ Net Reactance

$x_L > x_c \rightarrow x = x_L - x_c.$

$x_c > x_L \rightarrow x = x_c - x_L$

$|\cancel{Z}|.$

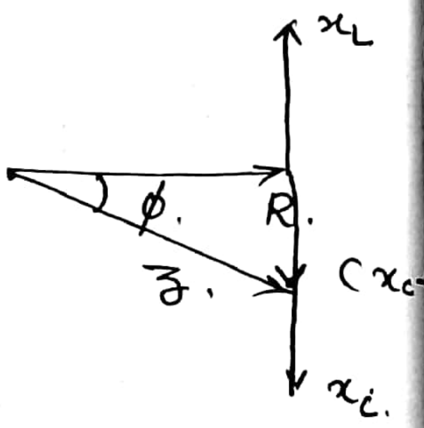
(i) $x_L > x_c.$



$$|Z| = \sqrt{R^2 + (x_L - x_c)^2}$$

$$\cos\phi = \frac{R}{Z}.$$

(ii). $x_c > x_L.$



$$|Z| = \sqrt{R^2 + (x_c - x_L)^2}$$

$$\cos\phi = \frac{R}{Z}.$$

Phasor Representation of an alternating quantity.

Phasor diagram

The diagram which gives the Information of Magnitude & phase angle, about the alternating quantity is known as phasor diagram or vector diagram.

Phasor Algebra

Any alternating voltage and current can be represented by a ~~vector~~ phasor.

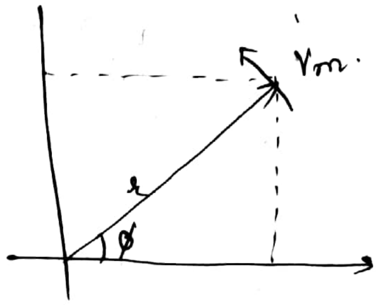
The instantaneous voltage $v(t) = V_m \sin(\omega t + \phi)$

① Polar Representation

$v(t) = V_m \sin(\omega t + \phi)$.

$v(t) = r \angle \phi$.

$r = V_m$;



2). Rectangular Representation

$v(t) = V_m \sin(\omega t + \phi)$

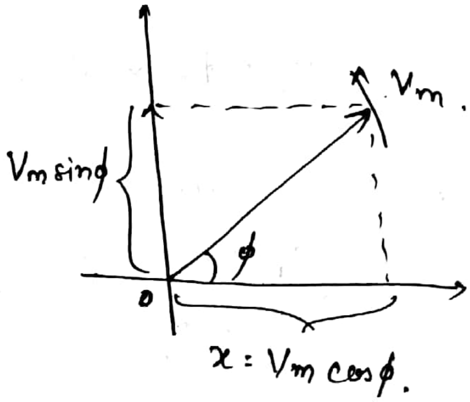
$v(t) = x + iy$.

x component of phasor

$x = V_m \cos\phi$

y component of phasor.

$y = V_m \sin\phi$.



$v(t) = V_m\cos\phi + i \times (V_m \sin\phi)$.

Eg: $v(t) = 20 \sin(100\pi t + 60^\circ)$

Comn..

4

Conversion of polar to Rectangular

$$v(t) = r \angle \phi.$$

Rectangular $v(t) = \cancel{r \sin\phi + i}$
$= r\cos\phi + i\, r\sin\phi.$

Conversion of Rectangullar to Polar conversion

$V = x + iy. \Rightarrow v = r\angle\phi.$

$r = \sqrt{x^2+y^2}$

$\phi = \tan^{-1}(y/x).$

Addition and Substraction in Rectangular form.

- Addition

$(x+iy) + (A+iB) = (A+x) + i\,(B+y).$

- Substraction

$(x+iy) - (A+iB) = \cancel{(A - c}$
$(x-A) + i(y-B)$

Multiplication & division in polar form

- Multiplication

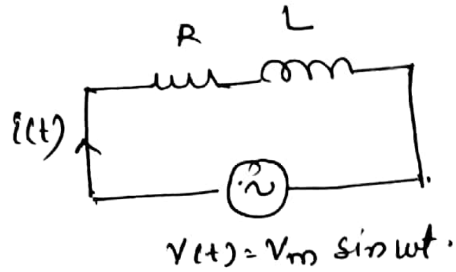
$E_1\angle\phi_1 \times E_2\angle\phi_2 = E_1 \times E_2 \angle(\phi_1+\phi_2).$

- Division

$$\frac{E_1\angle\phi_1}{E_2\angle\phi_2} = \frac{E_1}{E_2}\angle\phi_1-\phi_2$$

(2). A series RL circuit has following parameters $R = 200\ \Omega$, $L = 40\ mH$, Calculate Power factor and draw vector diagram. Assume $f = 50\ Hz$. and Source voltage of 200 V.

⇒ Solⁿ



given data :

$R = 200\ \Omega$,

$L = 40\ mH$.

$V = 200\ V$.

$f = 50\ Hz$.

Calculate : (1) Z =

(2). I =

(3) $\cos\phi$ =

(1) $Z = R + jX_L$.

$X_L = 2\pi f L = 2\pi \times 50 \times 40\ mH$.

$X_L = 12.56\ \Omega$

$Z = 200 + j\ 12.56$ —(Rectangular)

$= 200.39 \angle 3.59$ → polar.

$|Z| = 200.39$

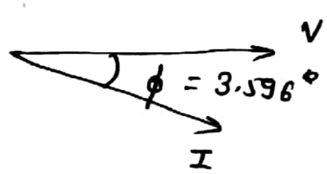
(2). $I = \frac{V}{|Z|} = \frac{200}{200.39} = 0.99\ A$.

(3) $\cos\phi = \cos 3.59 = 0.99$,

Or $\cos\phi = \frac{R}{Z} = \frac{200}{200.39}$.

$\cos\phi = 0.99$

Phasor diagram :



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series RLC circuit. 34

③ A series R-L circuit consist of 120 Ω & 0.15H inductance. The series circuit connected across a 100V, 50 Hz. Calculate current, voltage across individual Elements, Power factor, active Power, Reactive Power apparent power

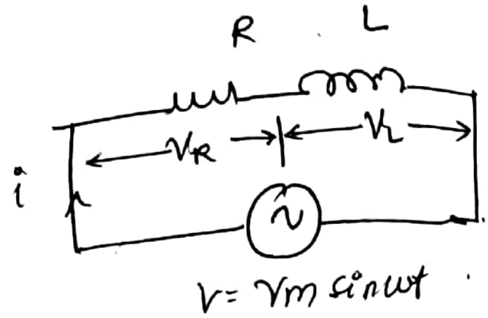
⇒ Given:

$R = 120\,\Omega$.

$L = 0.15H$.

$V = 100V$

$f = 50\,Hz$.



$X_L = 2\pi f L = 47.12\,\Omega$.

$\therefore Z = R + jX_L$

$= 120 + j\,47.12 = 128.92 \angle 21.43 \rightarrow$ polar

$\therefore |Z| = 128.92$

① $\therefore I = \dfrac{V}{|Z|} = \dfrac{100}{128.92} = 0.7757A.$

②. Voltage across R $= I \times R$.

$= 0.7757 \times 120 = 83$ volts.

$\therefore \boxed{V_R = 83 \text{ Volts.}}$

$\therefore$ Voltage across $L = I \times X_L$

$= 0.7757 \times 47.12$

$\boxed{V_L = 36.55 \text{ volts}}$

③ $\cos\phi = \cos 21.43 = 0.93$.

④ $\cdot P = I^2 R$.

$= (0.77)^2 \times 120$

$= 71.14$ Watts.

$Q = I^2 X_L$

$= (0.77)^2 \times 47.12$

$= 27.93$ VAR

$S = I^2 Z$. or

$S = V \times I$.

$= 100 \times 0.77$

$= 76.43$ VA

choke

(4) A Coil takes 10 A from 200 V, 50Hz supply. Its resistance is 5 Ω determine its Inductance also obtained real power, Reactive & apparent Power.

⇒ given:→ $I = 10A$, $V = 200V$

$R = 5\Omega$, $f = 50 Hz$.

Calculate: (1) $L = ?$

(2) $P, Q, S = ?$

(1) $|Z| = \dfrac{V}{I} = \dfrac{200}{10} = 20\,\Omega.$

$|Z| = \sqrt{R^2 + X_L^2}$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{20^2 - 5^2}$

$X_L = 19.36\,\Omega.$

$X_L = 2\pi f L.$

$L = \dfrac{X_L}{2\pi f} = \dfrac{19.36}{2\pi \times 50} = 0.061\,H$

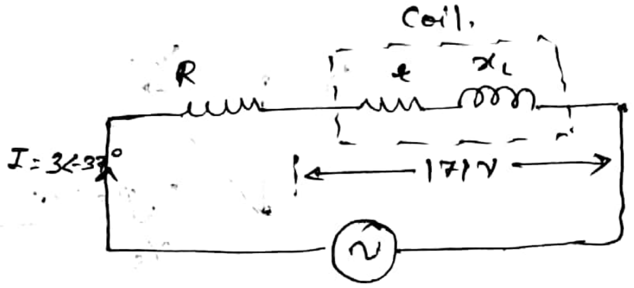
(2) $P = I^2 R = (10)^2 \times 5 = 500\,W.$

$Q = I^2 X_L = (10)^2 \times 19.36 = 1936\,VAR.$

$S = I^2 Z = (10)^2 \times 20$

$= 2000\,VA.$

Q. When resistor and an coil are connected in series to 240 V ; 50 Hz AC Mains, a current of 3 A flowing lagging 37° behind the supply voltage, while the voltage across the ~~inductor~~ coil is 171 V. Determine the Resistance of Resistor, & Resistance & Inductance of coil. Draw the Phasor diagram.



Soln.

$V_{rms} = 240\,V$

$f = 50\,Hz.$

$I = 3\angle -37°.$

$Z = \dfrac{240}{3\angle -37°} = 63.89 + j48.14 = \cancel{60.64\angle 52.53}\ 80\angle 37°.$

$Z = (R + r) + jx_L.$

$R + r = 63.89\,\Omega \qquad x_L = 48.14\,\Omega.$

<u>Coil.</u>

$Z_{coil} = \dfrac{V_{coil}}{I} = \dfrac{171}{3\angle -37}$

$Z = \dfrac{V}{I} = 80\,\Omega\angle 37° = 63.89 + j48.14.$

$Z_{coil} = \dfrac{V_{coil}}{I} = \dfrac{171}{3} = 57\,\Omega.$

$Z_{coil} = \sqrt{r^2 + x_L^2} = r^2 + x_L^2 = 3249 \quad -(1).$

$Z = \sqrt{(r+R)^2 + x_L^2} = (r+R)^2 + x_L^2 = 6400 \quad -(2).$

$\cancel{r^2+2}$, from Eqn (1) $\Rightarrow r^2 + 48.14^2 = 6400$

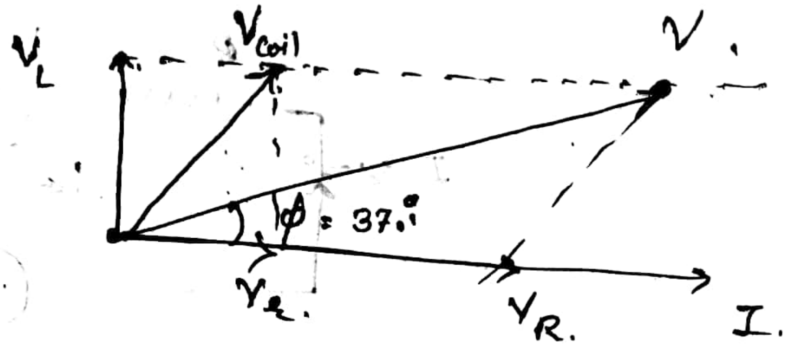
$r = 30.44\,\Omega.$

$R = 33.39\,\Omega.$

$$x_L = 2\pi f L.$$

$$L = \frac{48.15}{2\pi \times 50} = 0.153\ H.$$

Phasor diagram.



Q. A Choke coil & Resistor are connected in series across a 230V, 50 Hz ac supply. The circuit draws a current of 2A at 0.866 lagging P.f. The voltage drop across Resistor is 100V. Calculate Resistance & Inductance of coil; Power factor of coil; Power factor of Circuit.

⇒

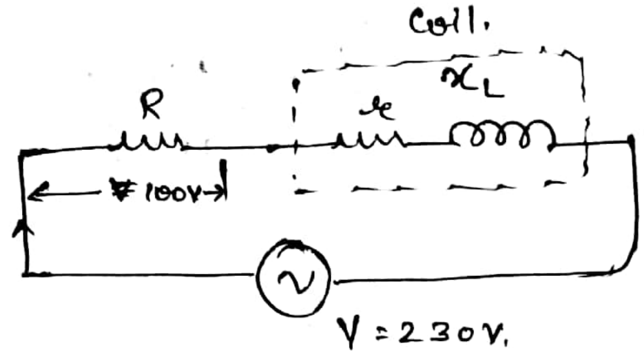
$V = 230V.$

$f = 50 Hz.$

$I = 2A.$

$\cos\phi = 0.866$ lagging

$\phi = 30°.$



To find ; ① r & L.

② P.f of coil.

③ P.f of circuit.

$$Z = \frac{V}{I} = \frac{230}{2} = 115 \angle 30°$$

$$Z = \underbrace{99.59}_{(R+r)} + \underbrace{57.5i}_{X_L} \Rightarrow \quad R+r = 99.59\,\Omega. \quad X_L = 57.5\,\Omega.$$

for resistor; $V_R = I \times R = R \Rightarrow \frac{100}{2} = 50\,\Omega.$

$50 + r = 99.59.$

① $r = 49.59$; $X_L = 57.5.$

$$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = 0.183\ H$$

② P.f of coil. $\rightarrow (\cos\phi)_{coil} = \frac{r}{Z_{coil}}.$

$$Z_{coil} = \sqrt{r^2 + X_L^2} = 75.93.$$

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$$(\cos\phi)_{coil} = \frac{49.59}{75.93} = 0.653 \text{ (lagging)}.$$

(3) P.f of circuit $\Rightarrow$ $\cos\phi = \frac{R+r}{Z}$.

$\cancel{Z = \sqrt{(R+r)^2 + X_L^2}}$.

$$\cos\phi = \frac{99.59}{115} = 0.865 \text{ (lagging)}.$$

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Example 4.31 *The voltage and current in a circuit are given by* $\bar{V} = 150\angle 30°$ *V and* $\bar{I} = 2\angle -15°$ *A. If the circuit works on a 50-Hz supply, determine the power factor, power loss, impedance, resistance, and reactance considering the circuit as a simple series circuit.*

Solution

Data

$$\bar{V} = 150\angle 30° \text{ V}$$

$$\bar{I} = 2\angle -15° \text{ A}$$

$$f = 50 \text{ Hz}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{150\angle 30°}{2\angle -15°}$$

$$= 75\angle 45° \ \Omega = 53.03 + j53.03 \ \Omega$$

Impedance $Z = 75 \ \Omega$

Resistance $R = 53.03 \ \Omega$

Reactance $X = 53.03 \ \Omega$

Power factor $\text{pf} = \cos\phi = \cos(45°) = 0.707$ (lagging)

Power loss $P = VI\cos\phi$

$$= 150 \times 2 \times 0.707 = 212.1 \text{ W}$$

Example 4.33 *A voltage v (t) = 177 sin (314t + 10°) is applied to a circuit. It causes a steady state current to flow, which is described by i (t) = 14.14 sin (314t – 20°). Determine the power factor an average power delivered to the circuit.*

Solution

Data

$$v(t) = 177 \sin(314t + 10°)$$

$$i(t) = 14.14 \sin(314t - 20°)$$

Current $i(t)$ lags behind voltage $v(t)$ by 30°.

$$\phi = 30°$$

Power factor $\quad \text{pf} = \cos(30°) = 0.866 \text{ (lagging)}$

Power consumed $\quad P = VI \cos \phi$

$$= \frac{177}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times 0.866 = 1083.7 \text{ W}$$

Example 4.34 *When a sinusoidal voltage 120 V (rms) is applied to a series R-L circuit, it is found that there occurs a power dissipation of 1200 W and a current flow given by i (t) = 28.3 sin (314 t – ϕ). Find the circuit resistance and inductance.*

Solution

Data

$$V = 120 \text{ V}$$

$$i(t) = 28.3 \sin(314t - \phi)$$

$$P = 1200 \text{ W}$$

$$I = \frac{28.3}{\sqrt{2}} = 20.01 \text{ A}$$

$$P = VI \cos \phi$$

$$1200 = 120 \times 20.01 \times \cos \phi$$

$$\cos \phi = 0.499$$

$$\phi = 60.02°$$

$$Z = \frac{V}{I} = \frac{120}{20.01} = 6\ \Omega$$

$$\overline{Z} = Z \angle \phi = 6 \angle 60.02° = 3 + j5.2\ \Omega$$

Resistance $R = 3\ \Omega$

Reactance $X_L = 5.2\ \Omega$

$$X_L = \omega L$$

$$5.195 = 314 \times L$$

Inductance $L = 0.0165$ H

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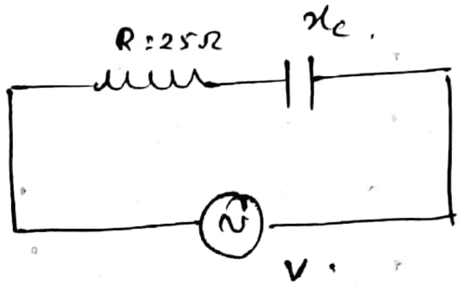
Q. ~~A series R-C. Circuit takes a~~

Q. A valtage $v = 100 \sin 314t$ is applied to a circuit consisting of $25\,\Omega$ resistor & an $80\,\mu f$ capacitor in series. Determine supply frequency, impedance, Peak value of current, Power factor & Power consumed by the circuit

$\Rightarrow$ $v = 100 \sin 314t$.

$R = 25\,\Omega$.

$C = 80\,\mu F = 80 \times 10^{-6}\,F$



To Find : (1) f ; (2) Z ; (3) I_{max} ; (4) $\cos\phi$. (5) P.

$\Rightarrow$ (1). $v = 100 \sin 314t$ $\{ v = V_m \sin \omega t \}$

$V_m = 100\,V \Rightarrow V_{rms} = 70.71\,V$.

$\omega = 314$; $2\pi f = 314$.

$f = \dfrac{314}{2\pi} = 49.97 \cong 50\,Hz$.

(2). $R = 25\,\Omega$.

$x_c = \dfrac{1}{2\pi f c} = 39.78\,\Omega$

$Z = \sqrt{R^2 + x_c^2} = \sqrt{25^2 + 39.78^2}$

$Z = 46.98 \cong 47\,\Omega$.

$Z = R - jx_c$

$= 25 - j\,39.78$.

$Z = \underbrace{46.98}_{Mag} \angle \underbrace{-57.85}_{\phi}$.

(3). $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{70.71}{47} = 1.525\,Amp$.

$I_{max} = 2.15\,Amp$.

(4) $\cos\phi = 0.53$ (lead).

(5) $P = V_{rms} \times I_{rms}$.

$P = 107.83\,Watts$

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R-C.

Q. A series circuit consumes 2000 W. at 0.5 leading P.F, when connected to 230 V, 50 Hz ac supply. Calculate (i) kVA (ii) kVAR & (iii) current

⇒ $V = 230V.$

$f = 50 Hz$

$P = 2000 W.$

$\cos\phi = 0.5$ leading.

$P = VI\cos\phi.$

$2000 = 230 \times I \times 0.5$

$I = 17.39A$

$S = VI = 230 \times 17.39$

$S = 4000 VA = 4 kVA.$

⇒ $\phi = \cos^{-1} 0.5 = 60°$

$Q = VI\sin\phi.$

$= 230 \times 17.39 \times \sin 60$

$Q = 3.464 kVAR.$

Example 4.50 *The voltage applied to a circuit is e = 100 sin (ωt + 30°) and the current flowing in the circuit is i = 15 sin (ωt + 60°). Determine impedance, resistance, reactance, power and power factor.*

Solution

Data

$$e = 100 \sin(\omega t + 30°)$$
$$i = 15 \sin(\omega t + 60°)$$
$$\bar{E} = \frac{100}{\sqrt{2}} \angle 30° \text{ V}$$
$$\bar{I} = \frac{15}{\sqrt{2}} \angle 60° \text{ A}$$

$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{\frac{100}{\sqrt{2}}\angle 30°}{\frac{15}{\sqrt{2}}\angle 60°} = 6.67 \angle -30° = 5.77 - j3.33\ \Omega$$

Impedance $Z = 6.67\ \Omega$

Resistance $R = 5.77\ \Omega$

Reactance $X_C = 3.33\ \Omega$

Power factor $= \cos\phi = \cos(30°) = 0.866$ (leading)

Power $P = VI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5$ W

$R = 157.5\ \Omega$

Example 4.54 *A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a capacitor. The current is 2.2 A. The power loss in the resistor is 96.8 W. Calculate the resistance and capacitance.*

Solution

Data $V = 125\text{ V}$ $\quad P = 96.8\text{ W}$

$I = 2.2\text{ A}$ $\quad f = 50\text{ Hz}$

$$Z = \frac{V}{I} = \frac{125}{2.2} = 56.82\text{ A}$$

$$P = I^2R$$

$$96.8 = (2.2)^2 \times R$$

$$R = 20\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2} = 53.18\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$53.18 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 59.85\ \mu\text{F}$$

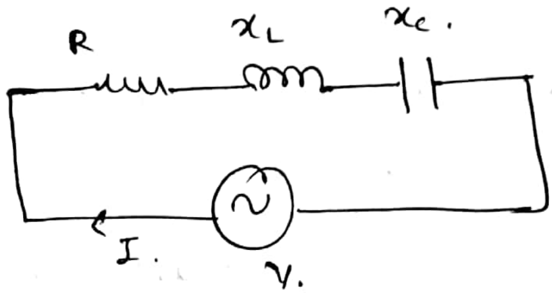
R-L-C Series Circuit.

(1) A series RLC circuit consisting of a Resistance of 20 Ω; inductance of 0.2 H and Capacitance of 150 μF. is connected across a 320 V; 50 Hz source. Calculate:

(a) Impedance ; (b) Current

(c) Magnitude & Nature of p.f.

⇒ Given :



$R = 20\ \Omega$.

$L = 0.2\,H$.

$C = 150\ \mu F. = 150 \times 10^{-6}\ F.$

$V_{rms} = 320\,V$.

$f = 50\ Hz$.

To find : (a) Z. (2) I_{rms} (3) $\cos\phi$ & Nature.

⇒ $R = 20\ \Omega$.

$x_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$.

$x_c = \dfrac{1}{2\pi f c} = 21.22$.

$x_L > x_c$. Inductive in Nature

(a) $Z = \sqrt{R^2 + (x_L - x_c)^2} = \sqrt{20^2 + (62.83 - 21.22)^2}$.

$Z = 46.16\ \Omega$.

$Z = R + j x_L - j x_c$

$Z = 20 + 62.83 j - 21.22 j$

$Z = 20 + 41.61 j$.

$Z = \underbrace{46.16}_{Mag} \angle \underbrace{64.32}_{\phi}$.

(b). Current (I_{rms}) $= \dfrac{V_{rms}}{Z} = \dfrac{320}{46.16}$.

$I_{rms} = 6.93$ Amp.

(c). $\cos\phi = \cos 64.32$.

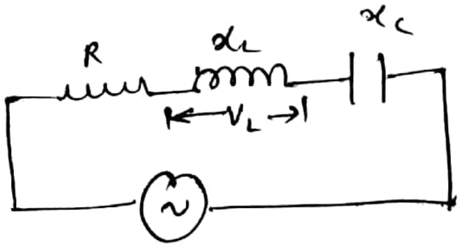
$\cos\phi = 0.433$ (lagging).

$\cos\phi = \dfrac{R}{Z}$.

$= \dfrac{20}{46.16}$.

$\cos\phi = 0.433$ (lagging).

Q. A R-L-C series circuit has a current which lags the applied voltage by 45°. The voltage across the inductance is equal to twice the maximum value of voltage across the capacitor. Voltage across the Inductance is $300 \sin(1000t)$ & $R = 20\,\Omega$. Find value of Inductance & Capacitance.



⇒ $V_L = 300 \sin(1000t)$

$R = 20\,\Omega$.

$\phi = 45°$.

To find : L & C.

$$V_L = 2V_C.$$

$$\cancel{I} \times x_L = 2\cancel{I} \times x_C$$

$$x_L = 2x_C.$$

$$\cos\phi = \frac{R}{Z} \Rightarrow Z = \frac{20}{\cos 45}$$

$\underline{Z = 28.28\,\Omega}$.

for a series R-L-C circuit,

$$Z = \sqrt{R^2 + (x_L - x_C)^2}$$

$$(28.28)^2 = \sqrt{(20)^2 + (2x_C - x_C)^2}$$

$$799.76 = 400 + x_C^2$$

$$x_C = 20\,\Omega.$$

$$x_L = 40\,\Omega.$$

$x_L = \omega L$

$40 = 1000\,L$

$L = 0.04\,H$

$x_C = \dfrac{1}{\omega C}$.

$20 = \dfrac{1}{1000 \times C}$.

$C = 50\,\mu F$.

Quality factor Q_0 of the R-L-C Circuit

It is the Ratio of the Resonant frequency to the Bandwidth.

It is a Measure of the Selectivity or Sharpness of tuning of the Series R-L-C Circuit

$$Q = \frac{\omega_0}{\text{Bandwidth}} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R}$$

$Q/=$

~~Quality~~

Quality factor relates the maximum or Peak Energy Stored in the circuit (the Reactance) to the Energy Dissipated (the Resistance) during each cycle of oscillation

Q. A circuit having Resistance of 5 Ω & Inductance of 0.4 H ; a variable capacitor in series is connected across supply of 110 V; 50 Hz
Calculate:

(1) Value of Capacitor to give Resonance
(2) Current at Resonance
(3) Voltage across inductor & Capacitor
(4) Q factor of circuit

(1) $x_L = x_C$ — (At Resonance)

$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times 50^2 \times 0.4}$$

$$C = 25.3\ \mu F.$$

(2) $I = \dfrac{V}{Z}$.

$$Z = \sqrt{R^2 + (x_L - x_C)^2}$$

$x_L = 2\pi f L$
$= 2\pi \times 50 \times 0.4$
$x_L = 125.66\ \Omega$

$x_C = 1/2\pi f C$
$x_C = \dfrac{1}{2\pi \times 50 \times 25.3 \times 10^{-6}}$
$x_C = 125.66\ \Omega$

At Resonance:
$x_L = x_C$
$\therefore\ Z = R.$

$$I = \frac{110}{5} = 22\,A$$

(3) $V_L = I \times x_L$
$= 22 \times 2\pi \times 50 \times 0.4$
$= 2764\ V$

$V_C = I \times x_C$
$= 22 \times 125.66$
$V_C = 2764\ V$

(4) $$Q = \frac{\omega_0 L}{R} = \frac{2\pi \times 50 \times 0.4}{5} = 25.13$$

Example 4.59 *A coil having a power factor of 0.5 is in series with a 79.57 μF capacitor and when connected across a 50-Hz supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil.*

Solution

Data

$$pf = 0.5$$
$$C = 79.57\ \mu\text{F}$$
$$f = 50\ \text{Hz}$$
$$V_{\text{coil}} = V_C$$
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 79.57 \times 10^{-6}} = 40\ \Omega$$
$$V_{\text{coil}} = V_C$$
$$I \,.\, Z_{\text{coil}} = I \,.\, X_C$$
$$Z_{\text{coil}} = X_C = 40\ \Omega$$
$$\text{pf of coil} = \cos\phi = \frac{R}{Z_{\text{coil}}}$$
$$0.5 = \frac{R}{40}$$

Resistance of coil $R = 20\ \Omega$

$$X_L = \sqrt{Z_{\text{coil}}^2 - R^2} = \sqrt{(40)^2 - (20)^2} = 34.64\ \Omega$$
$$X_L = 2\pi fL$$
$$34.64 = 2\pi \times 50 \times L$$

Inductance of coil = 0.11 H

Example 4.81 *A series R-L-C circuit is connected to a 200-V ac supply. The current drawn by the circuit at resonance is 20 A. The voltage drop across the capacitor is 5000 V at series resonance. Calculate resistance and inductance if capacitance is 4 μF. Also, calculate the resonant frequency.*

Solution

Data

$$V = 200\text{ V} \qquad I_0 = 20\text{ A}$$
$$V_C = 5000\text{ V} \qquad C = 4\ \mu\text{F}$$

Resistance

$$R = \frac{V}{I_0} = \frac{200}{20} = 10\ \Omega$$

$$X_{C_0} = \frac{V_{C0}}{I_0} = \frac{5000}{20} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi X_{C0} C} = \frac{1}{2\pi \times 250 \times 4 \times 10^{-6}} = 159.15\text{ Hz}$$

At resonance

$$X_{C_0} = X_{L_0}$$
$$X_{L_0} = 250\ \Omega$$
$$X_{L_0} = 2\pi f_0 L$$
$$250 = 2\pi \times 159.15 \times L$$

Inductance

$$L = 0.25\text{ H}$$