

①

Ans 1:  $j \quad i$   
 1 0  
 2 0+1  
 3 0+1+2  
 ...  
 K+1 0+1+2+...+K

Let the loop runs for  $K$  times and base condition is / termination condition is  $i = n$ ,

$$\Rightarrow 0+1+2+\dots+K = n$$

$$\frac{K(K+1)}{2} = n \Rightarrow K \approx \sqrt{n}$$

So time complexity is  $O(\sqrt{n})$

Ans 2: Recurrence relation for fibonacci series is

$$T(n) = T(n-1) + T(n-2) + 1$$

for finding upper bound let us assume that

$$T(n-2) \approx T(n-1), \text{ as } O(T(n-2)) = T(n-1)$$

$$\text{So, } T(n) = 2T(n-1) + 1$$

using master's theorem, for  $T(n) = aT(n-b) + O(g(n))$

here  $a, b > 0$  and  $f(n) = O(g(n))$

and  $a > 1$ , so, time complexity is  $O(2^{n/1} \times 1) = O(2^n)$

There will be  $n-1 \approx n$  function calls which are stored in stack, so space complexity is  $O(n)$ .

Ans 3: for  $n(\log n)$  complexity.

```
int fun(int arr[], int n)
{
    if (n == 1)
        return 0;
    int i, max = INT_MIN, mxind = -1;
    for (i = 0; i < n; i++)
        if (arr[i] > max)
        {
            max = arr[i];
            mxind = i;
        }
}
```

for  $n/2$  return

fun( $n/2$ , arr);

fun( $n/2$ , arr);

}

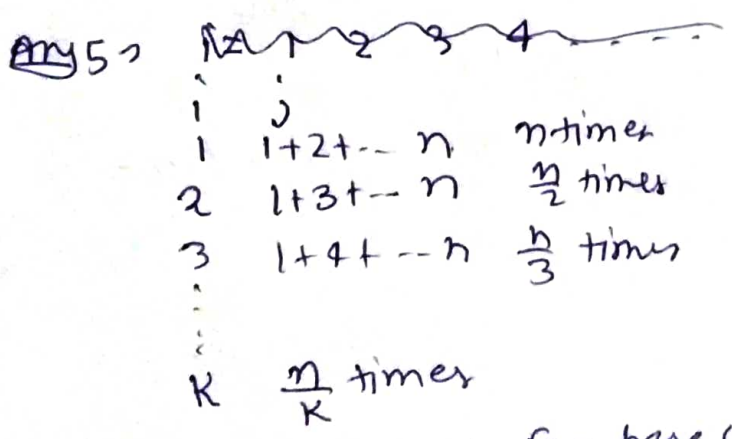
```
for ns,  
for (int i=0; i<n; i++)  
    for (j=0; j<n; j++)  
    { arr[i][j]=0;  
      for (k=0; k<n; k++)  
        arr[i][j] += arr[i][k] * arr[k][j];  
    }  
  
for log(log n)  
    p=0;  
    for (int i=0; i<n; i=i*2)  
    {  
        p++;  
    }  
    for (int j=0; j<p; j=j*2)  
    {  
        print(p) cout<<p;  
    }
```

Ans 4) Since  $T(n) \geq \frac{n}{2}$ , we can write  $T(\frac{n}{2}) = T(\frac{n}{2})$  for finding upper bound of relation.

So,  $T(n) = 2T(\frac{n}{2}) + cn^2$

Using master's theorem  $\log_2 2 = 1$

$n^1 < n^2$ , so time complexity will be  $O(n^2)$



$\frac{n}{K} = 1 \Rightarrow n = K$  for base case

so, time will be =  $n + \frac{n}{2} + \frac{n}{3} + \dots + \frac{n}{n}$   
=  $n + (\frac{n}{2}) + \frac{n}{3} + \dots + 1$   
=  $n (1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n})$   
=  $n \log n$

Q61

1  
2  
 $2^K$   
 $2^{2K}$   
 $2^{3K}$   
⋮  
 $2^{PK}$

base case,  $2^{PK} = n$

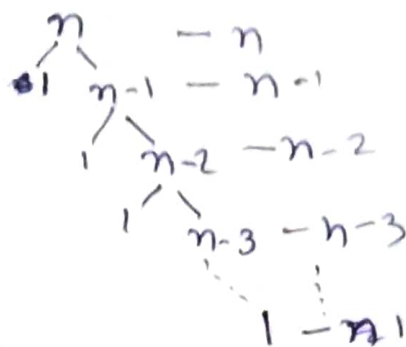
$$PK = \log_2 n$$

$$P = \frac{\log_2 n}{K}, \text{ where } K \text{ is a constant}$$

So time complexity =  $O(\log n) = O(\lg n)$

Q7?  $T(n) = T(n-1) + n$

recurrence tree,



$$\begin{aligned} \text{Time complexity} &= n + (n-1) + (n-2) + \dots + 3 + 2 + 1 \\ &= \frac{n(n+1)}{2} \approx n^2 \end{aligned}$$

so time complexity =  $O(n^2)$

Q8 a)  $100, \log(\log n), \sqrt{n}, \log n, \log^2 n, n, \log(n!), n \log(n), n^2, 2^n, 4^n, n!, 2^{2^n}$

b)  $1, \sqrt{\log n}, \log(\log n), \log(n), \log(2n), 2 \log(n), 3n, 2n, 4n, \log n!, n \log n, n^2, 2(2^n), n!$

c)  $96, \log_8 n, \log_2 n, 5n, n \log_6(n), \log(n!), n \log_2(n), 8n^2, 7n^3, 8^{2^n}, n!$