SUMMER 2025

1. Testing for sequential linear separability

In [CE24], we define what it means for a set of training data to be sequentially linearly separable (SLS). We are interested in finding out if common benchmark datasets, like MNIST and CIFAR10, are SLS.

It might be easier to start with MNIST, and perhaps choose a subset of the classes, say $N = 3$ or 4 classes.

Suggested initial plan:

- Find mean $\overline{x_{0,j}}$ for each class.
- Find barycenter $\overline{x}$.
- Pick an order
- For n in range(N):
  1. Use SVM to do one-vs-all classification and find a hyperplane that separates class $\mathcal{X}_{0,n}$ from all other ones.
  2. Find intersection p_n of this hyperplane with the line connecting the class mean $\overline{x_{0,n}}$ to $\overline{x}$.
  3. Send all points in $\mathcal{X}_{0,n}$ to p_n.

It's possible that, even if MNIST **is** SLS, the choice of hyperplanes will be suboptimal and at some point the separation will not be perfect, even for the correct order. If none of the experiments result in perfect separation, report back with some measure of the errors, and we can go from there.

2. Visualizing the cones

In [CE23, CE24, Ewa25] we define certain cones which are used to explicitly construct ReLU neural networks that classify data. We are interested in seeing how such cones might arise when training neural networks with gradient descent and its variants.

Suggested initial plan:

- Train a neural network to classify MNIST (perhaps only a subset of the classes), using the architecture suggested in the hyperplanes paper [CE24].
- Determine cumulative parameters $W^{(\ell)}, b^{(\ell)}$.
- Determine cones (can use polyhedral cones from [Ewa25] and determine the base point and edges).
- Try to visualize the cones by some form of dimensional reduction. For instance: Do principal component analysis (PCA) on training data, then use these coordinates and project cones (base point and edges) as well.

There may be other ways of gaining information about these cones that would be interesting.

Date: June 3, 2025.

References

[CE23] Thomas Chen and Patrícia Muñoz Ewald. Geometric structure of deep learning networks and construction of global $\mathcal{L}^2$ minimizers, 2023. arXiv:2309.10639, *submitted*.

[CE24] Thomas Chen and Patrícia Muñoz Ewald. Interpretable global minima of deep relu neural networks on sequentially separable data, 2024. arXiv:2405.07098, *submitted*.

[Ewa25] Patrícia Muñoz Ewald. Explicit neural network classifiers for non-separable data, 2025. arXiv:2504.18710, *preprint*.