

AI1110: Probability and Random Variable

Assignment-3

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Question: 11.16.4.6

Problem Statement:

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:

Let l_0, l_1, l_2 denote the three letters, and E_0, E_1, E_2 denote the three corresponding envelopes respectively.

'S' = Sample space = Randomly distributing the three letters in the three letters.

'A' = Event that atleast one letter is in correct envelope

Let X and Y be random variables such that,

Random Variable	Value of the random variable	Event
X	1	letter l_0 is inserted
	2	letter l_1 is inserted
	3	letter l_2 is inserted
Y	1	letter E_0 is used
	2	letter E_1 is used
	3	letter E_2 is used

TABLE I

WHERE $l_k E_k$ DENOTE LETTER l_k , AND ENVELOPE E_k FOR X AND Y RESPECTIVELY

Let XY denote that letter X is inserted into envelope Y. The sample space is,

- 1) (00, 21, 12)
- 2) (11, 20, 02)
- 3) (22, 01, 10)
- 4) (00, 11, 22)
- 5) (01, 12, 20)
- 6) (02, 10, 21)

There are only 2 ways in none of the 3 letters are correctly placed

Hence, the PMF

$$\Pr(X = x, Y = y \neq x) = \frac{2}{6} \quad (1)$$

$$= \frac{1}{3} \quad (2)$$

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Hence,

$$\Pr(\text{at least once } X = Y = k) = 1 - \Pr(X = x, Y = y \neq x) \quad (3)$$

$$= 1 - \frac{1}{3} \quad (4)$$

$$= \frac{2}{3} \quad (5)$$