

# AI1110: Probability and Random Variable

## Assignment-3

Mayank Parasramka\*  
AI22BTECH11018

### Question: 11.16.4.6

#### Problem Statement:

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

#### Solution:

Let  $l_0, l_1, l_2$  denote the three letters, and  $E_0, E_1, E_2$  denote the three corresponding envelopes respectively.

'S' = Sample space = Randomly distributing the three letters in the three letters.

'A' = Event that atleast one letter is in correct envelope

Let X and Y be random variables such that,

Random Variable	Value of the random variable	Event
X	1	letter $l_0$ is inserted
	2	letter $l_1$ is inserted
	3	letter $l_2$ is inserted
Y	1	letter $E_0$ is used
	2	letter $E_1$ is used
	3	letter $E_2$ is used

TABLE I

WHERE  $l_k E_k$  DENOTE LETTER  $l_k$ , AND ENVELOPE  $E_k$  FOR X AND Y RESPECTIVELY

Let XY denote that letter X is inserted into envelope Y. The sample space is,

- 1) (00, 21, 12)
- 2) (11, 20, 02)
- 3) (22, 01, 10)
- 4) (00, 11, 22)
- 5) (01, 12, 20)
- 6) (02, 10, 21)

There are only 2 ways in none of the 3 letters are correctly placed

Hence, the PMF

$$\Pr(X = x, Y = y \neq x) = \frac{2}{6} \quad (1)$$

$$= \frac{1}{3} \quad (2)$$

\*The student is with the Department of Artificial Intelligence, Indian Institute of Technology, Hyderabad, 502285, India.  
e-mail: ai22btech11018@iith.ac.in.

Hence,

$$\Pr(\text{at least once } X = Y = k) = 1 - \Pr(X = x, Y = y \neq x) \quad (3)$$

$$= 1 - \frac{1}{3} \quad (4)$$

$$= \frac{2}{3} \quad (5)$$