AI1110: Probability and Random Variable Assignment-3

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Question: 11.16.4.6

Problem Statement:
Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:
Let l_1, l_2, l_3 denote the three letters, and E_1, E_2, E_3 denote the three corresponding envelopes respectively.

Let $l_i E_i$ denote that i^{th} letter is inserted into the corresponding i^{th} envelope.

'S'= Sample space = Randomly distributing the three letters in the three letters.

'A'= Event that atleast one letter is in correct envelope

$$n(S) = 3! \tag{1}$$
$$= 6 \tag{2}$$

The sample space is,
1) $l_1 E_1, l_2 E_3, l_3 E_2$
2) $l_2 E_2, l_1 E_3, l_3 E_1$
3) $l_3 E_3, l_1 E_2, l_2 E_1$
4) $l_1 E_1, l_2 E_2, l_3 E_3$
5) $l_1 E_2, l_2 E_3, l_3 E_1$
6) $l_1 E_3, l_2 E_1, l_3 E_2$

There are 4 ways in which at least one letter is correctly placed,

$$\therefore n(A) = 4 \tag{3}$$

Hence,

$$\Pr(A) = \frac{n(A)}{n(S)} \tag{4}$$
$$= \frac{4}{6} \tag{5}$$
$$= \frac{2}{3} \tag{6}$$

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