

AI1110: Probability and Random Variable

Assignment-3

Mayank Parasramka*
AI22BTECH11018

Question: 11.16.4.6

Problem Statement:

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:

Let l_0, l_1, l_2 denote the three letters, and E_0, E_1, E_2 denote the three corresponding envelopes respectively.

'S' = Sample space = Randomly distributing the three letters in the three letters.

'A' = Event that atleast one letter is in correct envelope

Let X and Y be random variables such that,

R.V.	l_0E_0	l_1E_1	l_2E_2
X	0	1	2
Y	0	1	2

TABLE 1

WHERE l_kE_k DENOTE LETTER l_k , AND ENVELOPE E_k FOR X AND Y RESPECTIVELY

Let XY denote that letter X is inserted into envelope Y. The sample space is,

- 1) (00, 21, 12)
- 2) (11, 20, 02)
- 3) (22, 01, 10)
- 4) (00, 11, 22)
- 5) (01, 12, 20)
- 6) (02, 10, 21)

There are only 2 ways in none of the 3 letters are correctly placed

Hence, the PMF

$$\Pr(X = x, Y = y \neq x) = \frac{2}{6} \quad (1)$$

$$= \frac{1}{3} \quad (2)$$

Hence,

$$\Pr(\text{at least once } X = Y = k) = 1 - \Pr(X = x, Y = y \neq x) \quad (3)$$

$$= 1 - \frac{1}{3} \quad (4)$$

$$= \frac{2}{3} \quad (5)$$

*The student is with the Department of Artificial Intelligence, Indian Institute of Technology, Hyderabad, 502285, India.
e-mail: ai22btech11018@iith.ac.in.