AI1110: Probability and Random Variable Assignment-1

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Question: 12.13.2.5

Problem Statement:
A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event,'the number is even,' and B be the event, 'the number is red'. Are A and B independent?
Solution:
Given,
'S'= Sample space = $\{1,2,3,4,5,6\}$
'A'= Event that number is even = $\{2,4,6\}$
'B'= Event that number is red = $\{1,2,3\}$

$$n(A) = 3 \tag{1}$$
$$n(B) = 3 \tag{2}$$
$$n(S) = 6 \tag{3}$$
$$n(AB) = 1 \tag{4}$$

Let X and Y be two random variables such that,

$$X = \begin{cases} 0, & \text{if number is odd} \\ 1, & \text{if number is even} \end{cases} \tag{5}$$

$$Y = \begin{cases} 0, & \text{if number is green} \\ 1, & \text{if number is red} \end{cases} \tag{6}$$

Now,

$$\Pr(X = 1) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2} \tag{7}$$
$$\Pr(Y = 1) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2} \tag{8}$$
$$\Pr(X = 1, Y = 1) = \frac{n(AB)}{n(S)} = \frac{1}{6} \tag{9}$$

Now,

$$\Pr(X = 1) \times \Pr(Y = 1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \tag{10}$$
$$\implies \Pr(X = 1, Y = 1) \neq \Pr(X = 1) \times \Pr(Y = 1)$$

Hence, A and B are not independent.

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