Reduced Row Echelon form :→

$5a+b=17$
$4a-3b=6$

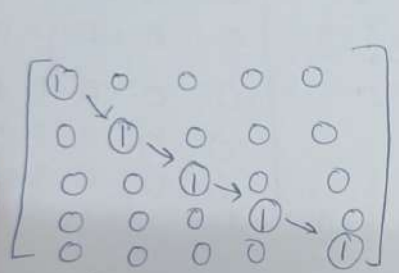
$\longrightarrow$ $a+.2b=3.4$, $b=2$ $\longrightarrow$ $a=3$, $b=2$

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & .2 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

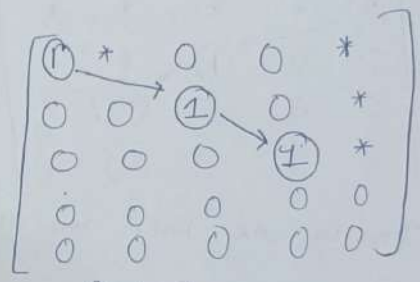
Row Echelon — Reduced Row Echelon

$$\begin{bmatrix} 1 & .2 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - .2R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Reduce Row Echelon form

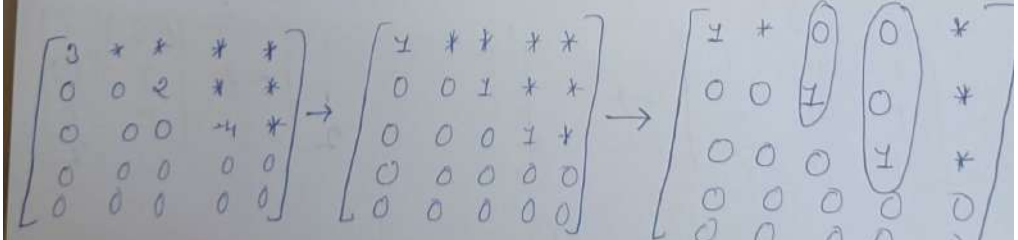


RANK=5

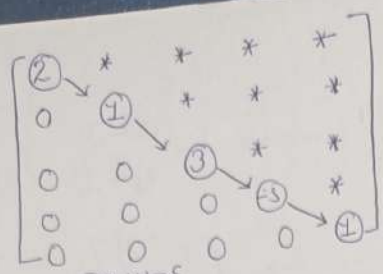


RANK = 3

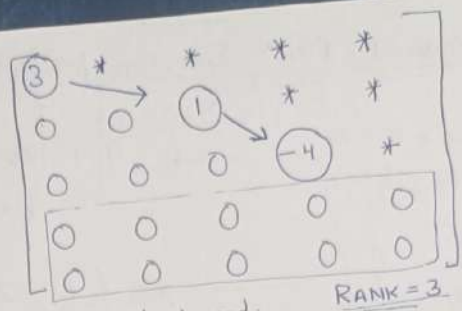
→ Reduce Row Echelon form is in Row Echelon form
→ Each pivot is 1
→ Any no. above pivot is 0
→ Rank = No. of Pivot



Row Echelon — Red. Row Echelon



RANK = 5



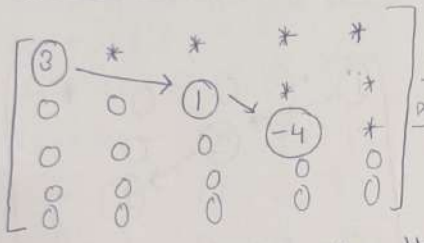
RANK = 3

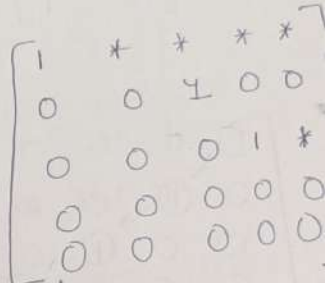
→ Zero Row at bottom may or may not be found

→ Each Row has pivot (Left Most non-zero Entry)

→ Every Pivot is in the Right of the Pivot on the above Row

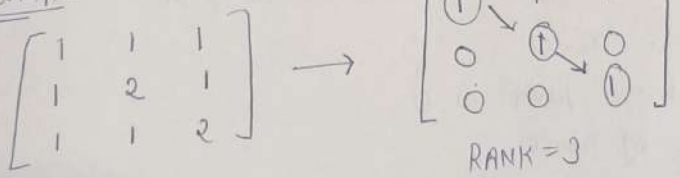
→ Rank = No. of Pivot



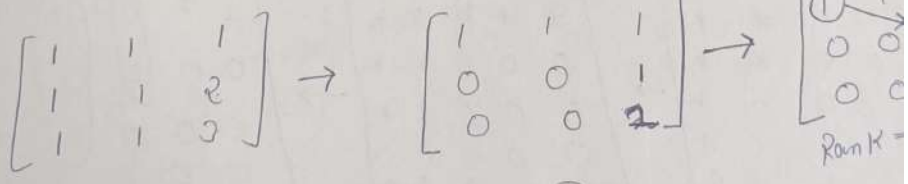


→ In Gomon Pivot other than 1 is Allowed

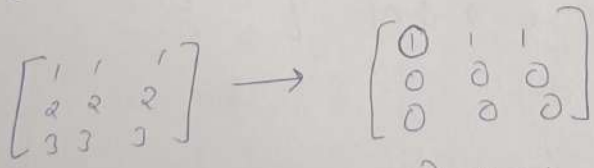
Example :→



RANK = 3



Rank =



Rank = 1

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Divide by 1st non zero $\rightarrow$ $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$

↓ Row Echelon

Row Echelon

$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} (1) & 2 \\ 0 & (1) \end{bmatrix}$ Rank = 2

$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} (1) & 2 \\ 0 & 0 \end{bmatrix}$ Rank = 1

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Rank = 0

Rank = No. of one's in Row Echelon form

ystem

$a + b + 2c = 12$

$3a - 3b - c = 3$

$2a - b + 6c = 24$

Convert $\longrightarrow$

Sysm

$a + b + 2c = 12$

$-6b - 7c = -33$

$6c = 18$

↓ Matrix

$\begin{bmatrix} 1 & 1 & 2 \\ 3 & -3 & 1 \\ 2 & -1 & 6 \end{bmatrix} \longrightarrow$

Row Eachelon form

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -6 & 7 \\ 0 & 0 & 6 \end{bmatrix}$

Rank = 2 − (Dimension of Sol^n space) → only for 2×2 Matrix

Non Singular → If Rank = No. of Row

Rank of Matrics :→

→ $a+b+c=0$
$a+2b+c=0$
$a+b+2c=0$
3 Eq^n, 3 unk 0
Rank = 3
$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

→ $a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$
3 Eq^n, 2 unk 0
Rank = 2
$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

→ $a+b+c=0$
$2a+2b+2c=0$
$3a+3b+3c=0$
1 unk 0
Rank = 1
$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

→ $0a+0b+0c=0$
$0a+0b+0c=0$
$0a+0b+0c=0$
0 unk 0
Rank = 0

Row Echelon form a Matrix :→

→ $\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix}$ Divide Each Row By left Most coff. $\begin{bmatrix} 1 & 1/5 \\ 1 & -3/4 \end{bmatrix}$ $R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & .2 \\ 0 & -.95 \end{bmatrix}$

Divide Sec By left Most coff

$\begin{bmatrix} 1 & .2 \\ 0 & 1 \end{bmatrix}$ Row Echelon form

Row Echelon

→ $\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix}$ Divide Each Row By left Most coff $\begin{bmatrix} 1 & .2 \\ 1 & .2 \end{bmatrix}$ $R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & .2 \\ 0 & 0 \end{bmatrix}$

Sec. Divide By left Most Non zero coff

$\begin{bmatrix} 1 & .2 \\ ? & ? \end{bmatrix}$

Rank ⟶ Use in Image Compression

- System of Information :→

Dog is Red Cat is Black	Dog is Red Dog is Red	Dog is Dog is
2 sentence, 2 Info	2 Sent., 1 info	2 Sent., 0 info
Rank = 2	Rank = 1	Rank = 0

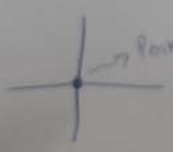
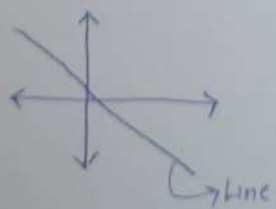
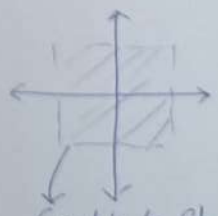
Rank ↓
No. of information carried by a system.

- System of Eqⁿ

$a+b=0$ $a+2b=0$	$a+b=0$ $2a+2b=0$	$0a+0b=0$ $0a+0b=0$
2 Sent, 2 info	2 Sent, 1 info	2 Sent, 0 info
RANK = 2	RANK = 1	RANK = 0
↳ No of Info.		
↓		
$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Rank of Mat = 2	Rank of Mat = 1	Rank of Mat = 0

- Relation b/wⁿ Solⁿ Space and Rank

Solⁿ Space = Set of Solⁿ when system of Equation is Eq to zero (0).

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Rank = 2		
		
0 Dimension	1 dimension	Complete Space 2d

$a+b=11$ ⟶ $a+2b=22$

$0a+2b=2$

$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix}$ ⟶ $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ ← { Gaussian Elimination or Matrix Row Reduction Methods }

Row Echelon

↓

Reducing matrix into Red: Row Echelon form using Row operation.

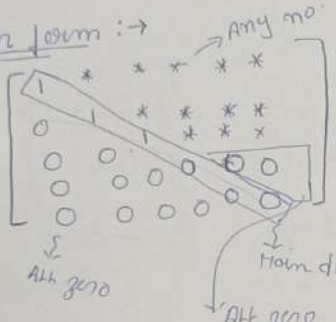
$0a+0b=0$ ⟶ $0a+0b=0$

$0a+0b=0$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ⟶ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Row Echelon form

Echelon form :→



→ Main diagnol 1 and 0 are allowed also all 1's and all's 0's.

→ Below Main dia. All zero

→ After one → Any no. come (x)

→ After 0 → All zero, or May Not be

2×2 matrix 3 case possible :→

$\begin{bmatrix} 1 & * \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ne Row operation :→ In this singularity can't change

i) Switching Row :→ Det get -ve.

$\begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$ ⟶ $\begin{bmatrix} 4 & 3 \\ 5 & 1 \end{bmatrix}$

$Det_1 = \Delta$ $Det_2 = -\Delta$

ii) Multiply by Non-zero (Scalar)

$\begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$ ⟶ $\begin{bmatrix} 50 & 10 \\ 4 & 3 \end{bmatrix}$

$Det = \Delta$ $Det = 10\Delta$

iii) Adding Row

$\begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$ ⟶ $\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}$ ← Det. Not change

$Det = \Delta$ $Det = \Delta$