Dividing Sequences

(IARCS OPC Archive, K Narayan Kumar, CMI)

This problem is about sequences of positive integers $a_1,a_2,\ldots,a_N$. A subsequence of a sequence is anything obtained by dropping some of the elements. For example, *3,7,11,3* is a subsequence of *6,**3**,11,5,**7**,4,3,**11**,5,**3*** , but *3,3,7* is not a subsequence of *6,3,11,5,7,4,3,11,5,3* .

A *fully dividing sequence* is a sequence $a_1,a_2,\ldots,a_N$ where a_i divides a_j whenever $i < j$. For example, *3,15,60,720* is a fully dividing sequence.

Given a sequence of integers your aim is to find the length of the longest fully dividing subsequence of this sequence.

Consider the sequence *2,3,7,8,14,39,145,76,320*

It has a fully dividing sequence of length *3*, namely *2,8,320*, but none of length *4* or greater.

Consider the sequence *2,11,16,12,36,60,71,17,29,144,288,129,432,993* .

It has two fully dividing subsequences of length *5*,

- ***2***,*11,16,**12**,**36**,60,71,17,29,**144**,**288**,129,432,993* and
- ***2***,*11,16,**12**,**36**,60,71,17,29,**144**,288,129,**432**,993*

and none of length *6* or greater.