* Random Variables
A random variable is a function that assigns a real number to each outcome in the Sample Space of a random experiment.

* Discrete Random Variable:
A random variable which takes on finite number of values is called as discrete random variable.
ex. Sum appearing on tossing two coins

* Continuous Random Variable:
A random variable which takes on non Countably infinite number of values is called as Continuous random variable.
ex. Arrival time of bus.

* Probability Mass Function
Let X be a discrete random variable which takes k different values,
The PMF of X is given by

$$F(x) = \begin{cases} P(X = x_i) = P_i & \text{for } i = 1, 2, \ldots k \\ 0 & \text{otherwise} \end{cases}$$

ex. Rolling a die.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$X : \Omega \rightarrow \mathbb{R}$$

$$f(x) = \begin{cases} 1/6 & \text{if } x \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases}$$

$f(x) \geq 0$

$\sum_{x} f(x) = 1$

* Cumulative Distribution Function.

The cumulative distribution function (CDF) describes the probability that a random variable X takes on a value less than or equal to a given number x.

$$F(x) = P(X \leq x)$$

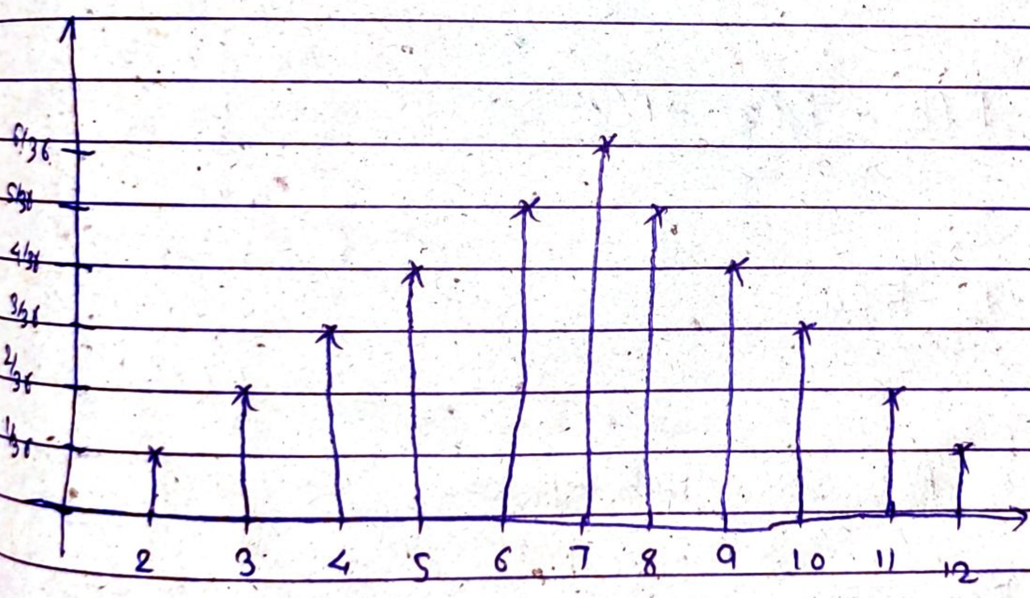
$0 \leq F(x) \leq x$ for all x.

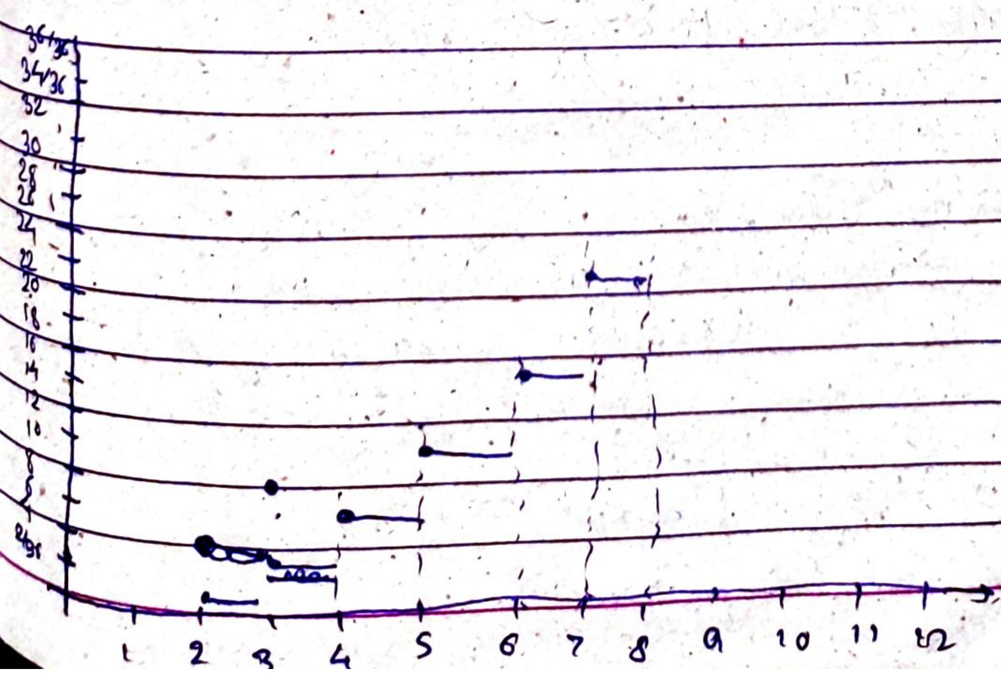
$$F(x) = \sum_{u \leq x} f(u)$$

Q A pair of dice is thrown. X denotes the sum

$$f(x) = \begin{cases} 0 & -\infty < x < 2 \\ 1/36 & 2 \leq x < 3 \\ 2/36 & 3 \leq x < 4 \\ 3/36 & 4 \leq x < 5 \\ 4/36 & 5 < x < 6 \\ 5/36 & 6 \leq x < 7 \\ 6/36 & 7 \leq x \leq 8 \\ 5/36 & 8 \leq x < 9 \\ 4/36 & 9 \leq x < 10 \\ 3/36 & 10 \leq x < 11 \\ 2/36 & 11 \leq x < 12 \\ 1/36 & 12 \leq x < \infty \end{cases}$$

$$F(x) = \begin{cases} 0 & -\infty < x < 2 \\ 1/36 & 2 \le x < 3 \\ 3/36 & 3 \le x < 4 \\ 6/36 & 4 \le x < 5 \\ 10/36 & 5 \le x < 6 \\ 15/36 & 6 \le x < 7 \\ 21/36 & 7 \le x < 8 \\ 26/36 & 8 \le x < 9 \\ 30/36 & 9 \le x < 10 \\ 33/36 & 10 \le x < 11 \\ 35/36 & 11 \le x < 12 \\ 36/36 & 12 \le x < \infty \end{cases}$$





* Formula

1. $P(X \leq a) = F(a)$

2. $P(x < a) = P(x \leq a - p(x=0$
$= F(a) - P(x=a)$

3. $P(X > a) = 1 - P(X \leq a)$
$= 1 - F(a)$

4. $P(X \geq a) = 1 - F(a) + P(X=a)$

5. $P(a \leq x \leq b) = F(b) - F(a) + P(x=a$

6. $P(a < x \leq b) = F(b) - F(a)$

7. $P(a \leq x < b) = F(b) - P(x=b) - F(a) + P(x=a)$

8. $P(a < x < b) = F(b) - F(a) - P(x=b)$

* Probability Density Function:

For a continuous random variable X with a probability density function f(x), the probability that X falls within an interval [a, b] is given by

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx.$$

ex. $f(x) = \begin{cases} cx^2 & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$

Compute $P(1<x<2)$

$$P(1<x<2) = \int_1^2 cx^2\,dx = \left[\frac{cx^3}{3}\right]_1^2 = \frac{8c}{3} - \frac{c}{3} = \frac{7c}{3}$$

We know

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^3 cx^2 = 1$$

$$\Rightarrow \left[\frac{cx^3}{3}\right]_0^3 = 1$$

$$\Rightarrow \frac{\cancel{27}^{\,9}c}{\cancel{3}} = 1$$

$$\boxed{c = \frac{1}{9}}$$

$P(1<x<2)$

$= \frac{7}{3} \times \frac{1}{9}$

$= \frac{7}{27}$

Q. $f(x) = 1.5 - 6(x-50.2)^2$ for $49.5 \le x \le 50.2$

$= 0$ otherwise

$$\int_{49.5}^{50.2} 1.5 - 6(x-50.2)^2\,dx = \left[1.5x - 2(x-50.2)^3\right]_{49.5}^{50.2}$$

$$= 1.$$

$\Rightarrow$ It is valid PDF.

* CDF for CRV.

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)\,dy.$$

* Expectation:

It is the weighted average of all possible outcomes.

CRV:

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

DRV:

$$E(x) = \sum_{i=1}^{k} x_i p_i$$

Q. CRV. X

$$f(X) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

find $E(X)$, $E(X^2)$.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{0}^{\infty} x \cdot 2e^{-2x}\,dx.$$

Using integration by parts

$$= x\int 2e^{-2x}dx - \int\left(1\cdot\int 2e^{-2x}dx\right)dx$$

$$= x\left(\frac{2e^{-2x}}{-2}\right) - \int \frac{2e^{-2x}}{-2}\,dx$$

$$= -xe^{-2x} - \int -e^{-2x}\,dx$$

$$= -x\left[e^{-2x}\right]_0^{\infty} - \left[\frac{e^{-2x}}{2}\right]_0^{\infty}$$

$$= 0 - \left[\frac{1}{2}e^{-2x}\right]_0^{\infty}$$

$$= \frac{1}{2}$$

$$E[X^2] = \int_0^{\infty} x^2\, 2e^{-2x}\,dx$$

$$= \cancel{x^2\left[-e^{-2x}\right]_0^{\infty}} - \int \cancel{2x\left[-e^{-2x}\right]_0^{\infty}}\,dx$$

$$= 0 -$$

$$= 2\int_0^{\infty} x^2 e^{-2x}$$

we know $\int x^n e^{-\lambda x} = \frac{n!}{\lambda^{n+1}}$

$$\Rightarrow E[X]^2 = 2\cdot\frac{2!}{2^3} = \frac{\cancel{2\times 2}}{\cancel{2\times 2}\times 2} = \frac{1}{2}$$

* When probabilities are equal

$$E(X) = \frac{x_1 + x_2 + \cdots x_n}{n}$$

1) $E[cX] = c\,E(X)$

$E(X+Y) = E(X) + E(Y)$

$E(XY) = E(X) \cdot E(Y)$

$Var(X) = E[(X-\mu)^2]$

For DRV.

$$Var(X) = \sum (x-\mu)^2 f(x)$$

For CRV

$$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

* $Var(cX) = c^2 Var(X)$

$Var(X+Y) = Var(X) + Var(Y)$

Q Find $E[x]$ $f(x) = 1.5 - 6(x-50)^2$

$$E[x] = \int_{49.5}^{50.5} x\,(1.5 - 6(x-50)^2)\,dx$$

let $y = x - 50$

$$E[X] = \int_{-0.5}^{0.5} (y+50)(1.5 - 6(y)^2)\,dy$$

$$\int_{-0.5}^{0.5} \left(-6y^3 - 300y^2 + 1.5y + 75 \right) dy$$

$$= \left[-\frac{3y^4}{2} - 100y^3 + 0.75y^2 + 75y \right]_{-0.5}^{0.5}$$

$$= \underline{\underline{50}}$$

Q A pair of dice is thrown find variance & std deviation of sum.

$$f(x) = \begin{cases} 1/36 & x = 2 \\ 2/36 & x = 3 \\ 3/36 & x = 4 \\ 4/36 & x = 5 \\ 5/36 & x = 6 \\ 6/36 & x = 7 \\ 5/36 & x = 8 \\ 4/36 & x = 9 \\ 3/36 & x = 10 \\ 2/36 & x = 11 \\ 1/36 & x = 12. \end{cases}$$

Let X denote the sum of numbers obtained.

$$\cancel{\text{Var}}\ E(X) = \sum x_i p_i = \frac{2 \times \frac{1}{36} + 3 \times \frac{2}{36} + \cdots}{36.}$$

$$= \frac{252}{36} = 7.$$

$$E(X^2) = 2^2 \times \frac{1}{36} + 3^2 \times \frac{2}{36} + \cdots$$

$$= \frac{1974}{36},$$

$$\text{Var} = E[X^2] - (E[X])^2 = 54.83 - 49 = \underline{5.83}$$

Q If $X = CRV$.

PDF $=$ $f(x) = \begin{cases} \alpha(2x - x^2) & 0<x<2 \\ 0 & \text{Otherwise} \end{cases}$

find 1) α

2) $P(X>1)$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^2 \alpha(2x - x^2) = 1$$

$$\alpha\left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^2 = 1$$

$$\alpha\left[4 - \frac{8}{3}\right] = 1$$

$$\boxed{\alpha = \frac{3}{4}}$$

$$P(x>1) = \int_1^{\infty} f(x)dx = \int_1^2 \left[\frac{3}{4}x^2 - \frac{x^3}{4}\right]_1^2$$

$$= \frac{1}{2}$$

Q If $X = CRV$.

PDF

$f(x) = \begin{cases} kx^2 & -3<x<3 \\ 0 & \text{Otherwise} \end{cases}$

1) Find k 2) $P(1 \le x \le 2)$ $P(x > 1)$ $P(x \le 2)$

$$\int_{-3}^{3} kx^2 = 1$$

$$\left[\frac{kx^3}{3}\right]_{-3}^{3} = 1$$

$$9k - (-9k) = 1$$

$$k = \frac{1}{18}$$

$P(1 \le x \le 2)$

$$= \int_1^2 \frac{1}{18} x^2 = \frac{1}{18}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{18}\left(\frac{8}{3} - \frac{1}{3}\right) = \frac{7}{18 \times 3}$$

$$= 0.1296$$

$P(x > 1)$

$$= \int_1^3 \frac{x^2}{18}$$

$$= \frac{1}{18}\left[\frac{27}{3} - \frac{1}{3}\right]$$

$$= \frac{26}{54} = 0.4815$$

$P(x \le 2$

$$= \int_{-3}^{2} \frac{x^2}{18} dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2}$$

$$= \frac{1}{18}\left[\frac{8}{3} - \frac{(-27)}{3}\right.$$

$$= \frac{1}{18} \times \frac{8+27}{3} = \frac{35}{54}$$

$$= 0.6481$$

* Joint Distribution
If X and Y are two discrete random variable we define the joint probability distribution of X & Y by $P(X = x, Y = y) = f(x,y)$
where $f(x,y) \ge 0$

2. $\sum_{x}\sum_{y} f(x,y) = 1$

The joint distribution of two or more random variables describes the probability that these variables take a specific values simultaneously.

$X = x_j$ is obtained by adding all entries in the row corresponding to x_j is given by

$$P(X = x_j) = f(x_j) = \sum_{k=1}^{n} f(x_j, y_k)$$

y	y_1	y_2	...	y_n	
x_1	$f(x_1, y_1)$	$f(x_1, y_2)$		$f(x_1, y_n)$	$f(x_1)$
x_2	⋮	⋮		⋮	$f(x_2)$
x_3	⋮	⋮		⋮	$f(x_3)$
x_4	$f(x_4, y_1)$	$f(x_4, y_2)$	---	$f(x_4, y_n)$	$f(x_4)$
x_n	$f(x_n, y_1)$	$f(x_n, y_2)$	---	$f(x_n, y_n)$	$f(x_n)$

* For CRV.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$P(a < X < b,\ c < Y < d) = \int_{a}^{b}\int_{c}^{d} f(x,y)\,dx\,dy$$

Suppose X and Y are discrete RV and the events $X=x$, $Y=y$ are independent events for all x & y then we say that RV are independent random variables.

$$P(X=x, Y=y) = P(X=x)P(Y=y)$$

If X & Y are independent CRV then.

$$P(X \le x, Y \le y) = P(X \le x)\,P(Y \le y)$$

* Change of Random Variable.

Let X be a discrete random variable whose probability function is $f(x)$
Suppose that a discrete random variable U is defined in terms of X by $U = \phi(X)$, where to each value of X there corresponds one and only one value of U and conversely, So that $X = \psi(U)$
Then the probability function for U is given by.

$$g(u) = f[\psi(u)]$$

Let X and Y be discrete random variables having join probability function $f(x,y)$. Suppose that two discrete random variables U and V are defined in terms of X and Y by $U = \phi_1(X,Y)$, $V = \phi_2(X,Y)$ Where to each pair of values of X and Y there corresponds one and only one pair of values of U and V.

So that $X = \psi_1(U,V)$, $Y = \psi_2(U,V)$

Then the joint probability function of U & V is given by

$$g(u,v) = f[\psi_1(U,V), \psi_2(U,V)]$$

* CRV : Let X be a CRV with prob densitity $f_1(x)$.

let $U = \phi(X)$ wher $X = \psi(U)$ then pdf of U is given by

$$g(u)\,|du| = f(x)\,|dx|.$$

or $$g(u) = f(x)\left|\frac{dx}{du}\right| = f[\psi(u)]\,|\psi'(u)|$$

The density function of the sum of two CRV X and Y i.e. of $U = X+Y$, having joint density function $f(x,y)$ is given by

$$g(u) = \int_{-\infty}^{\infty} f(x, u-x)\,dx.$$

If X and Y are independent

$$g(u) = \int_{-\infty}^{\infty} f_1(x)\, f_2(u-x)\,dx.$$

Which is called as convolution of f_1 and f_2 written as $f_1 * f_2$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

* Conditional Distribution

If X and Y are discrete random variables and we have events A and B then as.

$(A : X = x)$

$(B : Y = y)$ then.

$$P(Y=y|X=x) = \frac{f(x,y)}{f_1(x)}$$

where $f(x,y) = P(X=x, Y=y)$ is the joint probability function and $f_1(x)$ is the marginal probability function for X.

$$f(y|x) = \frac{f(x,y)}{f_1(x)} \qquad f(x|y) = \frac{f(x,y)}{f_2(y)}$$

* Binomial Distribution.

The binomial distribution is a discrete prob distribution that describes the number of successes in a fixed number of independent trials each having same probability of success.

$$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}$$

$E(X) = np$

$Var(X) = npq = np(1-p)$.

$\sigma = \sqrt{npq}$.

Ex. Prob of getting exactly 2 head in 6 tosses is

$$P(X=2) = \binom{6}{2}\left(\frac{1}{2}\right)^6 = \frac{6!}{2!\,4!} \times \frac{1}{2^6} = \frac{15}{64}$$

Q On 5 question MCQ test there are 5 possible ans of which 1 is correct. If student guesses randomly, what is probability she is correct only on two questn

$$P(X=2) = \binom{5}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3$$

$$= \frac{5}{3!\,2!} \times \frac{1}{25} \times \frac{4^3}{25 \times 5}$$

$$= 0.1024$$

Q Find the prob that in family of 4 children that there will be
a) At least 1 boy
b) at least 1 boy and at least 1 girl.

a) $P(1\text{ boy}) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = \frac{1}{4}$

$P(2\text{ boy}) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

Let X be the no of boys in family with 4 children.

$p(3 \text{ boys}) = \left(\frac{4}{3}\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = \frac{1}{4}$

$p(4 \text{ boys}) = \left(\frac{4}{4}\right)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = \frac{1}{16}$

$P(\text{atleast 1 boy}) = P(1 \text{ boy}) + P(2 \text{ boys}) + P(3 \text{ boys}) + P(4 \text{ boys})$
$= \frac{15}{16}$

b) $P(\text{at least 1 boy and at least 1 girl})$

$= 1 - P(\text{no boy}) - P(\text{no girl})$
$= 1 - \frac{1}{16} - \frac{1}{16} = \frac{7}{8}$

Q Out of 2000 familes with 4 children, how many would you expect to have (a) at least 1 boy.
b) 2 boys
c) 1 or 2 girls
d) no girls.

a. Expected no of families with at least 1 boy
$= 2000 \times \frac{15}{16} = 1875$

b). Expected no of familes with 2 boy $= 2000 \times \frac{3}{8} = 750$

$P(1 \text{ or } 2 \text{ girls}) = \frac{1}{4} + \frac{3}{8} = \frac{5}{8}$

Expected no $= 2000 \times \frac{5}{8} = 1250$

d) Expected no of families with no girl $= \frac{2000}{16} = 125$

Q Given that 2% of the fuses manufacture by firsm are defective find probability of
1) Box containing 200 fuses has atleast 1 defect
2) No defective

Let X be no of defective fuses in box of 200.

$b(x;\ 200,\ 0.02)$

$P(X=0) = \binom{200}{0} (0.02)^0 (1-0.02)^{200}$

$= 0.98^{200}$

$= 0.0176$

Prob

$P(X \geq 1) = 1 - P(X=0)$

$= .9824$

If 20% of the bolts produced are defective, determine the prob that out of 4 bolts chosen at random

a) 1

b) 0

c) less than 2 will be def

Let X = No of defective

$b(X, 4, 0.2)$

a) $P(X=1) = \binom{4}{1}(0.2)^1(0.8)^3 = 0.4096.$

b) $P(X=0) = \binom{4}{0}(0.2)^0(0.8)^4 = 0.4096$

c) $P(X<2) = P(X=0) + P(X=1).$

$= 0.4096 + 0.4096 = 0.8192.$

* Poisson distribution:

The probability of observing exactly k occurences in a poisson distribution is given b.

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

where λ is a given positive constant

$E(X) = \lambda$

$Var(X) = \lambda$

$\sigma = \sqrt{\lambda}$

In a certain factory ~~there~~ ~~are~~ manufacturing blades there is a small chance of 0.002 the blade to be defective.
The blades are supplied in packets of 10 using poisson dist as well as binomial find.
approximate no of packs containing
1) No defective blade
2) One defective in 10,000 pack Consm

No of.

$P = 0.002 \qquad P' = 99.998$

$$\text{mean} = np = 10 \times 0.002 = 0.02$$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{0.02^x e^{-0.02}}{x!}$$

a) No defective $\Rightarrow x = 0$

$$P(0) = \frac{0.02^0 e^{-0.02}}{0!} = e^{-0.02} = 98 \times .98 = 0.9801$$

$$P(1) = \frac{e^{-0.02} \times 0.02}{1} = 0.0196$$

$\Rightarrow$ No dfa $= 0.9801 \times 10000 = 9801$

1 defect $= 196$.

Using binomial.

$b(x, n, p)$

$n = 10$

$p = 0.002$

X : No of defective in a packet

$$P(X=0) = \binom{10}{0} \cancel{(0.002)^0} \cancel{(1-0.002)}^{10}$$

$$= 0.998^{10}$$

$$= 0.9802$$

No of packets in consignment of 10,000

$= 0.9802 \times 10,000$

$= 9802$

$$P(X=1) = \binom{10}{1} (0.002)^1 (1-0.002)^9$$

$$= 0.0196$$

No of packet $= 0.0196 \times 10,000$

$= 196$

Q If a bank receives on the average 6 bad cheques per day. Find the probability that it will receive 4 bad cheques on any day

Given $\lambda = 6$ (Average bad Cheques)

By Poisson distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{6^x e^{-6}}{x!}$$

$$P(X=4) = \frac{6^4 e^{-6}}{4!}$$

$$= 0.1338$$

~~Using~~

Q. A Web Server receives on average of 1,000 queries per day.
How much excess capacity should it build into be 95% sure that it can meet the demand.

$\lambda = np = 1000$

$\sigma = \sqrt{1000} = 31.62$

For 95% probability we use Z-Score of 95%.

$Q_{95} = 1000 + (1.645 \times 31.62)$

$= 1052$

$\Rightarrow$ 52 queries per day excess

The mean weight of 500 male students at a certain c'g is 75
Assuming that the weights are normally distributed find the no of students between 60 and 78 kg.
and more than 92 kg.
Given that deviation is 7kg.

Given $\mu = 75$
$n = 500$
$\sigma = 7$.

Let X denote the weight of student.

Convert to Standard normal.

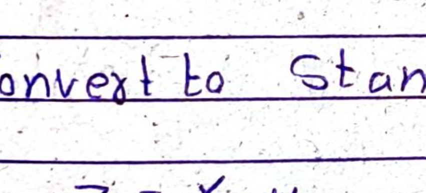
$$z = \frac{X - \mu}{\sigma}$$

For 60:

$$z = \frac{60-75}{7} = -2.14 \qquad \text{Use } 59.5$$

For 78

$$z = \frac{78-75}{7} = 0.43 \qquad \text{Use } 78.5$$



$$P(60 < X < 78) = P(Z < 0.43) - P(Z < -2.14)$$
$$= 0.6664 - 0.0162$$
$$= 0.6502$$

$\Rightarrow 0.6502 \times 500$
$= 325.1$ Students.
≈ 325.

b) Above 92.

$$z = \frac{92-75}{7} = \frac{17}{7} \simeq 2.43.$$

Use 91.5

$z = 2.357$

$P(X > 92) = 1 - P(Z < 2.43)$

$= 0.0075$

$0.0075 \times 500 = 3.75$

$\approx$ 4 Students

* Normal Distribution:

A normal distribution is a probability distribution that is symmetric around mean. mean most of the data points cluster around the central value.

It follows a bell shaped curve defined by pdf.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

μ = mean

σ = Standard deviation

σ^2 = Variance

68 $\mu \pm \sigma$

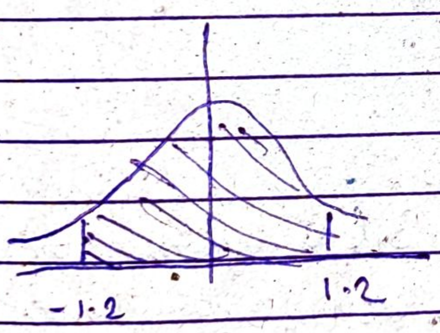
95 $\mu \pm 2\sigma$

99.7. $\mu \pm 3\sigma$

Q. The mean inside diameter of a sample of 200 washers produced by a machine is 0·502 inches and the standard deviation is 0·005 inches.
The purpose for which these washers are intended allow 0·496 to 0·508 inches.
otherwise they are considered defective.
Determne the % of defective washers.

$$0.496 \text{ in Standard Unit} = \frac{0.496 - 0.502}{0.005} = -1.2$$

$$0.508 \text{ in Standar Unit} = \frac{0.508 - 0.502}{0.005} = 1.2$$



$$P(0.496 < X < 0.508)$$
$$= P(-1.2 < Z < 1.2)$$
$$= P(1.2 > Z) - P(-1.22 < Z)$$
$$= 0.8849 - 0.1151$$
$$= 0.7898.$$

$$= 77\%$$

$\Rightarrow$ % of defective washers $= 23\%$

Q $\mu = 100, \quad \sigma = 15$

$X \sim N(100, 15)$

$\mu \pm \sigma$

$= 100 \pm 15$

$= 18$ to 115

$P(85 < X < 115)$

$Z_1 = \frac{85-100}{15} = -1, \quad Z_2 = \frac{115-100}{15} = 1$

$P(-1 < Z < 1)$

$= 0.8413 - 0.1587$

$= 0.6826$

$= 68.26\%$

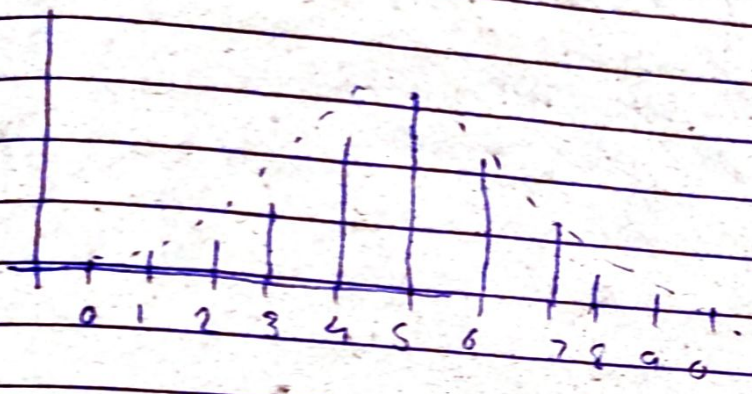
Q Find the prob of getting betw 3 & 6 heads in 10 tosses of fair coin by using binomial & normal approximation to the binomial distribution

$P(X=3) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128}$

$P(X=4) = \frac{105}{512}$ $\quad P(X=5) = \frac{63}{256}$

$P(X=6) = \frac{105}{512}$

$P(3 \leq X \leq 6) = 0.7734$



The prob distribution of no of heads in 10 tosses is similar to normal.

$\mu = np = 10 \times \frac{1}{2}$

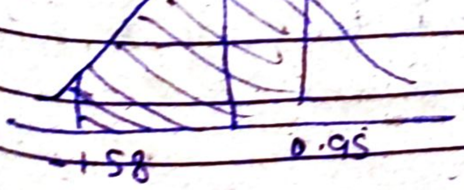
$\sigma = \sqrt{npq} = \sqrt{10 \frac{1}{2} \times \frac{1}{2}} = 1.58$

P(between 3 & 6) = $P(2.5 < X < 6.5)$

In standard unit

$z_1 = \frac{2.5 - 5}{1.58} = -1.58$

$z_2 = \frac{6.5 - 5}{1.58} = 0.95$



$P(2.5 < X < 6.5) = P(-1.58 < Z < 0.95)$

$= .8289 - 0.0571$

$= 0.7718$

Q A fair coin is tossed 500 times
Find prob of no of heads will not deffer from 250 by
a) More than 10
b) More than 30.

$\mu = np = 250$
$\sigma = \sqrt{npq} = 11.18$

a) We need prob of no of head lie in between 240 and 260.

$$z_1 = \frac{239.5 - 250}{11.18} = -0.94$$

$$z_2 = \frac{260.5 - 250}{11.18} = 0.94$$

$$P(240 \leq X \leq 260) = 2\,P(-0.94 < Z \leq 0)$$
$$= 2\,(0.5 - P(Z < -0.94$$
$$= 2\,(0.5 - 0.1788)$$
$$= 0.6528$$

Q A die is tossed 120 times. Find the prob that the face 4 will turn up 18 times or less

Let x is no of 4's obtained

$P = \frac{1}{6}$

$\mu = np = 120 \times \frac{1}{6} = 20$

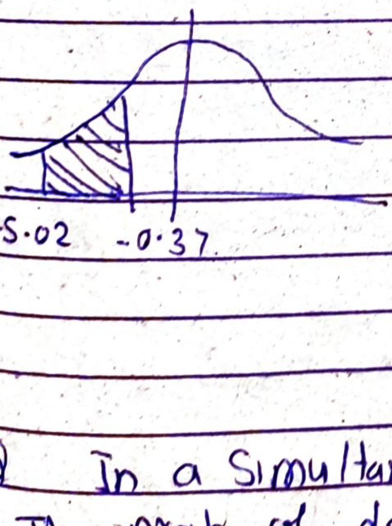
$\sigma = \sqrt{npq} = 4.08$.

P (18 times or less)

$P(-0.5 \leq X \leq 18.5)$

$Z_1 = \frac{-0.5 - 20}{4.08}$ $\qquad$ $Z_2 = \frac{18.5 - 20}{4.08}$

$= -5.02$ $\qquad$ $= -0.37$



$= P(Z \leq -0.37) - P(Z \leq -5.02)$

$= .3557 - 0$

$= 0.3557$

Q In a Simultaneous inspection of 10 units. The prob of defect is equal to non defect. Find prob at least 7 nondefective

Let x be no defective

$P(X \geq 7)$

$\mu = np = 10 \times \frac{1}{2} = 5$.

$\sigma = \sqrt{npq} = 1.58$.

$z = \frac{7-5}{1.58}$

$1 - P(0.95 \leq z) = 0.1711$

Q. The mean inside diameter of a Sample of 200 washers produced by a machine is 0·502 inches.
S.D = 0·05.
0·496 to 0·508 is allowed
What % of washers are defect

$\mu = 0.502$
$\sigma = 0.005.$

Assuming Normal distribution,

$$z_1 = \frac{0.496 - 0.502}{0.005} = -1.2$$

$$z_2 = \frac{0.508 - 0.502}{0.005} = 1.2.$$

$$\begin{array}{r} 0.8849 \\ -0.5000 \\ \hline 0.3849 \end{array}$$

$0.3449 \times 2.$

$\Rightarrow 77\%.$

Q. The marks of 1000 students are normally distributed with mean 78% and S.D 11%
1. How many student got marks above 90%.
2. What are the highest Score by the lowest 10% of studen.

$\mu = 0.78$
$\sigma = 0.11$
$N = 1000.$

Let X be marks of Students (%ages)

$P(X \geq 0.90)$

$$z = \frac{.90 - .78}{.11} = 1.09$$

$P(Z \leq 1.09) = 0.8621$

$P(Z > 1.09) = 1 - 0.8621$
$= 0.1378$

No of Stud $= \underline{\underline{138}}$

b).

$P(X < x_1) = 0.1$ (i.e 10%)

$P(Z < z_1) = 0.1$

$$-1.28 = \frac{x - 0.78}{0.11}$$

$x = \underline{\underline{64\%}}$

Q In a normal distribution 7% of the items are under 35 and 89% of items are under 63 determine mean and variance.

$P(X < 35) = 7\% = 0.07$

$P(X < 63) = 89\% = 0.89$

$P(Z < -1.47) = 0.07$

$P(Z < 1.22) = 0.89$

$\Rightarrow \quad Z_1 = -1.475 \qquad Z_2 = 1.227$

$$Z = \frac{X - \mu}{\sigma}$$

$$\frac{35 - \mu}{\sigma} = -1.475 \qquad \frac{63 - \mu}{\sigma} = 1.227$$

$35 - \mu = -1.475\,\sigma$

$-\;63 - \mu = 1.227\,\sigma$

$-28 = -2.702\,\sigma$

$\boxed{\sigma = 10.37}$

$\boxed{\mu = 50.3}$