In a normal distribution
7% of the items < 35
89% of items are under 63
determine mean and
variance of the
distribution

1.54	Prob
	bayes
	ind even
	Random Var
	Descr
	Con
	Joint
	Change of
	Pd varn
	Condtl dist
	Binom
	Norm
	Poisn
	Relationsh
	betw Binom
	Norm & Poissn.

0.9382.

Poisson distribution is given
as $X \sim P_o(\lambda)$

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}, x=0,1,2 \ldots$$

A poisson dist arises when
the events being counted
a independently. Such that
the probability that two or
more event occur simultaneously

Given length of material
accidents on a particular
part of road in a given
time or telephone calls
made to a call centre
in one day.
and the key parameter in
fitting the
is the mean value.

2% of the fuses, ...
200 fuses has atleast 1 defect.

$\lambda = np$
$= 200 \times 0.002$
$= 4$

20% of items from a factory are
defective find the prob that in a
Sample of 5 chosen at random
none defective item is chosen.
2. One is defective
3. $P(1 < X < 4)$

$P(X=0)$

P&S by
Mrrey Spizel
Schaum Series.

1, 2, 4

Sampling Theory.

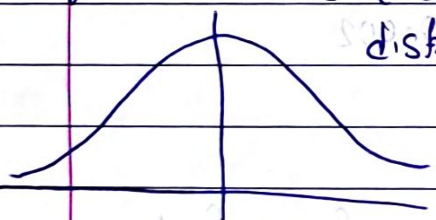
* Population:
* Sample.

* Central Limit theorem:

	Mean
$S_1 (x_1, x_2 \dots x_{50})$	$\bar{x}_1$
$S_2 (x_1, x_2 \dots x_{50})$	$\bar{x}_2$
$\vdots$	$\bar{x}_3$
$S_n (x_1, x_2 \dots x_{50})$	$\bar{x}_n$

n Samples are taken from a population of Size N
Each Sample is of Size 50.

On plotting the means we get normal (Gaussian) distribution



It may or may not be a guassian distribution

If taken means of these large Samples and plotted histogram then mean ($\bar{x}$) will be approximate equal to

$$\bar{X} \approx GD\left(\mu, \frac{\sigma^2}{n}\right)$$

Obx.

Let $X_1, X_2, X_3 \dots X_n$ be a Sequence of independent and identically distributed random variable ~~with~~ having mean μ and Varian σ^2 then for large n the distribution of X_1, X_2, X_n is approximately normal & Variance σ^2.

A coin is tossed 200 time find approximate prob that number of heads obtained is between 80 to 120.

$n = 200$

$p = q = \frac{1}{2}$

To find:
$P(80 \le X \le 120)$

$\mu = np = 200 \times \frac{1}{2} = 100$

$\sigma^2 = npq = 100 \times \frac{1}{2} = 50$

~~$Z_1 = \frac{200 - 70.5}{\sqrt{50}}$~~

$\bar{X} = G.D\left(\mu, \frac{\sigma^2}{n}\right)$

$z_1 = \frac{80 - \mu}{\sigma} = \frac{80 - 100}{\sqrt{50}}$

$\underline{\underline{2.88}}$ $= -2.888$

$$Z_2 = \frac{120-100}{\sqrt{50}}$$

$= 2.888$

9980 -

$P = 0.0010$

Work on #26 (Z Value)

Civil engineers believe that the amount of weight W (in units of 1000 pounds) That a certain bridge can withstand without structural damage is normally distr with mean 400 and standard dev 40. Suppose that the weight (in units of 1000 pound) of a car is random variable with mean 3 and Std dev 3 how many car would have to be on bridge for the probability of structural damage exceed 0.1.

let x denote weight on which we get 0.1 damage.

$P(X=x) = 0.1$

$$Z_1 = \frac{x-400}{40}$$

Z Score of $0.1 = -2.325$

$$\frac{x-400}{40} = -2.325$$

$x = 307.$

$\Rightarrow$ 307 thousand pounds.

A random sample of Size 100 is taken from an infinite population Having mean $\mu = 156$ $\sigma^2 = 256$. what is the probability that $\bar{X}$ will be between 75 and 78.

* Sampling

Probabilistic / Non Probabilistic.

- Simple Random Sampling
- Startified " "
- Systematic Random Sampling.
- Cluster.

- purposive
- convnient
- Quota
- Snowball.

Population
Parameter

Sample
Statistic.

Parameter
It is a Statistical measure.

Statistic.
It is a measure based on Sample

Sampling distribution of a Statistic

- To estimate unknown population parameter from known statistic
- To set Confidence limits of parameter within which the parameter values are expc
- To test a hypothesis
- To get/draw a Statistical inference

A populat consist of 5, 10, 14, 18, 13, 24, Consider all possible Samples of Size 2 which can be drawn without replacement from the population
Find mean of the population
S.D of po
Mean of Sampling distrib of mean
· SD of the sampling distribution of means.

$^6C_2$ Samples will be there. (15).

$\bar{x} = \frac{5+10+14+18+13+24}{6}$

$= 14$ $\sigma = \sqrt{35.67}$

mean of Sampling distribution of means.

(5,10) (5,14) (5,18) (5,13) (5,24)
(10,14) (10,18), (10,13), (10,24),
(14,18) (14,13) (14,24)
(18,13) (18,24)
(13,24)

7.5 + 9.5 + 11.5 + 9 + 14.5,
12 + 14 + 11.5 + 17.
16 + 13.5 + 19
15.5 + 21 + 18.5

= 14

$\sigma = 3.78$

No of Samples with replacement

A population consist of 18, 5, 10, 14, 13, 24 Consider of Size 2 which is drawn with replacem fn mean, sd, $\bar{x}$ of Samplgh SD of Sam

No of Samples = 6×6
= 36

An insurance has 25,000 atomobile policy holders. If the yearly claim of a policy holder is represented by a random variable with mean 320 and SD: 540

Approximate the prob that total yearly claim exceeds 8.3 Million

X: Yearly Claim of policy holders

$\mu = 320$

$\sigma = 540$

$P(X > 8.3 \text{ Million})$

$\frac{8.3\ Mn - 320}{540}$

We have C.L.T.

with $\mu = 320$

$\sigma = 540$.

$X = \sum_{i=0} X_i$

$\mu = 320 \times 25000 = 8 \times 10^6$

$\sigma = 540 \times \sqrt{25000} =$

$Z = 3.512$

$= 0.0003$.

If 1 Gallon can of paint Covers on 0.5 Sq 513.3 Sq feed of SD of 31.5 Sq feet

What is Prob that the sample mean area covered via a sample of 40 of these 1 Gallon cans will be anywhere from 510.0 520.0 Sq. feet.

Lecture 24 Sampling distributions

Orthagonal transformation

Lec No 23, 24, 25
Someshwar kgp.

Chi Square
Student
Standard Normal

Sampling distribution of means, sum & difference

★ Statistical Inference:

- Estimation
- Hypothesis Testing.

Point Estimation:

Confidence Interval

Sampling distribution of means
" " sums
" " differences

The mean of the Sampling distribution of means is ~~denoted by~~

$$\mu_{\bar{x}} = E(\bar{x}) = \mu$$

If a population is infinite and the sampling is random or if the population is finite and Sampling is with replacement; then the variance of the Sampling distribution of means

$$Var = \sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = E\left[(\bar{x}-\mu)^2\right]$$

If the population is of size n and Sampling is without replacement and if the size of the sample is n then

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{N}\left(\frac{N-n}{N-1}\right)$$

If the population from which the samples are taken are normally distributed with mean μ and var σ^2 then the Sample mean is distributed with mean μ and Variance $\frac{\sigma^2}{n}$

Suppose the population from which Samples are taken having a prob distribution with mean μ and var σ^2 it is not necessary

to have normal distribution Then the Standardised random variable associated by $\bar{x}$

$$\boxed{Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}}$$

Q.

Assuming the heights that 3000 male Students from are normally distribute with mean 68.0 inch. and σ 3.0 inches.

If 80 Samples consisting of 25 Students each are obtained What would be the mean & S.D of resulting Sample of means of Sampling has been done

1. With replacement
2. Without replacement

$\mu = 68.0 = \mu_{\bar{x}}$

a) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = 0.6$ inches

b) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{1}{\sqrt{25}}\sqrt{\frac{3000-25}{3000-1}}$ 0.5

No of Samples with Out replac $= 3000^{25}$.

No of Samples with reple $= 3000C_{25}$

Mean of Sample

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - 68.0}{0.6}$$

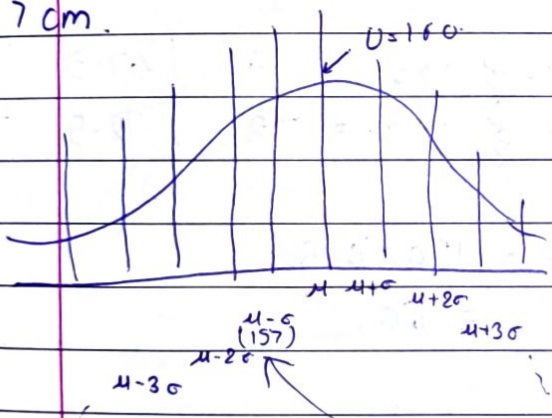
0.0885

8.69%

Q Suppose it is known that the heights of all premiere league players follows normal dst with mean of 160 cm and S.D of 7 cm.
What is the prob that the average height of 10 random big players is less than 157 cm.

$\mu = 160$ cm

$\sigma = 7$ cm.



$z = 157$

Sample Size $= 10$.

$n = 10$

$\mu = 160$ cm

$\sigma = 7$cm.

$\mu_{\bar{x}} = \mu = 160$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{10}} = 2.21.$$

$P(X < 157)$

$$Z = \frac{157 - 160}{2.21} = -1.35$$

Prototype automobile tyre has a design life of 38,500 miles with S.D of 2,500 miles.
5 Such tyres are manufactured and tested on the assumption that the actual population mean 38,500 miles.
Actual populat. SD is 2500 miles find the prob that the Sample mean will be less than 36,000 miles
Assume that the distribution of the lifetime of the tires is normal.

23, 24, 25 27, 28, 29

Least Square estimation
Method of moments
Maximum likelihood

Let $x_1, \ldots x_n$ be random Sample from a population with pdf (pmf) $f(x, \theta)$
The joint pdf (pmf) of $x_1, \ldots x_n$

$$f(x, \theta) = \prod_{i=1}^{n} f(x_i, \theta)$$

$$= L(\theta, x) \rightarrow \text{likelihood fn}$$

Q 1. Let $X_1 \ldots X_n \sim P(\lambda), \lambda > 0$

$$L(\lambda, x) = \frac{e^{-\lambda} \lambda^{x_i}}{x_i!}$$

$$= \frac{e^{-n\lambda} \lambda^{\sum x_i}}{\prod_{i=1} (x_i!)}$$

$l(\lambda) = \ln L(\lambda)$

Sampling Theory

1) Sampling distribution of means
2) Sampling distribution of differences and sums.
3) Sampling distributions of variances.
4. & other statistics.

Suppose the population is infinite and binomially distributed with P & $q = 1-P$ find the respective probabilities for any given member that exhibits or not.

* Sampling Distribution of Properties

The population may be of tossing a coin with probabil 1/2 then

$\mu = np$

P could be the proportion of heads to the tossing of n tosses.

Then the sampling distribution whose mean

$\mu_p = P$

Find the probability in 120 tosses of a pair coin 40% to 60%

40% of 120 = 48	47.5
60% of 120 = 72	72.5

$np = 120 \times 0.5$
$= 60.$

$\sqrt{npq} = \sqrt{120 \times 0.5 \times 0.5}$
$= \sqrt{30}$
$= 5.2$

$Z_1 = \frac{47.5 - 60}{5.2}$ -2.28

$Z_2 = \frac{72.5 - 60}{5.2}$ $= 2.28$

$.9918 - .0082$

$0.9857 - 0.113$
$= 0.8727$

The election returns shows a certain candidate recen 46 % of votes determn the prob that a poll of 200 & poll of 1000 people selected at random from voting population would have showed majority in favour of candidate

Case 1.

99.5

100.5

$n = 200$

$p = .46$

$q = .54$

7.05

$\mu = 92$

$\sigma = 7.05$

$$z_1 = \frac{46\% \text{ of } 200 - 92}{7.05}$$

* Sampling distribut of diff

Let U_1 for the population whose mean is μs_1 and S.D is σs_1. Similarly for each sample of size n_2 drawn from second population with mean μ_{s_2} and S.D σs_2. We can obtain the distribution of differences $S_1 - S_2$ which is called as Sampling distribution of Statistics with the mean

10+2

13

$$\mu_{s_1 - s_2} = \mu_{s_1} - \mu_{s_2}.$$

$$\sigma_{s_1 - s_2} = \sqrt{\sigma^2_{s_1} + \sigma^2_{s_2}}.$$

has mean $\mu_{s_1} + s_2 =$

$\sqrt{\sigma^2_{s_1} + \sigma^2_{s_2}}$.

Assuming ....

....

200 hours and for company B with a mean life time of 1200 hours with S.D. of 150 hrs
A random sample of 125 bulbs of each brand is tested
What is the prob that the brand A bulbs we have mean life time that is atleast 160 hours and > 250 hours for bulbs of brand B

Mean of U_1, mean of U_2

Mean of $U_1 - U_2$.

S.D U_1, S.D U_2, S.D $_{U_1 - U_2}$

* Estimation Theory:

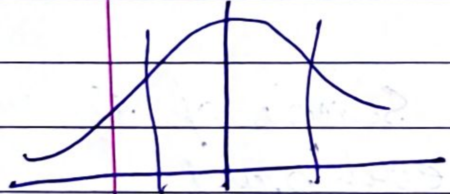
Null Hypothesis. : H_0
Alternate Hypothesis : H_1

ex. A bottle of water contain's 1000 ml water.

H_0 = bottle has exact 1000 ml water
H_1 = bottle does not have 1000 ml water

(Two tailed
↳ 998 ml - 1002 ml

ex.
Point estimat - 5.3 inches
Interval estimate = 5.1 - 5.6



$\mu \pm s$
$\mu \pm 2s$
$\mu \pm 3s$

Confidance interval estimates of a population parameter.

Let μ_s and σ_s be the mean and S.D of sampling distribution of statistic S

if the sampling distribution of S is approximately normal. we can find S lying between
$\mu_s \pm \sigma_s \rightarrow 68.27\%$
$\mu_s \pm 2\sigma_s \rightarrow 95.4\%$
$\mu_s \pm 3\sigma_s \rightarrow 99.73\%$

~~With a probability of 68.~~

Or with a confidence interval of 68.27%, 95.4%, 99.73%

Confidence intervals for means, variances & different statistics

Confidence interval of means
If the statistic S is the with the sample mean $\bar{x}$, then the 95% and 99% confidence limits for estimation of these population is given by.
$\bar{x} \pm 1.96\,\sigma_{\bar{x}} \rightarrow 95\%$
$\bar{x} \pm 2.58\,\sigma_{\bar{x}} \rightarrow 99\%$

In general the confidence intervals are given by

$$\boxed{\bar{x} \pm Z_c\,\sigma_{\bar{x}}}$$

The Z_c depends on the

particular level of confidence desired.

The confidence limits for the population mean are given by

$$\boxed{\bar{x} \pm Z_c \frac{\sigma}{\sqrt{n}}}$$

This is for the case the sampling is from an infinite population or Sampling is with replacement

$$\bar{x} \pm Z_c \frac{\sigma}{\sqrt{2}} \sqrt{\frac{N-n}{N-1}}$$

is unknown
So that, to obtain the confidence limits we use the estimator $\bar{S}$ or S

A Sample of Size 64 and mean 60 was taken from a population whose S.D is 10 construct 95% of Confidence interval for the mean.

$n = 64.$

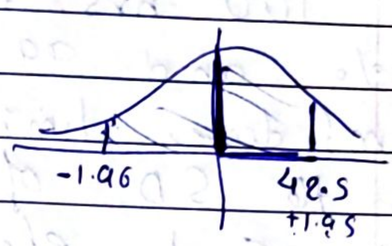
$\mu = 60$

$\sigma = 10$

$$\bar{x} \pm Z_c \frac{\sigma}{\sqrt{n}}$$

$60 \pm$

$2\overline{)95}$ 42.5

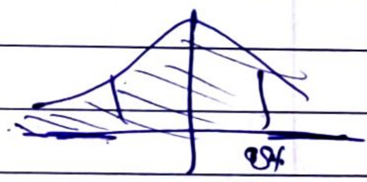


$0.5 - Z$

Suppose
Heights of all 1546 students from the university. determine the 98% and 99% interval for estimating the mean height of the

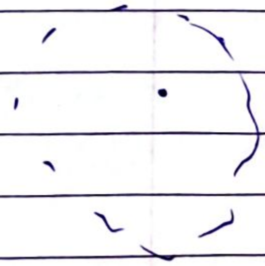
$\sigma = \frac{10}{\sqrt{64}}$

$\mu = 60.$



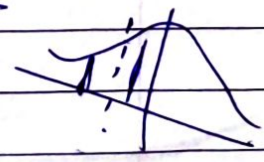
97.5

$$Z = \frac{42.5 - 60}{\frac{10}{\sqrt{64}}} = -14$$



$$\bar{x} \pm Z_c \frac{\sigma}{\sqrt{n}}$$

$$60 \pm 1.96 \times \frac{10}{\sqrt{64}}$$



$$60 \pm 3.36$$

$$[56.64, 63.36]$$

1.98
0.96

The Standard deviated of the life tm of 200 light bulbs was computed or 100 hrs find 95% and 99% Comfidence limit for the S.D. of all such electric bulbs.

256 what is the prob that Sample mean will be between 75 and 78.