~~Data Communication and Networking~~

Combinometrics Probability Statistics

Syllabus

Basic combinatorial nos, selection with repetition pigeon hole principle, catalan nos, double counting, recurence relation generating function, partition nos, inclusion exclusion principle.

Q Arrange 5 dashes and 8 dots.

$^{13}C_5 \cdot {}^{13}C_8$ Let there are n objects that are not distinct specifically

Let there be q_1 object of first type, q_2 objects of second type, ... q_r of t types.

Then the number of n permutation of these n objects is given by $\frac{n!}{q_1! q_2! q_n!}$

$13!$

* Selecting n objects from r objects

$^nC_r = \frac{n!}{r!(n-r)!}$

* Arrangement of r objects from n objects

$^nP_r = \frac{n!}{r!}$ $P(n,r) = \frac{n!}{(n-r)!}$

Q Arrange 7 from these dashes and dots.

$\cancel{^5C_5 \cdot {}^8C_2 \cdot \frac{7!}{5!2!} + {}^5C_4 \cdot {}^8C_3 \cdot \frac{7!}{4!3!}}$ $^5P_2 \cdot {}^8P_2 \, {}^{13}P_4$

$\frac{7!}{5!2!} + \frac{7!}{4!3!} + \frac{7!}{3!4!} + \frac{7!}{2!5!} + \frac{7!}{4!6!} + \frac{7!}{\ldots}$

In how many ways 2 books of different languages from 5 latin books 7 greek and 10 french book

$$\frac{2!}{2!} + \frac{2!}{2!} + \frac{2!}{2!} + 2! + 2! + 2!$$

latin	gree	fr
1	1	0
2	0	1
0	1	1
1	0	1

$= \cancel{1} + \cancel{1} + \cancel{1} + 2 + 2 + 2$

155

$$5c_1 \cdot 7c_1 + 5c_1 \cdot 10c_1 + 7c_1 \cdot 10c_1$$

$$= \frac{5!}{1!\,4!} \cdot 7 + 5 \cdot 10 + 7 \cdot 10$$

$= 35 + 50 + 70$

$= 155$

Q 5 persons how many different ways to sit of 5 chairs

$5! = 5 \times 4 \times 3 \times 2 = 120$

Q How many 3 letter words without repeatation

$26P_3 = 26 \times 25 \times 24$

Q There are 5 numbers 1,2,3,4,5, how many 3 digit numbers greater than 200 can be formed if repeatation is allow.
if repeatatn is not allw

1) Repeat allow = $4 \cdot 5 \cdot 5$ + = 100

2) " Not " $4 \cdot 4 \cdot 3 = 48$

Q 6 person can be seated in linear arrangments and a circular arrangement.

1). $6!$

2). $(n-1)!$

$= 120.$

Page No.:
Date:
YOUVA

6 digit Alphanumeric password
first 2 digits are alphabet
next 4 are ~~are~~ digit

* Repeatation Allowed:
$= 26 \times 26 \times 10 \times 10 \times 10 \times 10$
$= 6760000$

* Repeatation Not allowed.
$= 26 \times 25 \times 10 \times 9 \times 8 \times 7$
$= 3376800$

Q 10 player participate in raffle
$= 10!$

* $^nP_0 = \frac{n!}{(n-0)!} = 1.$

* $^nP_1 = \frac{n!}{(n-1)!} = n$

* $^nP_n = \frac{n!}{(n-n)!} = n!$

* $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$ $\quad \frac{(n+1)!}{r!(n+1-r)!}$

$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$

$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$

$= \frac{(n-r+1)n! + r.n!}{r(r-1)!(n-r+1)(n-r)!} = \frac{n!((n-\cancel{r}+1+\cancel{r})}{r(r-1)!(n-r+1)(n-r)!}$

$= \frac{n+1(n!)}{r(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!} = {^{n+1}C_r}$

Rough work (right margin):
$326 \times 26 = 156 + 520 = 676$
0 1 2 3 4 5 6 7 8 9
$26 \times 25 = 150 + 520 = 650$
$90 \times 8 = 720$
$720 \times 7 = 5040$
$504 \times 67 = 3528 + 30240 = 33768$

$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$

$\frac{n!(r-1)!(n-r+1)! + n!r!(n-r)!}{r!(n-r)!(r-1)!(n-r+1)!}$

$n!((r-1)!(n-r+1)! + r!(n-r)!)$

* Pigeon Hole Principle:

- There are 13 students prove that at least two students have birthday in same month.

- 101 integers that are to be selected from 1-200 then we have selected two numbers a and b such that a divides b (a/b gives same remainder)

- There are n numbers, if we divide these numbers by n-1 then at least two of the numbers should give same numbers. (remainder)

a number divided by n-1 will give remainder in range [0, n-2]

⇒ There are (n-1) possible remainders.

⇒ No of pigeon holes = n-1
No of pigeon = n.

⇒ as no of pigeon > No of holes
⇒ At least two numbers will have same remainder.

- 5 lakh words, 4 letter or fewer, will they be distinct or same.

Total possible words of 4 letter or fewer

$= 26\times26\times26\times26 + 26\times26\times26 + 26\times26 + 26$

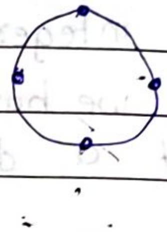
$= 26^4 + 26^3 + 26^2 + 26^1$

$= 475254$

No of pigeon holes = ~~5,00,000~~ 475254
No of pigeon = ~~475254~~ 5,00,000
⇒ They will be ~~distinct~~ repeatations

Given n points on a Circle, how many lines can be drawn.

~~(n-1) (n-2) (n-3)~~



directed : $^{n}P_2$

undirected : $^{n}C_2$.

$\frac{(n-1)(n-2)}{2}$

$(x+y)^n$

$= {}^{n}C_0 x^n y^0 + {}^{n}C_1 x^{n-1} y^1 + {}^{n}C_2 x^{n-2} y^2 + \ldots {}^{n}C_n x^0 y^n$

$= \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \ldots \binom{n}{n} x^0 y^n$

Q Prove

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots \binom{n}{n} = 2^n.$

LHS $= 1 + \frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} + \frac{n!}{3!(n-3)!} + \ldots = 2^n$

$= 1 + n! \left[\frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} + \frac{n!}{3!(n-3)!} + \ldots \right] = 2^n$

RHS $= 2^n = (1+1)^n$

$= \binom{n}{0} 1^0 1^n + \binom{n}{1} 1^1 1^{n-1} + \binom{n}{2} 1^2 1^{n-2} + \ldots \binom{n}{n} 1^n 1^0$

$= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots \binom{n}{n}$

$k=0$ ... $0!\,n! \times n!$

$RHS = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots \binom{n}{n}^2$

$= 1^2 + \left(\frac{n!}{1!(n-1)!}\right)^2 + \left(\frac{n!}{2!(n-2)!}\right)^2 + \dots \left(\frac{n!}{n!(0)!}\right)$

$LHS = \binom{2n}{n} = \frac{2n!}{n!\,n!} \times \cancel{n!} = \frac{2}{n!}$

$RHS = 1^2 + \left(\frac{n\cancel{(n-1)!}}{1!\cancel{(n-1)!}}\right)^2 + \left(\frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}}\right)^2 + \left(\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}}\right) + \dots$

$= 1^2 + \left(\frac{n}{1!}\right)^2 + \left(\frac{n\times(n-1)}{2!}\right)^2 + \left(\frac{n(n-1)(n-2)}{3!}\right)^2 + \dots$

$= \cancel{1^2 + \frac{n^2}{1!^2} + \frac{n^2-n}{2!^2} + \frac{n^3-3n^2+2n}{3!^2} + \dots}$

$n^2 - n \; (n-2)$

$n^3 - 2n^2 - n^2 + 2n \; (n-3)$

$n^3 - 3n^2 + 2n \; (n-3)$

$n^4 - 3n^3 - 3n^3 + 9n^2 + 2n^2 - 6n$

$n^4 - 6n^3 - 9n^2 - 6n$

$\cancel{\text{But } \binom{n}{0} = \binom{n}{n},\ \binom{n}{1} = \binom{n}{n-1} \dots}$

$\cancel{\Rightarrow RHS = 2\times 1^2 + 2\left(\frac{n!}{1!(n-1)!}\right)^2 + \dots 2\binom{n}{n/2}^2}$

$\cancel{= 1^2 + n^2\left[\frac{1}{1!} + \left(\frac{n-1}{2!}\right)^2 + \left(\frac{(n-1)(n-2)}{3!}\right)^3 + \dots\right]}$

45 - 2d - 2 both

Page No.:
Date:

* Principle of Inclusion and exclusion.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$
$+ n(A \cap B \cap C)$.

Determine the number of positive integers $n < 100$ which are divisible by 2, 3, 5

let A = Elements divisible by 2
B = " " 3
C = " " 5.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$
$+ n(A \cap B \cap C)$.

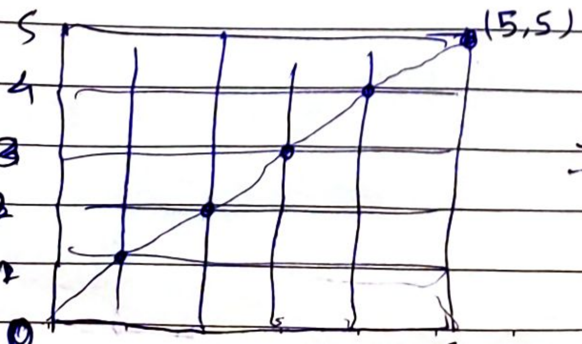
$= 50 + 33 + 20 - 16 - 10 - 6 + 3$
$= 103 - 32 + 3$
$= 74.$

At a certain university all second year science students may choose either mathematics or physics or both. math course is attended by 50, Physics by 30, 15 attend both what is St

$= 50 + 30 - 15$
$= 65.$

* Catalan no.

No of ways to travel from (0,0) to (5,5) without rising above line $x = y$.



5 left, 5 up.

1 1 1 1 1 x x x x x

$\frac{10!}{5!\,5!}$

RURURURUR

$C_n - 1$ ways

Find the ... different ways to parethatise x_1, x_2, x_3, x_4.

$((x_1\ x_2)(x_3\ x_4))$ $(x_1, x_2, 2, 4))$

$((x_1\ x_2)\ x_3)\ x_4)$

5 $((x_1(x_2\ x_3))\ x_4)$ $(((x_1)\ x_2)\ x_3)\ x_4)$

$(x_1(x_2(x_3\ x_4)))$

$$\boxed{b_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}}$$

$C_0 = 1$ 1

$C_1 = 1$ 1

$C_2 = C_0 C_1 + C_1 C_0$ 2

$C_3 = C_0 C_2 + C_1 C_1 + C_2 C_0$ 5

$C_4 = C_0 C_3 + C_1 C_2 + C_2 C_1 + C_3 C_0$ 14

$C_5 = C_0 C_4 + C_1 C_3 + C_2 C_2 + C_3 C_1 + C_4 C_0$ 42

$C_n = C_0 C_{n-1} + C_1 C_{n-2} + \cdots C_{n-1} C_0$

Catalan Numbers

0	1	1	0				
1	1	2	2				
2	1	3	5	5			
3	1	4	9	14	14		
4	1	5	14	28	42	42	
5	1	6	20	48	90	132	132

$1+5+4+5$ 16

216 + 16 + 5 5 4 = 46

Q How many triangles can be formed with 4 points.

$\boxed{C_{n-1}}$

$C_2 = 2$ (Triangulation)

$\boxed{C_{n-1}}$

if one each table atleast
1 must be seated.

$$\left(\left({}^6C_1 \times {}^5C_4 \,(5-1)!\; + (2-1)! \times (4-1)! +\right) \times 2 + (3-1)! \times (3-1)!\right)$$

5	1
4	2
3	3
2	4
1	5

$= 2(1 \times 4! + 1! \times 3!) + 2!\,2!)$
$= 2(24 + 6 + \cancel{6}) + 4$
$= 64$

$$\begin{array}{r} 236 \\ -144 \\ \hline \end{array}$$

$$\left(\begin{array}{l} {}^6C_1 = (1) \times (5-1)! \\ \quad + \\ {}^6C_2 \; (2-1)! \times (4-1)! \\ \quad + \\ {}^6C_3 \; (3-1)! \times (3-1)! \end{array} \right)$$

$= 144 + 90 + 80$ ${}^6C_1 = {}^6C_5$
$= 314$

274.

6 persons are to be seated on 3 indistiguishable. Atleast 1 is seated on each table

Cases

	Table 1	Table 2	Table 3.
	4	1	1
	3	2	1
	2	2	2

${}^6C_4 \times 3! + {}^6C_3 \times 2!\,{}^3C_2 + {}^6C_2 \times {}^4C_2.$
$90 + \cancel{120} + 225.\; 90 \times \frac{1}{3!}$
$= \cancel{400}\;\cancel{200} = \boxed{240}$

$P_1P_2 \quad P_3P_4 \quad P_5P_6$
$P_1P_3 \quad P_2P_4 \quad P_5P_6$

Given R and N two natural numbers. Let $S(r,n)$ denote the number of ways to arrange r distinct objects around n circles such that each circle has at least one object. $0 \le r \le r$.
These numbers S, R, N are Sterlings numbers of first kind.

Case:
$S(r, 0) = 0$
$S(r, r) = 1$
$S(r, 1) = (r-1)!$
$S(r, r-1) = {}^rC_2$

Prove that:
$S(r,n) = S(r-1, n-1) + (r-1)S(r-1, n)$

Consider first object 1. 1st object may be available on circle or may not available.

1) '1' is only object in circle
2) 1 is mixed with other objects.

$r-1 \quad n-1$

Case 1: Object 1 is only object in circle

⇒ ~~1. S(r-1, n-1)~~ ways arrangements.

Case 2: Object 1 is mixed with other objects.

~~(r-1) S(r-1, n)~~

Adding both

$S(r,n) = S(r-1,n-1) + (r-1)S(r-1,n)$

Q. 6 person gathering 3 persons know each other

$S(6,3)$

$= S(5,2) + (5)\,S(5,3)$

$= S(4,1) + 4S(4,2) + 5 \times S(5,3)$

$= 6 + 4[S(3,1) + 3(3,2)] + 5 \times S(5,3)$

$= 6 + 4[2 + 3[S(2,1) + 2(2,2)] + 5 \times S(5,3)$

$= 6 + 4[2 + 3[1+2] + 5 \times S(5,3)$

$= 50 + 5[S(4,2) + 4(S(4,3))]$

$= 50 + 5[S(3,1) + 3(S(3,2)) + 4(S(4,3))]$

$= 50 + 5[2 + 3[S(2,1) + 2(S(2,2)) + 4(S(4,3))]$

$= 50 + 5[2 + 3[1+2] + 4(S(4,3))]$

$= 50 + 5[2 + 9 + 4(S(3,2) + 3(S(3,3))]$

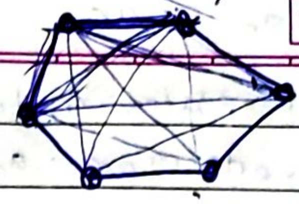
$= 50 + 5[11 + 4[S(2,1) + 3(S(2,2) + 3]]$

$= 50 + 5[11 + 4[1 + 12]]$

$= 50 + 5[63]$

13
144
52

$\frac{n!}{r!(n-r)!}$ $\quad {}^6C_3 = \frac{6\times5\times4}{3\times2} = 20$

Q.  15

Find No of line Segments

${}^6C_2 = \frac{6!}{2!4!} = \frac{6\times5}{2} = 15$

Ramsey theory

If m & n are positive integers >=2 then the minimum number of people at a gathering such that there are either m are mutual friends or n mutual strangers assuming every pair of people at gathering are either friends or strangers are called ramsey

Ramsey theory deals with unexpected patterns arising in sufficiently large datasets

R of 1,1 is 1

R of 2,2 is 2

$R(1,1) = 1$

$R(2,2) = 2$

$R(3,3) = 6$

$R(4,4) = 18$

$R(5,5)$ 43 - 48.

There are 17 vertices and all edges are coloured with 3 types blue, red, & yellow

${}^{17}C_{...} = 17 \times 16 \times 15$

★ Recurrence Relations:

A recurrence relation for the sequence $a_0, a_1, \dots a_n$ is an equation that relates a_n to certain of its predecessors $a_0, a_1, \dots a_{n-1}$.

A person invest Rs. 10,000/- @15% intrest compounded anually. At the end of n^{th} year.

$$A_n = A_{n-1} + 0.15\, A_{n-1}$$
$$= (1.15) A_{n-1}$$
↳ recurrence relati

$$A_3 = 1.15 \times A_2$$
$$= 1.15^2 A_1$$
$$= 1.15^3 A_0$$
$$= 1.15^3\, 10000$$

$$\Rightarrow A_n = (1.15)^n \times 10000$$
↳ Explicit Formula

Let S_n be the number of subsets of an n-element set.

S_{n-1} is no of subsets of an $(n-1)$ elem set

Let, we start with the set having $(n-1)$ elements

No of subsets of a set having n-elements

= No of subsets having $(n-1)$ elements (without a)

+ No of subsets having $(n-1)$ elements (with a)

where a is n^{th} element

$$S_{n-1} + S_{n-1}$$
$$= 2S_{n-1}$$

$$S_n = 2S_{n-1}$$
$$= 2 \cdot 2\, S_{n-2}$$
$$= 2 \cdot 2 \cdot 2\, S_{n-3}$$
$$\vdots$$
$$2^n S_{n-n}$$
$$= 2^n S_0$$
$$= 2^n (1) \quad (\because S_0 = 1)$$

$$S_n = 2^n$$
↑ explicit formula

Find the number of n bit string that do not contain '11'

Let

S_n = No of n-bit strings not contain '11'

S_{n-1} = No of $n-1$ bit strin not contain '11'

We get S_n

1. no of strin begin with '0'
2. '' '' '10'
3. '' '' '110'

$$S_n = S_{n-1} + S_{n-2} + S_{n-1}$$

$S_1 = 2$ (0, 1)

$S_2 = 4$ (00, 01, 10, 11)

$S_3 = 7$ (000, 001, 010, 011, 100, 101, 110)

Codeword Enumeration.

A valid codeword is some n bit word having decimal digits with even number of 0's

Let a_n be the no of valid codeword

1. first digit can be other than 0, 9 way.

$9a_{n-1}$.

2. total no of Codewords with $(n-1)$ bits $- a_{n-1}$.

= the total number of invalid codewords with $(n-1)$ bit

~~Now if we add one digit n^{th}, as "0".~~

$= (10^{n-1} - a_{n-1})$ — invalid Code words with $(n-1)$ bit

$a_n = 9a_{n-1} + (10^{n-1} - a_{n-1})$

$= 8a_{n-1} + 10^{n-1}$

$a_1 = 9$

$a_2 = 8 \cdot 9 + 10 = 82$.

A recurrence relation for sequence $a_0, a_1, a_2 \ldots a_n$ is an equation that refers a_n to certain of its predecessors like a_0, a_1, a_{n-1}.

A Set has n elements what number of Subsets will it have — 2^n

Q. $a_n = a_{n-1} + F(n)$ for $(n) \geq 1$.

Substitute method.

$a_n = a_{n-1} + F(n)$

$a_1 = a_0 + F(1)$

$a_2 = a_1 + F(2)$

$= a_0 + F(1) + F(2)$

$a_3 = a_2 + F(3)$

$= a_0 + F(1) + F(2) + F(3)$

$$\boxed{a_n = a_0 + \sum_{i=1}^{n} F(i)}$$

Q $a_n = a_{n-1} + n^2$

$a_0 = 7$.

by substitution Method.

$a_1 = a_0 + n^2$

$= 7 + n^2 = 8$

$a_2 = a_1 + n^2$

$= 8 + 2^2 = 12$

$a_3 = a_2 + n^2$

$= 12 + 3^2 = 21$

$= 7 + 1^2 + 2^2 + 3^2$

$\Rightarrow a_n = 7 + \sum_{i=1}^{n} i^2$

$$\boxed{a_n = 7 + \frac{n(n+1)(2n+1)}{6}}$$

Q. $a_n = a_{n-1} + \frac{1}{n(n+1)}$

$a_0 = 1$.

$a_n = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots \frac{1}{n(n+1)}$

$a_n = 1 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots$

$$\boxed{a_n = 1 + \sum_{i=1}^{n} \frac{1}{i(i+1)}}$$

$\frac{1}{i^2+i}$

$a_n = 1 + \sum_{i=1}^{n} \frac{(i+1) - (i)}{i(i+1)}$

$a_n = 1 + \sum_{i=1}^{n} \frac{1}{i} - \sum_{i=1}^{n} \frac{1}{(i+1)}$

$a_n = 1 + \sum_{i=1} \frac{1}{(i+1)}$

$a_n = 1 + \frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \frac{1}{3} + \dots \frac{1}{n} - \frac{1}{n+1}$

$a_n = 1 + 1 - \frac{1}{n-1} = 2 - \left(\frac{1}{n-1}\right)$

Q $a_n = a_{n-1} + 3^n$

$a_0 = 1$

$a_n = 1 + 3^1 + 3^2 + 3^3 + \dots 3^n$

$a_n = 1 + \sum_{i=1}^{n} 3^i$

It is infinite G.P.

$= 1 + 1\left(\frac{3^n - 1}{2}\right)$ $\quad S_\infty = \frac{a}{1-r}$

$= 1 + \frac{3^{n} - 1}{n}$ $\quad \frac{3}{2}$

$a\left(\frac{1-r^n}{1-r}\right)$

Tower of Hanoi :

let C_n be the no of moves required to transfer n disks from Peg 1 to Peg (using intermediate Peg 2)

let C_{n-1} ~~moves~~ bes the no of moves to transfer $(n-1)$ disks from Peg 1 to Peg 2.

C_{n-1} = no of moves to transfer $(n-1)$ disks from Peg 2 to Peg 3.

to 3 (la...

$C_n = C_{n-1} + C_{n-1} + 1$

$C_n = 2C_{n-1} + 1$

$C_1 = 2C_0 + 1$

$C_2 = 2C_1 + 1$

$= 4C_0 + 2 + 1$

$= 4C_0 + 3$

$C_3 = 2C_2 + 1$

$= 8C_0 + 6 + 1$

$= 8C_0 + 7$

$\boxed{C_n = 2^n C_0 + 2^n - 1}$

$= 2^n 0 + 2^n - 1$

$1 + 3 + 7 + \dots$

$C_0 = 0$

$C_1 = 1$

$C_2 = 3$

$C_2 = 7.$

$\boxed{C_n = 2^n - 1}$

* Generating Function :

Let the sequence $\langle a_0, a_1, \dots a_n \rangle$ be the sequence of number the the generating funct (ordinary) for

$A(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \dots a_r x^r$

$A(x) = \sum_{r=0}^{\infty} a_r x^r$

Find the G.F.
$<1, 1, 1, 1, \ldots>$
$a_0 \; a_1 \ldots$

$$\sum_{r=0}^{\infty} a_r x^r = a_0 + a_1 x^1 + a_2 x^2 + \ldots a_n x^n$$
$$= 1x^0 + 1x^1 + 1x^2 + \ldots 1x^4.$$

It is in GP. with common ratio r.

$$\left(\frac{1}{1-x}\right)$$

Find generating function for the sequence
$<1, 2, 3 \ldots>$

$$\sum_{r=0}^{\infty} a_r x^r = 1x^0 + 2x^1 + 3x^2 + \ldots$$

It is AGP.

$$= a_0 + a_1 x^1 + a_2 x^2 + \ldots$$
differentiating w.r.t.

let $A(x) = 1x^0 + 1x^1 + 1x^2 + \ldots$

$\frac{1}{(1-x)^2}$

~~$A'(x) = 0 + x + 2x + 3x^2 + \ldots$~~

$\Rightarrow$ Generating function

$$= \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{-1}{(1-x)^2}$$

Solve the recurrence relation.
$$a_n - 7a_{n-1} + 10a_{n-2} = 0$$
~~By using the generating~~ function for $n \geq 2$

$a_2 = a_2 - 7a_1 + 10a_0$

Multiplying ~~by x^n and sum~~ from 2 to ∞

$$\sum_{n=2}^{\infty} a_n x^n - 7a_{n-1} x^n + 10a_{n-2} x^n = 0$$

$$= \sum_{n=2}^{\infty} a_n x^n - 7\sum_{n=2}^{\infty} a_{n-1} x^n + 10\sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$= [a_2 x^2 + a_3 x^3 + \ldots] - 7[a_1 x^2 + a_2 x^3 + \ldots] + 10[a_0 x^2 + a_1 x^3]$$

$$= A(x) - a_0 - a_1 x - 7x[A(x) - a_0] + 10x^2[A x]$$

$$[A x - a_0 - a_1 x] - 7[A(x) - a_0] + 10x^2 [A(x)] = 0$$

$$= A(x) - a_0 - a_1 x - 7xA(x) + 7a_0 x$$
$$+ 10x^2 A(x).$$

$$A(x)[1 - 7x + 10x^2] + a_0[-1 - 7x]$$
$$+ a_1[a_1 - 1]$$

$$A(x)[1 - 7x + 10x^2] = a_0 + a_1 x - 7a_0 x.$$

$$A(x) = \frac{a_0 + a_1 x - 7a_0 x}{[1 - 7x + 10x^2]}$$

$(x-2)(x-5)$

$$A(x) = \frac{a_0 + a_1 x - 7a_0 x}{(1-2x)(1-5x)}$$

$1 - 2x - 5x + 10x^2$
$(1-2x) - 5x(1-2x)$

$$A(x) = \frac{a_0[1 - 2x - 5x] + a_1 x}{(1-2x)(1-5x)}$$

$(1-2x)(1-5x)$

$$A(x) = \frac{a_0}{1-5x} + \frac{a_1 x - 5x a_0}{(1-5x)(1-2x)}$$

$a_0 + a_1 - 7$

$$A(x) = \frac{C_1}{(1-2x)} + \frac{C_2}{(1-5x)}$$

$$\boxed{A(x) = C_1\sum_0^{\infty} 2^n x^n + C_2 \sum_0^{\infty} 5^n x^n}$$

By Using Partial Fraction.

Q By Using generating function

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

$n \geq 2$.

$A(x) \Rightarrow \sum_2^{\infty} a_n x^n - 5a_{n-1}x^n + 6a_{n-2}x^n = 0$

Q Find the G.F. for the Sequence

$\left(\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \binom{n}{n}, 0, 0, 0\right)$

$A(x) \Rightarrow A(x) - a_0 - a_1 - x5[A_x - a_0] + 6x^2[A(x)] = 0$

$\Rightarrow$

$A(x) - a_0 - a_1 - 5A(x)x + 5a_0 x$
$+ 6A(x)x^2$

$A(x)[1 - 5x + 6x^2] = a_0 + a_1 - 5a_0 x$

$$A(x) = \frac{a_0 + a_1 - 5a_0 x}{[1 - 5x + 6x^2]}$$

3 2

$$A(x) = \frac{a_0 + a_1 - 5a_0 x}{(x-3)(x-2)}$$

$(x-3)(x-2)$

$$= \frac{C_1}{\ldots} + \frac{C_2}{(x-2)}$$

* Some equivalent expressions

$A(x) = \sum_{n=0}^{\infty} a_n x^n$ then

1) $\sum_{n=k}^{\infty} a_n x^n = A(x) - a_0 - a_1 x \ldots a_{k-1} x^{k-1}$

2) $\sum_{n=k}^{\infty} a_{n-1} x^n = x[A(x) - a_0 - a_1 x \ldots a_{k-2} x^{k-2}]$

3) $\sum_{n=k}^{\infty} a_{n-2} x^n = x^2[A(x) - a_0 - a_1 x \ldots a_{k-3} x^{k-3}]$

4) $\sum_{n=k}^{\infty} a_{n-3} x^n = x^3[A(x) - a_0 - \ldots a_{k-4} x^{k-4}]$

Recurrence Relation (Cont. Nptel)

A linear homogeneous recurrence relation of order k with constant co-efficients is an equation of the following form

$a_n = C_1 a_{n-1} + C_2 a_{n-2} + \ldots C_k a_{n-k}$, $C_k \neq 0$

k - initial conditions

$a_0 = C_1'$

$a_1 = C_1'$

⋮

Q $a_n - 5a_{n-1} - 6a_{n-2}$, $a_0 = 7$, $a_1 = 18$

Let the Solution is on the form

$V_n = t^n$

we have $S_n = 2^n$

~~For recurrence~~

relatn $S_n = 2S_{n-1}$

$$\boxed{\begin{array}{l} S_n = 2S_{n-1} \\ S_n = 2^n \end{array}}$$

$V_n =$

* Probability *

A probability experiment is an action through which specific results are obtained.

ex: Rolling a die and observing number is a probability experiment

- The result of a single trial in a probability experiment is the outcome.
- Set of all possible outcomes for an experiment is the sample space.

- An event consists of one or more outcomes and is a subset of the sample space.

A simple event is an event that consists of a single outcome.

* Classical (theoretical) probability is used when each outcome in a sample space is equally likely to occur. The classical probability for event E is given by.

$$P(E) = \frac{\text{No of Outcomes}}{\text{Total no of Outcomes in Sample Space}}$$

* Empirical Probability:

Empirical (or statistical) probability is based on observations obtained from probability experiments. The empirical frequency of an event E is the relative frequency of event E.

$$P(E) = \frac{\text{Frequency of Event E}}{\text{Total Frequency}}$$

$$= \frac{f}{n}.$$

ex.
A travel agent determines that in every 50 reservation. She makes 12 will be for a cruise.
what is prob of next reservation she makes will be for a cruise.

$$P(\text{cruise}) = \frac{12}{50} = 0.24$$

* Law of large Numbers:
As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.

* Subjective Probability:
Subjective probability results from intuition, educated guesses and estimate.
ex. Prob of worker strike is 0.15.

* Range of Probabilities Rule
The Probability of an event E is between 0 and 1 inclusive.
That is $0 \le P(A) \le 1$

★ Complementary Event

The complement of Event E is the set of all outcomes in the sample space that are not included in event E (denoted by E' and read as "E prime")

$$\boxed{P(E) + P(E') = 1}$$

There are 5-red, 4-blue, 6-white chips in a basket. Find the probability of randomly selecting a chip that is not blue.

$P(\text{Blue}) = \frac{4}{15}$

$P(\text{Blue}') = 1 - \frac{4}{15} = \boxed{\frac{11}{15}}$

★ Conditional Probability

A conditional probability is the probability of an event occurring, given that another event has already occurred.

$P(B|A) \longrightarrow$ Probability of B given A.

find prob of second chip is red given that first chip is ble (not replaced)

A: Chip is ~~blue~~ red.

B: first chip is blue.

$P(\text{red second}) = \frac{4}{14}$

	Less than 5	5 to 10	More than 10	Tot
Male	11	22	16	49
Femal	13	24	14	51
	24	46	30	100

Probability of studen studn more than 10 hours given that he is male.

$p(\text{more than } 10|\text{male}) = \frac{16}{49}$

★ Independent Events

two events are independent if the occurance of one of the events does not affect the probability of the other event.

Two event A and B are independ- if

$P(B|A) = P(B)$ or if $P(A|B) = P(A)$

★ Multiplication Rule

The probabily that two events A and B will occur in sequ is

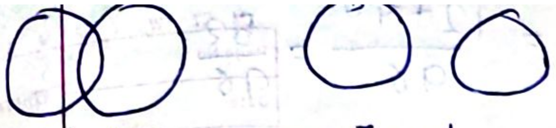
$P(A \text{ and } B) = P(A) \cdot P(B|A)$

If A & B are independ

$P(A \text{ and } B) = P(A) \cdot P(B)$

★ Mutually Exclusive Events.

Two event A and B, are mutually exclusive if they cannot occur at the same time.



A: Roll a number less than 3 or
B: Roll a num More than 3

$A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$ } exclusive

A: Selecting a Jack
B: Selecting a heart
Not exclu. as Jack of heart.

* Addition rule

The probability that event a or B will occur is given by

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If they are mutually exclusive

$P(A \text{ or } B) = P(B) + P(B)$

Q Find proba that a card is Jack or heart

$P(\text{Jack or heart}) = P(\text{Jack}) + P(\text{heart}) - P(\text{Jack \& heart})$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{16}{52}$

* Fundamental Counting Principle

If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is m.n. This rule can be extended for any number of events occurring.

ex. Access code to a house.

* Distinguishable permutations.

$\frac{n!}{n_1! \, n_2! \, n_3! \ldots}$

The number of distinguishable permutations of n object where n_1 are one type, n_2 are another type, and so on.

ex A want to plant 10 plants. She has 3 rose 4 daffodils and 3 lilies. In how many diff

$\frac{10!}{3!4!3!} = 4200.$

In a State lottery you must select 6 numbers (in any order) out of 44 to win the grand prize.

a) How many was can 6 number be chosen from the 44 num

b) If you purch one lottery ticket, what is probability of winning

a) ${}^{44}C_6 = \frac{44!}{6! \cdot 38!} = 7059052$

b) $= \frac{1}{{}^{44}C_6} = \frac{1}{7059052}$

As there is only one winning ticket.

A bag contain 4W and 2B balls. Another bag contains 3W and 5B balls.
If one ball is drawn from each bag find the probability that both are white both are black. One black and One white.

Bag 1 → W → 4, B → 2

Bag 2 → W → 3, B → 5

a> A: Both white

$P(A) = \frac{\cancel{4}^1}{\cancel{6}_2} \times \frac{\cancel{3}^1}{\cancel{8}_2} = \frac{1}{4}$.

b> B: Both are black

$P(B) = \frac{\cancel{2}^1}{6} \times \frac{5}{\cancel{8}4} = \frac{5}{24}$.

c) C: One black and One wh.

$P(c) =$ First black Second white + First whit Second bla.

$= \frac{\cancel{2}^1}{\cancel{6}_2} \times \frac{\cancel{3}^1}{\cancel{8}4} + \frac{\cancel{4}^1}{6} \times \frac{5}{\cancel{4}2} = \frac{1}{8} + \frac{5}{12}$

$= \frac{12+40}{96} = \frac{\cancel{52}}{\cancel{96}}$

$= \frac{26}{48} = \boxed{\frac{13}{24}}$

*A 75% Cases and B 50% Cases. In what percentage of aces they likely to contradict Each other in stating the same fact.

% A is Speaking truth = 75%

% B " = 50.

C: A and B contradict.

$P(C) = P(A\text{ speak truth}) \cdot P(B\text{ Speaks}$ ...
$+ P(A\text{ pseak false}) \cdot P(B\text{ Speaks}$ ...

$P(A\text{ Speak truth}) = 75\% = \frac{75}{100}$

$P(B\text{ Speaks lie}) = \frac{50}{100}$

$P(A\text{ Speak lie}) = \frac{25}{100}$

$P(B\text{ Speak tru}) = \frac{50}{100}$

$P(C) = \frac{\cancel{75}^3}{\cancel{100}_4} \times \frac{\cancel{50}^2}{\cancel{100}_4} + \frac{\cancel{25}^1}{\cancel{100}_4} \times \frac{\cancel{50}^2}{\cancel{100}_4}$

$= \frac{6}{16} + \frac{2}{16}$

$= \frac{8}{16} = \frac{1}{2} \Rightarrow 50\%$

E_1: A is Speaking truth $E_1 = \frac{75}{...}$

$C = E_1 \cap E_2'$

$D = E_1 \cap E_2$.

$P(X) = C + D$.

Properties
- Associate
- Distributive

* Random Variables *

- Discrete Random Variable
- Continuous Random Variable
- Expectation of Random Variable
- Variance of a random Variable.
- Jointly distributed Random Variables
- Combinations and functions of Random variables.

For each point of a Sample Space. we assign a number and define a function on the Sample Space. This function is random variable.

A numerical value to each outcome of a particular experiment

Ex 1: Machine Break downs.
- Sample Space: S = {electrical, mechanical, misuse}
- Each of these failures may be associated with a repair cost.
- State Space: {50, 200, 350}.
- Cost is a random variable 50, 200 and 350

a pair of fair dice are tossed an let the random variable X denote the Sum of points

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2) . . . . .
(6,1) . . . . (6,6)}

Possible Sums.

X = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$\frac{1}{36}$, $\frac{2}{36}$, $\frac{3}{36}$, $\frac{4}{36}$, $\frac{5}{36}$, $\frac{6}{36}$, $\frac{5}{36}$, $\frac{4}{36}$, $\frac{3}{36}$, $\frac{2}{36}$, $\frac{1}{36}$

A random variable which takes on finite number of values is a discrete random variable and one which takes on non countably infinite number of values is called as continuous random variable.

6.1 5.2 4.3

* Probability Mass Function

- A Set of probability value p_i assigned to each of the values taken by discrete random variable x_i
- $0 \le p_i$ and $\sum_i p_i = 1$.
- Probability : $P(X = x_i) = p_i$

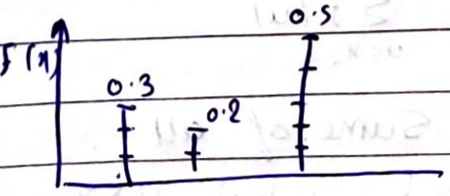
(Machine Breakdown Problem)

$p(\text{cost} = 50) = 0.3$

$p(\text{cost} = 200) = 0.2$

$p(\text{cost} = 350) = 0.5$

$- 0.3 + 0.2 + 0.5 = 1$.



Let X be a discrete random variable and suppose that possible values which it can assume are given by x_1, x_2 arrange in increasing order of Magnitude Suppose some values are also associated with probabilities given by

$$P(X = x_k) = f(x_k).$$

In general the probability function • $f(x) \geq 0$

• $\sum_x f(x) = 1.$

(Probability Function).

* Cumulative Distribution Function

$$F(x) = P(X \leq x).$$

$$F(x) = \sum_{y \leq x} P(X = y).$$

CDF for a random variable x is defined as.

$$P(X \leq x) = F(x)$$

where x is any real number

$$-\infty < x < \infty$$

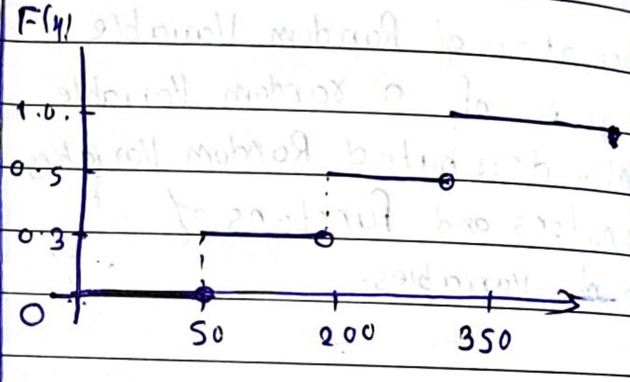
The distribution function can be obtained from distribution function by noting.

$$F(x) = \sum_{u \leq x} f(u)$$

where the sume of all values of u for which $u \leq x$ and the probabil... function

be obtained from distribution function if X taken only finite number of values. $x_1, x_2 \ldots x_m$ then the distribution function is given by

$$F(x) = \begin{cases} 0 & -\infty < x < x_1 \\ f(x_1) & x_1 < x < x_2 \\ f(x_1 + x_2) & x_2 < x < x_3 \\ f(x_1 + x_2 + \ldots) & x_n < x < \infty \end{cases}$$

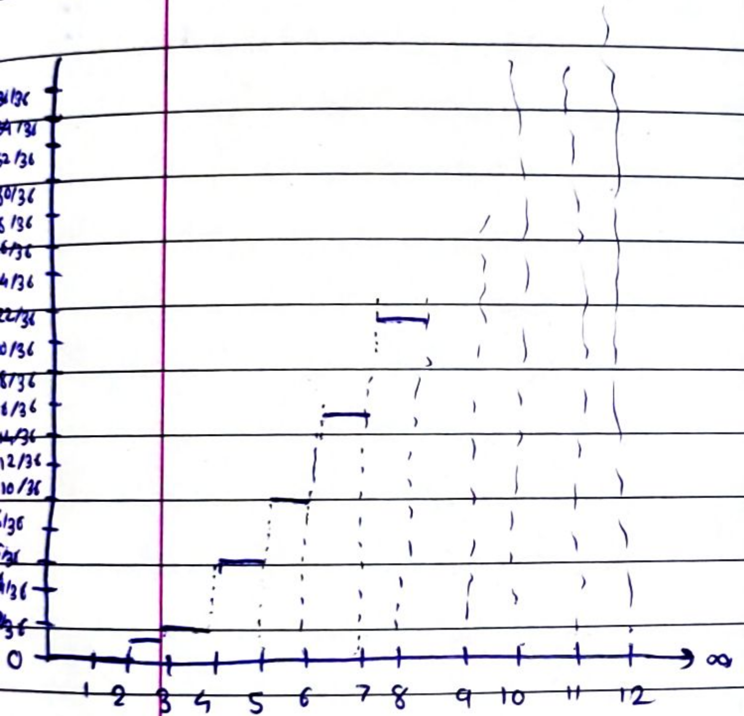


A pair of dice thrown X denotes Sum of no find CDF

Construct graph for ths distribution.

$$f(x) = \begin{cases} 0 & -\infty < x < 2 \\ 1/36 & 2 \leq x < 3 \\ 2/36 & 3 \leq x < 4 \\ 3/36 & 4 \leq x < 5 \\ 4/36 & 5 \leq x < 6 \\ 5/36 & 6 \leq x < 7 \\ 6/36 & 7 \leq x \leq 8 \\ 5/36 & 8 \leq x < 9 \\ 4/36 & 9 \leq x \leq 10 \\ 3/36 & 10 \leq x \leq 11 \\ 2/36 & 11 \leq x \leq 12 \end{cases}$$


1/36
2/36
3/36
4/36
2 3 4 5 6 7 8 9 10 11 12


0
1 2 3 4 5 6 7 8 9 10 11 12
∞
10/36
12/36