Noise Analysis of Variational Algorithms for Weighted MAX-CUT

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Abstract. In this work, we investigate the performance of quantum-classical hybrid algorithms for finding solution to the MAX-CUT problem for a weighted square graph Sq_2 in the presence of various kinds of noise. To simulate the noisy quantum logic circuits, we have used the density matrix simulator[1] implemented as an extended back-end in the Qiskit platform [2]. Our analysis shows the effect of memory and gate-precision errors on the fidelity of states prepared by quantum routine, and the effect of measurement errors on the success probability of these algorithms.

Keywords: Quantum Computing, Ising Model, Combinatorial Optimization, Variational Quantum Algorithms, NISQ Devices

1 Motivation

Near-term quantum processors provide methods to approximately solve optimization problems through a hybrid quantum-classical approach, but these methods are highly dependent on the robustness and capabilities of concerned quantum computational device. Since these devices require extreme execution environments and are prone to noise, it becomes crucial to characterize the behaviour of quantum hardware on the particular problem, in the presence of noise. As the variational algorithms require precise estimates of quantum state measurement, it is necessary to develop techniques which minimize the effect of noise.

Hence, through our simulations of Variational Quantum Eigensolver (VQE) [3] and Quantum Approximate Optimization Algorithm (QAOA) [4], we provide insightful data about the effect of amplitude decay, decoherence, thermalization, depolarization and logic gate imprecision on their success. We hope that this will help in the improvement of designs of near-term quantum hardware.

2 Implementation

In our simulations, we solve the weighted MAX-CUT problem by VQE and QAOA by minimizing its standard Ising Hamiltonian, $H = \sum_i w_i Z_i + \sum_{i<j} w_{ij} Z_i Z_j$ [5], for a weighted square graph Sq_2 on the the density matrix simulator [1].

2.1 QAOA

In QAOA, we discretized the adiabatic pathway in $p = 2$ steps. This is equivalent to discretizing the time-dependent Hamiltonian, $H(t)$, as $H(t) = (1-t)H_0 + tH_1$ using the following reference Hamiltonian H_0 and target Hamiltonian H_1.

$$H_0 = -\sum_{i=0}^{3} \sigma_i^x \tag{1}$$

$$\begin{aligned} H_1 = {} & w_{01}(\sigma_1^z \otimes \sigma_0^z) + w_{12}(\sigma_2^z \otimes \sigma_1^z) \\ & + w_{23}(\sigma_3^z \otimes \sigma_3^z) + w_{03}(\sigma_3^z \otimes \sigma_0^z) \end{aligned} \tag{2}$$

For the variational parameters $\{\beta, \gamma\}$, we first prepare a state $|{+}{+}\rangle$ and then evolve it using unitary U approximated by a third order Trotter-Suzuki decomposition of original unitary e^{-iHt} with the number of steps = 1.

$$U' = U(H_0, \beta_0) \cdot U(H_1, \gamma_0) \cdot U(H_0, \beta_1) \cdot U(H_1, \gamma_1) \tag{3}$$

$$|\beta, \gamma\rangle = e^{-iH_0\beta_1} e^{-iH_1\gamma_1} e^{-iH_0\beta_0} e^{-iH_1\gamma_0} |{+}{+}\rangle \tag{4}$$

The optimal values of $\{\beta, \gamma\}$ is obtained by classical minimization of the following objective function C:

$$C = \langle \beta, \gamma | H | \beta, \gamma \rangle \tag{5}$$

2.2 VQE

In VQE, we use 3-layers each of the R_y, R_z gates followed by CZ as entangling gates for constructing our ansatz to prepare the variational state $|\psi(\vec{\theta})\rangle$. Creating ansatz require 32 randomly initialized parameters, $(\{\theta_0 \dots \theta_{31}\} | 0 \leq \theta_i \leq 2\pi)$, which are updated by the following objective function C until it reaches a minimum:

$$C = \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle \tag{6}$$

3 Results

To study the fidelity F, of quantum states, ρ and ρ_0, prepared by the quantum routine in the presence and absence of errors, we use the metric: $F = Tr(\rho_0 \rho)$.

Our analysis showed that the fidelity of states prepared by the quantum routine and consequently the success probability of variational algorithms is affected by both memory errors: thermalization, decoherence, amplitude decay and gate-precision errors. The off-diagonal coefficients are contracted by decoherence factor $f \in [0, 1]$ and by $\sqrt{g}$, where $g \in [0, 1]$ is the decay factor. The diagonal coefficients decay with rate g towards the thermal state specified by thermal factor p. For QAOA, we see that decoherence and thermalization have a far more significant effect on the success probability, in comparison to the amplitude decay. This is certainly due its lower contribution in the variation of the off-diagonal elements of the quantum state. Whereas, the scenario in VQE was just the opposite. This is because in QAOA the more

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than 50% of the gates were entangling ones whereas in VQE these gates were less than 20%.

When considering gate-precision, in both cases the errors in $R_z(\theta)$ affects state fidelity relatively more than error in $R_y(\theta)$ because of Euler decomposition, $R_z(\phi)R_y(\theta)R_z(\lambda)$, implemented in our density matrix simulator for the single unitary gates, $U_3(\theta,\phi,\lambda)$.

Measurement errors for $\langle\hat{H}\rangle$ were modeled as depolarization (equivalent to the bit flip error) during single qubit measurement operations. We see that in both cases there was an exponential decay in the value of $\langle\hat{H}\rangle$ with respect to parameter update iteration.

Overall, the effect of noise in VQE was lesser than what we encountered in QAOA due to shorter circuit depth ($42 < 1576$) with fewer entangling gates ($18 < 844$).

4 Figures

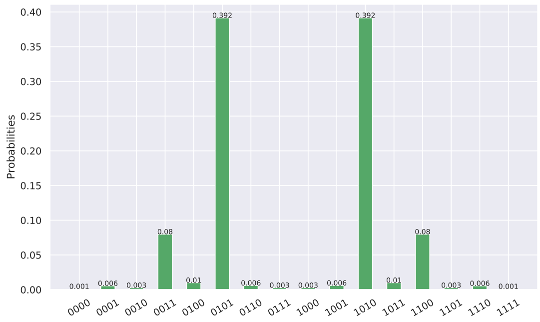

Figure 1: Probability distribution for optimized $\{\beta,\gamma\} = [1.784, 5.308], [3.665, 2.732]$, in QAOA

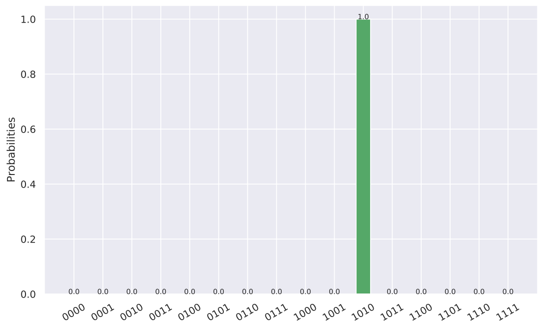

Figure 2: Probability distribution for optimized $\{\theta\} = [2.191, 2.047, \ldots, 3.142, 5.932, 0, 0.771]$, in VQE.

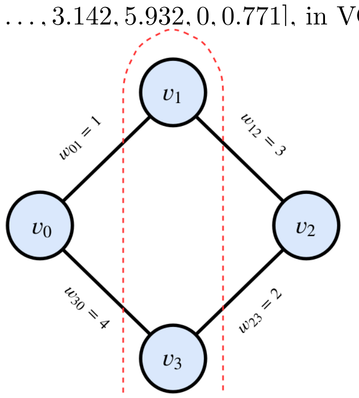

Figure 3: Weighted MAX-CUT solution: 1+2+3+4 = 10

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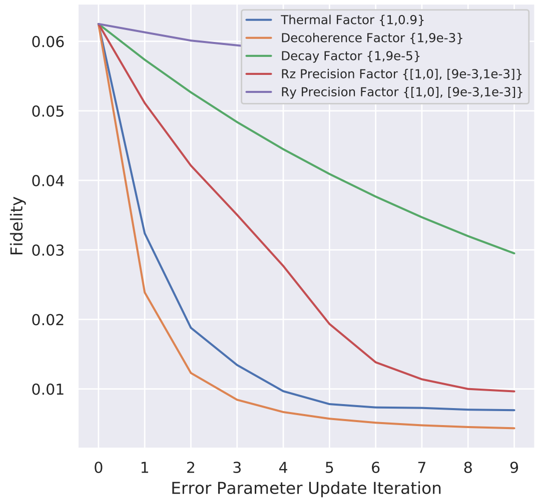

Figure 4: Effect of errors on state fidelity in QAOA.

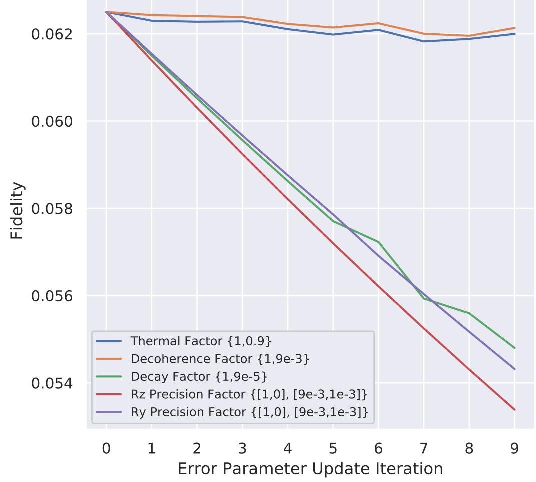

Figure 5: Effect of errors on state fidelity in VQE.

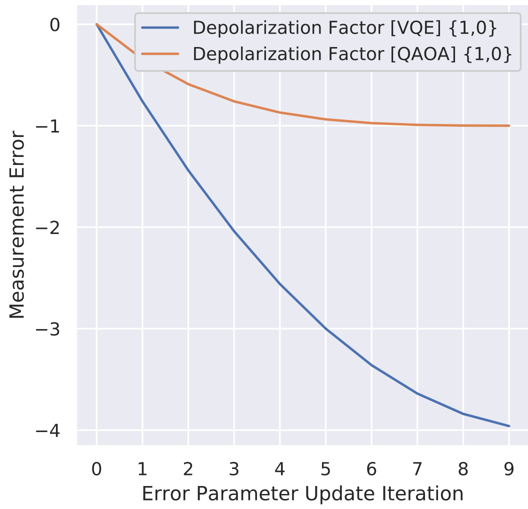

Figure 6: Effect of depolarization on $\langle\hat{H}\rangle$.
Legend denotes: {Initial Value, Final Value}

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