Noise Analysis of Variational Hybrid Quantum-Classical Algorithms on Weighted MAX-CUT

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Outline

Introduction

Quantum bit (qubit): two-dimensional complex Hilbert space $\mathbb{C}^2$

- Computational basis states (classical states):

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- General states (superposition):

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha\,|0\rangle + \beta\,|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1 \qquad \alpha, \beta \in \mathbb{C}$$

- Denisty matrix (statistical state):

$$\rho = \sum_i p_i\,|\psi_i\rangle\,\langle\psi_i|$$

Unitary evolution: deterministic, continuous, reversible

- Transformation on quantum register is determined by unitary matrices (operators) which preserve l_2-norm.

$$|\psi\rangle \xrightarrow{U} U\,|\psi\rangle \qquad |\psi\rangle \in (\mathbb{C}^2)^{\otimes n}, \quad U \in \mathcal{U}((\mathbb{C}^2)^{\otimes n})$$

$$\rho \xrightarrow{U} \sum_i p_i U\,|\psi_i\rangle\,\langle\psi_i|\,U^\dagger = U\rho U^\dagger$$

Introduction

Measurements: projection to an eigenstates of measurement operators **M**

- The probability of obtaining the outcome m is given by:

$$Pr(m) = ||M_m |\psi\rangle ||^2 \qquad Pr(m) = tr(M_m^\dagger M_m \rho)$$

- The post-measurement state:

$$|\psi_m\rangle = \frac{M_m |\psi\rangle}{\sqrt{\langle\psi| M_m^\dagger M_m |\psi\rangle}} \qquad \rho_m = \frac{M_m \rho M_m^\dagger}{tr(M_m^\dagger M_m \rho)}$$

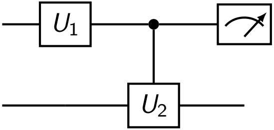
Quantum circuit

- Each transformations $U \in \mathcal{U}((\mathbb{C}^2)^{\otimes n})$ has to be implemented by a quantum circuit, i.e., a temporal sequence of elementary gates.
- Universal gate set: $CNOT$ gates + arbitrary single qubit gates

$$\mathbb{G}_0 = \{X_\theta, Y_\theta, Z_\theta, Ph_\theta, CNOT\}$$

$$\mathbb{G}_1 = \{H, S, T, CNOT\}$$

Quantum computation in the circuit model



$$U_1 = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix} \quad CU_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u'_{00} & u'_{01} \\ 0 & 0 & u'_{10} & u'_{11} \end{pmatrix}$$

Action of unitary gate on the basis states of $\mathbb{C}^2 \otimes \mathbb{C}^2$

$$\begin{aligned} |0\rangle \otimes |0\rangle &\mapsto U_1 |0\rangle \otimes |0\rangle \\ |0\rangle \otimes |1\rangle &\mapsto U_1 |0\rangle \otimes |1\rangle \\ |1\rangle \otimes |0\rangle &\mapsto U_1 |1\rangle \otimes |0\rangle \\ |1\rangle \otimes |1\rangle &\mapsto U_1 |1\rangle \otimes |1\rangle \end{aligned}$$

Action of controlled unitary gate on the basis states of $\mathbb{C}^2 \otimes \mathbb{C}^2$
control: first qubit
target: second qubit

$$\begin{aligned} |0\rangle \otimes |0\rangle &\mapsto |0\rangle \otimes |0\rangle \\ |0\rangle \otimes |1\rangle &\mapsto |0\rangle \otimes |1\rangle \\ |1\rangle \otimes |0\rangle &\mapsto |1\rangle \otimes U_2 |0\rangle \\ |1\rangle \otimes |1\rangle &\mapsto |1\rangle \otimes U_2 |1\rangle \end{aligned}$$

Hybrid quantum-classical algorithms

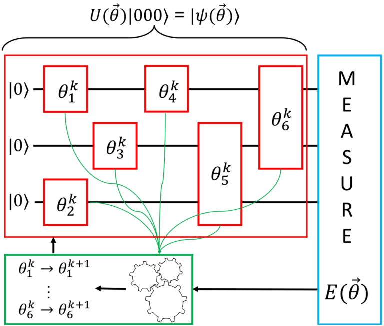


Figure: The classically intractable state preparation and measurement subroutines (red and blue) are performed on the small quantum computer. The current energy and parameter values are fed into a classical optimisation routine (green), which outputs new values of the parameters. The new parameters are then fed back into the quantum circuit. The gates acting on the qubits can be any parametrised gates, e.g. single qubit rotations or controlled rotations. Non-parametrised gates (e.g. X, Y, Z, CNOT) are also allowed.

Quadratic Unconstrained Binary Optimization problems and the Ising problems

QUBO Problem

$$H(x) = \sum_i Q_{i,i} x_i + \sum_{i<j} Q_{i,j} x_i x_j$$

$$H(X) = X^T Q X \qquad x_i, x_j \in \{0, 1\}$$

Ising Problem

$$H(s) = -\mu \sum_i h_i s_i - \sum_i \sum_{i<j} J_{i,j} s_i s_j$$

$$s_i, s_j \in \{-1, 1\}$$

QUBO and Ising problems

- The major difference between Ising and QUBO is that Ising deals with spin variables $\{-1, 1\}$, while QUBO uses binary variables $\{0, 1\}$.
- The choice of spin or binary can effect the way the problem can be expressed; namely, expanded and matrix forms.
- Ising and QUBO problems are *isomorphic*, i.e., they can be mapped to each other in a one-to-one relation in terms of their solutions and the moves in the problem solving trajectories.

The idea

Hamiltonian ground-states

- An important application for a quantum computer is to compute the ground-state $|\psi\rangle$ of a Hamiltonian $\hat{H}$.
- However, solutions cannot be guaranteed for all Hamiltonians, as it is a QMA-hard problem.

Importance

- One can encode any optimization problem in the Hamiltonian $\hat{H}$[1].
- It can then be used simulate evolution, for example, of the electronic structure of molecules and materials, as well as in more general optimization problems.
- Proposals: We use the Ising formulation of the optimization problem to encode the problem as an Ising Hamiltonian and solve it using a quantum-classical variational hydbrid quantum-classical algorithms such as the Quantum Approximate Optimization Algorithm (QAOA), and Variational Quantum Eigensolver (VQE).

[1] Lucas A (2014) Ising formulations of many NP problems. Front. Physics 2:5. doi: 10.3389/fphy.2014.00005

Weighted MAX-CUT

Given: (V, E) w: $E \mapsto R^+$ $S \cup S' = V$
Objective: Find the cut which maximizes the following:

$$\sum_{u \in S, v \in S', (u,v) \in E} w_{uv}$$

QUBO Formulation:

$$max \sum_{(u,v) \in E} w_{ij}(x_i - x_j)^2 \quad x_i \in \{0, 1\}$$

Ising Formulation:

$$max \sum_{v_i, v_j \in E} w_{ij} \frac{(1 - x_i x_j)}{2} \quad x_i \in \{-1, 1\}, \forall v_i \in V$$

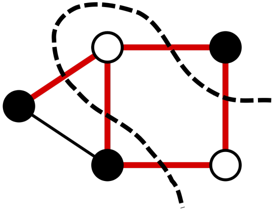


Figure: **MAX-CUT**: Cut (dotted line) partitions the nodes into two sets S (white) and S' (black).

Weighted Max-CUT problem on a SQ2

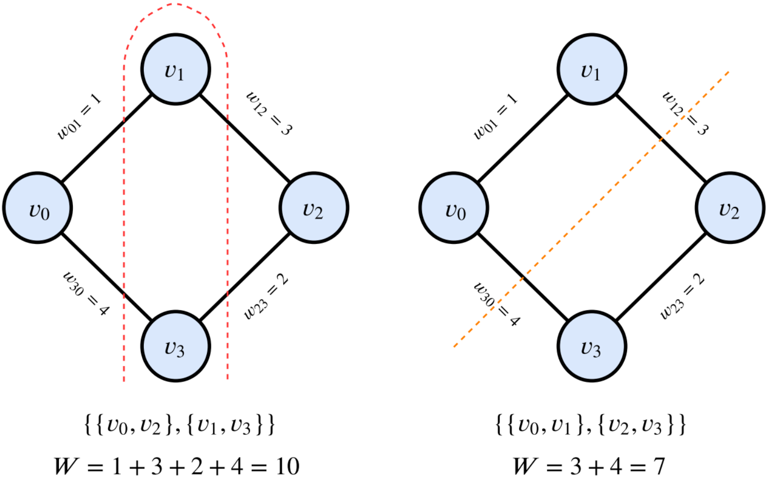


Figure: Partitions "0101" (left) and "0011" (right)

Quantum Approximate Optimization Algorithm[2]

Methodology

1. $C(z) = \sum_{\alpha=1}^{m} C_\alpha(z) = \sum_{i<j} w_{ij}(\sigma_i^z \otimes \sigma_j^z)$
2. $U(C,\gamma) = e^{-\iota\gamma C} = \prod_{\alpha=1}^{m} e^{-\iota\gamma C_\alpha}$
3. $B = \sum_{j=1}^{n} \sigma_j^x \qquad U(B,\beta) = e^{-\iota\beta B} = \prod_{j=1}^{n} e^{-\iota\beta\sigma_j^x}$
4. $|s\rangle = \frac{1}{\sqrt{2^n}} \sum_z |z\rangle$
5. $|\gamma,\beta\rangle = U(B,\beta_p)U(C,\gamma_p)\ldots U(B,\beta_1)U(C,\gamma_1)\,|s\rangle$
6. $F_p(\gamma,\beta) = \langle\gamma,\beta|\, C\, |\gamma,\beta\rangle$
7. $M_p = \max_{\gamma,\beta} F_p(\gamma,\beta)$

[2] Edward Farhi, Jeffrey Goldstone, Sam Gutmann "A Quantum Approximate Optimization Algorithm," JACM Volume 42 Issue 6, Nov. 1995 Pages 1115-1145

Variational Quantum Eigensolver[3]

Methodology

1. $U(G) = e^{-\iota\sigma(G)}$ $\quad\sigma(G) = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}^{\otimes N}$
2. $|s\rangle = |0\rangle^{\otimes N}$
3. $|\psi(\theta)\rangle = U(\theta)U(G)\,|s\rangle$
4. $F_p(\theta) = \frac{\langle\psi(\theta)|C|\psi(\theta)\rangle}{\langle\psi(\theta)|\psi(\theta)\rangle}$
5. $M_p = \min_\theta F_p(\theta)$

[3] Peruzzo, A., McClean, J., Shadbolt, P. et al. A variational eigenvalue solver on a photonic quantum processor. Nat Commun 5, 4213 (2014)

Probability distribution for QAOA

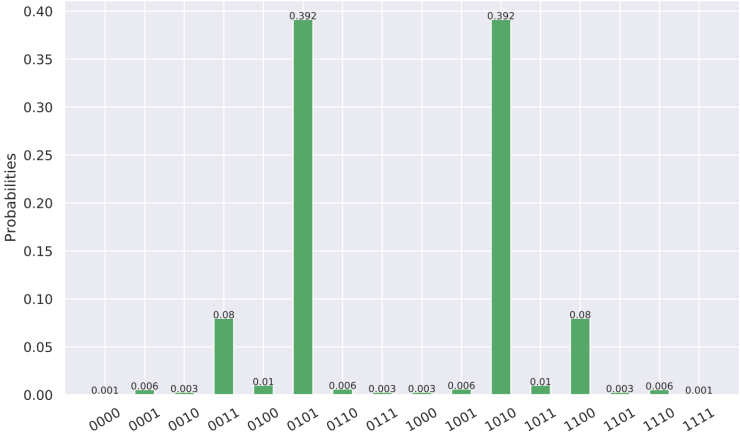


Figure: Probability distribution for optimized $\{\beta, \gamma\} = [1.784, 5.308], [3.665, 2.732]$, in QAOA

Probability distribution for VQE

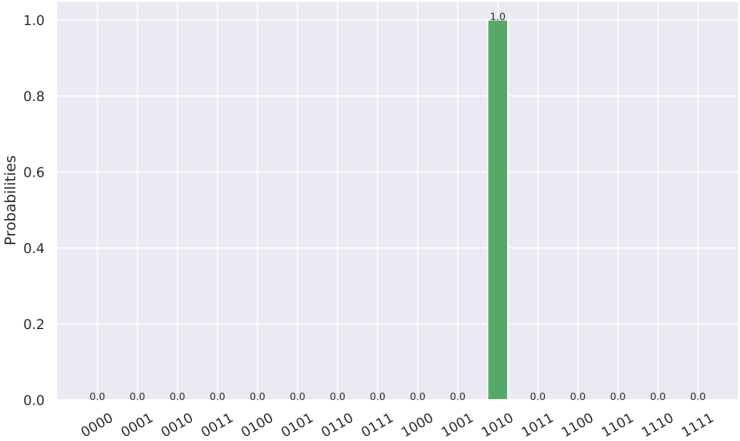


Figure: Probability distribution for optimized $\{\theta\} = [2.191, 2.047, \ldots, 3.142, 5.932, 0, 0.771]$, in VQE

Errors in QAOA and VQE

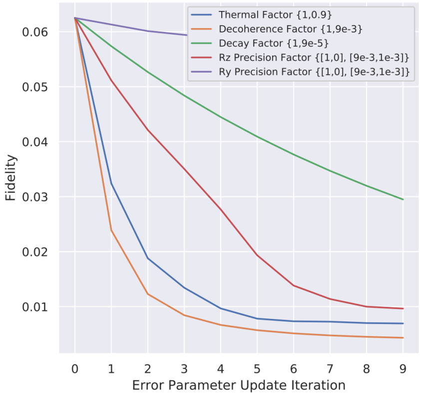


Figure: Effect of errors on state fidelity in QAOA.

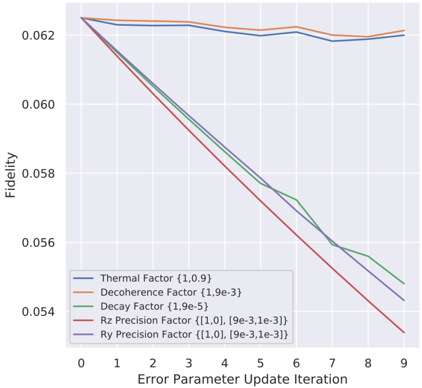
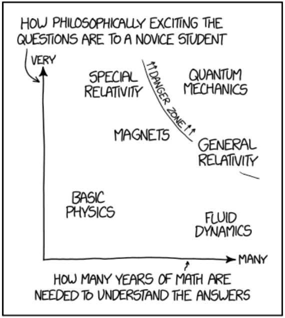


Figure: Effect of errors on state fidelity in VQE.

Conclusions

Analysis

1. Our analysis showed that the fidelity of states prepared by the quantum routine and consequently the success probability of variational algorithms is affected by both memory errors: thermalization, decoherence, amplitude decay and gate-precision errors.
2. Overall, the effect of noise in VQE was lesser than what we encountered in QAOA due to shorter circuit depth ($42 < 1576$) with fewer entangling gates ($18 < 844$)
3. Design exploration of NISQ processors is dedicated to executing hybrid quantum-classical algorithms and determining the effects of noise while retrieving the solutions.
4. With the development of error codes we will be able to generate optimized circuit designs for NISQ hardwares.



WHY SO MANY PEOPLE HAVE WEIRD IDEAS ABOUT QUANTUM MECHANICS

Questions?

Thank you.