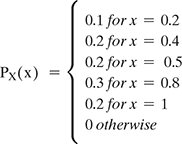
1. Given X be a discrete random variable with the following PMF



1. Range RX of the random variable X:

* The range RX is the set of all possible values that X can take.
* Based on the given PMF, RX = {0.2, 0.4, 0.5, 0.8, 1}.

2. P(X ≤ 0.5):

* This is the probability that X takes a value less than or equal to 0.5.
* P(X ≤ 0.5) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) = 0.1 + 0.2 + 0.2 = 0.5

3. P(0.25<X<0.75):

* This is the probability that X takes a value strictly between 0.25 and 0.75.
* P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5) = 0.2 + 0.2 = 0.4

4. P(X = 0.2 | X < 0.6):

* This is the conditional probability that X equals 0.2 given that X is less than 0.6.
* We use the formula for conditional probability: P(A | B) = P(A ∩ B) / P(B)
* P(X = 0.2 ∩ X < 0.6) = P(X = 0.2) = 0.1
* P(X < 0.6) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) = 0.5
* P(X = 0.2 | X < 0.6) = 0.1 / 0.5 = 0.2

2. Two equal and fair dice are rolled, and we observed two numbers X and Y.

1. Ranges RX and RY, PMFs of X and Y:

* Both X and Y represent the face value of one die, so RX = RY = {1, 2, 3, 4, 5, 6}.
* Since the dice are fair, each outcome is equally likely. Therefore, the PMF for both X and Y is:
  + P(X = k) = P(Y = k) = 1/6 for k in RX = RY.

2. P(X = 2, Y = 6):

This represents the probability of rolling a 2 on the first die and a 6 on the second die. Since the rolls are independent, we simply multiply the individual probabilities:

* P(X = 2, Y = 6) = P(X = 2) \* P(Y = 6) = 1/6 \* 1/6 = 1/36.

3. P(X > 3 | Y = 2):

This is the conditional probability of rolling a value greater than 3 on the first die given that the second die is a 2. Since the rolls are independent, the event on the second die (Y = 2) doesn't affect the probability of the first die. Therefore:

* P(X > 3 | Y = 2) = P(X > 3) = 1/2 (since there are 3 outcomes greater than 3 and 6 total possible outcomes).

4. Range and PMF of Z = X + Y:

* The possible values of Z are the sums of any two values in RX and RY.
* These range from 2 (1 + 1) to 12 (6 + 6). Therefore, RZ = {2, 3, 4, ..., 11, 12}.
* To find the PMF of Z, we need to consider all possible combinations of X and Y that lead to each value of Z. For example:
  + P(Z = 3) = P(X = 1, Y = 2) + P(X = 2, Y = 1) = 2 \* (1/6 \* 1/6) = 1/18.
* You can calculate the PMF for all other values of Z similarly.

5. P(X = 4 | Z = 8):

This is the conditional probability of rolling a 4 on the first die given that the sum of both dice is 8. There are two ways to achieve a sum of 8: (4, 4) and (2, 6). However, only one of these combinations has X = 4. Therefore:

* P(X = 4 | Z = 8) = P(X = 4, Y = 4) / P(Z = 8) = (1/6 \* 1/6) / (2 \* 1/36) = 1/2.

3. Student's Score PMF and P(X > 15):

PMF of X:

* X represents the student's score (correct answers).
* Since he knew 10 answers and guessed randomly for the rest, we can consider two cases:
  + P(X = k) = 1 for k = 10 (if guessed all unknown answers correctly).
  + P(X = k) = (43/44)^(10-k) \* (1/44)^k for 0 < k < 10 (probability of getting k correct guesses out of 10 unknowns).

P(X > 15):

This represents the probability of getting more than 15 correct answers. Since he can only get exactly 10 from known answers, this requires all 10 guesses to be correct:

* P(X > 15) = (43/44)^10 ≈ 0.00028.

4. Poisson Distribution and Student Arrivals:

P(10 < Y ≤ 15):

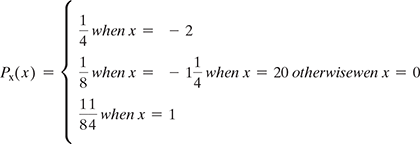
* Y represents the number of students arriving between 10 am and 11:30 am.
* Since Y is a Poisson distribution with an average of 10 students per hour, the average for 1.5 hours is 1.5 \* 10 = 15 students.
* We want the probability of having between 11 and 15 students arrive (inclusive).
  + P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15).
* You can calculate each probability using the Poisson PMF formula with λ = 15.

5. Independent Poisson and Sum Distribution:

PMF of Z:

* X and Y are independent Poisson variables with parameters α and β, respectively.
* Z = X + Y represents the sum of their values.
* When you add two independent Poisson distributions, the resulting distribution is another Poisson distribution with the sum of the original parameters: Z ~ Poisson(α + β).
* Therefore, the PMF of Z is:
  + P(Z = k) = e^(-(α+β)) \* ((α+β)^k) / k! for k = 0, 1, 2, ...

6. There is a discrete random variable X with the pmf.

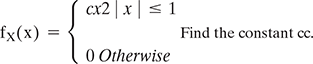


If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.

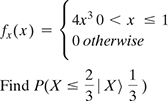
2. Find the pmf of Y.

2.Assuming X is a continuous random variable with PDF

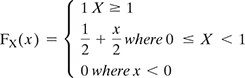


* + 1. Find EX and Var(X).
    2. Find *P*(*X* ≥ img).

1. If *X* is a continuous random variable with pdf

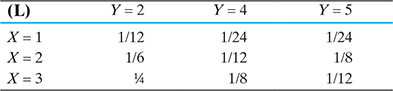


1. If *X*~Uniformimg and *Y* = sin(*X*), then find *fY*(*y*).
2. If X is a random variable with CDF



* + 1. What kind of random variable is *X*: discrete, continuous, or mixed?
    2. Find the PDF of *X*, f*X*(*x*).
    3. Find E(eX).
    4. Find P(*X* = 0|X≤0.5).

1. There are two random variables *X* and *Y* with joint PMF given in Table below
   * 1. Find *P*(*X*≤2, *Y*≤4).
     2. Find the marginal PMFs of *X* and *Y*.
     3. Find *P*(*Y* = 2|*X* = 1).
     4. Are *X* and *Y* independent?



6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

7.If A and B are two jointly continuous random variables with joint PDF

images

a. Find fX(a) and fY(b).

b. Are A and B independent of each other?

c. Find the conditional PDF of A given B = b, fA|B(a|b).

d. Find E[A|B = b], for 0 ≤ y ≤ 1.

e. Find Var(A|B = b), for 0 ≤ y ≤ 1.

8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF

If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.