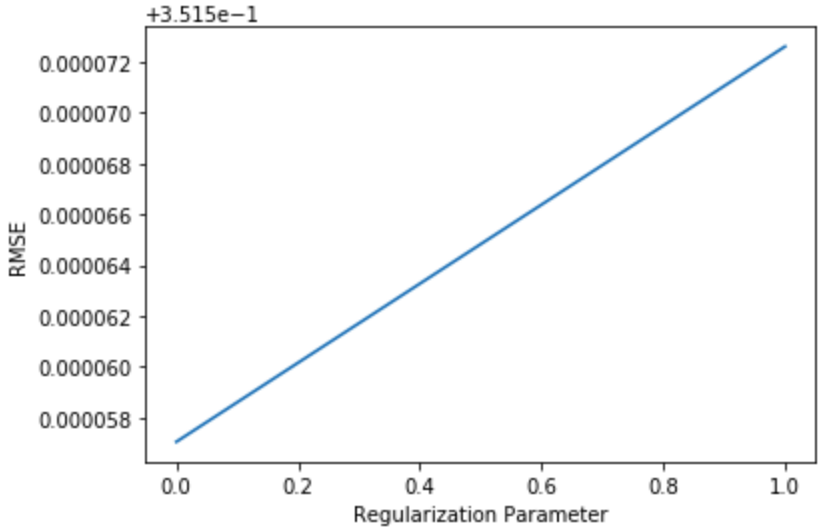
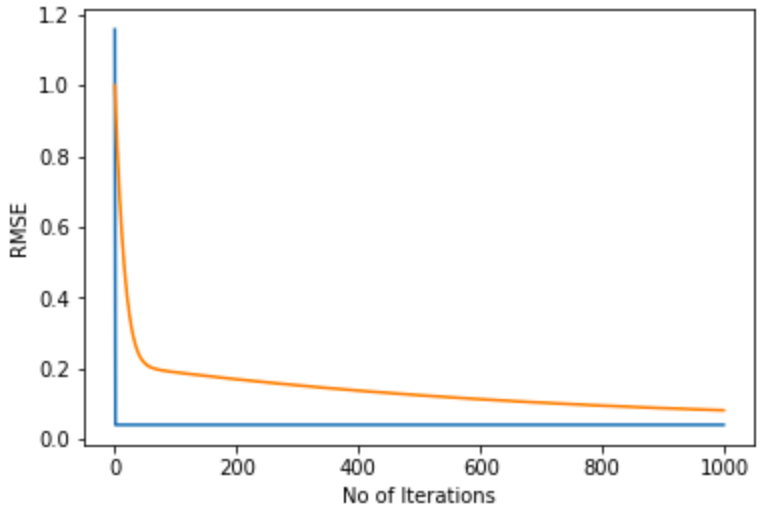
1 a.




We can see from the graph that Root Mean Square Error increases linearly with Regularisation Parameter.

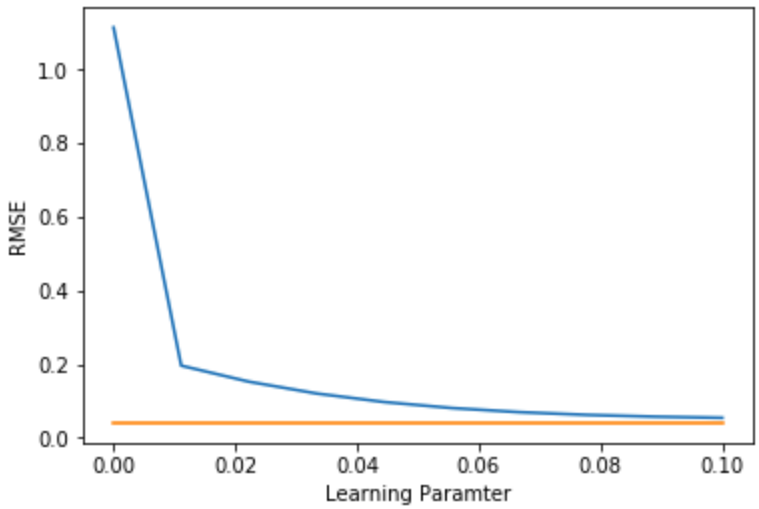
```
Reg Param :  0.0  , Thetas :  [ 0.5610846  -0.39982792 -2.67734432 -1.19423063 -0.40261742]
Reg Param :  0.111111111111  , Thetas :  [ 0.56108801 -0.39982535 -2.6773271  -1.19422295 -0.40261483]
Reg Param :  0.222222222222  , Thetas :  [ 0.56109142 -0.39982278 -2.67730987 -1.19421528 -0.40261223]
Reg Param :  0.333333333333  , Thetas :  [ 0.56109483 -0.39982021 -2.67729264 -1.1942076  -0.40260963]
Reg Param :  0.444444444444  , Thetas :  [ 0.56109824 -0.39981764 -2.67727541 -1.19419992 -0.40260703]
Reg Param :  0.555555555556  , Thetas :  [ 0.56110165 -0.39981507 -2.67725818 -1.19419225 -0.40260444]
Reg Param :  0.666666666667  , Thetas :  [ 0.56110506 -0.3998125  -2.67724095 -1.19418457 -0.40260184]
Reg Param :  0.777777777778  , Thetas :  [ 0.56110847 -0.39980993 -2.67722372 -1.1941769  -0.40259924]
Reg Param :  0.888888888889  , Thetas :  [ 0.56111188 -0.39980737 -2.67720649 -1.19416922 -0.40259665]
Reg Param :  1.0  , Thetas :  [ 0.56111528 -0.3998048  -2.67718926 -1.19416154 -0.40259405]
```

1 b.




For learning rate = 0.05

Orange - Gradient Descent
Blue - IRLS

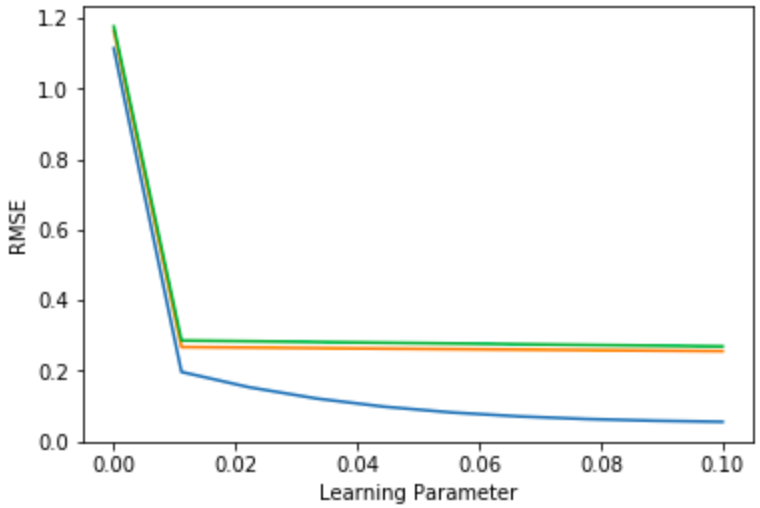



Orange - IRLS
Blue - Gradient Descent
We should prefer IRLS over gradient descent as it gives the solution in one step. It is much faster than gradient descent and also it doesn't depend on the learning rate.

```
Learning Rate GD:  0.0  , Thetas :  [ 1.  1.  1.  1.  1.]
Learning Rate IRLS:  0.0  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0111111111111  , Thetas :  [ 0.01797919  0.99027491  0.75013572  0.97289086  0.85434827]
Learning Rate IRLS:  0.0111111111111  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0222222222222  , Thetas :  [ 0.01355606  0.98155933  0.54840738  0.95016849  0.73413147]
Learning Rate IRLS:  0.0222222222222  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0333333333333  , Thetas :  [ 0.01020334  0.97337028  0.39656252  0.93096343  0.63855517]
Learning Rate IRLS:  0.0333333333333  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0444444444444  , Thetas :  [ 0.00765588  0.96560207  0.282626    0.91449815  0.56203391]
Learning Rate IRLS:  0.0444444444444  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0555555555556  , Thetas :  [ 0.00572191  0.95817342  0.19747303  0.90017634  0.50029064]
Learning Rate IRLS:  0.0555555555556  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0666666666667  , Thetas :  [ 0.00425523  0.95102166  0.13415281  0.88754011  0.45004758]
Learning Rate IRLS:  0.0666666666667  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0777777777778  , Thetas :  [ 0.00314436  0.94409839  0.08737226  0.8762374   0.40879017]
Learning Rate IRLS:  0.0777777777778  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.0888888888889  , Thetas :  [ 0.00230436  0.93736612  0.05310222  0.865997    0.37458697]
Learning Rate IRLS:  0.0888888888889  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
Learning Rate GD:  0.1  , Thetas :  [ 0.00167046  0.93079572  0.02827702  0.85660957  0.34595212]
Learning Rate IRLS:  0.1  , Thetas :  [ 0.0008177   0.08097741  0.00915992  0.07491422  0.24162522]
```

1 c.
Run for 1000 iterations

```
RMSE Linear:  0.054529679687960735
RMSE Quadratic:  0.25503363685054387
RMSE Cubic:  0.26857669195938977
```




Blue - Linear combination of features
Orange - Quadratic combination of features
Green - Cubic combination of features

I would prefer Linear combination of features as it gives the least RMSE on test data set approximately 0.0545. Also the RMSE error for different learning parameter is least for linear combination of features.

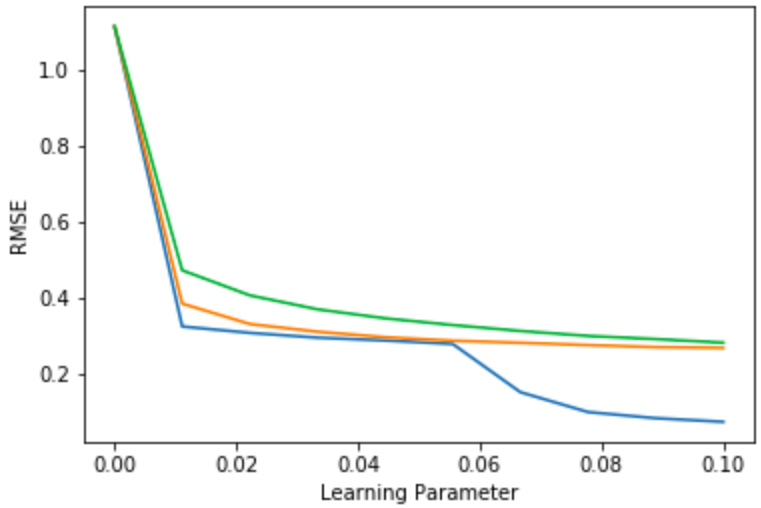
```
Alpha :  0.0  Thetas(Linear) :  [ 1.  1.  1.  1.  1.]
Alpha :  0.0  Thetas(Quadratic) :  [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
Alpha :  0.0  Thetas(Cubic) :  [ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.]
Alpha :  0.0111111111111  Thetas(Linear) :  [ 0.01797919  0.99027491  0.75013572  0.97289086  0.85434827]
Alpha :  0.0111111111111  Thetas(Quadratic) :  [-0.03370015  0.99715572  0.96981639  0.99831947  0.9894495   0.9980
5726
  0.99554273  0.99924712  0.99945893]
Alpha :  0.0111111111111  Thetas(Cubic) :  [-0.03520045  0.99615294  0.96701686  0.99791527  0.98827226  0.99724502
  0.99410121  0.99890795  0.9987449   0.99724502  0.99410121  0.99890795
  0.9987449 ]
Alpha :  0.0222222222222  Thetas(Linear) :  [ 0.01355606  0.98155933  0.54840738  0.95016849  0.73413147]
Alpha :  0.0222222222222  Thetas(Quadratic) :  [-0.03352836  0.99497235  0.96195146  0.99747149  0.98824139  0.9963
3633
  0.99215832  0.99854945  0.99923471]
Alpha :  0.0222222222222  Thetas(Cubic) :  [-0.03495085  0.99297259  0.95641974  0.99666522  0.98590762  0.99471593
  0.98929306  0.9978718   0.99781035  0.99471593  0.98929306  0.9978718
  0.99781035]
Alpha :  0.0333333333333  Thetas(Linear) :  [ 0.01020334  0.97337028  0.39656252  0.93096343  0.63855517]
Alpha :  0.0333333333333  Thetas(Quadratic) :  [-0.0333434   0.99279373  0.95414194  0.99662432  0.98703911  0.9946
19
  0.98879937  0.99785233  0.99901332]
Alpha :  0.0333333333333  Thetas(Cubic) :  [-0.03468889  0.98980174  0.94591078  0.99541683  0.98355519  0.99219417
  0.98452627  0.9968368   0.9968818   0.99219417  0.98452627  0.9968368
  0.9968818 ]
Alpha :  0.0444444444444  Thetas(Linear) :  [ 0.00765588  0.96560207  0.282626    0.91449815  0.56203391]
Alpha :  0.0444444444444  Thetas(Quadratic) :  [-0.0331597   0.99061984  0.94638714  0.99577794  0.98584249  0.9929
0525
  0.9854657   0.99715575  0.99879475]
```

```
Alpha :  0.0444444444444  Thetas(Cubic) :  [-0.03442902  0.98664035  0.93548893  0.99417009  0.98121476  0.98967972
  0.97980046  0.99580297  0.99595921  0.98967972  0.97980046  0.99580297
  0.99595921]
Alpha :  0.0555555555556  Thetas(Linear) :  [ 0.00572191  0.95817342  0.19747303  0.90017634  0.50029064]
Alpha :  0.0555555555556  Thetas(Quadratic) :  [-0.03297725  0.98845068  0.93868666  0.99493235  0.9846515   0.9911
951
  0.98215711  0.99645971  0.99857897]
Alpha :  0.0555555555556  Thetas(Cubic) :  [-0.0341712   0.98348841  0.92515347  0.99292499  0.97888626  0.98717254
  0.97511531  0.99477029  0.99504254  0.98717254  0.97511531  0.99477029
  0.99504254]
Alpha :  0.0666666666667  Thetas(Linear) :  [ 0.00425523  0.95102166  0.13415281  0.88754011  0.45004758]
Alpha :  0.0666666666667  Thetas(Quadratic) :  [-0.03279605  0.98628623  0.93104014  0.99408756  0.98346612  0.9894
8851
  0.97887343  0.99576423  0.99836598]
Alpha :  0.0666666666667  Thetas(Cubic) :  [-0.03391544  0.98034587  0.91490368  0.99168153  0.97656961  0.98467263
  0.97047046  0.99373877  0.99413175  0.98467263  0.97047046  0.99373877
  0.99413175]
Alpha :  0.0777777777778  Thetas(Linear) :  [ 0.00314436  0.94409839  0.08737226  0.8762374   0.40879017]
Alpha :  0.0777777777778  Thetas(Quadratic) :  [-0.03261608  0.98412649  0.92344717  0.99324355  0.9822863   0.9877
8549
  0.9756145   0.99506929  0.99815574]
Alpha :  0.0777777777778  Thetas(Cubic) :  [-0.03366171  0.9772127   0.90473883  0.9904397   0.97426475  0.98217995
  0.96586559  0.99270841  0.99322681  0.98217995  0.96586559  0.99270841
  0.99322681]
Alpha :  0.0888888888889  Thetas(Linear) :  [ 0.00230436  0.93736612  0.05310222  0.865997    0.37458697]
Alpha :  0.0888888888889  Thetas(Quadratic) :  [-0.03243733  0.98197143  0.91590739  0.99240033  0.98111202  0.9860
8604
  0.97238012  0.99437489  0.99794825]
```

```
Alpha :  0.0888888888889  Thetas(Cubic) :  [-0.03340999  0.97408889  0.89465823  0.9891995   0.97197161  0.97969449
  0.96130036  0.99167919  0.99232767  0.97969449  0.96130036  0.99167919
  0.99232767]
Alpha :  0.1  Thetas(Linear) :  [ 0.00167046  0.93079572  0.02827702  0.85660957  0.34595212]
Alpha :  0.1  Thetas(Quadratic) :  [-0.0322598   0.97982105  0.90842042  0.9915579   0.97994325  0.98439013
  0.96917013  0.99368104  0.99774349]
Alpha :  0.1  Thetas(Cubic) :  [-0.03316027  0.9709744   0.88466117  0.98796093  0.96969011  0.97721623
  0.95677444  0.99065113  0.9914343   0.97721623  0.95677444  0.99065113
  0.9914343 ]
```

1 d.

```
RMSE Mean Absolute Error Method :  0.07478588726810699
RMSE Mean Squared Error Method :  0.2690695508433692
RMSE Mean Cubed Error Method :  0.2828743834816575
```




Blue - Mean Absolute Error Method
Orange - Mean Square Error Method
Green - Mean Cubed Error Method

I would prefer the Mean Absolute Error Method as it gives the least error on test data set. Also the RMSE error for different learning parameter is least for Absolute error method.

```
Learining Rate(MAE) :  0.0  , Thetas :  [ 1.  1.  1.  1.  1.]
Learining Rate(MSE) :  0.0  , Thetas :  [ 1.  1.  1.  1.  1.]
Learining Rate(MCE) :  0.0  , Thetas :  [ 1.  1.  1.  1.  1.]
Learining Rate(MAE) :  0.0111111111111  , Thetas :  [ 0.14621168  0.99921675  0.99981346  0.99724858  0.99286725]
Learining Rate(MSE) :  0.0111111111111  , Thetas :  [ 0.22930894  0.99837791  0.97197502  0.99600929  0.98173921]
Learining Rate(MCE) :  0.0111111111111  , Thetas :  [ 0.33432989  0.99752972  0.95401444  0.99483235  0.97313584]
Learining Rate(MAE) :  0.0222222222222  , Thetas :  [ 0.12487372  0.9989869   0.99221498  0.99619029  0.98779759]
Learining Rate(MSE) :  0.0222222222222  , Thetas :  [ 0.16387222  0.99803665  0.96344912  0.99506     0.97673143]
Learining Rate(MCE) :  0.0222222222222  , Thetas :  [ 0.26073162  0.99703394  0.94475808  0.99393478  0.96805191]
Learining Rate(MAE) :  0.0333333333333  , Thetas :  [ 0.10927704  0.99871439  0.98267352  0.99500421  0.98188499]
Learining Rate(MSE) :  0.0333333333333  , Thetas :  [ 0.13779479  0.99786385  0.95895059  0.99457188  0.97411533]
Learining Rate(MCE) :  0.0333333333333  , Thetas :  [ 0.21812702  0.99671035  0.93896342  0.9933944   0.96492325]
Learining Rate(MAE) :  0.0444444444444  , Thetas :  [ 0.10041771  0.99850537  0.97491485  0.99412749  0.97725161]
Learining Rate(MSE) :  0.0444444444444  , Thetas :  [ 0.1184403   0.99771284  0.95493342  0.99414184  0.97179109]
Learining Rate(MCE) :  0.0444444444444  , Thetas :  [ 0.18969959  0.99648106  0.93502608  0.99303822  0.96282608]
Learining Rate(MAE) :  0.0555555555556  , Thetas :  [ 0.09106741  0.9981113   0.95965843  0.992459    0.96846356]
Learining Rate(MSE) :  0.0555555555556  , Thetas :  [ 0.10589004  0.99760277  0.95196668  0.99382681  0.97007982]
Learining Rate(MCE) :  0.0555555555556  , Thetas :  [ 0.16779508  0.99629849  0.93201731  0.99277336  0.96124309]
Learining Rate(MAE) :  0.0666666666667  , Thetas :  [ 0.03585117  0.98547099  0.49520719  0.94280348  0.69975867]
Learining Rate(MSE) :  0.0666666666667  , Thetas :  [ 0.09828034  0.99753287  0.95007341  0.99362638  0.968989  ]
Learining Rate(MCE) :  0.0666666666667  , Thetas :  [ 0.14797327  0.99612865  0.92933406  0.99254384  0.95984939]
Learining Rate(MAE) :  0.0777777777778  , Thetas :  [ 0.01618534  0.97827787  0.27814583  0.91602486  0.5622105 ]
Learining Rate(MSE) :  0.0777777777778  , Thetas :  [ 0.08968371  0.99744349  0.94762567  0.99336903  0.96758232]
Learining Rate(MCE) :  0.0777777777778  , Thetas :  [ 0.13065515  0.99597859  0.92707578  0.99235677  0.95869316]
Learining Rate(MAE) :  0.0888888888889  , Thetas :  [ 0.01092218  0.97531513  0.20163356  0.9051259   0.5096584 ]
Learining Rate(MSE) :  0.0888888888889  , Thetas :  [ 0.08195839  0.99735449  0.9451682   0.99311195  0.96617265]
Learining Rate(MCE) :  0.0888888888889  , Thetas :  [ 0.11933961  0.99588507  0.92575448  0.99225098  0.95802747]
Learining Rate(MAE) :  0.1  , Thetas :  [ 0.00726431  0.97323273  0.15354238  0.89756344  0.47455495]
Learining Rate(MSE) :  0.1  , Thetas :  [ 0.07890994  0.9973246   0.94435229  0.99302597  0.9657034 ]
Learining Rate(MCE) :  0.1  , Thetas :  [ 0.10619418  0.99577335  0.92423247  0.99213295  0.95727091]
```