5. Compute the value of $e$ approximately by evaluating $\int_0^1 e^{-x}\,dx$ using trapezoidal rule with sub-intervals.

6. Evaluate $\int_1^2 \frac{x}{y}\,dx$ and $\int_1^4 \frac{y}{x}\,dx$ from the following data using Simpson's one-third rule.

| $x$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 1.0414 | 1.0792 | 1.1139 | 1.1461 | 1.1761 | 1.2041 | 1.2304 | 1.2553 | 1.2788 | 1.3011 |

7. Evaluate $\int_0^2 \log_e \sqrt{1+x}\,dx$ using Simpson's one-third rule with 8 sub-intervals.

8. Evaluate $\int_0^2 \frac{x^3}{e^x - 1}\,dx$, using Simpson's one-third rule with 10 sub-intervals.

**Hint:** $\left[\left(\frac{x^3}{e^x-1}\right)_{x=0} = \lim_{x\to 0}\left(\frac{x^3}{e^x-1}\right)\right]$

9. Use Simpson's one-third rule with 6 sub-intervals to find the area contained between the x-axis and the curve $4x^2 + 9y^2 = 36$.

10. Compute $\int_0^6 \frac{1}{y}\,dx$ from the following data using Simpson's three-eight's rule.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | 0.146 | 0.161 | 0.176 | 0.190 | 0.204 | 0.217 | 0.230 |

11. A river is 45 m wide. The depth $d$ in metres at a distance $x$ metres from one bank is given by following data.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| $dy$ | 0 | 3 | 6 | 8 | 7 | 7 | 6 | 4 | 3 | 0 |

Find the cross-section of the river by Simpson's three-eight's rule.

12. A curve passes through the points below.

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0.7 | 1.0 | 1.3 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 | 2.4 |

Find the volume of the solid generated by revolving the area between the curve, the x-axis, the ordinates $x = 1$ and $x = 10$ about the $x$-axis using Simpson's three-eight's rule.

3. Evaluate $\int_1^2 \frac{\sin x}{x}\,dx$ using Simpson's $\frac{3}{8}^{\text{th}}$ rule with 6 sub-intervals.

Evaluate $\int_0^{\pi/2} e^{\sin x}\,dx$ using Simpson's $\frac{3}{8}^{\text{th}}$ rule with 6 sub-intervals.

Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule and Simpson's $\frac{1}{3}^{\text{rd}}$ rule with $h = \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$.

Evaluate $\int_0^1 \cos x^2\,dx$ using trapezoidal rule and Simpson's $\frac{1}{3}^{\text{rd}}$ rule with $h = \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$.

Evaluate $\int_0^1 \frac{dx}{\sqrt{x^4+1}}$ using Simpson's one-third rule with $h = \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$.

Evaluate $\int_0^{1/2} \frac{x\,dx}{\cos x}$ using Simpson's $\frac{1}{3}^{\text{rd}}$ rule with $h = \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{16}$.

Evaluate $\int_0^{10} \left(x + \frac{1}{x}\right) dx$ by Trapezoidal rule with 10 sub-intervals and find the error.

Evaluate $\int_0^{10} \left(x^2 + \frac{1}{x^2}\right) dx$ using Simpson's $\frac{1}{3}^{\text{rd}}$ rule with 10 sub-intervals and find the error.

## SUMMARY

After going through this chapter, you should be able to:

Learn Different methods of numerical differentiation

- **Values of Derivatives of $y$ Based on Newton's Forward Interpolation Formula**

$$\left(\frac{dy}{dx}\right)_{x=x_0} = Dy_0 = \frac{1}{h}\left(\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \ldots\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = D^2y_0 = \frac{1}{h^2}\left(\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \ldots\right)$$

$$\left(\frac{d^3y}{dx^3}\right)_{(x=x_0)} = D^3y_0 = \frac{1}{h^3}\left(\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \frac{7}{4}\Delta^5 y_0 + \ldots\right)$$

- **Values of Derivatives of $y$ Based on Newton's Backward Interpolation Formula**

$$\left(\frac{dy}{dx}\right)_{x=x_n} = Dy_n = \frac{1}{h}\left(\nabla y_n - \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \ldots\right)$$