Identities and Formulas

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even and Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then:

$$\frac{\pi}{180^\circ} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180^\circ} \quad \text{and} \quad x = \frac{180^\circ t}{\pi}$$

Half Angle Formulas

$$\sin\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\tan\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

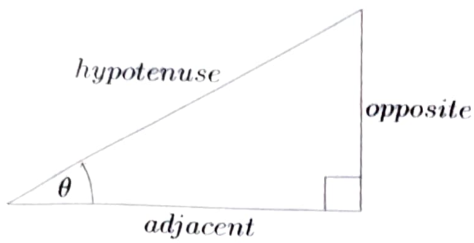
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Trigonometric Formula Sheet
Definition of the Trig Functions

Right Triangle Definition

Assume that:

$0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$



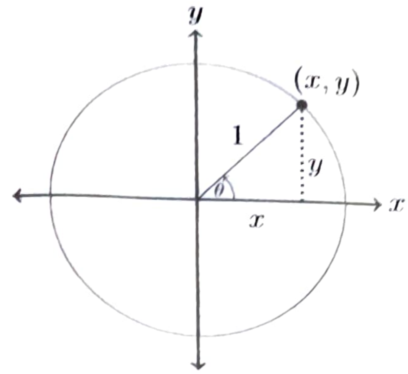
$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Unit Circle Definition

Assume θ can be any angle.



$$\sin\theta = \frac{y}{1} \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Domains of the Trig Functions

$\sin\theta,\quad \forall\, \theta \in (-\infty, \infty)$

$\cos\theta,\quad \forall\, \theta \in (-\infty, \infty)$

$\tan\theta,\quad \forall\, \theta \neq \left(n + \frac{1}{2}\right)\pi,\ where\ n \in \mathbb{Z}$

$\csc\theta,\quad \forall\, \theta \neq n\pi,\ where\ n \in \mathbb{Z}$

$\sec\theta,\quad \forall\, \theta \neq \left(n + \frac{1}{2}\right)\pi,\ where\ n \in \mathbb{Z}$

$\cot\theta,\quad \forall\, \theta \neq n\pi,\ where\ n \in \mathbb{Z}$

Ranges of the Trig Functions

$-1 \leq \sin\theta \leq 1$

$-1 \leq \cos\theta \leq 1$

$-\infty \leq \tan\theta \leq \infty$

$\csc\theta \geq 1\ and\ \csc\theta \leq -1$

$\sec\theta \geq 1\ and\ \sec\theta \leq -1$

$-\infty \leq \cot\theta \leq \infty$

Periods of the Trig Functions

The period of a function is the number, T, such that f $(\theta + T) = $ f (θ) .
So, if ω is a fixed number and θ is any angle we have the following periods.

$\sin(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$

$\cos(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$

$\tan(\omega\theta) \ \Rightarrow\ T = \frac{\pi}{\omega}$

$\csc(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$

$\sec(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$

$\cot(\omega\theta) \ \Rightarrow\ T = \frac{\pi}{\omega}$

LIST OF BASIC INTEGRAL FORMULAE

The basic integral formulas are given below:

- $\int 1\, dx = x + C$
- $\int a\, dx = ax + C$
- $\int x^n\, dx = ((x^{n+1})/(n+1)) + C$; $n \neq 1$
- $\int \sin x\, dx = -\cos x + C$
- $\int \cos x\, dx = \sin x + C$
- $\int \sec^2 x\, dx = \tan x + C$
- $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
- $\int \sec x (\tan x)\, dx = \sec x + C$
- $\int \operatorname{cosec} x (\cot x)\, dx = -\operatorname{cosec} x + C$
- $\int (1/x)\, dx = \ln |x| + C$
- $\int e^x\, dx = e^x + C$
- $\int a^x\, dx = (a^x/\ln a) + C$; $a>0$, $a \neq 1$
- $\int \tan x.dx = \log|\sec x| + C$
- $\int \cot x.dx = \log|\sin x| + C$
- $\int \sec x.dx = \log|\sec x + \tan x| + C$
- $\int \operatorname{cosec} x.dx = \log|\operatorname{cosec} x - \cot x| + C$

INTEGRATION FORMULAE OF INVERSE TRIGNOMETRIC FUNCTIONS

- $\int 1/(1 + x^2).dx = -\cot^{-1}x + C$
- $\int 1/x\sqrt{(x^2 - 1)}.dx = \sec^{-1}x + C$
- $\int 1/x\sqrt{(x^2 - 1)}.dx = -\operatorname{cosec}^{-1}x + C$
- $\int 1/\sqrt{(1 - x^2)}.dx = \sin^{-1}x + C$
- $\int /1(1 - x^2).dx = -\cos^{-1}x + C$
- $\int 1/(1 + x^2).dx = \tan^{-1}x + C$

1→ $\int \frac{f'(x)}{f(x)}\, dx = \log f(x)$

2→ $\int \frac{f'(x)}{\sqrt{f(x)}}\, dx = 2\sqrt{f(x)}$

3→ $\int \frac{f'(x)}{1+[f(x)]^2}\, dx = \tan^{-1}[f(x)]$

4→ $\int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1}$

5→ $\int e^{f(x)} f'(x)\, dx = e^{f(x)}$

6→ $\int e^x [f(x) + f'(x)]\, dx = e^x f(x)$

7→ $\int f(x) \cdot g(x) =$

$f(x) \times \int g(x) - \int [f'(x) \times \int g(x)\, dx]\, dx$

I L A T E

I ↓ Inverse

L ↓ Log

A ↓ Algebra

T ↓ Trig

E ↓ Exponential

DIFFICULT INTEGRAL FORMULAE

- $\int \sqrt{(x^2 + a^2)}.dx = 1/2.x.\sqrt{(x^2 + a^2)} + a^2/2 . \log|x + \sqrt{(x^2 + a^2)}| + C$
- $\int 1/(x^2 + a^2).dx = 1/a.\tan^{-1}x/a + C$
- $\int 1/\sqrt{(x^2 - a^2)}dx = \log|x + \sqrt{(x^2 - a^2)}| + C$
- $\int \sqrt{(x^2 - a^2)}.dx = 1/2.x.\sqrt{(x^2 - a^2)} - a^2/2 \log|x + \sqrt{(x^2 - a^2)}| + C$
- $\int 1/\sqrt{(a^2 - x^2)}.dx = \sin^{-1}x/a + C$
- $\int 1/(x^2 - a^2).dx = 1/2a.\log|(x - a)(x + a| + C$
- $\int 1/(a^2 - x^2).dx = 1/2a.\log|(a + x)(a - x)| + C$
- $\int 1/\sqrt{(x^2 + a^2)}.dx = \log|x + \sqrt{(x^2 + a^2)}| + C$
- $\int \sqrt{(a^2 - x^2)}.dx = 1/2.x.\sqrt{(a^2 - x^2)}.dx + a^2/2.\sin\text{-}1\ x/a + C$

V. Important formulas →

① $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_a^b f(x)dx$ where $a < c < b$

② $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

③ $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

④ $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ → if $f(x)$ is even $[f(-x) = f(x)]$

→ 0, if $f(x)$ is odd, $f(-x) = -f(x)$

⑤ $\int_0^{2a} f(x)dx = 2\int_0^a f(x)\,dx$, if $f(2a-x) = f(x)$

$= 0$, if $f(2a-x) = -f(x)$

⑥ $\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)(n-5)\dots}{n(n-2)(n-4)\dots} \times K$

$K = 1$, if n is odd.

$K = \frac{\pi}{2}$, if n is even.

⑦ $\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{(m-1)(m-3)\dots(n-1)(n-3)}{(m+n)(m+n-2)} \times K$

$K = 1$, if either m or n or both are odd

$K = \frac{\pi}{2}$, if both m & n are even