3.1 From Tensors Mathematical Object to a Data Model

Most Used tensor products

- Kronecker product
- Khatri-Rao product
- Hadamard product
- External product
- N-mode

Sub-space or models of lower dimension, latent factors Results of data analysis algorithms Tucker other operators **CPD** External product **Analytics** operators Kronecker product Time serie Adjacency matrix Data Khatri-Rao product Transition matrix Data manipulation produced Multi-layer network Data operators other tensor Mode-n product other operators

Figure 2: Tensor Model and operations

3.2 Architecture

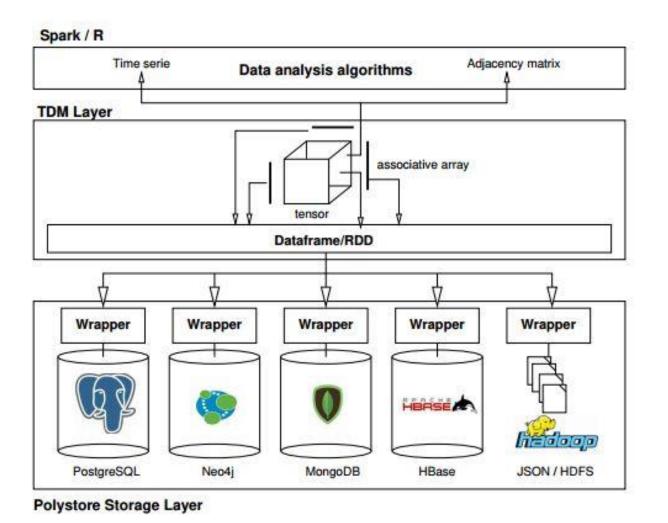
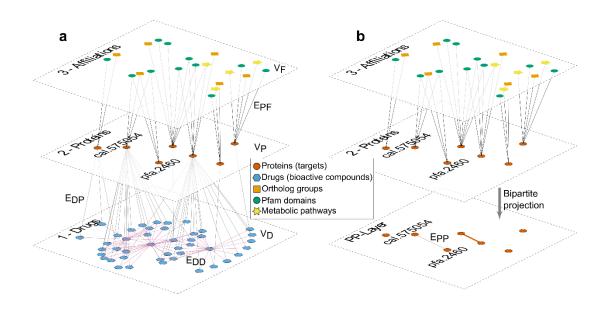


Figure 3: Tensor and associative arrays roles in the architecture of a polystore system

4 TDM: A TENSOR DATA MODEL

4.1 TDM Motivations

- Drawbacks of adjacency matrices.
- Allows to model complex relationships without fine knowledge and understanding.



4.2 TDM Formalization

- Associative array
- Named Typed Associative Array
- Typed Tensor

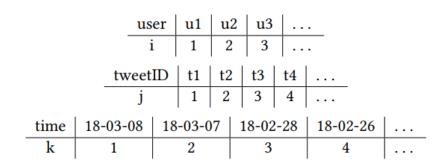


Figure 4: Named Typed Associative Arrays representing tensor dimensions

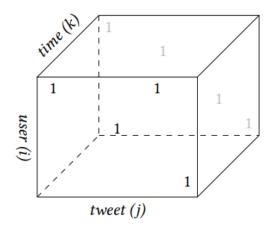


Figure 5: Associated tensor ${\cal X}$

4.3 Examples with Twitter Data

- Nodes are heterogeneous (users, tweets, hashtags, etc.)
- Relationships too are heterogeneous(retweet, publish, follow, mention, etc.)
- Different Dimensions like user interactions (retweet, follow), user actions (publish, like), tweet structure (content, mention, hashtag)

Let us denote the set of users by V_1 , the set of tweets by V_2

- mention, $R_1 : V_1 \times V_1 \rightarrow \mathbb{N}$
- retweet, $R_2 : V_1 \times V_2 \rightarrow \mathbb{N}$
- retweet_U, $R_3 : V_1 \times V_1 \rightarrow \mathbb{N}$
- publish, $R_4: V_1 \times V_2 \rightarrow \mathbb{N}$
- follow, $R_5: V_1 \times V_1 \rightarrow \mathbb{N}$

Using the null value tensor can model both simple and complex relationships, which can contribute to gain in performance by allowing materialized joins.

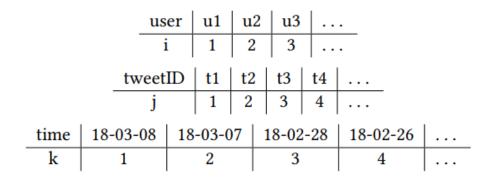


Figure 4: Named Typed Associative Arrays representing tensor dimensions

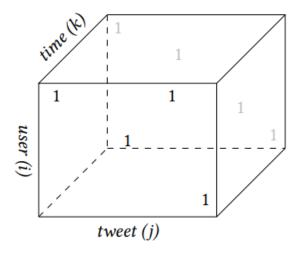


Figure 5: Associated tensor ${\cal X}$

5 TDM'S OPERATORS

- Data Manipulation Operators
- Analytical Decomposition Operators

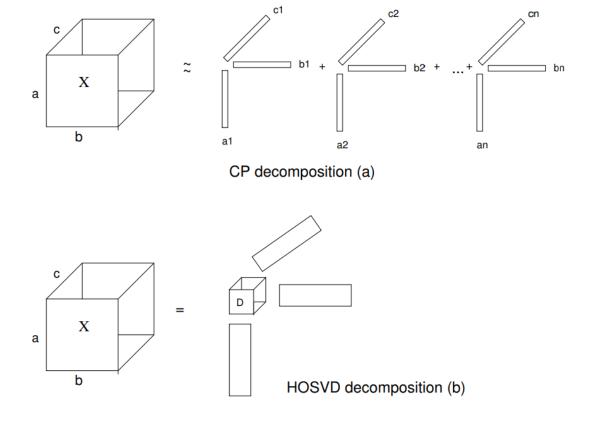


Figure 6: CP and HOSVD decompositions