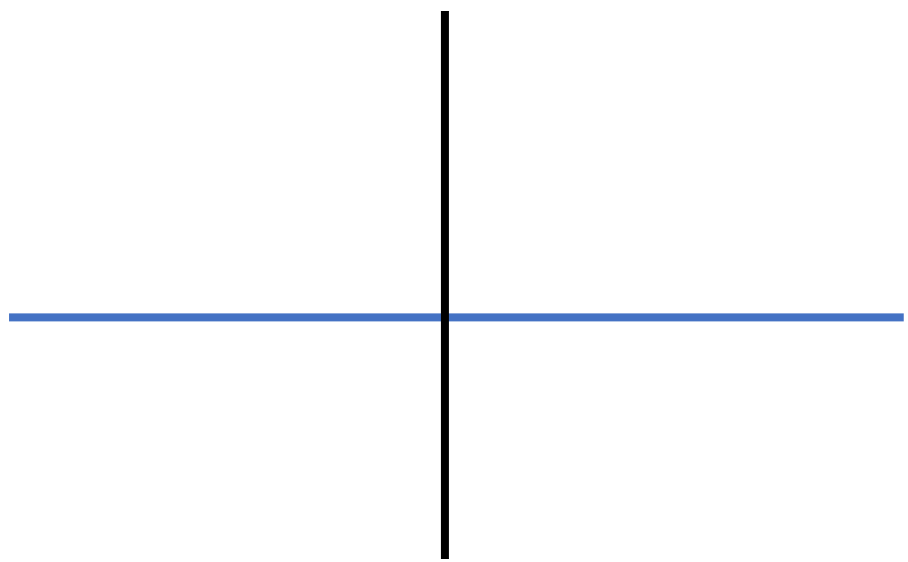
- Consider the following system:

$$\frac{dx}{dt} = x^2 + y^2 - 6 \qquad \frac{dy}{dt} = x^2 - y$$

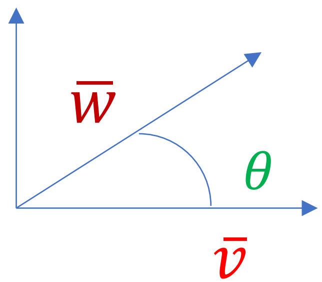
- Determine the roots as √2,2 with eigen values as $-2.96, 4.78$ and -√2,2 with eigen values as -$1.91 \pm 3.24i$ resulting in one saddle and one stable spiral



Vector space:

- Consists of a set of vectors V and a set of scalars S.
- Sum of any two vectors must be an element of V. Any vector multiplied by a scalar is also an element of V.
- S is the set of all the real numbers – "real vector space". Similarly define complex vector space
- Smallest number of the vectors required to define a vector in a vector space is the basis.
- Dimension of a vector space is the no. vector in the basis.

- Norm of a vector is the dot/inner product with itself $\|v\| = \sqrt{\bar{v}\bar{v}} \equiv v^T v$
- For any vector $\bar{v}$ and $\bar{w}$ $\quad cos\theta = \frac{\bar{v}.\bar{w}}{\|v\|\|w\|}$
- $\bar{v}$ and $\bar{w}$ are orthogonal if $\bar{v}.\bar{w}$=0
- They are orthonormal if magnitude of each vector is $\frac{\bar{v}}{\|v\|}$ and $\frac{\bar{w}}{\|w\|}$



- $\widehat{n_1}, \widehat{n_2}$……form the orthonormal basis if they are normal to each other i.e. projection onto one another is zero magnitude and projection onto itself is unit magnitude.

- A vector is usually decomposed into its components $\bar{v} = a_1\widehat{n_1} + a_2\widehat{n_2} \dots \dots \dots \dots \dots.$
- $\widehat{n_1}$ is unit vector in particular direction, a_1 is projection of $\bar{v}$ on $\widehat{n_1}, \widehat{n_2}$.
- Eg in 3D space: $\hat{\imath}, \hat{\jmath}, \hat{k}$ form orthonormal basis

$$\bar{v}=\begin{bmatrix}3\\4\\5\end{bmatrix}=3\underset{\hat{\imath}}{\begin{bmatrix}1\\0\\0\end{bmatrix}} + 4\underset{\hat{\jmath}}{\begin{bmatrix}0\\1\\0\end{bmatrix}} + 5\underset{\hat{k}}{\begin{bmatrix}0\\0\\1\end{bmatrix}}$$

- 3,4,5 are projections of the vector on the basis vectors i.e. $\hat{\imath}, \hat{\jmath}, \hat{k}$
- Likewise a function can also perhaps be decomposed into its components