Introduction

- ME 673: Mathematical Methods in Engineering
- Slot 3, Mon: 10:35 – 11:30, Tues: 11:35 – 12:30

  Thurs: 8:30 – 9:30

rpv@iitb.ac.in 9930034829,

puranik@iitb.ac.in 9867339443

Attendance: Not compulsory. No DX will be awarded. Will be taken.

- Texts: No specific texts but several available. Most material from “Advanced Engineering Mathematics” Peter O’Neal

Assessment

- 2 class tests :7.5 % each (approx. beg. Feb., end Mar.) Assgn. : 5%
- Mid-sem(Feb. 22-02 Mar.): 30% End sem (>21 Apr.): 50%
- All tests and examinations open book/notes. Online/Internet tools are permitted.
- All dates for exams will be announced in class and that will be considered as official.

Course Outline

- Eigen Value Problem: Solution procedure and applications; Scalar and Vector Field Theory: Divergence, Gradient, Curl, Laplacian, Divergence and Stoke theorems; Linear differential equation of second order and higher : Solution of homogenous and nonhomogeneous equations with and without constant coefficient;Power Series Solutions: Method of Frobenius, Legendre, Gamma and Bessel functions; Laplace Transform: Properties and application to solution of differential equations; Heaviside and Dirac-delta functions;Fourier Transform: Fourier series of a periodic function, Sturm-Liouville theory, Fourier integral, Fourier TransformDiffusion Equation: Separation of variables, Fourier and Laplace transforms;Wave Equation: Separation of variables, d`Alembert`s solution; Complex Integral Calculus: Complex integration, Cauchy`s theorem, Cauchy`s integral formula; Residue Theorem: Complex series and Taylor series, Laurent series, Classification of singularities, Residue theorem.

Ordinary Differential Equations

- In general an ordinary differential equation can be represented in the following form:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{d q_o}{dt} + a_o q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{d q_i}{dt} + b_o q_i$$

- Typically the RHS has qi term only and not its derivatives.
- Variables q_0, q_i depend only on one independent variable, i.e. t
- The highest derivative in the equation is its order and this is an n^{th} order differential equation.

First Order

- We shall look at first and second order equations in detail for which analytical solutions are possible
- We will look at methodologies to solve higher order equations but as a special case of multiple first order equations
- A first order equation is typically given as:

$$a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i$$

- Look at a special case where the RHS=0

$$a_1 \frac{dq_0}{dt} + a_0 q_0 = 0$$

- Such an equation is called a homogeneous differential equation. If RHS exists, it is a non-homogeneous equation.
- a1, a0 etc. in general can be functions of the independent variable ‘t’ and dependent variable q0. In this course we restrict to functions that are constant or functions of t for most part.

- Rewrite the equation as:

$$\frac{a_1}{a_o}\frac{dq_o}{dt} + q_o = 0$$
$$(\tau D + 1)q_o = 0$$

- (τD+1) is an operator that operates on the variable q0. (D+1/τ) can also be the operator.
- An operator is linear if a linear combination of solutions to the equation is also a solution of the Differential Equation

- Let q01 and q02 be solutions to the (τD+1)=0 i.e. the operator for the homogeneous equation.

$$\tau\frac{\mathrm{d}(\mathrm{q}_{o1})}{\mathrm{dt}}+q_{o1}=0;\ \tau\frac{d(q_{o2})}{dt}+q_{o2}=0$$

$$\tau\frac{d(q_{o1}+q_{o2})}{dt}+q_{o1}+q_{o2}=0$$

- The second equation follows from the first only if τ is either constant or a function of 't' only.

- An operator in an equation is called linear if a linear combination of the variables within the operator is a linear combination of the same operator with each variable.
- Also can be thought of as linear combination of solutions of the homogeneous portion(RHS=0) is also a solution. The above operator is therefore shown to be a linear one
- This would not be possible if τ had been a function of q_0
- A differential equation with linear operators usually is amenable to analytical solutions

- Generalize a little. Variables are functions of the independent variable.

$$\ell(q_o) = q_i;$$
$$\ell(q_{oh} + q_{oPI}) = q_i$$
$$\ell(q_{oh}) + \ell(q_{oPI}) = q_i + 0$$
$$\ell(q_{oh}) = 0;\ \ell(q_{oPI}) = q_i$$

- Essentially the equation is split into two equations using linearity property. Can also be thought of as superposition of solution of the homogenous part with a particular solution.

- Solution to a differential equation with linear operators is in general represented as the sum of the solutions of the corresponding homogeneous equation and a particular solution for the given equation
- Particular solution is ANY solution to the non homogeneous equation
- Solving the homogeneous equation is an important step towards getting the overall solution.
- Consider the following with τ , Kq_i as constant

$$\tau \frac{\mathrm{dq_0}}{\mathrm{dt}} + \mathrm{q_0} = Kq_i$$

First order equation

- Solution to homogeneous part:

$$\tau \frac{dq_o}{dt} + q_o = 0$$

$$\frac{dq_o}{q_o} = -\frac{dt}{\tau} \Rightarrow \mathrm{q}_0 = Ce^{-\frac{t}{\tau}}$$

- Need to determine the constant based on other information i.e. initial condition but first need the full solution
- Particular integral is Kq_i since it satisfies the equation.

- The final solution is therefore:

$$\mathrm{q}_0 = Ce^{-\frac{t}{\tau}} + Kq_i$$

- A first order differential equation requires a condition to complete the solution i.e. a condition that can be used to evaluate the constant C in the equation above.
- Suppose the RHS is not a constant but ‘t’

$$\tau\frac{\mathrm{dq_0}}{\mathrm{dt}} + \mathrm{q}_0 = t$$

- Particular integral is t- τ