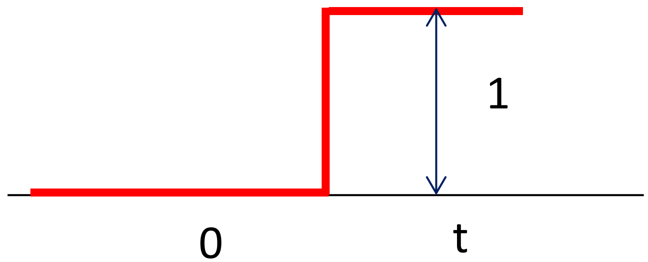
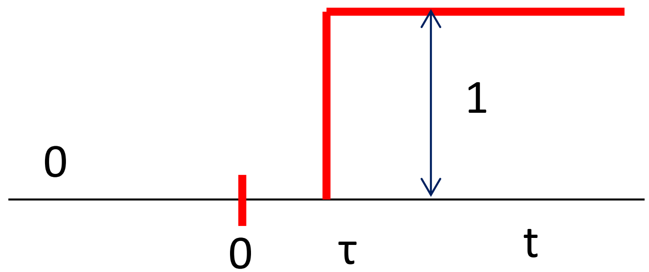
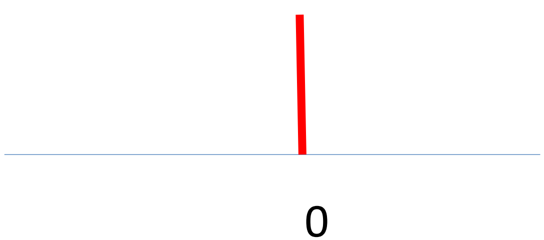
Heaviside/step function

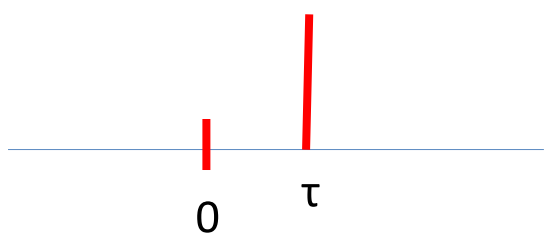




$$H(0) = \begin{pmatrix} 0\ for\ t \leq 0 \\ 1\ for\ t \geq 0 \end{pmatrix} H(t-\tau) = \begin{pmatrix} 0\ for\ t \leq \tau \\ 1\ for\ t \geq \tau \end{pmatrix}$$

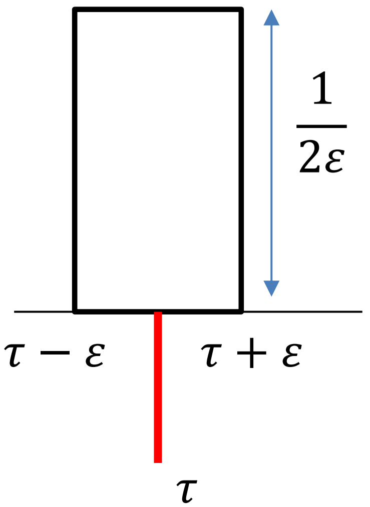
- Derivative of the Heaviside function is the Dirac delta function.





- Unit impulse function

$$\delta_\varepsilon(t-\tau) = \begin{cases} 0 \\ \dfrac{H(t-(\tau-\varepsilon)) - H(t-(\tau+\varepsilon))}{2\varepsilon}; \\ 0 \end{cases}$$



- As ε tends to zero unit impulse function approaches the Dirac Delta function

- It is appropriate to comprehend the delta function in an integral sense

$$\int_{-\infty}^{\infty} \delta(t)dt = H(0)_{-\infty}^{\infty} = 1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0); \int_{-\infty}^{\infty} f(t)\delta(t-\tau)dt = f(\tau)$$

- Our interest is in the use of the Laplace transform for solving differential equations

$$\mathcal{L}\left[\frac{df}{dt}\right] = \int_0^\infty e^{-st}\frac{df}{dt}$$

$$= [e^{-st}f(t)]_0^\infty - \int_0^\infty (-s)e^{-st}f(t)$$

$$= -f(0) + sL(f)$$

$$\mathcal{L}[f''] = -f'(0) + sL(f')$$

$$= -f'(0) + s(-f(0) + sL(f))$$