.

Solve and submit Q-1,4,6,13,14.

1. $y'' - 4y' = 6e^{3t} - 3e^{-t};$ $\quad y(0) = 1 \quad y'(0) = -1$

2. $y'' - y' = e^{t}\, cost\,;$ $\quad y(0) = y'(0) = 0$

3. $y'' + 5y' + 6y = f(t);$ $\quad y(0) = y'(0) = 0, wit$

$$f(t) = \begin{cases} -2 & for\ 0 \le t < 3 \\ 0 & for\ t \ge 3 \end{cases}$$

4. $y'' + 16y = f(t);$ $\quad y(0) = 0y'(0) = 1, with$

$$f(t) = \begin{cases} cos4t & for\ 0 \le t < \pi \\ 0 & for\ t \ge \pi \end{cases}$$

5. $y' + 6y + 9\int_0^t y(\tau)d\tau = 1,$ $\quad y(0) = 0$

6. Find the general solution, using the method of variation of parameters for a particular solution.

$$y'' + y' = \tan(x)$$

7. Find the general solution, using the method of undetermined coefficients for a particular solution.

$$y'' - y' - 2y = 2x^2 + 5$$

8. Consider the differential equation:

$$y'' - 2y' - 3y = f(t),$$

with initial conditions:

$$y(0) = 1, y'(0) = 0,$$

where $f(t)$ is given by:

$$f(t) = \begin{cases} 0, & for\ 0 \le t < 4 \\ 12, & for\ t \ge 4 \end{cases}$$

9. Consider the differential equation:

$$y'' + 5y' + 6y = f(t),$$

with initial conditions:

$$y(0) = 0, y'(0) = 0,$$

where $f(t)$ is given by:

$$f(t) = \begin{cases} -2, \; for \; 0 \le t < 3 \\ 0, \; for \; t \ge 3 \end{cases}$$

10. Solve the equation:

$$y'' + 3y' + 2y = e^{2x}, \quad y(0) = 1, \; y'(0) = 0$$

11. Find the general solution of the second-order differential equation:

$$y'' - 4y' + 4y = \sin(x) + x^2$$

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$$y'' + y = \begin{cases} 1, & \text{if } 0 \le x < \pi \\ 0, & \text{if } x \ge \pi \end{cases}, \qquad y(0) = 0,\ y'(\pi) = 1$$

13. $\frac{d}{dx}y(x) + \int_0^x y(t)dt = x^2 + 1, \qquad y(0) = 0$

14. Consider the Euler Cauchy equation $x^2y'' + xy' - y = 0$. Show that when the roots are imaginary the solution is $y = x^a(c_1\cos(b\ln x) + c_2\sin(b\ln x))$ and when the roots are equal the solution is $y = c_1x^r + c_2\,(\ln x)\,x^r$