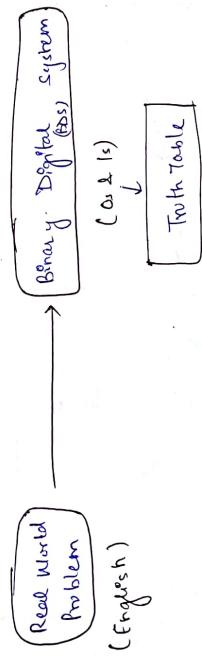


DLD Chapter 1: Representation of Boolean Functions:



→ A BDS is used to solve/implement real world problems. Moreover, whenever any real world problem has to be solved using a digital system, the most necessary requirement is to first convert the problem from our familiar English language representation into the language understood by a BDS, i.e., the language of 0s and 1s.

→ Furthermore, a truth table is the name given to the most basic representation of a real world problem in the language of 0s and 1s.

Min terms approach of designing a BDS:

$$f = (\bar{x} \cdot \bar{y}) + (x \cdot \bar{y})$$

known as minterms.

obtained by performing AND operation of all the variables in such a way that:

bit 0 is represented by complemented variable. E.g. \bar{x}

bit 1 is represented by uncomplemented variable. E.g. x

→ The function 'f' is said to be in the sum of minterms form.

→ They are also represented as m_j , where m_j is also represented as the decimal equivalent of the binary input.

$$f = \Sigma(0, 2)$$

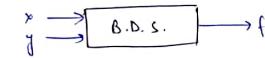
implies the OR operation.

→ After simplifying the function, the function is said to be in the sum of products form (SOP form).

E.g. An integer number N ranges from 0 to 3. Find out if N is divisible by 2; then output = 1 or else, output = 0

	x	y	f
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	0

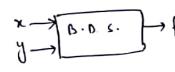
0 1 2 3	0 1
$2^n: 4$	$2^n: 2$
$n=2$	$n=1$



NOTE

- For simplifying a function, there are 2 types of simplification techniques:
- Boolean algebra simplification \rightarrow Karnaugh Map Simplification

Max terms approach of designing a BDD:



x	y	f
0	0	1
0	1	0
1	0	1
1	1	0

$$f = (x + \bar{y}) \cdot (\bar{x} + y)$$

Known as maxterms

Obtained by performing OR operation of all the variables of the function in such a way that:
bit 0 is represented by uncomplemented variable.
bit 1 is represented by complemented variable.

The func "f" is said to be in the product of maxterms form.
The func "f" is also represented as:

$$f = M_1 \cdot M_3$$

$$f = \pi(1, 3)$$

represents AND operation.

After simplification, the func "f" is said to be in Product of sums (POS) form.

Variables, Literals and Terms:

$$\text{E.g. } f(x, y) = (\bar{x} \cdot \bar{y}) + (\bar{x} \cdot y)$$

Variables = 2
Literals = 4
GR No. occurrence of variables

Canonical Forms:

\rightarrow A function that is represented in either the sum of minterms or the product of maxterms form. is said to be in the canonical form.

* Conversion between canonical forms:

$$\text{E.g. } \begin{array}{c|c|c} x & y & f \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array} \quad f = \Sigma(0, 2) \\ = \pi(1, 3)$$

\rightarrow The steps of conversion are as follows:

- ① Interchange the symbol " Σ " and " π ".
- ② write the missing numbers of the function.

$$\text{E.g. } f(x, y, z) = \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + xyz \\ = \Sigma(2, 4, 6, 7) \\ = \pi(0, 1, 3, 5) \\ = (x\bar{y}+z)(\bar{x}+y+\bar{z})(x+y+\bar{z})(\bar{x}+y+z) \quad \textcircled{a}$$

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$$\text{E.g. } f(x, y, z) = \Sigma(1, 2, 5, 6, 7) \\ = \pi(0, 3, 4) \\ = (x+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+z) \quad \textcircled{b}$$

Total Possible functions: $= 2^n$

x	y	f_0	f_1	f_2	\dots	f_{15}
0	0	0	0	0	\dots	0
0	1	0	0	0	\dots	0
1	0	0	0	0	\dots	0
1	1	0	1	0	\dots	1

→ With 'n' boolean variables, a total of 2^{2^n} number of different Boolean functions can be formulated.

$$\text{e.g. } \begin{array}{|c|c|c|} \hline \text{No. of variables} & & \text{Total possible functions} \\ \hline 2 & 2^2 = 16 & \\ \hline 3 & 2^3 = 256 & \\ \hline 4 & 2^4 = 65536 & \\ \hline & 2^{16} = 65536 & \\ \hline \end{array}$$

Logic Gates

Chapter - 2

- ① NOT / Inverter
- ② AND
- ③ OR
- ④ NAND
- ⑤ NOR
- Basic logic gates

Universal Logic Gates

different

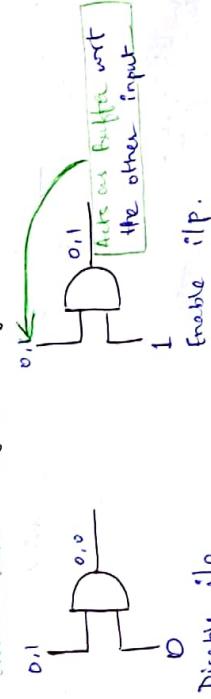
* AND Gate:



$$\begin{array}{|c|c|c|} \hline x & y & f \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ \hline \end{array} \quad f = x \cdot y = xy \quad \text{If there is no operator mentioned, then it is AND operation.}$$

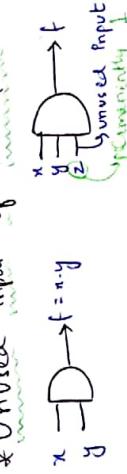
→ It obeys the commutative law. [$xy = yx$]
→ It obeys the associative law. [$(x \cdot (y \cdot z)) = ((x \cdot y) \cdot z)$]

* Control Inputs of AND gate:



Disable IP.
Enable IP.

* Unused Input of AND gate:



$$f = \overline{x} \cdot y \quad \left| \begin{array}{l} f = x \cdot \overline{y} \\ f = \overline{x} \cdot y \end{array} \right.$$

Unused Input

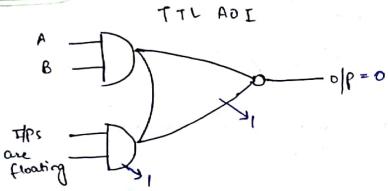
→ The unused input or inputs of an AND gate can be connected in one of the following ways:

- ① Logic 1 / Pull up method
- ② One of the used inputs
- ③ In case of TTL, it can be left unconnected (open/floating).

NOTE

→ In case of TTL, open/floating input acts as logic 1.

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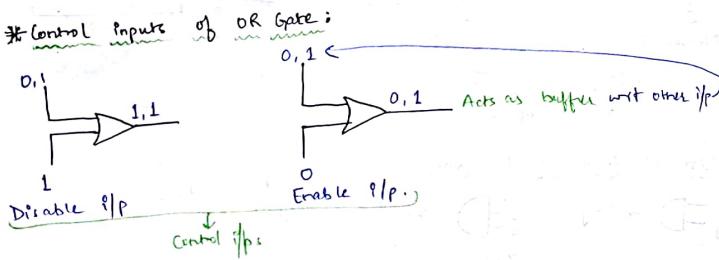


OR Gate: $y \Rightarrow f$

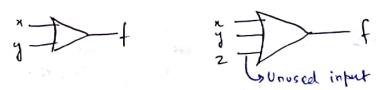
x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

$f = x + y$
 $= x \vee y$

→ obeys the commutative law
 → obeys the associative law



* Unused I/P of "OR" Gate:



$$f = x + y + z$$

$$= x + y$$

$$= x \vee y$$

→ Unused input/inputs of an "OR" gate can be connected in one of the following ways:

- ① Logic 0 (Pull down method) → Best method
- ② One of the used inputs.

NAND Gate: $x \overline{\wedge} y \Rightarrow f$

x	y	f
0	0	1
0	1	0
1	0	0
1	1	0

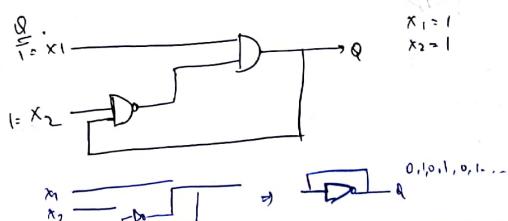
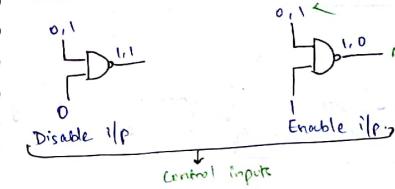
$f = \bar{x} \bar{y}$
 $= \bar{x} \cdot \bar{y}$
 $= x + y$

→ obeys commutative law.

→ Does not obey associative law.
 $(\bar{x} \bar{y}) \cdot z \neq x \cdot (\bar{y} \cdot z)$

NOTE: The universal logic gates (NAND & NOR) do not obey the associative law.

* Control Inputs of NAND Gate:



- ① at logic 1
- ② at logic 0
- ③ at its initial
- ④ Unstable

* Unused input of NAND gate:

→ The unused input(s) of a NAND gate can be connected in exactly the same way as the unused input of AND gate.

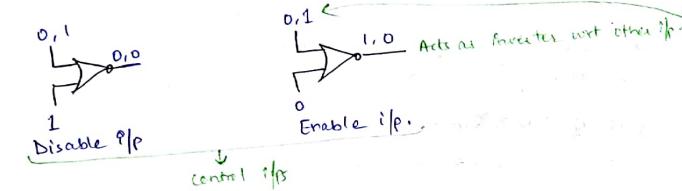
NOR Gate: $x \rightarrow y \rightarrow f$

x	y	f
0	0	1
0	1	0
1	0	0
1	1	0

$f = \overline{x} \cdot \overline{y}$
 $= \overline{x+y}$
 $= x \oplus y$
 $(x+y)+z \neq x+(y+z)$

→ obeys commutative law
→ Does NOT obey associative law.

* Control pins for NOR Gate:



* Unused i/p of NOR Gate:

→ The unused input(s) of a NOR gate can be connected in exactly the same way as the unused input of OR gate.

EXOR Gate / XOR Gate: $x \rightarrow y \rightarrow f$
Exclusive OR

x	y	f
0	0	0
0	1	1
1	0	1
1	1	0

$f = (\bar{x} \cdot y) + (x \cdot \bar{y})$
 $= (x+y) \cdot (\bar{x}+\bar{y})$
 $= x \oplus y$

$\rightarrow f=1$; when $x \neq y$
 $=0$; when $x=y$

→ EXOR acts as an odd function.

* In an odd function, the toggle output is equal to logic 1 when odd number of inputs are equal to 1.

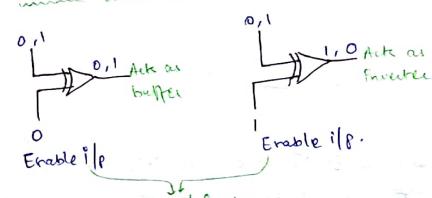
x	y	z	$x \oplus y$	$x \oplus y \oplus z$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

NOTE:

OR gate is also known as inclusive OR gate.

→ XOR gate obeys the commutative law.
→ XOR gate obeys the associative law.

* Control inputs of XOR Gate:



i/p In this case, phase inversion is not taking place. This implies buffer operation

o/p.

i/p. In this case phase inversion is taking place. This is the same as inverter operation.

$* A \oplus A = 0$
 $A \oplus \bar{A} = 1$
 $A \oplus 0 = A$
 $A \oplus 1 = \bar{A}$

If $A \oplus B = C$
 then, $\rightarrow A \oplus C = B$
 $B \oplus C = A$
 $A \oplus B \oplus C = 0$
 $C \oplus C = 0$

$A \oplus A \oplus A = A$
 $A \oplus A \oplus A \oplus A = 0$

$\bullet A \oplus A \oplus A \oplus A \dots - n \text{ times}$
 $= A$, when n is odd
 $= 0$, when n is even

EXNOR gate | XNOR gate:

x	y	f
0	0	1
0	1	0
1	0	0
1	1	1

$f = \bar{x}\bar{y} + xy$
 $= \bar{x}y(x+y) \cdot (\bar{x}+y)$
 $= x \oplus y$

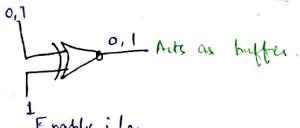
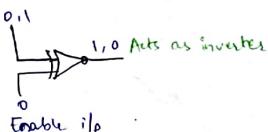
$f = 1$ when $x=y$
 $= 0$ when $x \neq y$

Because of this reason

\rightarrow XMR logic is also known as Equivalence logic / Coincidence logic.
 \rightarrow EXNOR is used as a Equality Detector circuit.

\rightarrow Obey's the commutative law.
 \rightarrow Obey's the associative law.

* Control I/p's of EXNOR gate:



$A \oplus A = 1$
 $A \oplus \bar{A} = 0$
 $A \oplus 0 = A$
 $A \oplus 1 = \bar{A}$

If $A \oplus B = C$
 then, $A \oplus C = B$
 $B \oplus C = A$
 $A \oplus B \oplus C = 0$

$\bullet A \oplus A \oplus A \oplus A \dots - n \text{ times}$
 $= A$; when n is odd
 $= 1$; when n is even

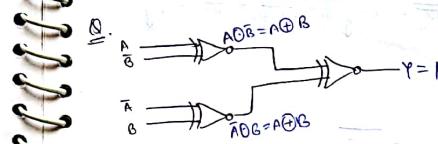
$A \oplus B = \bar{A} \cdot B + A \cdot \bar{B}$
 $= A \cdot B + \bar{A} \cdot \bar{B}$
 $= A \oplus B$

$A \oplus B = \bar{A} \oplus B$
 $= \bar{A} \oplus B$
 $A \oplus B \oplus C = \bar{A} \oplus B \oplus C$
 $= A \oplus B$

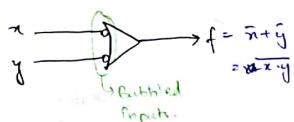
Let $x = A \oplus B$, $y = C$
 $= \bar{x} \oplus y$
 $= x \oplus y$
 $= A \oplus B \oplus C$

NOTE

\rightarrow In case of EXOR and EX NOR;
 ① for even no. of variables, EXOR and EX NOR are complements of each other.
 ② for odd no. of variables, EXOR and EXNOR are equal to each other.



Bubbled Logic:



→ NAND gate is equivalent to a bubbled OR gate.

Similarly:

\Rightarrow NOR gate = Bubbled AND gate

→ AND gate = bubbled NOR gate

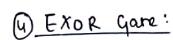
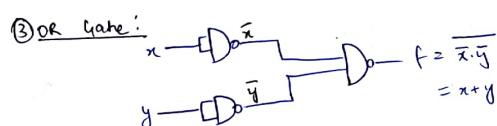
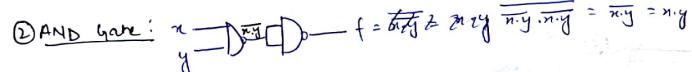
→ OR gate = bubbled NAND gate

Universal logic gates:

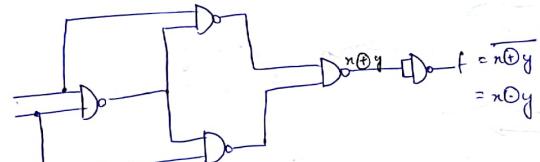
Universal Logic gates:
 → With the help of only ~~multiple~~^{1 or more} NAND gates, or with the help of only 1 or more than 1 NOR gates, it is possible to implement any other logic gate.

→ It is because of this reason NAND gate and NOR gates are known as universal logic gates.

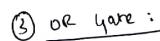
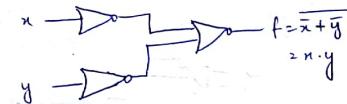
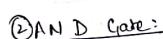
* NAND Gate as a Universal logic gate:



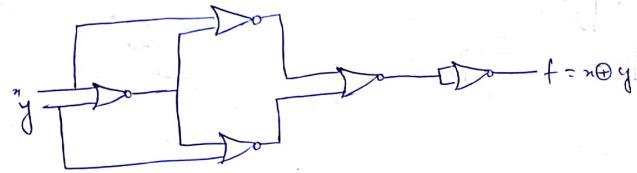
⑤ EXNOR Gate:



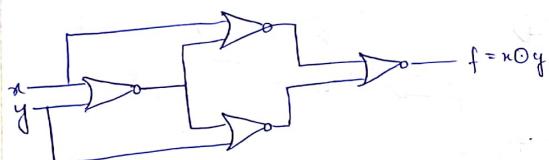
NOR Gate as a universal logic gate:



④ EXOR Gate:



⑤ EXNOR Gate:



NOTE:

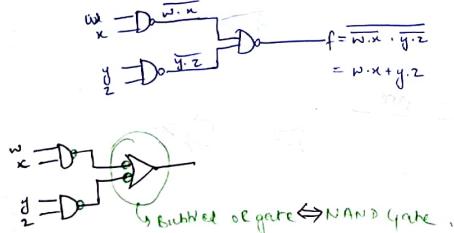
To implement the NOR gate from NAND gate and vice versa, a total of 4 logic gates are needed.

Q. What is the minimum number of:

2-input NAND gates needed to implement the boolean function $f = w \cdot x + y \cdot z$?

$$f = w \cdot x + y \cdot z = (\overline{w} \cdot \overline{x}) \cdot (\overline{y} \cdot \overline{z}) \rightarrow \overline{\overline{w} \cdot \overline{x}} + \overline{\overline{y} \cdot \overline{z}} = w \cdot x + y \cdot z$$

- Ⓐ 2
- Ⓑ 3
- Ⓒ 4
- Ⓓ 5



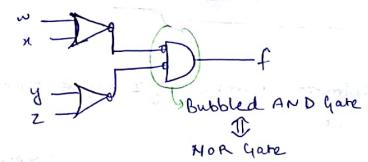
NOTE

1. The previous circuit is termed as a 2-level AND-OR implementation.
2. A 2-level AND-OR implementation is equivalent to a 2-level NAND-NAND implementation.
3. A 2-level AND-OR implementation is obtained when a digital circuit is implemented from a SOP expression.

Q. What is the minimum no. of 2-input NOR gates needed to implement the boolean function:

$$f = (w+x) \cdot (y+z)$$

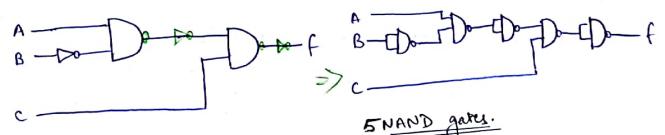
- Ⓐ 2
- Ⓑ 3
- Ⓒ 4
- Ⓓ 5



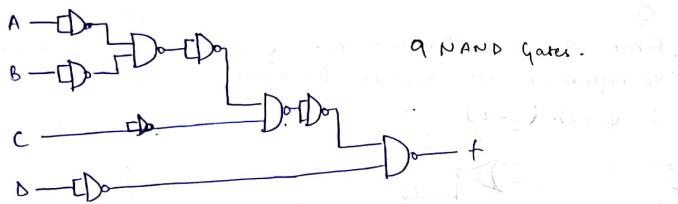
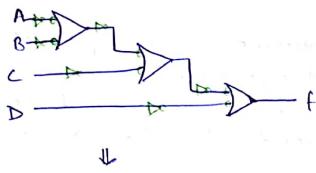
NOTE

1. The above circuit is termed as a 2-level OR-AND implementation.
2. A 2-level OR-AND implementation is equivalent to a 2-level NOR-NOR implementation.
3. A 2-level OR-AND implementation is obtained when a digital circuit is implemented from a POS expression.

$$\text{Q. 6. } f(A, B, C) = A \bar{B} C = \bar{A} \bar{B} \bar{C} + \bar{A} \bar{B} C + \bar{A} B \bar{C} + A \bar{B} \bar{C}$$

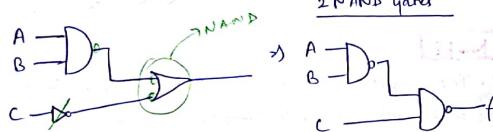


Q.55. $f = A + B + C + D$



Q. what is the minimum number of 2-input NAND gates needed to implement:

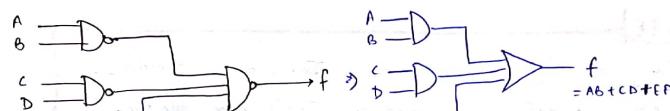
$$f = AB + \bar{C}$$



Q. AND-OR equivalent to

- NAND-NAND
- NAND-NOR
- NOR-NAND
- NOR-NOR

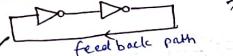
Q. The d/p of the ckt shown below is:



+ NOT Gate / Inverter:

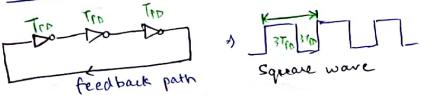
$$\begin{array}{c} x \\ \text{---} \\ f \end{array} \quad f = \bar{x}$$

Even NOT Gates:



The above circuit acts as the most basic memory element.
→ If even no. of NOT gates are connected in a feedback path, the circuit acts as a memory element/bistable multivibrator.

Odd NOT Gates:



$$T = 3T_{PD} + 3T_{RD}$$

$$= 2 \cdot 3T_{PD}$$

In general, the time period of the square wave generated,

$$T = 2 \times N T_{PD}$$

Odd no. of NOT gates connected in the feedback path.

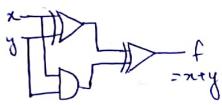
Frequency of square wave = $\frac{1}{T}$

If odd no. of NOT gates are connected in a feedback path, then the circuit acts as a square wave generator/astable multivibrator/ring oscillator.

NOTE

- $\bar{x}y$ and $x\bar{y}$ are known as Inhibition functions.
- $\bar{x}+y$ and $x+\bar{y}$ are known as Implication functions.

$$Q. 6. f = x \oplus y \oplus xy$$



x	y	xy	$x \oplus y$	$x \oplus y \oplus xy$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1

(6)

$$Q51. f_1(A, B, C) = \Sigma(2, 3, 4)$$

$$f_2(A, B, C) = \pi(0, 1, 3, 6, 7) \\ = \Sigma(2, 4, 5)$$

A	B	C	$f_1 f_2 f_3$
0	0	0	000
1	0	0	100
2	0	1	110
3	0	1	111
4	1	0	010
5	1	0	011
6	1	1	001
7	1	1	100

Total maximum no. of possible minterms = 6

$$f_1 = \bar{A}BC + \bar{B}AC + A\bar{B}C$$

$$f_2 = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$$

$$f_3 = \bar{A}BC + \bar{A}B\bar{C} + \bar{A}B\bar{C}$$

 f_3 **# KARNAUGH MAP SIMPLIFICATION: Chapter - 3**

2 variable k-map:

$$f(x, y)$$

MSB LSB

x	y	0	1
0	0	$\bar{x}\bar{y}$	$\bar{x}y$
0	1	$x\bar{y}$	$x\bar{y}$
1	0	$x\bar{y}$	$x\bar{y}$

Minterm Map

x	y	0	1
0	0	$x+y$	$x+\bar{y}$
0	1	$\bar{x}+y$	$\bar{x}+\bar{y}$
1	0	$\bar{x}+y$	$\bar{x}+\bar{y}$

Maxterm Map

3-variable k-map:

$$f(x, y, z)$$

MSB LSB

x	y	z	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$y\bar{z}$
0	0	0	1	0	1	1	0
0	0	1	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}z$	$x\bar{y}\bar{z}$	$x\bar{y}z$	$x\bar{y}\bar{z}$
0	1	0	$\bar{x}y\bar{z}$	$\bar{x}yz$	$x\bar{y}\bar{z}$	$x\bar{y}z$	$x\bar{y}\bar{z}$

Minterm map

x	y	z	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$y\bar{z}$
0	0	0	1	0	1	1	0
0	0	1	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}z$	$x\bar{y}\bar{z}$	$x\bar{y}z$	$x\bar{y}\bar{z}$
0	1	0	$\bar{x}y\bar{z}$	$\bar{x}yz$	$x\bar{y}\bar{z}$	$x\bar{y}z$	$x\bar{y}\bar{z}$

Maxterm Map

NOTE:

- In a k-map, any 2 cells are considered to be adjacent, if their binary numbering differs by only 1-bit.
- Furthermore, while performing simplification, any 2 cells that are adjacent to each other can be grouped together to obtain the simplified expression.

NOTE:

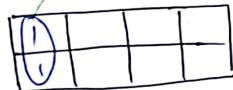
- In a k-map, the no. of cells that can be grouped is always a power of 2 value.
- E.g. 1, 2, 4, 8, 16, ... cells.

NOTE:

valid grouping



valid grouping



valid grouping



Invalid grouping



4-variable k-map : [24/10/2017] point ④.

$$\begin{aligned} \text{eg } f(w, x, y, z) &= \sum(5, 7) \\ &= \bar{w} \cdot x \cdot \bar{y} \cdot z + \bar{w} \cdot w \cdot y \cdot z \\ &= \bar{w} \cdot x \cdot \bar{y}^2 + \bar{w} \cdot w \cdot y \cdot z \\ &= \bar{w} \cdot x \cdot z \end{aligned}$$

(No. of cells) Grouping	No. of variables eliminated
2 (Pair)	1
4 (Quad)	2
8 (Octet)	3
16	4
2^n	n

Priority of grouping.

Octet > Quad > Pair

w y z	\bar{y}^2	\bar{y}^2	y^2	y^2	y^2
w y z	00	01	11	12	10
w y z	00				
w y z	00	1	1	0	0
w y z	01	0	0	1	1
w y z	12	1	1	1	0
w y z	11	0	0	0	0
w y z	10	0	0	0	0

* Remove redundant group(s).

→ It is a group whose all the cells are covered by adjacent groups.

Q. 31. $f = \cancel{\bar{w}xy} + \bar{w}yz + w\bar{y}z + wxy$ (b)

w y z	\bar{y}^2	\bar{y}^2	y^2	y^2	y^2
w y z	00	01	11	12	10
w y z	00				
w y z	00	1	1	0	0
w y z	01	0	0	1	1
w y z	11	0	0	0	0
w y z	10	0	0	0	0

This group is a redundant group.
Therefore, it is eliminated.

NOTE:

→ In a k-map, while forming the groupings, the biggest possible groupings should be formed.

→ In order to do so, if overlapping of 2 groups is required, then this overlapping is also allowed.

Ques what is the simplified expression for the boolean function?

$$f(x, y, z) = \sum(1, 2, 5, 6, 7)$$

w y z	\bar{y}^2	\bar{y}^2	y^2	y^2	y^2
w y z	00	01	11	12	10
w y z	00				
w y z	00	1	1	0	0
w y z	01	0	0	1	1
w y z	12	0	0	0	0
w y z	11	0	0	0	0
w y z	10	0	0	0	0

$$\begin{aligned} f &= \bar{y}^2 + \bar{y}\bar{z} + xz \\ &= \bar{y}^2 + \bar{y}^2 + xy \end{aligned}$$

NOTE:

→ In a k-map, sometimes, it may be possible to obtain more than 1 simplified expressions.

Semi-Simplified boolean function:

$$\text{E.g. } f(w, x, y, z) = \bar{w}\bar{y} + w\bar{y}z + \bar{w}yz$$

$$= \bar{w} + \cancel{\bar{w}y} + \cancel{wz}$$

$$\begin{aligned} & y(w+x+\bar{w}) \\ & = y(\bar{w}+\bar{x})(\bar{w}+x) \\ & = y\bar{w} + xy + \bar{w}y + \bar{w}(y+\bar{y}) + xy \\ & = \bar{w} + xy \end{aligned}$$

$$\text{T4. } f = \pi(0, 1, 2, 4, 5, 7)$$

$$= \cancel{B+C} \\ B \cdot (A+C) \cdot (\bar{A}+\bar{C}) \quad @$$

	$\bar{w}x$	$w\bar{x}$	wx	$\bar{w}\bar{x}$
\bar{w}	0	1	3	2
w	0	0	1	1
x	0	1	1	0
\bar{x}	1	0	0	1

5-variable K-map [Point-5]

→ In a 5-variable K-map, a cell in the $v=0$ map is adjacent to the corresponding cell in the $v=1$ map.

	v	w	x	y	z
v	0	0	0	0	0
w	0	1	1	1	1
x	0	1	1	1	1
y	0	0	0	0	0
z	1	1	1	1	1

	$\bar{w}x$	$\bar{y}z$	$\bar{y}z$	yz	yz
\bar{w}	0	1	1	2	2
w	0	0	0	1	1
x	0	1	1	1	1
y	0	0	1	1	1
z	1	1	1	1	1

$$\text{E.g. } f(v, w, x, y, z) = \sum(4, 5, 12, 13, 14, 15, 26, 27, 28, 29, 30, 31)$$

$$\bar{v} \quad v$$

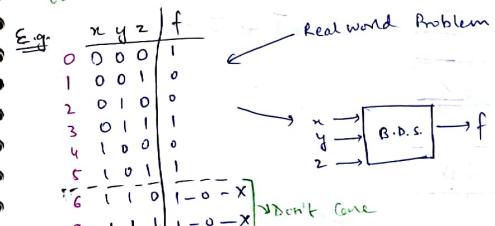
	$\bar{w}x$	$\bar{y}z$	$\bar{y}z$	yz	yz
\bar{w}	0	1	1	2	2
w	0	0	0	1	1
x	0	1	1	1	1
y	0	0	1	1	1
z	1	1	1	1	1

	$\bar{w}x$	$\bar{y}z$	$\bar{y}z$	yz	yz
\bar{w}	16	17	19	18	19
w	20	21	23	22	23
x	24	25	27	26	27
y	28	29	31	30	31
z	32	33	35	34	35

$$f = \bar{w}x + \bar{v}x\bar{y} + vwy$$

Don't Care Conditions:

$$\text{E.g. } \begin{matrix} x & y & z \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \\ 4 & 1 & 0 \\ 5 & 1 & 0 \\ 6 & 1 & 0 \\ 7 & 1 & 1 \end{matrix} / f$$



→ A function in which, depending upon the real world problem statement, the output is not specified for certain input combinations [i.e., unused input combinations] is known as an incompletely specified function.

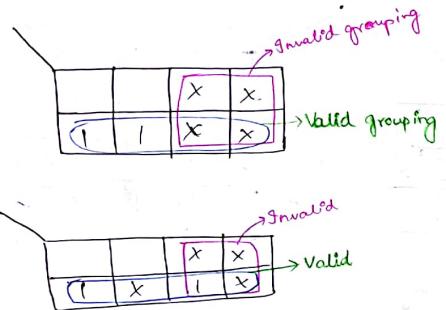
→ While simplifying such a function on the K-map, for the unused inputs we assume that the output of the function is either logic 1 or logic 0.

→ The advantage of doing so is that, it may provide additional amount of simplification on the K-map.

→ These unused input combinations are known as don't care conditions.

→ While simplifying a function on the k-map, it is not necessary to include the don't care conditions (DCCs) in a grouping.

E.g:



Q. What is the simplified expression.

$$\begin{array}{cccc} ab\bar{cd} & \bar{c}\bar{d} & \bar{c}d & cd \\ \bar{ab} & \boxed{1} & x & x \\ \bar{ab} & 1 & & \\ ab & & & \\ ab & 1 & & x \end{array} \quad f = \bar{b}\bar{d} + \bar{a}\bar{d}$$

T18

x	y_2	y_1	f
0	0	0	X
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	X

$$f = \sum(2, 3, 4, 5)$$

$$\begin{array}{ccccc} y_2 & \bar{y}_2 & \bar{y}_1 & y_2 & \bar{y}_2 \\ \bar{x} & 0 & x & 1 & 3 \\ n & 4 & 5 & x & 6 \end{array}$$

$$f = x\bar{y} + \bar{x}y$$

~~Implicants~~, Prime Implicants, Essential Prime Implicants

Non-essential prime implicants Point - 7 :

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & & \\ 1 & 0 & & \\ 1 & 1 & & \end{array}$$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & & \\ & 1 & & \\ & 1 & & \end{array}$$

$$EPI = 2$$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & & \\ 1 & 1 & & \\ 1 & 1 & & \end{array}$$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & & \\ & 1 & & \\ & 1 & & \end{array}$$

$$NPI = 2$$

NOTE

In an n -variable boolean function, the maximum possible no. of prime implicants is equal to 2^{n-1} .

Q.42. $f(a, b, c) = \bar{a}c + a\bar{c} + \bar{a}\bar{b}c$

$$\begin{array}{cccc} \bar{a} & \bar{b} & \bar{b}c & bc \\ a & b & \boxed{1} & 1 \\ a & b & \boxed{1} & 1 \\ a & b & 1 & 1 \end{array}$$

$$PI = 4 \quad EPI = 2 = \bar{a}c \text{ and } a\bar{c}$$

$$\begin{array}{cccc} \bar{b}c & \bar{b}c & bc & bc \\ a & \boxed{1} & 1 & 1 \\ a & 1 & 1 & 1 \\ a & 1 & 1 & 1 \end{array}$$

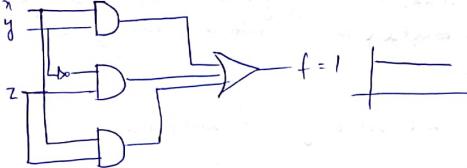
@

Q. 59. $f(w, u, y, z) = \sum(0, 2, 6, 7, 8, 9, 13, 15)$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
$w\bar{u}$	00	01	11	10
wu	00	01	11	10
$w\bar{u}$	11	01	11	10
wu	10	01	01	00

PI = 8

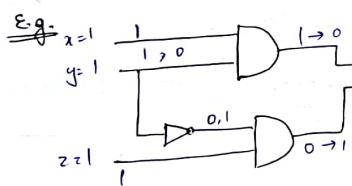
EPPI = 0
NPI = 8



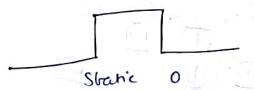
→ In a digital circuit, hazards are eliminated by including redundant groups on the K-map.

Hazards: (Point - 9)

→ It is a problem that occurs in a digital circuit because of the finite (non-zero) propagation delay of logic gates in a digital circuit.



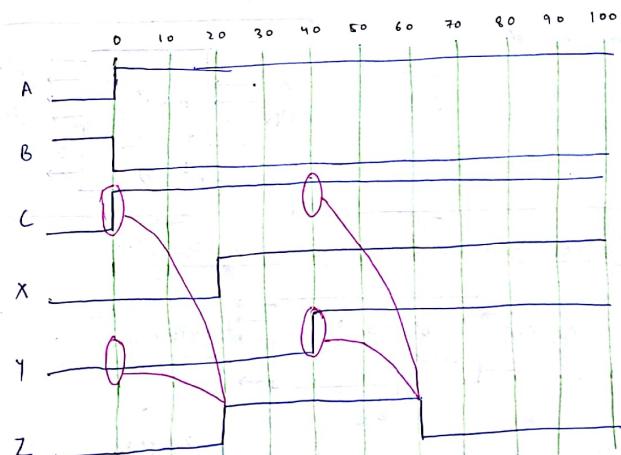
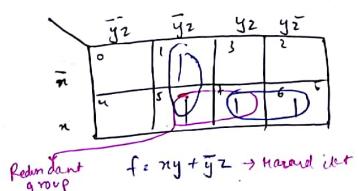
Static 1



Elimination of Hazards

$f = xy + \bar{y}z + xz$

(Hazard free circuit)



→ While solving a problem, the propagation delay of the logic gates is not considered in the initial value of the waveforms.
 ↗ while drawing the initial value of the waveforms.

CHAPTER-4

Combinational Circuits

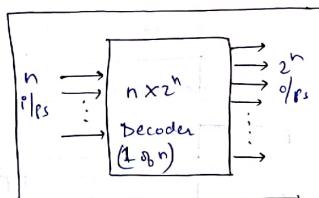
General design procedure of a combinational circuit Point-10.

Decoder :

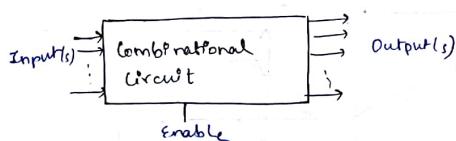
→ A decoder is a combinational circuit that converts binary coded information into familiar representations like octal, decimal, hexadecimal, etc.

→ A decoder has n inputs and $\leq 2^n$ outputs.

↗ Minterm generator.

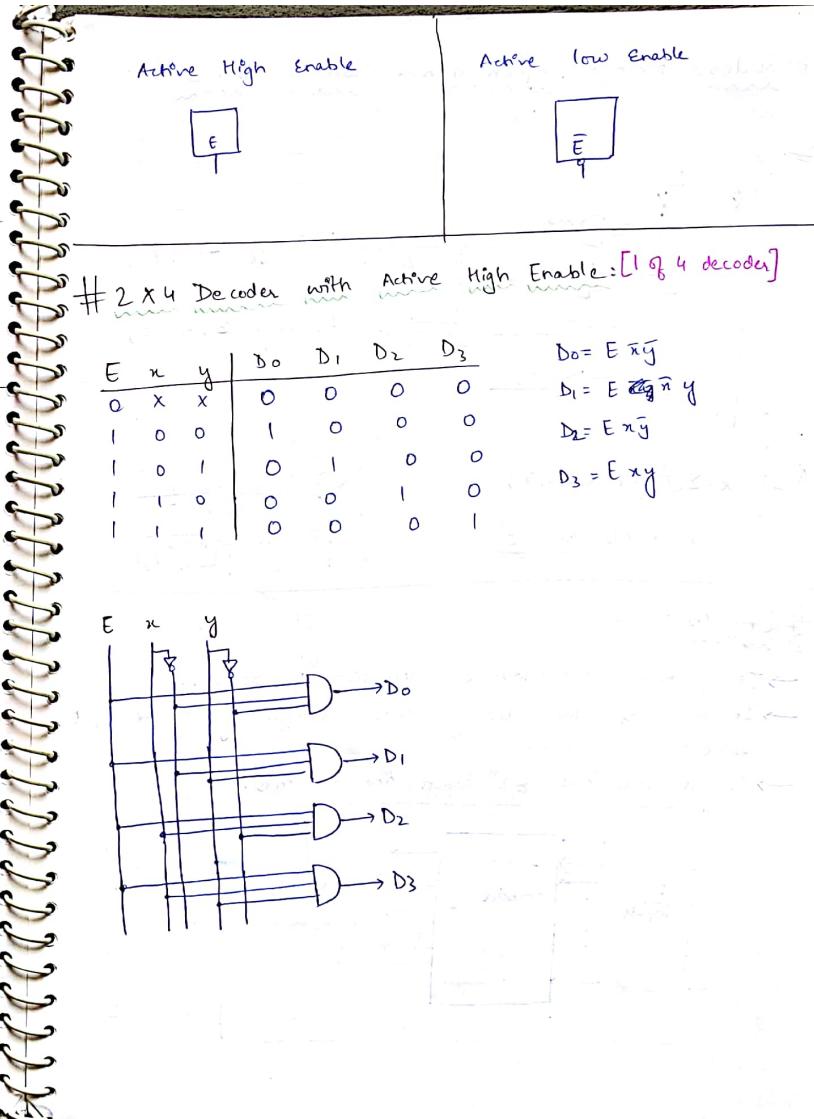


Note → Enable Input in a combinational circuit.

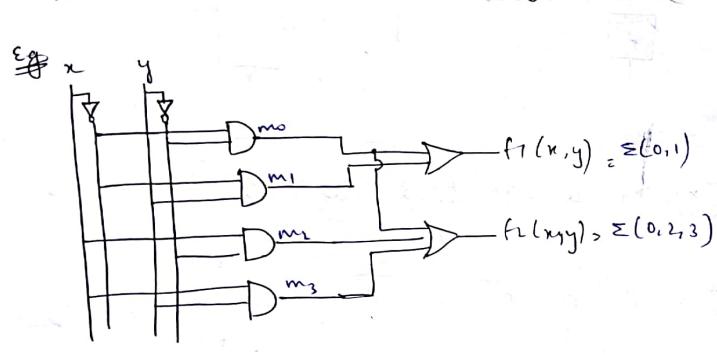


→ Enable input is an activating signal
 → It is an optional input.

2 types
 ↗ Active High Enable (E)
 ↗ Active Low Enable (\bar{E})



Boolean function implementation using a decoder:

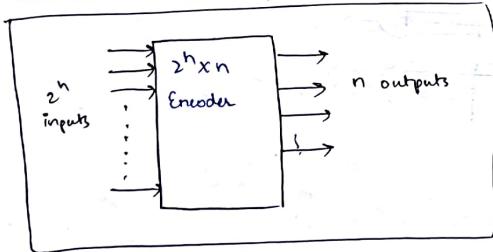


$$\text{Q13. } x = \Sigma(0, 1, 3, 5, 6, 7) \\ = AC + AB + BC \quad (\textcircled{a})$$

\bar{A}	\bar{B}	\bar{C}	\bar{D}	\bar{E}	\bar{F}	\bar{G}	\bar{H}
0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	0	0	1	0	0
0	0	0	1	1	1	1	1

Encoder:

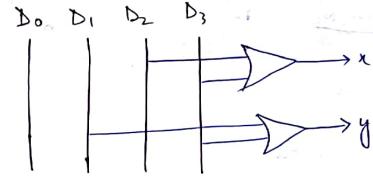
- It performs the inverse operation of decoders.
- It generates the binary code corresponding to the input value.
- An encoder has $\leq 2^n$ inputs and n outputs.



4x2 Encoder: (Basic Encoder)

D_0	D_1	D_2	D_3	x	y
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

$x = D_2 + D_3$
 $y = D_1 + D_3$



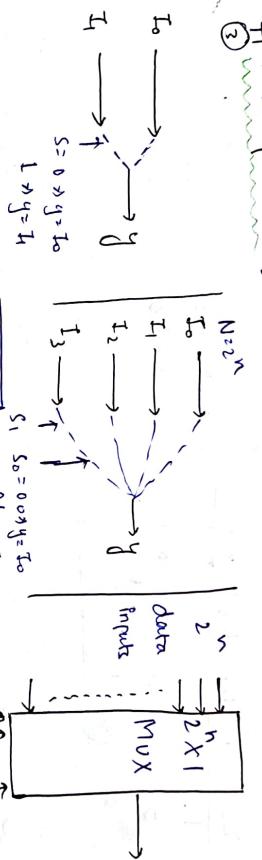
4x2 Encoder: (Priority Encoder)

D_0	D_1	D_2	D_3	x	y
1	0	0	0	0	0
(X)	1	0	0	0	1
(X)	X	1	0	1	0
X	X	X	1	1	1

$$x = D_2 + D_3 \\ y = D_1 \bar{D}_2 + D_3$$

The 'X' may either be logic 0 or logic 1.

Multiplexer :



→ MUX is also known as

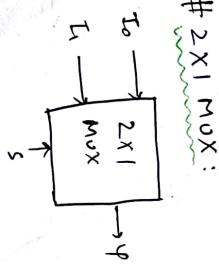
- Data selector
- Parallel to serial converter
- Many to one circuit
- Universal logic circuit

NOTE

In general, in a truth table,

$$\begin{array}{c|cc} S & 0 & 1 \\ \hline I_0 & 0 & 1 \\ I_1 & 0 & 1 \\ \hline f & 0 & 1 \end{array}$$

$f = \bar{S} \cdot \bar{I}_0 + S \cdot I_1$
 $= \bar{S} \cdot \bar{I}_0 + \bar{S} \cdot \bar{I}_1 + S \cdot I_1$



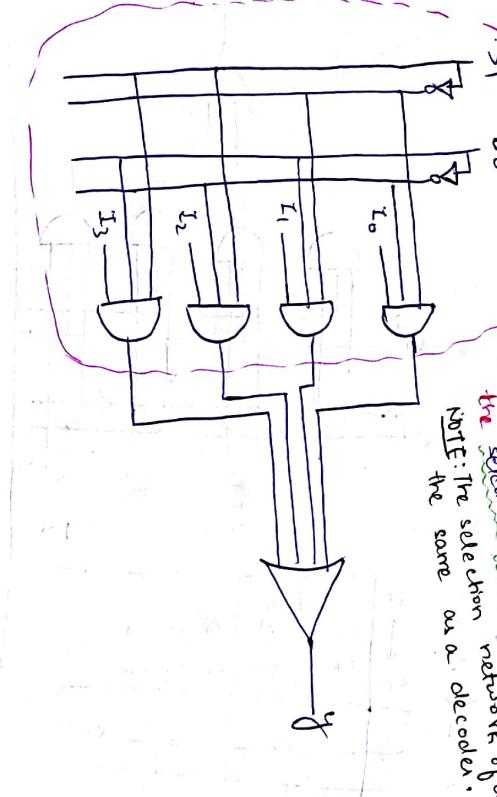
$$y = S \cdot I_0 + \bar{S} \cdot I_1$$

Boolean Expression

S	I_0	I_1	y
0	0	0	0
0	0	1	0
1	0	0	1
1	0	1	1

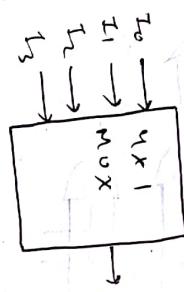
Block Diagram

Truth Table

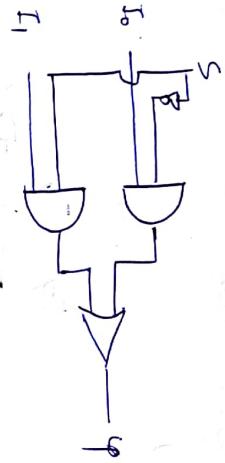


$$Y = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3$$

This part of the MUX can be termed as the selection network of a MUX. The selection network of a MUX is exactly the same as a decoder.

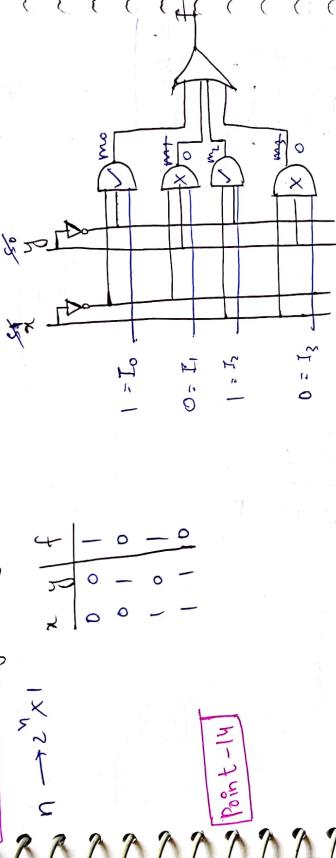


S_1	S_0	Y
1	0	I_0
1	1	I_1
0	1	I_2
0	0	I_3



Boolean function implementation using MUX:

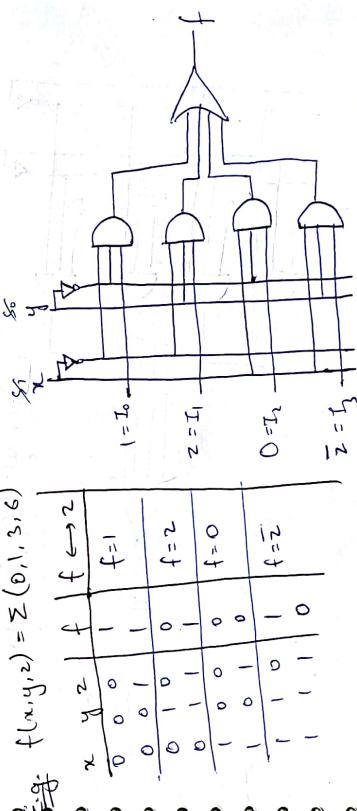
Approach 1 e.g. $f(x,y) = \sum(0,2)$



Approach 2:

An n-variable boolean function can also be implemented using a MUX having only $(n-1)$ select input, i.e., $2^{n-1} \times 1$ MUX.

e.g. $f(x,y,z) = \sum(0,1,3,6)$



Limitation:

The limitation of the above approach is that, while finding the relation between the function 'f' and any 1 variable of the function, if the complemented term ($F_g -$) appears in the relation, then in addition to the MUX circuit, 1 NOT gate is also needed to implement the function.

$$\begin{array}{ccccccc} & & & f & & & \\ a & b & c & & & & \\ 0 & 0 & 0 & & & & \\ 0 & 0 & - & 0 & & & \\ 0 & 0 & - & 0 & - & 0 & \\ 0 & 0 & - & 0 & - & 0 & - \\ 0 & 0 & - & 0 & - & 0 & - \\ 0 & 0 & - & 0 & - & 0 & - \end{array}$$

(a)

	$\bar{b}c$	$\bar{b}c$	bc	$b\bar{c}$
\bar{a}	1	-	1	-
a	-1	1	1	-
	a	-a	-a	a

$$f = \bar{a}\bar{b}\bar{c} + a\bar{b}\bar{c} + ab\bar{c} \\ + \bar{a}bc + abc$$

$$\begin{array}{ccccc} f & P & Q & R \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{aligned} f &= S_1 S_0 I_0 + S_1 S_0 I_1 + S_1 S_0 I_2 + S_1 S_0 I_3 \\ &= \bar{P} \bar{Q} R + \bar{P} Q \bar{R} + P \bar{Q} \bar{R} + P Q R \\ &= \bar{P} (\bar{Q} R + Q \bar{R}) + P (\bar{Q} \bar{R} + Q R) \\ &= \bar{P} (Q \oplus R) + P (Q \odot R) \\ &= \bar{P} (Q \oplus R) + P (Q \oplus R) \\ &= P \oplus Q \oplus R \end{aligned}$$

$$\text{eg: } f(x_1, y_1, z) = \sum (0, 1, 3, 6)$$

$$\begin{array}{ccccc} f & I_0 & I_1 & I_2 & I_3 \\ 1 & 0 & 2 & 4 & 6 \\ 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{ccccc} f & I_0 & I_1 & I_2 & I_3 \\ 1 & 0 & 1 & 4 & 5 \\ 0 & 2 & 3 & 6 & 7 \\ 0 & 2 & 3 & 6 & 7 \\ 0 & 1 & 4 & 5 & 0 \end{array}$$

$$\begin{array}{ccccc} f & I_0 & I_1 & I_2 & I_3 \\ 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 6 \\ 0 & 1 & 2 & 5 & 6 \\ 0 & 1 & 2 & 5 & 6 \end{array}$$

$$\begin{array}{ccccc} f & I_0 & I_1 & I_2 & I_3 \\ 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 6 \\ 0 & 1 & 2 & 5 & 6 \\ 0 & 1 & 2 & 5 & 6 \end{array}$$

ratio

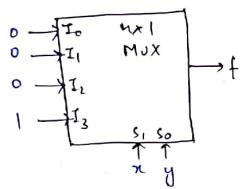
23

Implementation of standard boolean functions (logic gates) using MUX:

Approach 1 Implementation of logic gates using a MUX having n -select lines.

AND Gate:

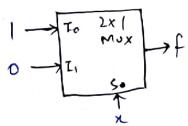
x	y	f
0	0	0
0	1	0
1	0	0
1	1	1



* Similarly, the OR, NAND, NOR, EXOR, ENOR logic gates can be implemented.

NOT Gate:

x	f
0	1
1	0

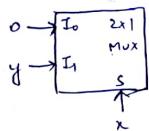


Approach 2 Implementation of logic gates using a MUX having $(n-1)$ select lines.

* This approach is not applicable for NOT gate.

AND Gate:

x	y	f	I ₀	I ₁
0	0	0	0	0
0	1	0	0	1
1	0	0	1	0
1	1	1	0	1



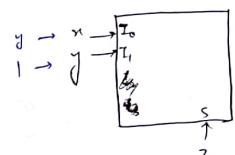
* Point - 15

NOTE

The uncomplemented variable (e.g. x) is also sometimes known as True form/ Basic form.
→ The complemented variable (e.g. \bar{x}) is also known as false form.

Q11. $f = T + \bar{T}R$

- R to x
- 1 to y
- T to 2

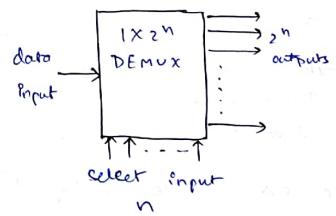
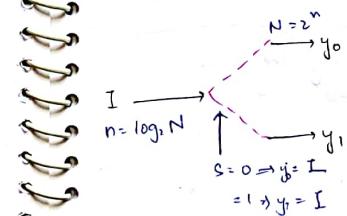


Q12. Tristate Buffer:



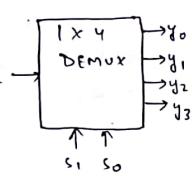
→ Generally, in a digital circuit, there are 2 states (0 & 1). However, sometimes, with the help of control input, a third state can be introduced in the system. This is known as high impedance state (high Z state).

#4 De Multiplexer (DE MUX)

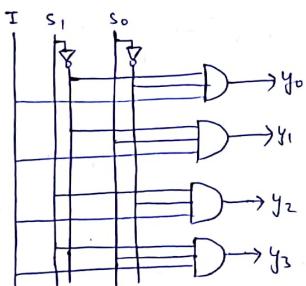


- A DEMUX is also known as
- Data distributor
 - Serial to Parallel converter
 - One to many circuit.

1x4 DEMUX:



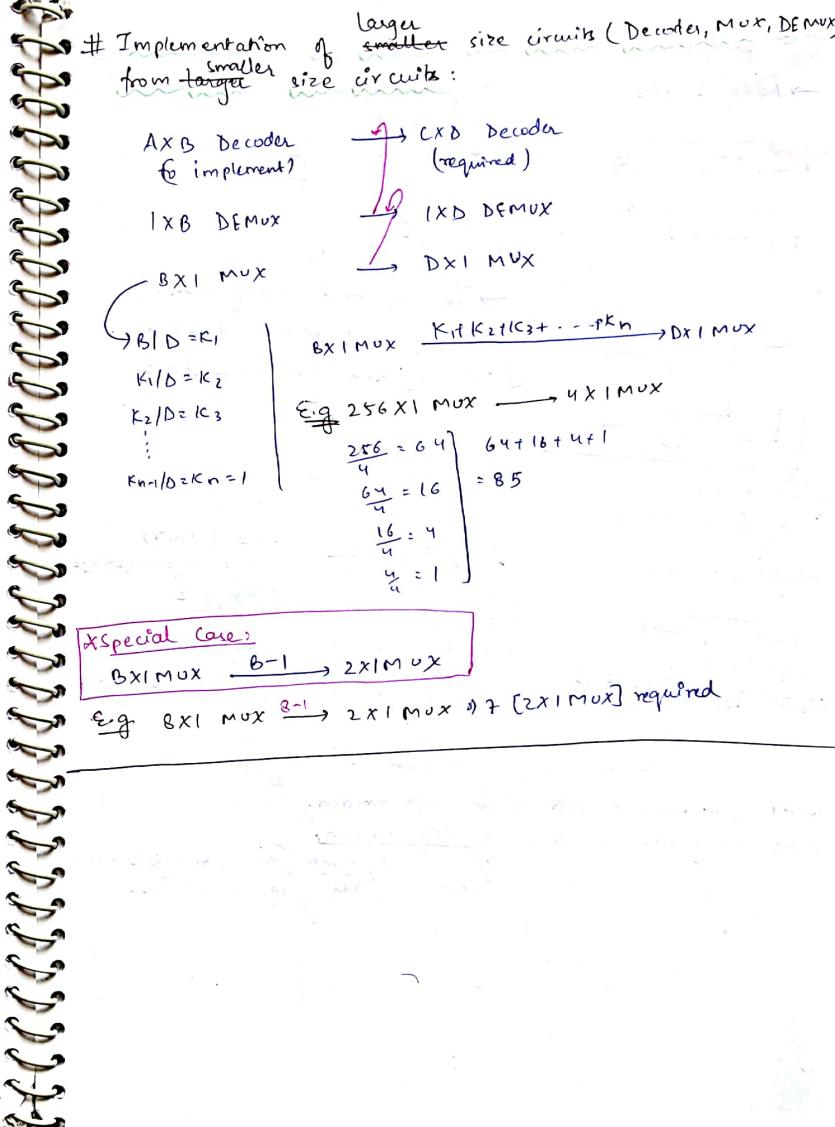
$$\begin{aligned} y_0 &= \bar{S}_1 \bar{S}_0 I \\ y_1 &= \bar{S}_1 S_0 I \\ y_2 &= S_1 \bar{S}_0 I \\ y_3 &= S_1 S_0 I \end{aligned}$$



S_1, S_0	y_0	y_1	y_2	y_3
0 0	I	0	0	0
0 1	0	I	0	0
1 0	0	0	I	0
1 1	0	0	0	I

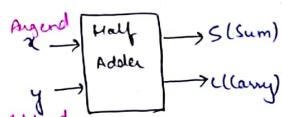
NOTE
A decoder with enable input is exactly the same as a DEMUX. Therefore, the circuit is known as decoder-demultiplexer circuit.

* A $n \times 2^n$ decoder is equivalent to a 1×2^n DEMUX.



#⑤ Half-Adder:

→ Adds 2-bits.



<u>x</u>	<u>y</u>	<u>S</u>	<u>C</u>
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

→ To implement a half adder:

- (16) • No. of NAND gates needed = 5
- (17) • No. of NOR gates needed = 5

#⑥ Full-Adder:

→ Adds 3 bits



x	y	z	S	C
0	0	0	0	0
1	0	1	1	0
1	1	0	1	0
2	0	1	0	1
1	1	0	1	0
2	1	0	0	1
2	1	1	0	1
3	1	1	1	1

$S = \Sigma(1, 2, 4, 7)$
 $= \bar{x}yz + \bar{x}\bar{y}z + x\bar{y}\bar{z} + xy$
 $= x \oplus y \oplus z$

$C = \Sigma(1, 5, 6, 7)$
 $= \bar{x}yz + x\bar{y}z + x\bar{y}\bar{z} + xyz$

The sum function acts as an odd function.

- * The carry acts out as a majority function.

majority function:
In a majority function, the output is logic 1, when majority number of inputs are logic 1.

* K-map simplification of sum and carry:

Sum

A diagram consisting of two parts. On the left, there is a vertical stack of four ovals, each containing a smaller oval inside it. On the right, there is a grid of four columns and three rows. The first column contains three circles, the second column contains two circles, the third column contains one circle, and the fourth column contains one circle. A line connects the bottom oval from the left stack to the first column of the grid.

* Simplification is not possible for the sum output.

Carry

\bar{y}_1	\bar{y}_2	y_2	y_1
\bar{x}			(1)
x		(1) (0) (1)	

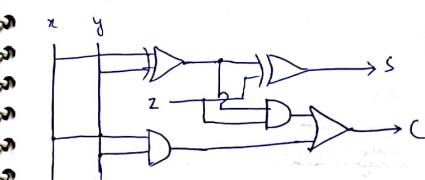
$$c = x^2 + y^2 + xy$$

$$= xy + y^2 + x^2$$

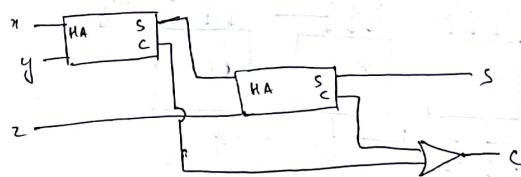
Implementation of Full adder from two half adders:

$$\begin{aligned}
 S(x, y, z) &= \Sigma(1, 2, 4, 7) \\
 &= \bar{x}yz + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z} \\
 &= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz) \\
 &= \bar{x}(y \oplus z) + x(y \ominus z) \\
 &= \bar{x}(y \oplus z) + x(\overline{y \ominus z}) \\
 &\quad \bar{A} \cdot B \qquad \quad A \cdot \bar{B} \\
 &= x \oplus y \ominus z
 \end{aligned}$$

$$\begin{aligned} c(x,y,z) &= \Sigma(3,5,6,7) \\ &= \bar{x}\bar{y}z + x\bar{y}z + xy\bar{z} + xyz \\ &= z(\bar{x}y + x\bar{y}) + xy(\bar{z} + z) \\ &= z(x \oplus y) + xy \end{aligned}$$



* One full adder is equivalent to 2 half adders + 1 OR gate



NOTE

→ To implement a full adder:
 No. of NAND gates needed = 9
 No. of NOR gates needed = 9

#(7) Parallel Adder (Ripple carry adder):

→ Adds 2 n-bit binary numbers.

$$\begin{array}{r} x_3 \ x_2 \ x_1 \ x_0 \\ + y_3 \ y_2 \ y_1 \ y_0 \\ \hline c_4 \ s_3 \ s_2 \ s_1 \ s_0 \end{array}$$

→ Requirements for n-bit addition:

→ 3 FA and 1 HA

→ 4 FA

→ 7 HA and 3 OR gates

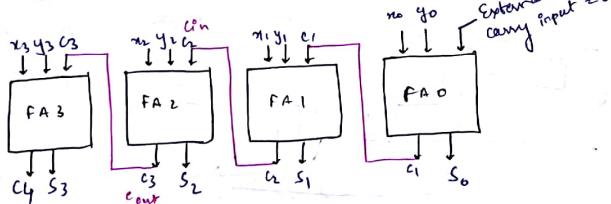
∴ In general, for n-bit addition

→ $(n-1)$ FA and 1 HA

→ n FA

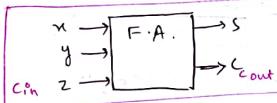
→ $(2n-1)$ HA and $(n-1)$ OR gates

→ n-bit parallel adder (Ripple carry adder):



→ In the above circuit, the carry bit propagates from the LSB to the MSB position in a ripple like manner.

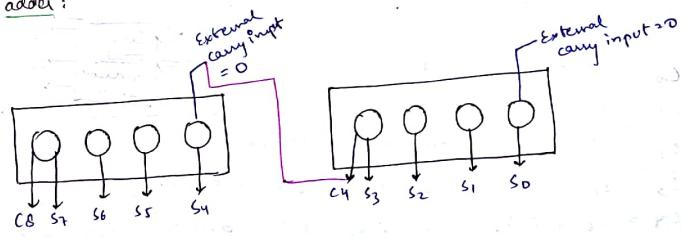
→ It is because of this reason, this circuit is known as ripple carry adder.



ci_in → Carry coming from previous state
 ci_out → Carry going to the next stage.

→ Implementation of larger size parallel adder from smaller size parallel adder.

→ Implementation of 8-bit parallel adder from 2 4-bit parallel adder:



Q32. 1 FA = $75 \text{ ns} \times 5$

(= 50ns)

4-bit parallel adder

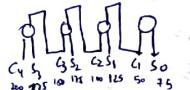
$\frac{1 \times 10^6}{50} = 200$



75 + 50

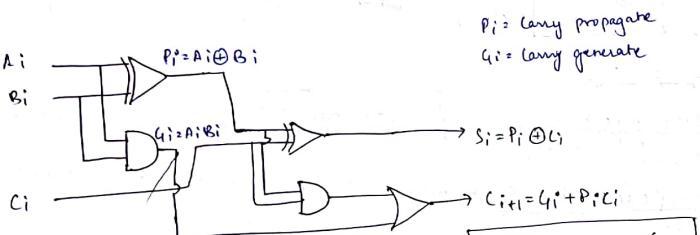
C4 S3 S2 S1 S0
 200 225 175 125 75

$$\frac{1000 \times 10^6}{225} = 4.45 \times 10^6$$



Parallel Adder: Carry Lookahead Adder:

* Disadvantage of Ripple carry adder: Point - 18



$$C_0 = \text{external carry input}$$

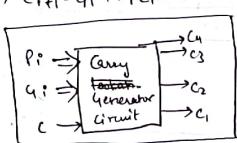
$$C_1 = G_0 + P_0 C_0$$

$$C_2 = G_1 + P_1 C_1 = G_1 + P_1 G_0 + P_1 P_0 C_0$$

$$C_3 = G_2 + P_2 C_2 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$$

$$C_4 = G_3 + P_3 C_3 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0$$

$$\begin{aligned} P_i &= \text{Carry propagate} \\ G_i &= \text{Carry generate} \end{aligned}$$

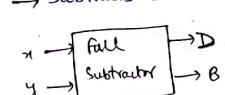


NOTE

To implement a half subtractor: no. of NAND gates needed = 5.
→ No. of NOR gates needed = 5

Full Subtractor:

→ Subtracts 3 bits



x	y	z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$D = \sum(1, 2, 4, 7)$$

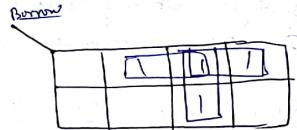
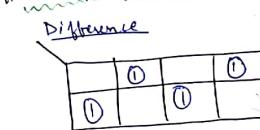
$$= \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z}$$

$$B = \sum(1, 2, 3, 7)$$

$$= \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xy\bar{z} = \bar{x}(y \oplus z) + yz$$

* Difference acts as an odd function.

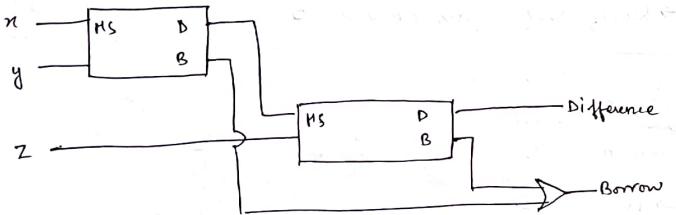
K-map simplification for difference and borrow:



Simplification is not possible for difference output

$$B = \bar{x}y + yz + \bar{x}z$$

Implementation of full subtractor from 2 half subtractors



NOTE

- To implement a full subtractor:
- No. of NAND gates needed = 9
- No. of NOR gates needed = 9

#⑨ Magnitude Comparator:

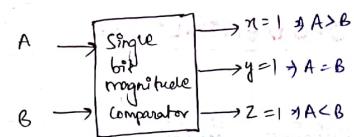
→ It determines the relative magnitude of 2 binary numbers.

→ It is of 2 types:

- ① Single bit magnitude comparator
- ② Multiple bit magnitude comparator

* Single Bit Magnitude Comparator:

→ The 2 inputs are of 1 bit each.



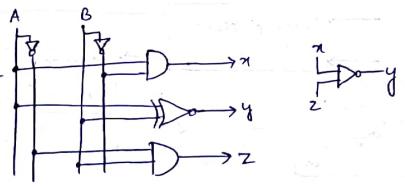
A	B	X	Y	Z
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

$$x = A \bar{B}$$

$$y = A \rightarrow DB = AB + \bar{A}\bar{B}$$

$$z = \bar{A}B$$

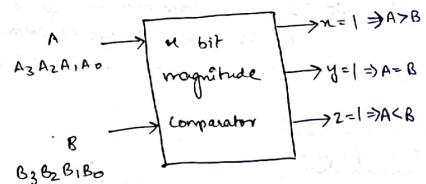
$$y = \overline{A \oplus B} \\ = \bar{A}\bar{B} + \bar{A}B$$



26/11/2017

Multiple Bit Magnitude Comparator:

4-bit magnitude comparator:



$$x = A_3\bar{B}_3 + y_3 \cdot A_2\bar{B}_2 + y_3y_2 \cdot A_1\bar{B}_1 + y_3y_2y_1 \cdot A_0\bar{B}_0$$

$$z = \bar{A}_3B_3 + y_3 \cdot \bar{A}_2B_2 + y_3y_2 \cdot \bar{A}_1B_1 + y_3y_2y_1 \cdot \bar{A}_0B_0$$

$$y_3 = A_3 \oplus B_3$$

$$y_2 = A_2 \oplus B_2$$

$$y_1 = A_1 \oplus B_1$$

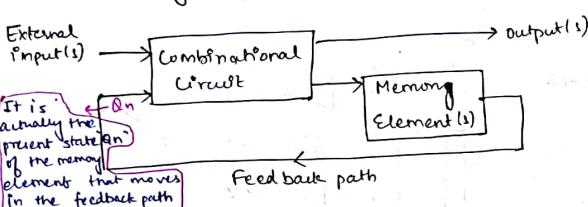
$$y_0 = A_0 \oplus B_0$$

$$y_3y_2y_1y_0$$

Chapter-5

SEQUENTIAL LOGIC

Block Diagram of a sequential circuit:



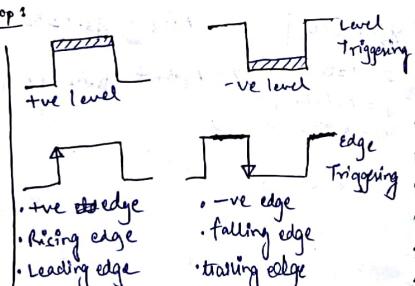
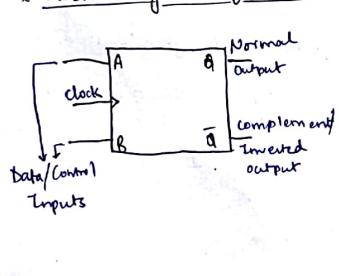
Types of sequential circuits:

(1) Synchronous Sequential Circuit: In a synchronous sequential circuit, all the memory elements are activated (triggered) simultaneously by a common activating signal known as a clock pulse.

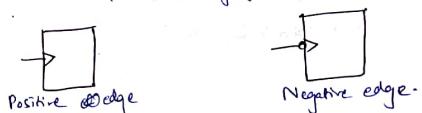
(2) Asynchronous sequential circuit: In an asynchronous sequential circuit, the different memory elements are triggered at random instants of time independent of each other.

Memory Elements: Flip-Flops:

* Block diagram of a flip-flop:



- Level triggering implies that the digital circuit is activated for a relatively significant amount of time.
- Edge triggering implies that the digital circuit is activated for a negligible amount of time.



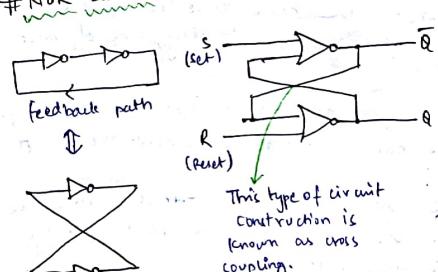
[NOTE] → It is only under normal/valid operation that the Q and \bar{Q} outputs of a memory element are complements of each other, whereas $Q = \bar{Q} = 0$ and $Q = \bar{Q} = 1$ ⇒ Invalid state

S-R Latch / SC Latch (Set, clear):

→ It consists of two cross-coupled NOR gates or NAND gates.

[NOTE] → The term latch, by default, refers to NAND latch.

NOR Latch: [Q input of S; \bar{Q} input of R]



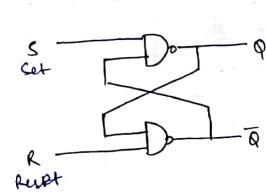
S	R	Q _{in}	\bar{Q}_{in}
0	0	0	0
0	1	0	1
1	0	1	0
1	1	0	0

Function Table (\Rightarrow)

Truth Table

Characteristic Table

#NAND Latch: [Q in front of S ; \bar{Q} in front of R]

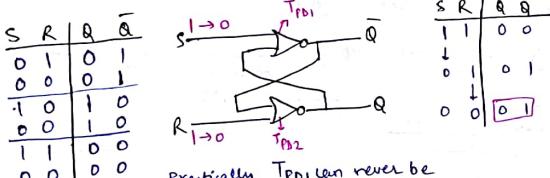


S	R	Q_{n+1}
0 0	0 0	$Q = \bar{Q} = 1$ invalid state
0 1	1	Set state
1 0	0	Reset/Clear state
1 1	1 1	Qn hold state

→ A NAND latch is also known as SR Latch.
Because for set state $\rightarrow \bar{S}$
reset state $\rightarrow \bar{R}$

* Special Case: Hold state applied immediately after invalid state in a SR latch (Applicable for both NOR & NAND latches)

→ Before, we understand this special case by considering the example of NOR latch.



Practically, TPD_1 can never be exactly equal to TPD_2 .

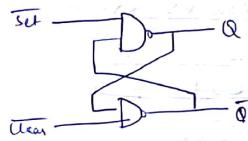
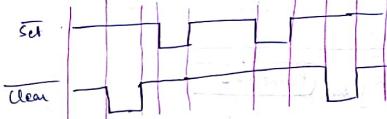
→ In the above circuit, when the hold state is applied immediately after the invalid state, ideally, the op must be equal to '00'.

→ In the very first place, we are not able to get this desired op.

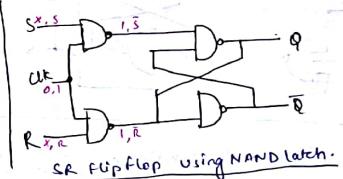
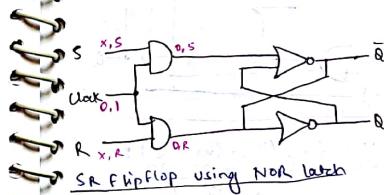
→ Furthermore, we are also not able to determine the exact value of the output.

→ The only thing, that we can conclude is that either the output is '01' or it is '10'. Hence, it is said that the latch enters into an Indeterminate state.

S	R	Q	\bar{Q}
1 1	0	0	1
1 0	0	1	0
0 1	1	1	0
0 0	0	0	1



Clocked Latches: Flip Flop:



Block diagram of SR Flip Flop

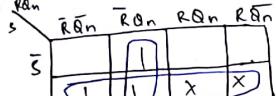
clock	S	R	Q_{n+1}
0 X	X X	X	Q_n
↑	0 0	0	Q_n
↑	0 1	0	0
↑	1 0	1	1
↑	1 1	1	Invalid

Function Table

S	R	Q_n	Q_{n+1}
0 0	0 0	0	0
0 0	1	1	1
0 1	0	0	0
0 1	1	0	0
1 0	0	1	1
1 0	1	1	1
1 1	0	X	X
1 1	1	X	X

Characteristic Table

Since the input "SR=11" drives the flip flop into invalid state, hence, practically "SR=11" is never applied to a SR flip flop. Hence, it is treated as don't use conditions.



$$Q_{n+1} = S + \bar{R} \cdot Q_n$$

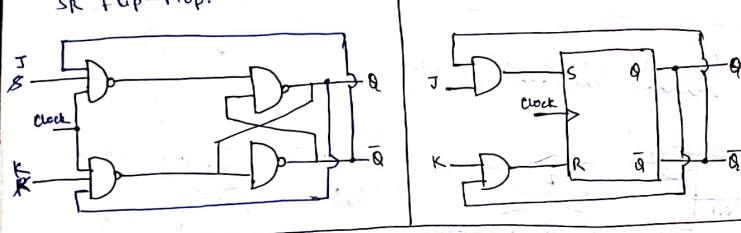
Characteristic Equation

Q_n	Q_{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

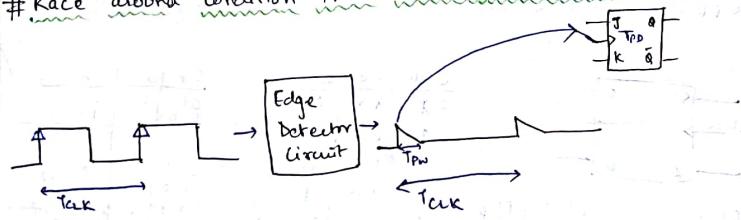
Excitation Table

JK flip flop (Jack Kilby): Point-22

→ The JK Flip Flop resolves the problem of invalid state of SR flip flop.



Race around condition in a JK flip flop (Disadvantage):



Let,

T_{PD} = flip flop propagation delay

T_{PW} = clock pulse width

T_{CLK} = clock time period

→ During the race around condition, the output of the JK flip flop toggles more than once, whereas, ideally, it must toggle only once.

→ It occurs when:

- $J=K=1$ and
- $T_{PW} > T_{PD}$

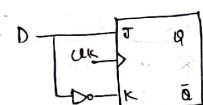
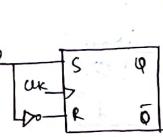
→ This problem is eliminated by ensuring the following conditions:

$$T_{PW} < T_{PD} < T_{CLK}$$

Master Slave → Point-23

D-Flip Flop (Data Delay):

→ In a D flip flop, the output follows the input (only on the active edges of the clock pulse).



Clock	D	Q_{n+1}	D	Q_{n+1}
X	X	X	0	0
↑	0	0	0	1
↑	1	1	1	0

Clock	D	Q_{n+1}
X	X	X
↑	0	0
↑	1	1

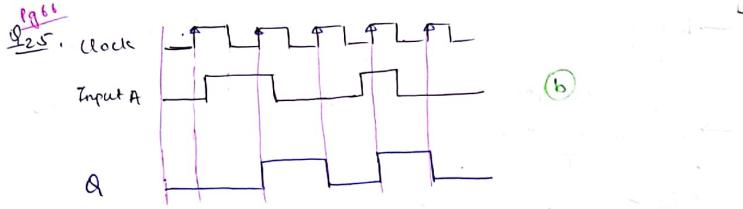
Function Table

D	Q_n	Q_{n+1}
0	0	0
0	1	0
1	0	1
1	1	1

Characteristic Table

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

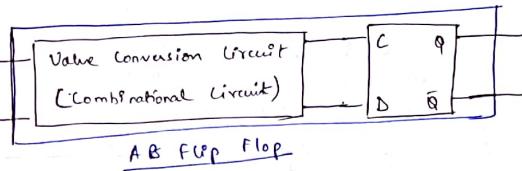
Excitation Table



NOTE

- Generally, in a problem related to memory elements, the initial state of the memory elements is given in the problem.
- However, if it is not given, then we assume the initial state to be the reset state.

Conversion Between flip flops: Point 24



A	B	Q _n	Q _{n+1}	C	D
*	X	Q _n			
↑ 0	0	Q _n			
↑ 1	1	Q̄ _n			

⇒ Excitation Table of CD

Characteristic table of AB flip flop

A	B	Q _n	Q _{n+1}	J	K
0	0	0	1	1	X
0	0	1	0	X	1
0	1	0	1	1	X
1	0	1	0	X	0
1	1	0	1	1	0

A	B	Q _n	Q _{n+1}
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

Q26.7:

$$J = \bar{A}$$

$$K = \bar{B} + AB$$

$$= A \oplus B$$

A	B	Q _n	Q _{n+1}	J	K
0	0	0	1	1	X
0	0	1	0	X	1
0	1	0	1	1	X
0	1	1	1	X	0
1	0	0	0	0	X
1	0	1	1	X	0
1	1	0	0	0	X
1	1	1	0	0	1

A	B	Q _n	Q _{n+1}	J	K
0	0	0	1	1	X
0	0	1	0	X	1
0	1	0	1	1	X
0	1	1	1	X	0
1	0	0	0	0	X
1	0	1	1	X	0
1	1	0	0	0	X
1	1	1	0	0	1

a) $\begin{array}{|c|c|c|c|c|} \hline & \bar{B} & \bar{B} & \bar{B} & \bar{B} \\ \hline A & | & X & X & 1 \\ \hline \bar{A} & X & | & 1 & X \\ \hline A & | & X & | & X \\ \hline \bar{A} & X & | & 1 & X \\ \hline \end{array}$ $J = \bar{A}$

b) $\begin{array}{|c|c|c|c|c|} \hline & \bar{B} & \bar{B} & \bar{B} & \bar{B} \\ \hline A & | & X & 1 & X \\ \hline \bar{A} & X & | & 1 & X \\ \hline A & | & X & | & X \\ \hline \bar{A} & X & | & 1 & X \\ \hline \end{array}$ $J = \bar{AB} + AB = A \oplus B$

Chapter - 6 Registers And Counters

- A register is a group of flip flops.
 - A register consists of a series connection of D-flip flops or other flip flops converted into D-flip flops.
 - A register is always designed as a synchronous sequential circuit.

Types of registers: (Depending upon the method of giving the input and taking output in a register)

- ④ SISO (serial in serial out)
 - ⑤ SIPO (serial in parallel out)
 - ⑥ PISO (parallel in serial out)
 - ⑦ PIPO (parallel in parallel out)

NOTE

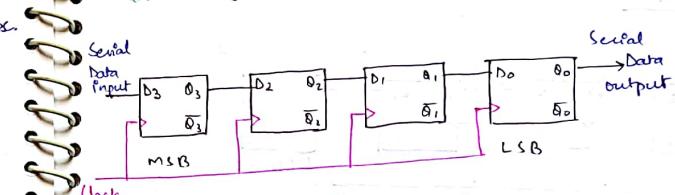
- Serial data is also known as Temporal code.
- Parallel data is also known as Spatial code.

Block pulse requirements in an n-bit register:

- for serial data
 - to input n bits n clock pulses.
 - to output n bits $n-1$ clock pulses.
 - for parallel data
 - to input n bits 1 clock pulse.
 - to output n bits 0 clock pulse.

S150 Register:

* 4 bit right shift register:



NOTE

E] A register in which the output of one flip flop is connected to the data input of an adjacent flip flop is known as a shift register.

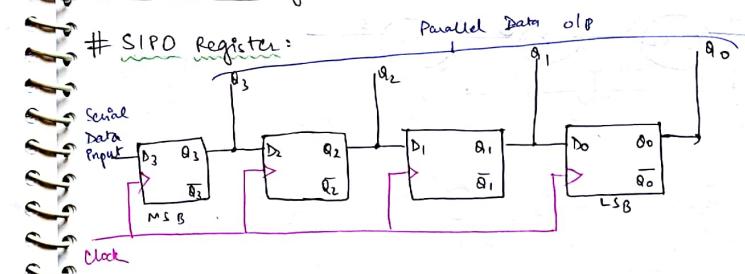
→ It is of 2 types:

- ① Right shift register: In this, the data shifts from MSB to ~~LSB~~ LSB.

② Left shift register: In this, the data shifts from LSB to MSB.

S180 800:7

SIPA requests: Parallel Data obj



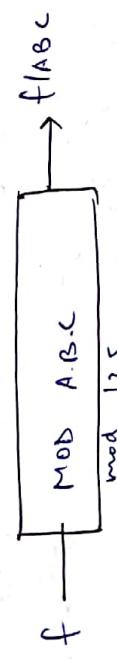
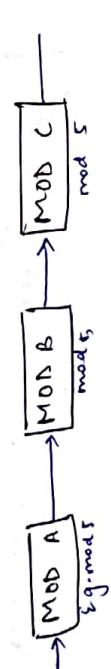
NOTE → Converts temporal code to spatial code.

Counter Applications:

- To count the no. of clock pulses.
- As a frequency divider.



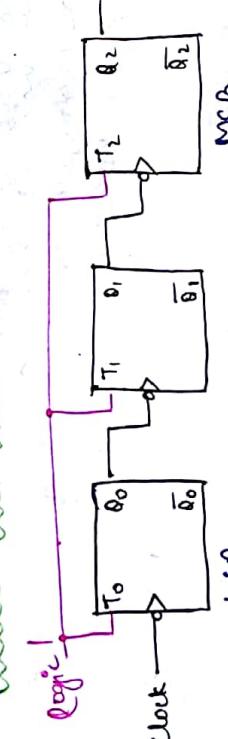
Cascading of counters:



Asynchronous Counter / Ripple Counter:

[Point-27]

3-bit UP counter: * Fix down counter take the edge trigger.



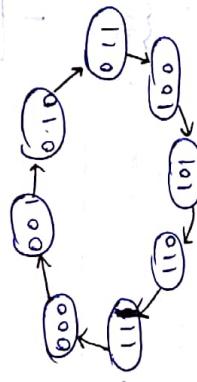
State Transition Conditions:

- Q₀ toggles on every clock pulse.
- Q₁ toggles when Q₀ goes from 1 to 0.
- Q₂ toggles when Q₁ goes from 1 to 0.

State Transition Table / State Table:

Clock	Q ₂	Q ₁	Q ₀
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8	0	0	0
9	0	0	1
10	0	1	0
11	1	0	0

State Transition Diagram / State Diagram:



Timing Diagram / Timing Diagram:

- In general, the total number of states in the circuit is equal to the state diagram of a sequential circuit.
- The total number of valid states in the sequential circuit is the same as the total number of states in the circuit.

[Point-28]

- * Input frequency is clock.
- * Output frequency is mod flip-flop.

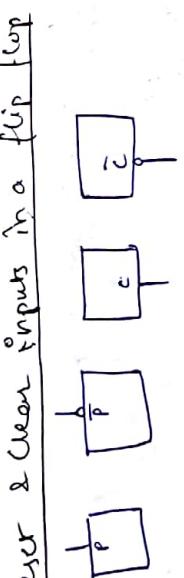


Possible combinations between flip flop output and clock

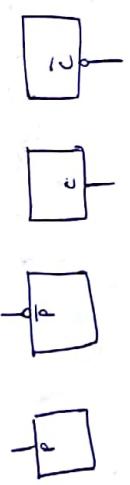
triggering in a ripple counter:

Flip Flop	Working	counter	Operation
			Down
\bar{T}	0	-ve	0
\bar{Q}	0	+ve	1
Q	1	-ve	0
Q	1	+ve	1

Implies Truth table of
EXOR gate.



Preset & Clear inputs in a flip flop : [Point 29]



Decade counter (ripple) counter:

→ It is also known as :

• Decimal counter

- BCD (Binary coded decimal) counter
- Mod 10 counter (or) divide by 10 counter (or)
10:1 scalar

→ It counts a total of 10 states from 0 to 9.
Therefore, total no. of flip flops needed: $n = 4$

∴ total no. of possible states $2^{2^4} = 16$

→ used valid states: $N = 10$

→ unused | Invalid states = 6.

[

]

h < 2ⁿ

First unused state

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Q. 16.

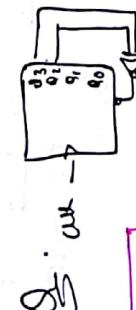
Q₄ Q₃ Q₂ Q₁ Q₀
1 1 0 0 0

(a)



(c)

	Q ₃	Q ₂	Q ₁	Q ₀	Q ₃ Q ₂ Q ₁ Q ₀
0	0	0	0	0	0 0 0 0
1	0	0	0	1	0 0 0 1
2	0	0	1	0	0 0 1 0
3	0	0	1	1	0 0 1 1
4	0	1	0	0	0 1 0 0
5	0	1	0	1	0 1 0 1
6	0	1	1	0	0 1 1 0
7	0	1	1	1	0 1 1 1
8	1	0	0	0	1 0 0 0
9	1	0	0	1	1 0 0 1
10	1	0	1	0	1 0 1 0
11	1	0	1	1	1 0 1 1
12	1	1	0	0	1 1 0 0
13	1	1	0	1	1 1 0 1
14	1	1	1	0	1 1 1 0
15	1	1	1	1	1 1 1 1



(b)

Q. 24. Generally, the preset and clear inputs are asynchronous in nature. In this case, the presetting/clearing of the flip flops is performed immediately.

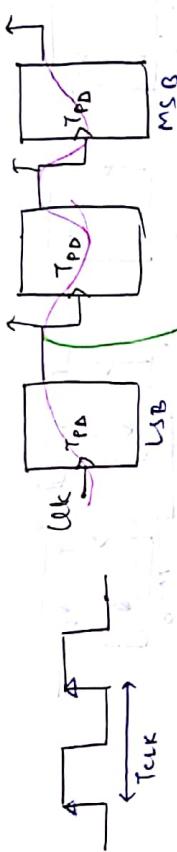
→ However, in the problem it is given that the clear input is synchronous.

→ Hence, in this case, when the counter enters into the first unused state, the clearing of flip flops is not performed immediately.

→ Rather, it is performed on the arrival of the next clock pulse. As a result of this, the first unused state also effectively becomes one of the used states.

→ Hence, the counter acts as a mod-5 counter and not as a mod-4 counter.

Propagation Delay in a Ripple Counter : [Point 30]



• T_{tp} is a ripple. It is because of this reason an asynchronous counter is also known as ripple counter.

• In a ripple counter, for valid operation.

T_{clock} \Rightarrow n · T_{PD}

$$\Rightarrow f_{clock} \leq \frac{1}{n T_{PD}}$$

n = Total no. of flip flops

T_{PD} = flip flop propagation delay

T_{clock} = clock time period
f_{clock} = clock frequency.

[NOTE]

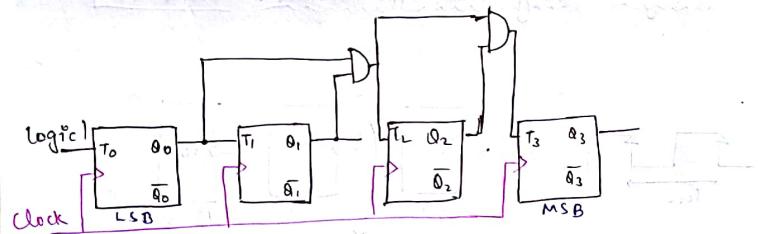
In a ripple counter, to eliminate decoding errors,

T_{clock} \geq n · T_{PD} + T_s \rightarrow strobe time

$$f_{clock} \leq \frac{1}{n T_{PD} + T_s}$$

Synchronous Counter :

- * Synchronous Up and Down Counter : [Point 31]
- * 4-bit Up counter:



State Transition Conditions:

- Q_0 toggles on every clock pulse.
- Q_1 toggles when $Q_0 = 1$.
- Q_2 toggles when $Q_1 \cdot Q_0 = 1$
- Q_3 toggles when $Q_2 \cdot Q_1 \cdot Q_0 = 1$

State Transition Table / State Table:

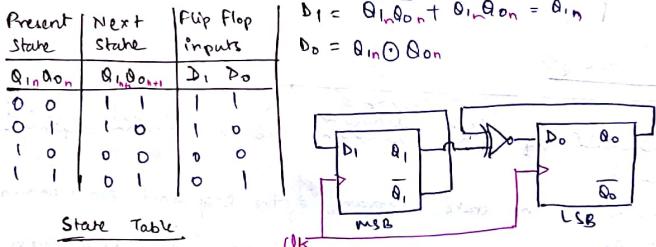
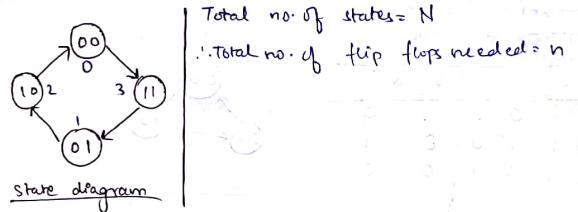
Clock	Q_3	Q_2	Q_1	Q_0	Present State	Next State	Flip Flop Inputs
0	0	0	0	0	0000	0000	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
1	0	0	0	1	0001	0011	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
2	0	0	1	0	0011	0111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
3	0	0	1	1	0111	1111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
4	0	1	0	0	0100	1000	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
5	0	1	0	1	1000	0100	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
6	0	1	1	0	0101	1011	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
7	0	1	1	1	1011	0111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
8	0	1	0	0	0100	1000	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
9	1	0	0	1	1001	0101	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
10	1	0	1	0	0101	1011	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
11	1	0	1	1	1011	0111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
12	0	1	0	0	1000	0100	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
13	1	1	0	1	0101	1011	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
14	1	1	1	0	1011	0111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
15	1	1	1	1	0111	1111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
16	0	0	0	0	0000	0000	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
17	0	0	0	1	0001	0011	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$
18	0	0	1	0	0011	0111	$D_1 = \bar{Q}_1 \bar{Q}_0$, $D_0 = Q_1 Q_0$

Synchronous Random Counter:

- A synchronous random counter can be easily/directly designed with the help of any flip flop.
- It is not necessary for the flip flop to be first converted into a T-flip flop.

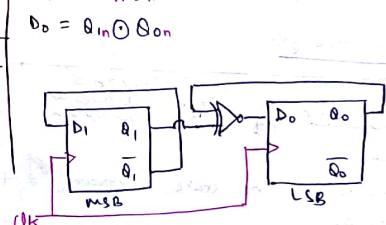
* Design of synchronous random counter:

E.g. with the help of D-flip flops, design a synchronous random counter



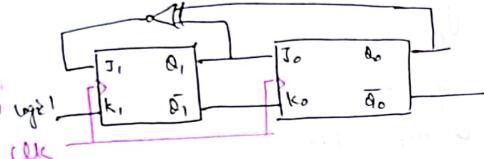
* Design of synchronous random counter having unused states:

Point 33



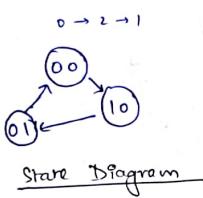
Analysis of synchronous random counter:

→ Obtain the count sequence of the counter circuit shown below:



J, K	Qn
0, 0	0
0, 1	1
1, 0	1
1, 1	0

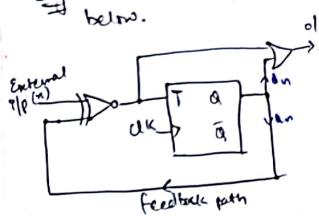
Present State	Next flip flop inputs	Next state
Q1, Q0	J, K, J0, K0	Q1, Q0
0, 0	1, 1, 0, 1	1, 0
1, 0	0, 1, 1, 0	0, 1
0, 1	0, 1, 0, 1	0, 0
0, 0		



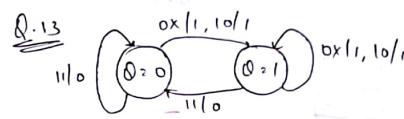
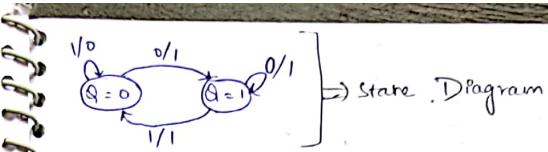
State Table

Construction of state diagram of a general sequential circuit:

E.g. Construct the state diagram of the sequential circuit shown below.



Present state	Input factors		Output factors	
	External Input	X	Q	f
1	0	0	1	1
0	0	1	0	0
0	1	0	1	1
1	1	1	0	1



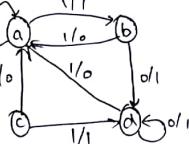
P	Qn	Q, K	Qn+1	f	
0	0	0x, 10	1	1	
0	0	11	0	0	
1	0	0x, 10	1	1	
1	1	11	0	0	

NAND (d)

28/10/2017 :-

State Reduction : Point 34

E.g. Reduce the state diagram shown below:



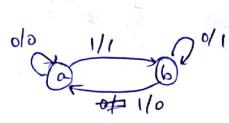
Present state	Next state	Output (f)
a	x=0	x=1
b	a	0
c	a	1
d	a	0

* States b and d are equivalent, ∴ state d is eliminated.

Present state	Next state	Output (f)
a	x=0	x=1
b	a	0
c	a	1

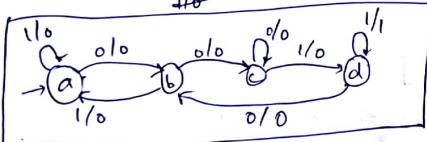
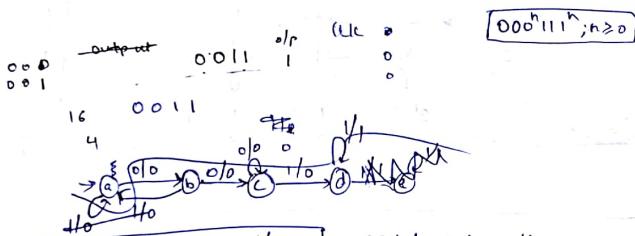
* a & c are equivalent.
∴ c is eliminated.

Present state	Next state		Output (f)	
	x=0	x=1	x=0	x=1
a	a	b	0	1
b	b	a	1	0



Shift Register Counter [Point 35]

Pg 68
Q40.



Total states = 4
Flip flops = 2

CHAPTER - 7

BOOLEAN ALGEBRA

[Point 38]

Chp 2.
I7.

$$Y = \bar{A}B + A(B+C) + (\bar{A}+\bar{B})C$$

$$= \bar{A}B + AB + AC + \bar{A}C + \bar{B}C$$

$$= (\bar{B}+\bar{B})(B+C) + A = A \oplus C$$

T8.

$$A \cdot (B + C \cdot D) \cdot E \cdot \bar{F} \cdot G \cdot \bar{H}$$

T9.

$$\overline{x+y} + y = x \cdot \overline{y} + y = x + y$$

T10.

$$\bar{A}B + \bar{B}A$$

$$A \cdot \bar{B} + \bar{A} \cdot B = \bar{A} \cdot B + \bar{A} \cdot B = (\bar{A} + \bar{B}) \cdot B$$

$$x \cdot \bar{y} = \bar{x} \cdot y = \bar{x} \cdot y = \text{NOR}$$

CHAPTER-8

NUMBER SYSTEMS

Radix/Base of a number system (r):

→ The radix of a number system defines the total no. of elements present in that number system.

→ The elements of any number system are derived from the decimal system and the English alphabets in such a way that:

* If " $r < 10$ ", the elements of the number system will be the first " r " decimal digits.

* If " $r = 10$ ", it becomes the decimal system itself and the elements will be the digits 0 to 9.

* If " $r > 10$ ", the elements will be the digits 0 to 9 + the first $(r-10)$ English letters.

Radix Point:

→ In the decimal number, 1104.0

known as Decimal Point.

→ In general, known as radix point.

→ for binary system ($r=2$), it's a binary point.

→ for octal system ($r=8$), it's a octal point.

→ for hexadecimal ($r=16$), it's a hexadecimal point.

Conversion of a no. from any radix (r) to Decimal system

Point 39

Conversion of a decimal number to any radix "r":

① Conversion of integer part:

r	I	R
2	Q	R
8	Q	R
10	Q	R
16	Q	R

2 53	1
2 26	1
2 13	0
2 6	1
2 3	0
2 1	1
	0

$(11010)_2$

$$\text{Eg. } (422)_{10} \rightarrow \begin{array}{r} 6 | 422 \\ 6 | 70 \quad 2 \\ 6 | 11 \quad 4 \\ 6 | 1 \quad 5 \\ \hline 0 \quad 1 \end{array} = (1542)_6$$

② Conversion of fractional part:

$$\begin{aligned} f \times r &= I \cdot f \\ f \times r &= I \cdot f \\ f \times r &= I \cdot f \\ \vdots & \vdots \\ &\text{until } f=0 \end{aligned}$$

$$\text{E.g. } (0.625)_{10} = (?)_4 = (0.22)_4$$

$$0.625 \times 4 = 2.500$$

$$0.500 \times 4 = 2.000$$

$$\begin{aligned} \text{E.g. } (0.840)_{10} &= (?)_2 = (0.110)_2 \\ 0.84 \times 2 &= 1.68 \\ 0.68 \times 2 &= 1.36 \\ 0.36 \times 2 &= 0.72 \\ 0.72 \times 2 &= 1.44 \\ 0.44 \times 2 &= 0.88 \\ 0.88 \times 2 &= 1.76 \\ 0.76 \times 2 &= 1.52 \\ 0.52 \times 2 &= 1.04 \\ 0.04 \times 2 &= 0.08 \\ 0.08 \times 2 &= 0.16 \\ 0.16 \times 2 &= 0.32 \\ 0.32 \times 2 &= 0.64 \end{aligned}$$

Octal to binary conversion:

→ It is obtained by writing the individual octal digits in its 3-bit binary equivalent form.

$$\text{E.g. } (76.21)_8 = (111 \ 110 \ . \ 010 \ 001)_2$$

Hexadecimal to binary conversion:

→ It is obtained by expressing the individual hexadecimal digits in its binary form.

$$\text{E.g. } (1F.2C)_{16} = (1101 \ 1111 \ . \ 0010 \ 1100)_2$$

Binary to Octal:

$$\text{Eg } (011 \ 010 \cdot 110 \ 010)_2 = (32.62)_8$$

Binary to Hexadecimal:

$$\text{Eg } (0110 \ 1010 \cdot 1110)_2 = (6A. \text{DE}6)_{16}$$

$$\text{Q6. } 4 \times 4096 + 9 \times 256 + 7 \times 16 + 5 \\ 4 \times 16^3 + 9 \times 16^2 + 7 \times 16^1 + 5 \times 16^0 = (4975)_{16} \\ 0100 \quad 1001 \quad 0111 \quad 0101 \Rightarrow 8 \ 15. @$$

$$\text{Q5. } (123)_5 \Rightarrow 25 + 2 \times 5 + 3 \times 1 \\ \Rightarrow 38 = 8 + xy \\ \Rightarrow xy = 30$$

(x,y)	
1	30
2	15
3	10
5	6

$x < y$	$y \geq 9$														
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1	30														
2	15														
3	10														
5	6														
1	30														
2	15														
3	10														

$$\text{Q7. } (E3)_{16} = 3+ \\ 1110 \ 0011$$

Octal:	
0	1
10	11
12	13
14	15
16	17
20	21
22	23
24	25
26	27
;	;
100	107
117	127
;	;
1000	1077

Hexadecimal:

0 1 2 3 4 5 6 7 8 9 A B C D E F
10 11 12 13 14 15 16 17 18 19 1A 1B 1C 1D 1E 1F

$$\text{Q8. } \begin{array}{r} \text{XFF} \\ \text{Zero} \\ \text{1111} \end{array}$$

$$\begin{array}{r} \text{F} \\ \text{1000} \\ -1110 \\ \hline \text{0E3} \end{array}$$

④

$$\begin{array}{r} \text{A} \\ \text{BA} \\ -\text{AB} \\ \hline 0 \end{array}$$

⑥

$$\begin{array}{r} \text{2.3} \\ + 1.2 \\ \hline 10.1 \end{array}$$

0 1 2 3 10 11 12 13
20 21 ⑤

Complements:

→ In general, in any number system, there are two types of complements:

- ① $(r-1)$'s complement (Radix-complement) (Diminished radix complement)
- ② r 's complement (Diminished radix complement) Radix Complement True complement

Binary	Octal	Decimal	Hexadecimal
1's	7's	9's	15's F's
2's	8's	10's	16's

→ In general in any radix ' r ', the r 's complement can be obtained by adding 1 to the $(r-1)$'s complement.

(r-1)’s Complement:

The (r-1)’s complement of a number N, of radix “r” and having “n” no. of digits is given by: $r^n - N - 1$.

[NOTE]:

As a shortcut, instead of applying the above formula, the (r-1)’s complement can be obtained by simply subtracting the individual digits of the given number from (r-1).

→ E.g. Decimal system (9’s complement)

$$N = \begin{array}{r} 9 & 9 & 9 & 9 & 9 & 9 \\ 6 & 0 & 9 & 8 & 3 & 0 \\ \hline 3 & 9 & 0 & 1 & 6 & 9 \end{array}$$

Special case:

Binary system - 1’s complement:

→ The 1’s complement of a binary number can be obtained by simply changing all the 1’s to 0’s and 0’s to 1’s.

$$\begin{aligned} \text{E.g. } N = 101100 &\rightarrow 010011 \\ N = 1001 &\rightarrow 0110 \end{aligned}$$

r’s complement:

→ The r’s complement of a no. N, of radix “r” and having ‘n’ no. of digits is given by, $r^n - N$; when $N \neq 0$.
0; when $N=0$.

Point 40

$$\text{E.g. } N = \begin{array}{r} 9 & 9 & 9 & 9 & 10 \\ 6 & 0 & 9 & 8 & 3 & 9 & 8 \\ \hline 3 & 9 & 0 & 1 & 7 & 0 & 0 \end{array} \text{ + 10's complement.}$$

$$\begin{aligned} \text{e.g. } N = 101100 &\xrightarrow{2's} 010100 \\ M = 1011 &\xrightarrow{2's} 0101 \\ N = 1000 &\xrightarrow{2's} 1000 \\ N = 000 &\xrightarrow{2's} 000 \end{aligned}$$

Subtraction of Positive Unsigned Numbers using complements-

$$\begin{array}{c} A \\ - B \\ \hline D \end{array} \xrightarrow{+X} \begin{array}{c} A \\ + X \\ \hline D \end{array} \rightarrow \text{r's complement of } B$$

$A - B = A + X$
 $= A + [\text{r's compl. of } B]$
 OR
 $= A + [(\text{r}-1)'\text{s comp. of } B + 1]$

Signed Binary numbers:

→ Unsigned Numbers

5

→ They are always +ve numbers by default.

→ The only exception is the digit 0.

→ Signed Number
+5, -5

→ They are both +ve as well as -ve numbers.

* Sign bit: In case of signed binary numbers, the MSB of the number is used to represent the sign of the number.

→ This bit is termed as the sign bit. Furthermore, as a standard notation:

① Sign bit = 0 → the binary number is +ve.

② Sign bit = 1 → the binary number is -ve.

[NOTE] → The above allocation always remains the same irrespective of the signed binary number representation used.

Signed Binary Number Representations:

for +ve no.

Signed Magnitude System

- for -ve no.
- signed Magnitude System
- signed Complement system
 - ↳ 1's complement
 - ↳ 2's complement

* Signed magnitude system of representation:

$$\text{E.g. } +5 \quad -5$$

$$\begin{array}{c} 0 \ 101 \\ \downarrow \text{Sign bit} \\ \text{Value bits} \end{array} \qquad \begin{array}{c} 1 \ 101 \\ \downarrow \text{Sign bit} \\ \text{Value bits} \end{array}$$

→ In the signed magnitude system, the value bits are simply the binary equivalent of the decimal number to be represented.

Signed complement system representation of a -ve number:

$$\text{E.g. } -5 \xrightarrow{\text{start with the corresponding +ve no.}} +5 \xrightarrow{\text{first find the signed mag. system representation of +5}} 0101$$

To find the signed 1's complement system representation of -5 :

$$0101 \xrightarrow{\text{simply find the 1's comp. of 0101}} 1010$$

To find the 2's complement repr. of -5 :

$$0101 \xrightarrow{\text{simply find the 2's comp. of 0101}} 1011$$

$$\text{Q.12. } 010001 \xrightarrow{\text{S.M.S.}} 101111 \quad (b)$$

$$\text{Q.12. } 10011 \xrightarrow{\text{1's comp.}} 01101 \xrightarrow{\text{+13}} 11101 = -13 \quad (b)$$

$$\text{Q.12. } -17 \xrightarrow{\text{S.M.S.}} 010001 \xrightarrow{\text{2's comp.}} 101111 \quad (b)$$

$$\text{Q.12. } 10011$$

Signed complement system representation of a positive number:

$$\begin{array}{c} \text{S.M.S.} \quad \text{S.1's.C.S.} \quad \text{S.2's.C.S.} \\ \text{E.g. } +5 \quad 0101 \quad 6101 \quad 0101 \end{array}$$

→ A positive no. is represented in the S.C.S. in exactly the same manner as it is represented in the S.M.S.

→ Two nos. x & y are represented in the signed 1's complement system as 00101 & 10100 . Find their decimal values.

$$00101 \rightarrow \text{out} +5$$

(sign bit is '0').

∴ it is already in S.M.S.
so we can directly find its decimal value.

$$\therefore x = +5$$

$$10100 \xrightarrow{\text{is } 01011} \text{out} +5 + 11000 = +2$$

∴ sign bit is '1'.
To find the decimal value, we need to find its complement once again
 $\therefore y = -11$

Bit Expansion in signed complement system:

→ To extend/increase the number of bits in a number represented in the S.C.S., simply rewrite the sign bit of the number repeatedly.

$$\text{E.g. } \text{true} \quad 0101 \oplus 00101$$

$$1101 \Rightarrow 111101$$

-ve

(2)

2 variable K-Map.

		y	\bar{y}
		y	\bar{y}
		0	1
\bar{x}	0	$\bar{x} \cdot \bar{y}$	$\bar{x} \cdot y$
	1	$x \cdot \bar{y}$	$x \cdot y$
x	0		
	1		

Minterms Map.

		y	\bar{y}
		y	\bar{y}
		0	1
x	0	$x + y$	$x + \bar{y}$
	1	$\bar{x} + y$	$\bar{x} + \bar{y}$
\bar{x}	0		
	1		

Maxterms Map.

3 variable K-Map.

(3)

		$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$y\bar{z}$
		00	01	11	10
		0	1	3	2
\bar{x}	0				
	1	4	5	7	6
x	0				
	1				

Minterms Map.

		$\bar{x}\bar{y}\bar{z}$	$\bar{x}y\bar{z}$	$\bar{x}yz$	$\bar{y}\bar{z}$	$\bar{y}z$
		00	01	11	10	
		0	1	3	2	
\bar{x}	0					
	1	4	5	7	6	
x	0					
	1					

Maxterms Map.

4 Variable K-Map

$\bar{w}\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$w\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
$\bar{w}\bar{x}$	00	01	11	10	$w\bar{x}$	00	01	11	10
$w\bar{x}$	01	11	10	00	$w+x$	00	01	11	10
0	1	3	2	4	5	7	6	12	13
4	5	7	6	13	15	14	11	8	9
12	13	15	14	15	16	17	16	24	25
11	10	11	10	14	13	15	14	29	31
8	9	11	10	10	9	11	10	25	27
10									26

Minterms Map.

Maxterms Map.

5 Variable K-Map

$g = 0 \text{ map}$

$\bar{w}\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$w\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$w\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	
$\bar{w}\bar{x}$	00	01	11	10	$w\bar{x}$	00	01	11	10	$w\bar{x}$	00	01	11	10	
$w\bar{x}$	01	11	10	00	$w+x$	00	01	11	10	$w+x$	01	11	10	00	
0	1	3	2	4	5	7	6	12	13	15	14	16	17	19	18
4	5	7	6	13	15	14	11	8	9	11	10	20	21	23	22
12	13	15	14	15	16	17	16	28	29	31	30				
11	10	11	10	10	11	10	9	24	25	27	26				
8	9	11	10												
10															

(5)

$g = 1 \text{ map}$

$\bar{w}\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$w\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	$w\bar{x}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz	
$\bar{w}\bar{x}$	00	01	11	10	$w\bar{x}$	00	01	11	10	$w\bar{x}$	00	01	11	10	
$w\bar{x}$	01	11	10	00	$w+x$	00	01	11	10	$w+x$	01	11	10	00	
0	1	3	2	4	5	7	6	12	13	15	14	16	17	19	18
4	5	7	6	13	15	14	11	8	9	11	10	20	21	23	22
12	13	15	14	15	16	17	16	28	29	31	30				
11	10	11	10	10	11	10	9	24	25	27	26				
8	9	11	10												
10															

Minterms Map.

5 Variable K-Map

$U = 0$ map.

$\bar{W}\bar{x}$	$\bar{Y}\bar{Z}$	$\bar{Y}\bar{Z}$	$\bar{Y}\bar{Z}$	$\bar{Y}\bar{Z}$
0	1	3	2	
$\bar{W}x$	4	5	7	6
12	13	15	14	
$W\bar{x}$	8	9	11	10
Wx				

(6)

$U = 1$ map.

$\bar{W}\bar{x}$	$\bar{Y}\bar{Z}$	$\bar{Y}\bar{Z}$	$\bar{Y}\bar{Z}$	$\bar{Y}\bar{Z}$
16	17	19	18	
$\bar{W}x$	20	21	23	22
28	29	31	30	
$W\bar{x}$	24	25	27	26
Wx				

Implicants (I), Prime Implicants (PI)

Essential Prime Implicants (E PI),

Non-Essential Prime Implicants (NPI)

$\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$				
$w\bar{x}$							
	$w\bar{x}$						
		$w\bar{x}$					
			$w\bar{x}$				
				$w\bar{x}$			
					$w\bar{x}$		
						$w\bar{x}$	

$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$				
$w\bar{x}$							
	$w\bar{x}$						
		$w\bar{x}$					
			$w\bar{x}$				
				$w\bar{x}$			
					$w\bar{x}$		
						$w\bar{x}$	

$w\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$				
$w\bar{x}$							
	$w\bar{x}$						
		$w\bar{x}$					
			$w\bar{x}$				
				$w\bar{x}$			
					$w\bar{x}$		
						$w\bar{x}$	

$w\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$				
$w\bar{x}$							
	$w\bar{x}$						
		$w\bar{x}$					
			$w\bar{x}$				
				$w\bar{x}$			
					$w\bar{x}$		
						$w\bar{x}$	

the product term obtained by considering any possible grouping on a K-Map is known as an Implicant (I).

→ the product terms obtained by considering all the biggest possible groupings on a K-Map are known as Prime Implicants (PI).

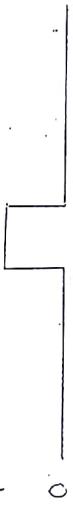
→ If a PI happens to be the only biggest possible grouping for a particular minterm, then that PI is known as

Essential Prime Implicant (EPI).

→ PI minus EPI gives the Non-Essential Prime Implicants (NPI).

→ If y goes from 1 to 0, the output (f) must remain at logic 1, but if momentarily goes to 0, this is known as Static - 1 Hazard. It occurs when the circuit is implemented as a 2-level AND - OR implementation.

Similarly,



If the output must remain at logic 0, but it momentarily goes to logic 1, it is known as Static - 0 Hazard. It occurs when the circuit is implemented as a 2-level OR - AND implementation.

General Design Procedure of a Combinational Circuit

(10)

Circuit

- Identify the number of inputs and outputs.
- Construct the truth table.
- Obtain the Boolean expression(s) of the output(s) using either the Minterms approach or the Maxterms approach.
- Simplify the Boolean expression(s) of the output(s) using either K-Map simplification or Boolean Algebra simplification.
- Implement the digital circuit using logic gates.

→ Note: In a decoder, at any instant, only one output is at logic 1 (active output) and all other outputs are at logic 0.
◦ An $n \times 2^m$ decoder is also known as 1 of 2^m decoder.

→ Note: A decoder can also be constructed using NAND gates. In that case, the active output is logic 0 and all other outputs are logic 1. Such It is known as a Decoder with Active Low Outputs.

(12) Boolean function implementation using Decoder
→ An $n \times 2^m$ decoder generates all the 2^m possible minterms for n input variables.
→ Any Boolean function can be represented in sum of minterms form.
⇒ any Boolean function can be implemented with a decoder by summing / ORing the desired minterms.

Implementation of Basic Encoder: In a Basic

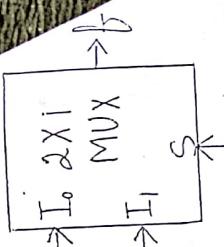
(13) Encoder, at any instant, only one input must be logic 1 and all other inputs must be logic 0; or else, the circuit produces invalid output. This problem is resolved in a Priority Encoder. In a Priority Encoder, at any instant, if more than one inputs are at logic 1, then the encoded output corresponds to the input having the highest decimal value.

(14) * To implement an n variable Boolean function, a MUX having m select inputs is needed, that is, a $2^m \times 1$ MUX.

→ Apply, → the variables of the function on the select inputs. → the truth table outputs of the function on the data inputs.

(15)

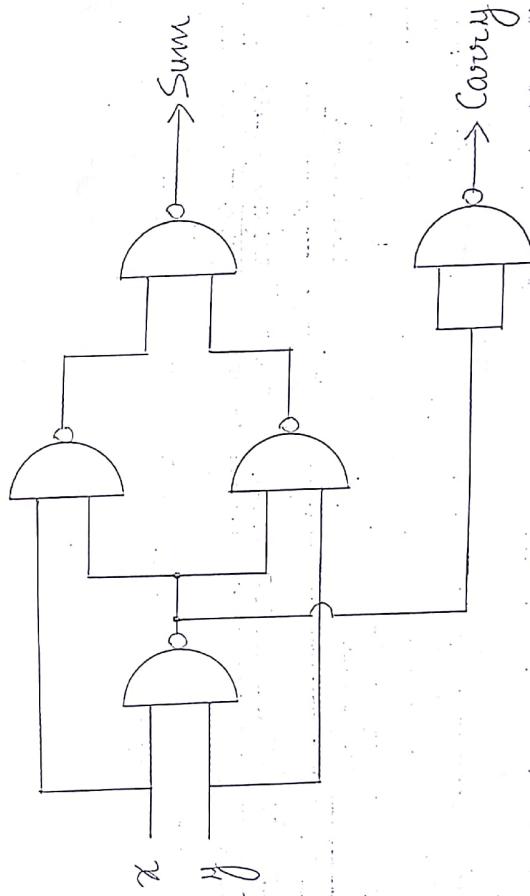
EXNOR	EXOR	NOR	NAND	OR	AND
\bar{y}	y	\bar{y}	1	y	0
y	\bar{y}	0	\bar{y}	1	y



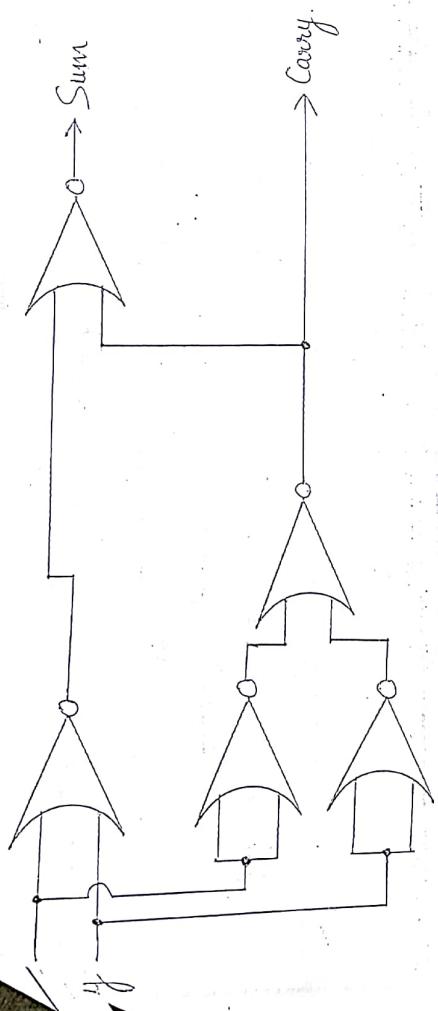
Note: To obtain the NAND, NOR, EXOR, EXNOR gates, two 2x1 MUXes are needed. The first 2x1 MUX is used as a NOT gate to obtain \bar{y} from y .

(16)

Implementation of Half Adder using NAND gates.



Implementation of Half Adder using NOR gates.



⑯

Disadvantage of Ripple Carry Adder: In a

Ripple Carry Adder, C_{i+1} depends upon C_i , and is generated after C_i passes through an AND gate followed by an OR gate; that is, after a propagation delay of 2 gate levels. The propagation delay in an n -bit Ripple Carry Adder is $2n$ gate levels. Thus problem is resolved in a Carry Lookahead Adder where C_{i+1} is directly generated from the P_i and G_i terms.

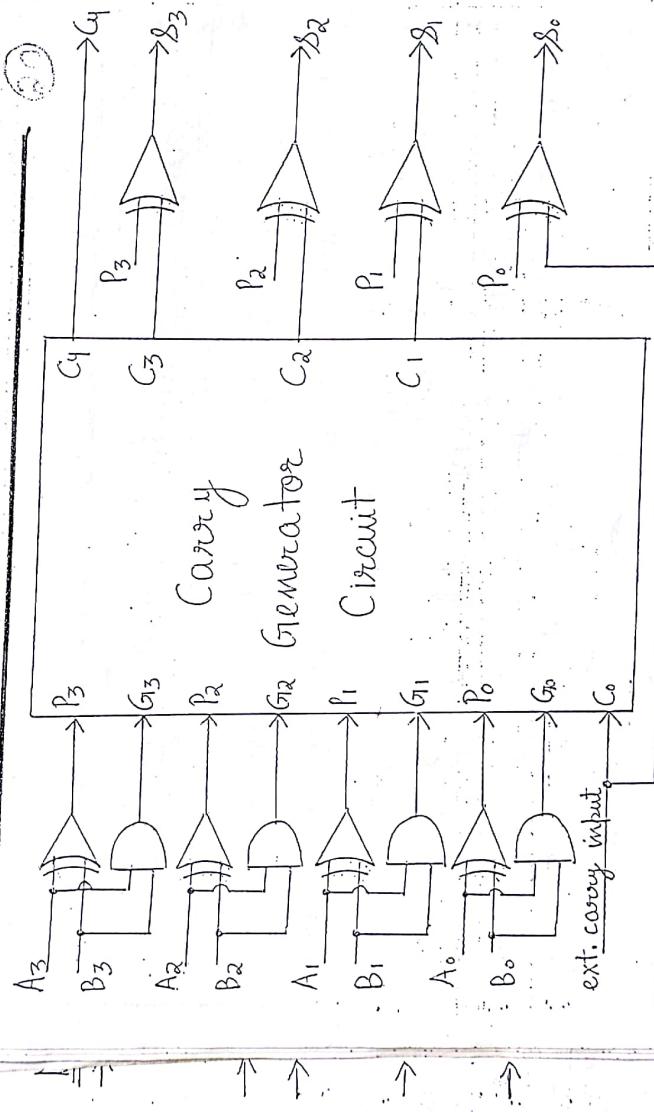
→ the carry lookahead logic increases speed of operation but the tradeoff is a highly complex circuit.

→ the carry generator circuit is a 2-level AND - OR implementation.

→ In an m -bit carry generator circuit,

$$\rightarrow \text{No. of AND gates} = \frac{m(m+1)}{2}$$

$$\rightarrow \text{No. of OR gates} = m$$



Note: the propagation delay of a carry lookahead adder is always a constant factor, and is independent of the number of bits in the adder.

SEQUENTIAL

ie binary data stored in the memory elements is known as the State of the Memory

Elements.

- Previous State $\rightarrow Q_{n-1}$
- Present State $\rightarrow Q_n$
- Next State $\rightarrow Q_{n+1}$

→ the Output(s) of a sequential circuit depends upon,
→ the Output(s) of a sequential Circuit
 \Rightarrow Moore Sequential Circuit

OR

→ Present State \Rightarrow Mealy Sequential Circuit
+
External Inputs)

→ the Next State of a sequential circuit depends upon,
→ Present State and/or External Input(s)

Note: the Moore and Mealy models of
sequential circuits are commonly known as
Finite State Machines (FSM).

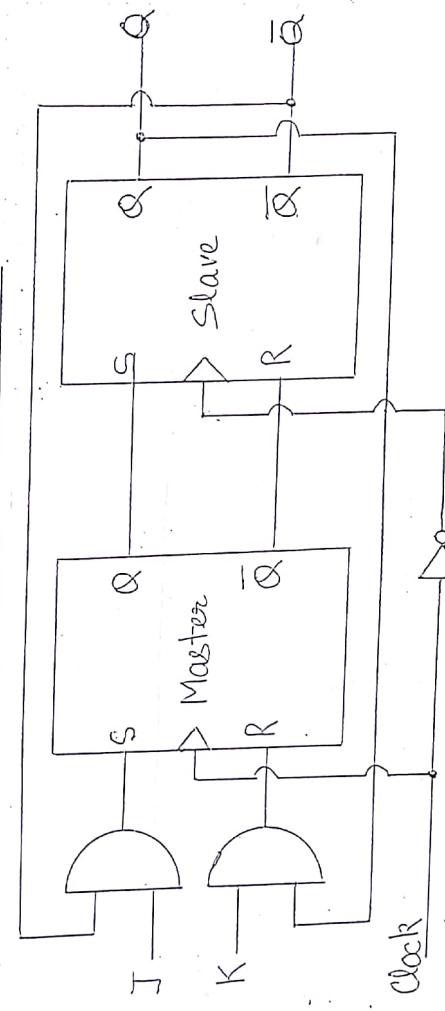
\Rightarrow Moore FSM
 \Rightarrow Mealy FSM

Clock	J	K	Q_{n+1}	\bar{Q}_{n+1}
X	X	X	Q _n	\bar{Q}_n
↑	0	0	Q _n	Q _n
↑	0	-	0	0
↑	-	0	0	0
↑	1	0	1	0
↑	1	-	1	0
↑	-	1	1	0
↓	Q _n	Q _n	Q _n	Q _n

Toggle state.

Function Table Characteristic Table Characteristic Equation Excitation Table.

Master Slave JK flip flop (MS JK FF)



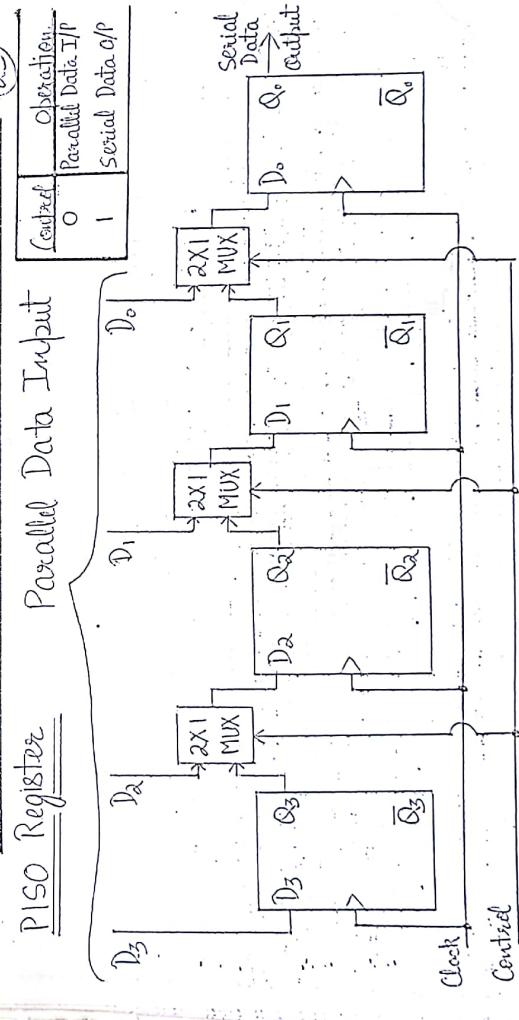
- the overall circuit acts as a single flip flop & stores 1 bit.
- At any instant, only one flip flop is triggered.
- the problem of race around condition is not present in a Master Slave JK flip flop.
- the function tables of Master Slave JK flip flop and JK flip flop are the same.

Standard flip flop conversions.

		S	R	T	K	D	T
SR	-	-	S	R	$S + \bar{R} \cdot Q_n$	$S \cdot \bar{Q}_n + R \cdot Q_n$	
JK	$J \cdot \bar{Q}_n$	$K \cdot Q_n$	-	-	$J \cdot \bar{Q}_n + \bar{K} \cdot Q_n$	$J \cdot \bar{Q}_n + K \cdot Q_n$	
D	D	\bar{D}	D	\bar{D}	-	$D \oplus Q_n$	
T	$T \cdot \bar{Q}_n$	$T \cdot Q_n$	T	T	$T \oplus Q_n$	-	

Note: To obtain any flip flop from the D flip flop, simply apply the characteristic equation of that flip flop on the D input.

PISO Register



Note: PISO converts Spatial Code into Temporal Code.

Types of Counters:

Synchronous Counter

- All the flip flops are triggered by a common clock pulse.
- Faster
- More complex circuit
- Up, Down and Random counters can be designed
- Decoding Errors are not present.

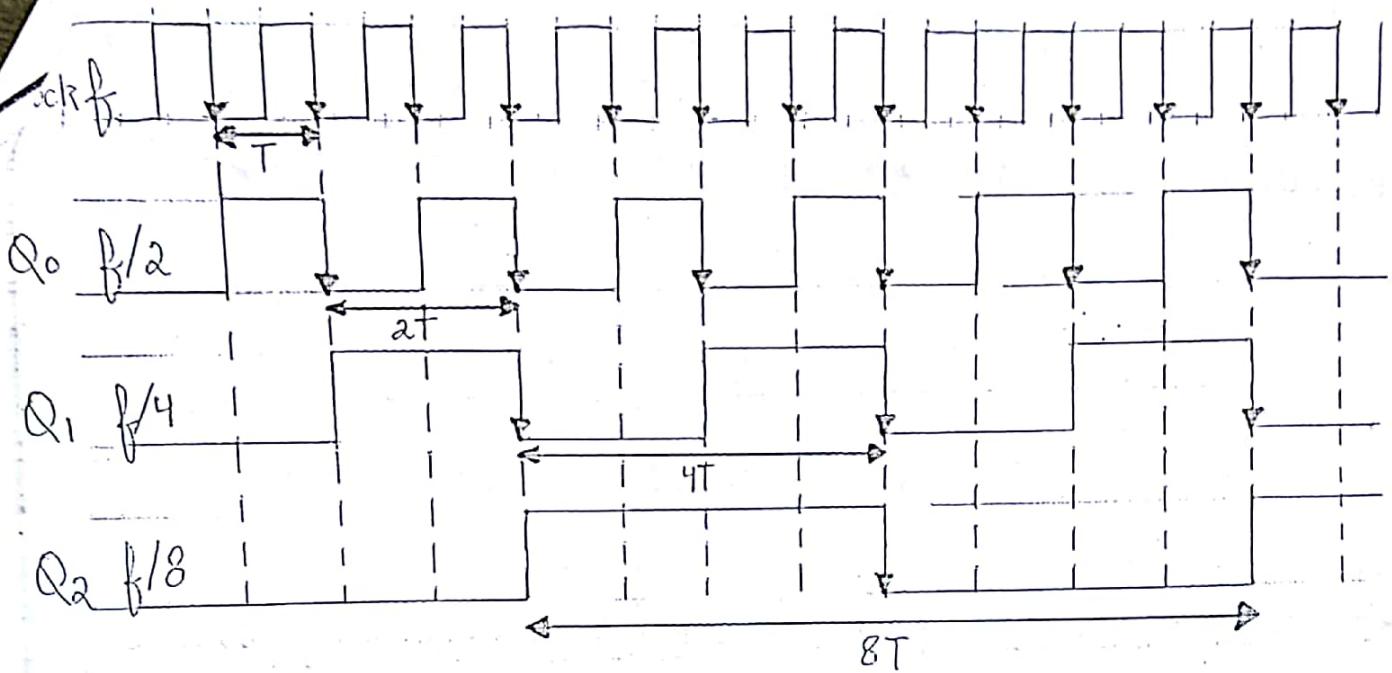
Asynchronous / Ripple Counter

- the flip flops are triggered either by clock pulse or by the outputs of adjacent flip flops.
- Slower
- Less complex circuit
- Only Up and Down counters can be designed
- Decoding Errors are present.

General Constructional Details of Ripple Counter:

- (27)
- It consists of a series connection of T flip flops (or other flip flops converted into T flip flops).
 - All the T flip flops operate in Toggle mode ($\Rightarrow T=1$).
 - the clock pulse is applied to only one flip flop. thus flip flop represents the LSB of the count sequence.
 - the output of each flip flop is connected to the clock input of the adjacent flip flop.

Timing Diagrams.



Note: $f \rightarrow f/2 \rightarrow f/4 \rightarrow f/8$

\Rightarrow frequency scaling by a factor of 2 takes place in a ripple counter as we move from LSB flip flop to MSB flip flop.

(29)

Preset and Clear Inputs in a Flip Flop.

Sets the flip flop.



Preset

2 types.

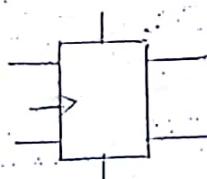
Direct Set

\rightarrow Active High Preset (P)

Asynchronous Set

\rightarrow Active Low Preset (\bar{P})

Irrespective of whatever is applied on the data inputs and the clock input of the flip flop.



Clear

2 types

Direct Reset

\rightarrow Active High clear (C)

Resets the flip flop.



Asynchronous Reset

\rightarrow Active Low clear (\bar{C})

→ In a ripple counter, the outputs of the flip flops are generated one bit at a time, starting from the LSB flip flop. This is because of the cascaded nature of flip flop interconnection.

In a synchronous counter, the outputs of all the flip flops are generated simultaneously.

∴ Ripple counters are slower than synchronous counters.

(31)

General Constructional Details of Synchronous Up and Down Counters.

→ It consists of a series connection of T flip flops (or other flip flops converted into T flip flops).

→ The T flip flop in the LSB position operates in Toggle mode ($\Rightarrow T=1$).

→ A flip flop in a higher significant position toggles when the outputs of all lower significant flip flops are equal to 1.

Note: In any synchronous counter (Up, Down, Random), the count sequence is independent of +ve or -ve edge triggering.

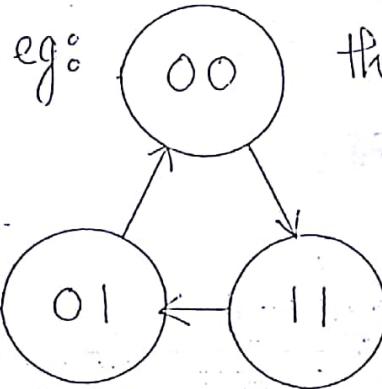
2 \therefore To obtain Down Counter, simply connect the complement output (\bar{Q}) as the input to the next stages.

Note: there are two types of Synchronous Up and Down Counters:

→ Series Type Counter : the input of an AND gate is connected to the output of the previous AND gate.

→ Parallel Type Counter : the inputs of all the AND gates are directly connected to the flip flop outputs.

Special Case: Design of Synchronous Random Counter having unused states. (33)



there are 2 methods to deal with unused states

Method 1: Assign any used state as next state for unused state

eg: Assign 00 as next state

Unused State = 10

PS	NS	FF I/P _s
00		
01		
10	00	
11		

Method 2: Treat them as Don't Care Conditions

PS	NS	FF I/P _s
00		
01		
10	--	XX
11		

Note: By default, Method 2 is considered.

(34)

State Reduction: It is the process of ~~keeping~~ reducing the total no. of states in a sequential circuit, while keeping the input-output relationship unchanged.

The steps of state reduction are as follows:

- Construct the state table.
- Find out if two states are equivalent states.
If so, then eliminate any one of them.
- Repeat the above step until there are no more equivalent states.

Note: 2 present states are considered to be equivalent states if they produce exactly same next state and same output.

Shift-Register Counter

(35)

- It consists of a series connection of D flip flops (or other flip flops converted into D flip flops.)
- the output of the last flip flop is connected back to the input of the first flip flop.
- 2 types.

→ Ring Counter

→ Johnson Counter

Ring Counter

→ Standard example of synchronous random counter.

(36)

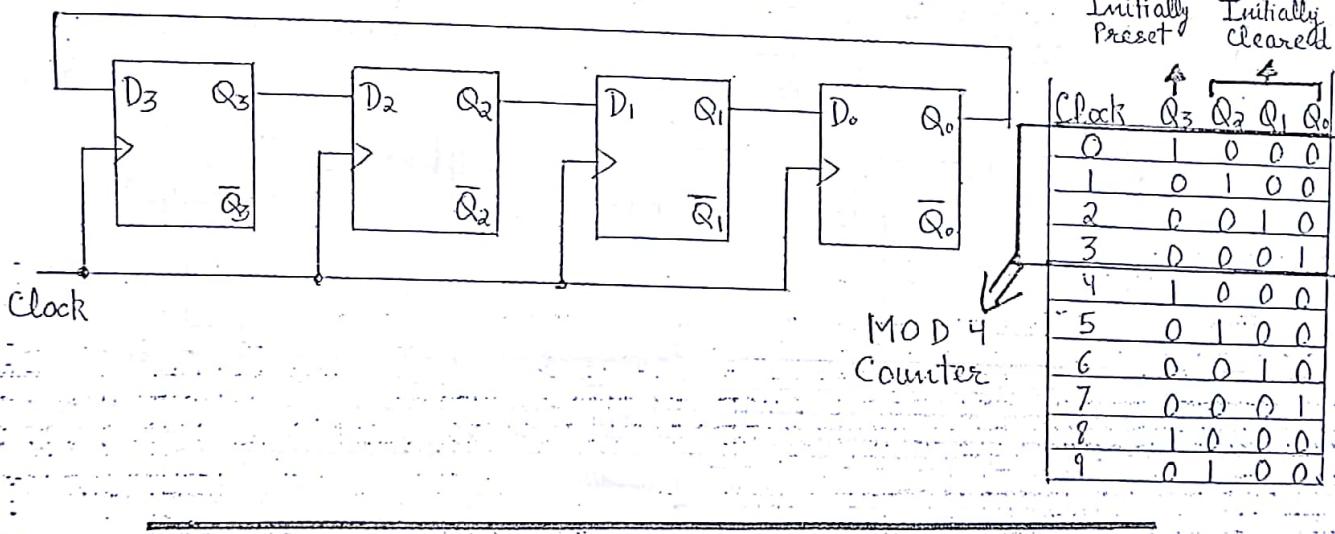
Also known as,

→ Straight Ring Counter

→ Overbeck Counter

→ One-Hot Counter

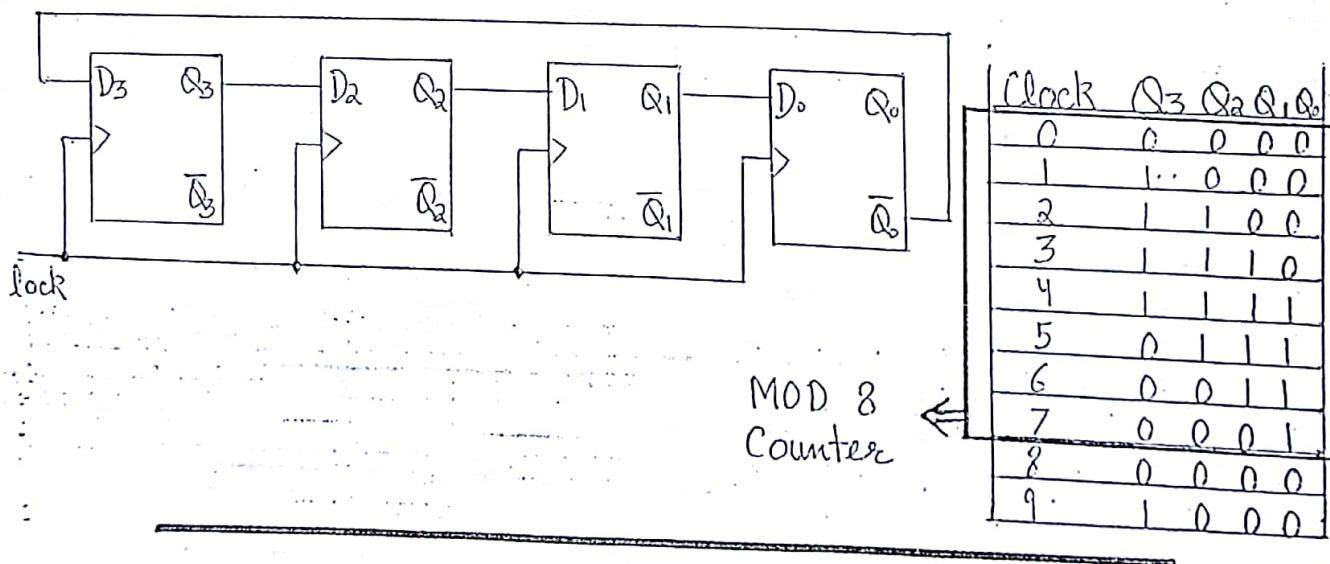
- An n -bit counter counts n states.
⇒ Used states = $N = n$.
- If the input frequency is f , then the output frequency of all the flip flops in the counter is $= f/N = f/n$.
- At any instant, only one flip flop is Set and all other flip flops are Cleared. (\therefore also known as One-Hot Counter).
- the Hamming Distance of the used states is 2.
- Decoding Circuit is not needed



Johnson Counter

(3)

- Also known as,
 - Twisted Ring Counter
 - Switch Tail Ring Counter
 - Walking Ring Counter
 - Creeping Ring Counter
 - Moebius / Möbius Counter
- An n -bit counter counts 2^n states
 - ⇒ Used States = $N = 2^n$.
- If the input frequency is f , then the output frequency of all the flip flops in the counter = $f/N = f/2^n$.
- the Hamming Distance of the used states is 1.
- Decoding Circuit is needed. This circuit consists of a total of 2^n logic gates. These logic gates are either 2-input AND gates or 2-input NOR gates.



Boolean Algebra

(38)

Introduced by George Boole in 1854

→ Basic operations of Boolean Algebra

→ NOT
 → AND
 → OR

∴ NOT gate, AND gate and OR gate are known as Basic Logic Gates.

→ Operator Precedence

→ Brackets

→ NOT

→ AND

→ OR

Note:

→ AND operation is also known as Logical Conjunction

∴ Conjunctive form of Boolean expression \Rightarrow POS expression

→ OR operation is also known as Logical Disjunction/Alternation

∴ Disjunctive form of Boolean expression \Rightarrow SOP expression.

Basic theorems of Boolean Algebra

Identity	Null Element / Annulment	Idempotent	Complement
$\rightarrow A \cdot 1 = A$ $\rightarrow A + 0 = A$	$\rightarrow A \cdot 0 = 0$ $\rightarrow A + 1 = 1$	$\rightarrow A \cdot A = A$ $\rightarrow A + \bar{A} = A$	$\rightarrow A \cdot \bar{A} = 0$ $\rightarrow A + \bar{A} = 1$
Involution / Double Negation	Absorption / Covering / Redundancy	De Morgan's	Distribution
$\rightarrow \bar{\bar{A}} = A$	$\rightarrow A + A \cdot B = A$ $\rightarrow A \cdot (A + B) = A$	$\rightarrow \overline{A \cdot B \cdot C} = \bar{A} + \bar{B} + \bar{C}$ $\rightarrow \overline{A + B + C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$	$\rightarrow A \cdot (B + C) = A \cdot B + A \cdot C$ $\rightarrow A + B \cdot C = (A + B) \cdot (A + C)$

Consensus theorem

It is applicable on a Boolean expression when,

→ 3 variables are present (eg: A, B, C)

→ each variable is used 2 times (\Rightarrow 6 literals)

→ only 1 literal is complemented (eg: \bar{A})

then the term which contains neither A nor \bar{A} is known as Consensus Term. this term is eliminated

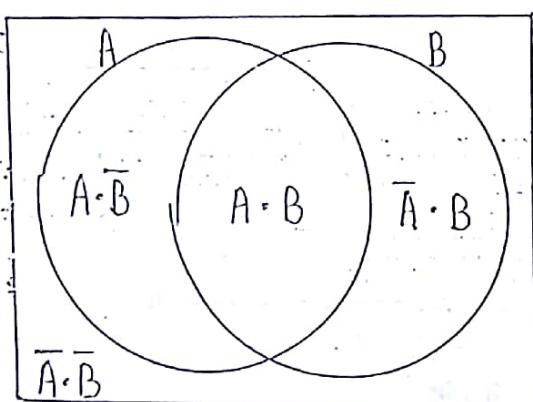
$$\text{eg: } AB + \underset{\substack{\downarrow \\ \text{Consensus Term}}}{BC} + \bar{A}C = AB + \bar{A}C$$

the Consensus theorem is also applicable to POS expressions

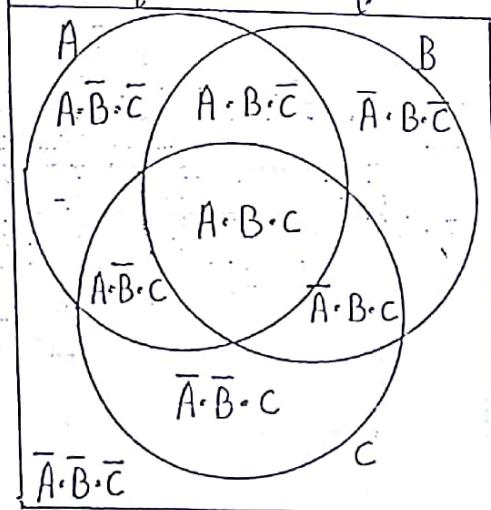
$$\text{eg: } (A + \bar{B}) \cdot (B + c) \cdot \underset{\substack{\downarrow \\ \text{Consensus Term}}}{(A + c)} = (A + \bar{B})(B + c)$$

Venn Diagram

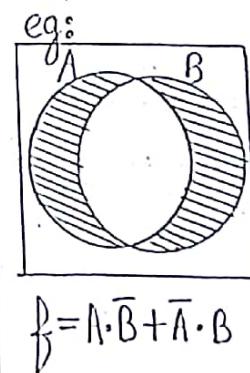
It is a graphical approach of Boolean function representation



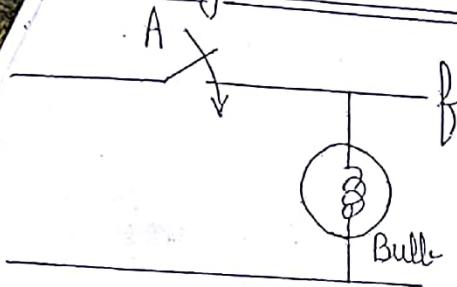
2 variable.



3 variable.

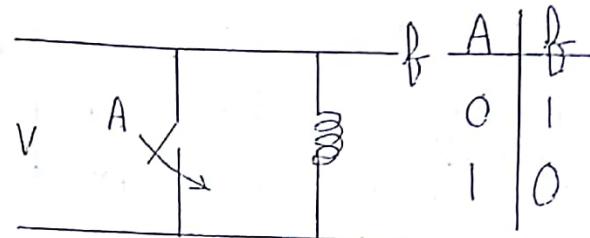


Building Circuits.

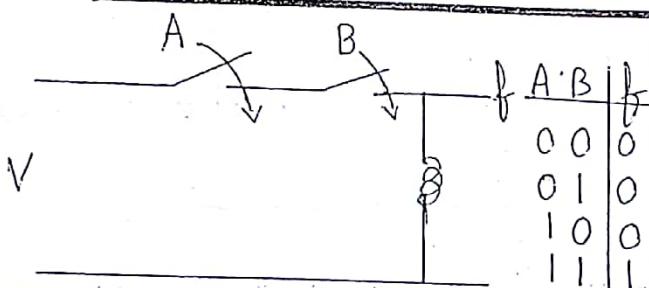


A	f
0	0
1	1

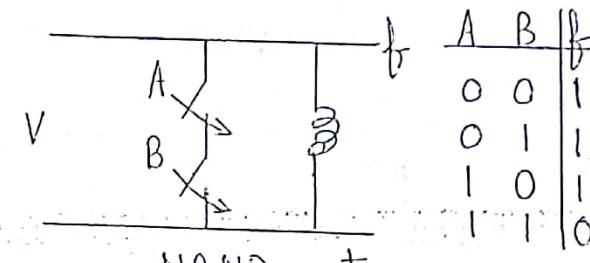
Buffer.



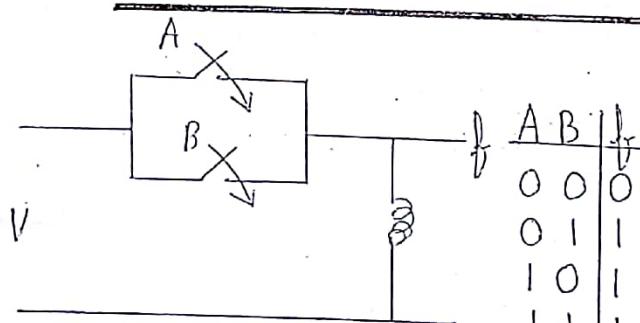
NOT gate



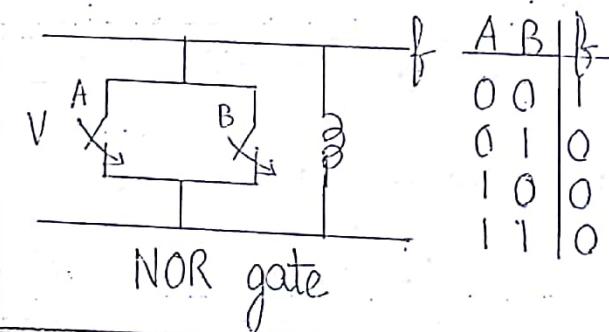
AND gate



NAND gate.



OR gate



NOR gate

Positive Logic and Negative Logic Systems.

+ve logic
system

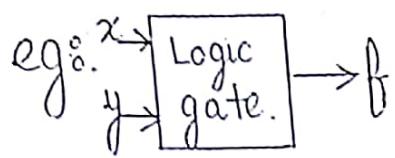
logic 0
logic 1

eg:

L (0V)	→	logic 1
H (5V)	→	logic 0

-ve logic
system.

Logic Gate/
Digital Circuit



x	y	f
L	L	L
L	H	L
H	L	L
H	H	H

Assuming
+ve logic

x	y	f
0	0	0
0	1	0
1	0	0
1	1	1

AND gate

Assuming
-ve logic

x	y	f
1	1	1
1	0	1
0	1	1
0	0	0

OR gate

→ +ve logic AND gate is equivalent to -ve logic OR gate.

Similarly,

+ve logic -ve logic

AND

OR

OR

AND

NAND

NOR

NOR

NAND

EXOR

EXNOR

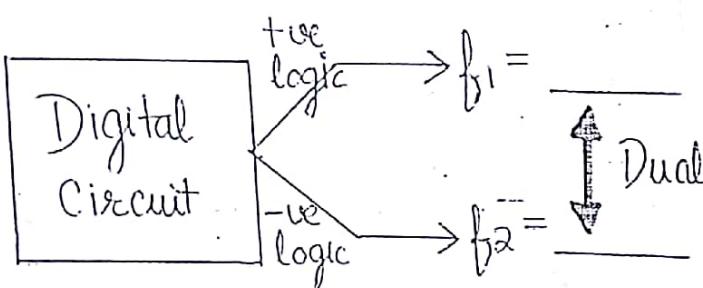
EXNOR

EXOR

NOT

NOT

Duality Principle / Dual.



Steps to find Dual:

→ Interchange the AND and OR operations.

→ Interchange the 0's and 1's.

$$\text{eg: } A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C + A \cdot \bar{B} \cdot C$$

$$A + 1 = 1$$

\downarrow Dual

\downarrow Dual

$$(A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (A+\bar{B}+C)$$

$$A \cdot 0 = 0$$

→ Finding the dual of an expression two times gives back the original expression.

→ Self Dual Expressions. (Not very imp.).

→ In self dual expressions, finding the dual only once gives back the original expression.

→ With n variables, a total of $2^{2^{n-1}}$ self dual expressions can be formulated.

Functionally Complete Boolean Operator

A Boolean operator (or a set of Boolean operators) is said to be functionally complete if all possible Boolean functions can be implemented from it.

$$\text{eg: } \{\text{NAND}\}$$

$$\{\text{NOR}\}$$

$$\{\text{AND, NOT}\}$$

$$\{\text{OR, NOT}\}$$

Number Systems:

Conversion of a no. from any radix (r) to Decimal System

(39)

$$\rightarrow (2206)_{10} = 2000 + 200 + 0 + 6 \\ = (2 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (6 \times 10^0)$$

in general, a decimal no $a_n \dots a_1 a_0 : a_{-1} \dots a_{-m}$ is a shorthand of $(a_n \times 10^n) + \dots (a_1 \times 10^1) + (a_0 \times 10^0) + (a_{-1} \times 10^{-1}) + \dots (a_{-m} \times 10^{-m})$

in general, for any radix r , $(a_n \times r^n) + \dots (a_1 \times r^1) + (a_0 \times r^0) + (a_{-1} \times r^{-1}) + \dots (a_{-m} \times r^{-m})$

the unique property of the power series expansion of a number (in any radix r) is that, upon solving back, it always gives the decimal equivalent of the no.

$$\text{eg: } (1101)_2 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$= 8 + 4 + 0 + 1 \\ = (13)_{10}$$

(40)

Note: the r 's complement of a no. in any radix can be obtained by applying the following 3 steps on the no.

- Leave all the least significant 0's unchanged.
- Subtract the first, least significant, non-zero digit from r .
- Subtract all higher significant digits from $(r-1)$.

(41)

Special Case: Binary System (2's complement.)

- Leave all the least significant 0's unchanged.
- Leave all the ^{first} least significant 1 unchanged.
- Change all higher significant 0's to 1's and 1's to 0's.

(42)

Addition of Signed Binary Numbers.

The steps of adding 2 signed binary numbers are as follows:

- The +ve operands are represented in Signed Magnitude System.

- The -ve operands are represented in Signed 2's Complement System.

- The 2 operands, including the Sign Bit, are added.

- A carry, if generated, is discarded.

- The result, if +ve (Sign Bit = 0) is automatically in the Signed Magnitude System.

- The result, if -ve (Sign Bit = 1) is automatically in the Signed 2's Complement System.

Detection of Overflow in Signed 2's Complement System

(43)

$C_4 \ C_3 \ C_2 \ C_1$
 $x_3 \ x_2 \ x_1 \ x_0$
 $\underline{y_3 \ y_2 \ y_1 \ y_0}$
 $s_3 \ s_2 \ s_1 \ s_0$

the following Boolean expressions detect the occurrence of overflow when 2 binary numbers are added in the Signed 2's Comp. System.

$$\Rightarrow C_4 \oplus C_3$$

44

Complement of a fractional me

$$(s_2 - 1) \quad (s_2 - 1) \quad (s_2 - 1) \quad (s_2 - 1) \quad (s_2 - 1)$$

$$+ \left(\begin{array}{cccc} _ & _ & _ & _ \\ _ & _ & _ & _ \end{array} \right) \xrightarrow{(2-i)^{\text{comp}}} \left(\begin{array}{cccc} _ & _ & _ & _ \\ _ & _ & _ & _ \end{array} \right)$$

→ $(2-1)$'s complement : Subtract the individual digits of the number (both integer and fractional parts) from $(2-1)$.

$\rightarrow r$'s complement: First find the $(r-1)$'s complement, then add 1 to the rightmost digit of the fractional part of the $(r-1)$'s complement.

Binary Codes.

BCD (Binary Coded Decimal)

e.g. $\begin{array}{ccc} 7 & 9 & 3 \\ \downarrow & \downarrow & \downarrow \\ 0111 & 1001 & 0011 \end{array}$

→ It is obtained by representing the individual digits of a decimal number in its 4 bit binary equivalent form.

→ It is a 4 bit code.

→ Used / Valid Codes : 0000 to 1001

→ Unused / Invalid Codes : 1010 to 1111

→ It is a Weighted Code. the weights of the bit positions are 8 4 2 1. ∴ BCD code is also known as 8421 BCD code / 8421 code

Note: In a weighted code, each bit position of the code is assigned a decimal number (known as weight); and the decimal value represented by a particular binary sequence in a weighted code is obtained by adding the weights of all those bit positions where a 1 is present in the binary sequence.

$$\text{e.g. } \begin{array}{cccc} 1 & 1 & 0 & 0 \\ \underline{2} & \underline{4} & \underline{2} & \underline{1} \end{array} \Rightarrow 6$$

Weights.

BCD Addition

→ Valid BCD codes are from 0000 to 1001

∴ if the sum of 2 BCD numbers is > 1001 , then it is considered Invalid sum.

→ (Invalid sum) + (0110)₂ gives the (Valid sum)

eg: 5 0101
+ 3 0011
$\frac{8}{1000}$
⇒ Valid sum
9 1001
+ 4 0100
$\frac{13}{1101}$
⇒ Invalid sum
0001 0011
$\frac{1}{3}$
⇒ Valid sum

21 code

It is a 4 bit code. The weights of the bit positions are $2-4-2-1$.

→ It is a Weighted Code.

→ Some decimal numbers can be coded in 2 ways.

$$\text{eg: } 5 = 0101 \\ = 1011$$

→ It is a Self-Complementing Code.

$$\text{eg: } 2206 \rightarrow 1000 \ 0010 \ 0000 \ 1100 \\ \text{9's comp} \uparrow \qquad \qquad \qquad \downarrow \text{1's comp.} \\ 7793 \leftarrow 0111 \ 1101 \ 1111 \ldots 0011$$

Note: In a self-complementing code, if a decimal number D is represented by a binary code B , then the 1's complement of B directly represents the 9's complement of D .

Note: the following codes are weighted as well as self-complementing:

$$\rightarrow 2421 \rightarrow 3321 \rightarrow 4341 \rightarrow 5211 \rightarrow 4221$$

Note: In codes that are weighted + self-complementing, the sum of weights is always 9.

Excess-3 Code/Excess-3 BCD Code

$$\begin{array}{r} \text{eg: } 7 \ 9 \ 3 \\ +3 \quad +3 \quad +3 \\ \hline 10 \quad 12 \quad 6 \\ \downarrow \quad \downarrow \quad \downarrow \\ 1010 \quad 1100 \quad 0110 \end{array}$$

It is obtained by first incrementing the individual digits of a decimal number by 3, and then representing them in their 4 bit binary equivalent form.

→ It is a 4 bit code.

→ Used/Valid Codes. : 0011 to 1100

→ Unused / Invalid Codes : (0000 to 0010) & (1101 to 1111)

→ It is an Unweighted Code.

→ It is a Self-Complementing Code.

Gray Code/Reflected Binary Code

→ Also known as,

→ Minimum Error Code.

→ Cyclic Permutation Code.

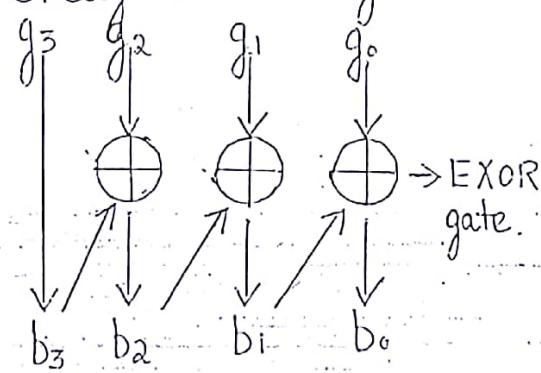
→ ~~It is~~ It is an Unweighted code.

→ It is a Unit Distance code.

If a code is weighted, it may not necessarily be self-complementary vice versa.

Note: In a Unit Distance code, the binary code corresponding to successive decimal numbers differs by only 1 bit.

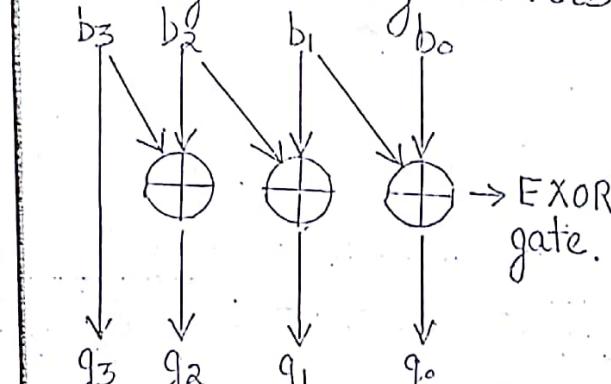
Gray to Binary Conversion



$$\text{eg: } G = 110101$$

$$\Rightarrow B = 100110$$

Binary to Gray Conversion



$$\text{eg: } B = 100101$$

$$\Rightarrow G = 110111$$

Note: the MSB of a gray code and its corresponding binary code is always the same.

Q8. Detecting Code: Parity Bit



→ Parity Bit is an additional bit added to the message bits at the MSB position.

→ It is set to either logic 0 or logic 1 depending upon the no. of 1's in the message bits.

→ 2 types.

→ Even Parity.

→ Odd Parity.

Even Parity: Total no. of 1's in parity bit + message bits is an even no.

$$\text{eg: } 1101 \rightarrow \underline{1} \quad 1101$$

$$1001 \rightarrow \underline{0} \quad 1001$$

↓
Parity Bit

Odd Parity: Total no. of 1's in parity bit + message bits is an odd no.

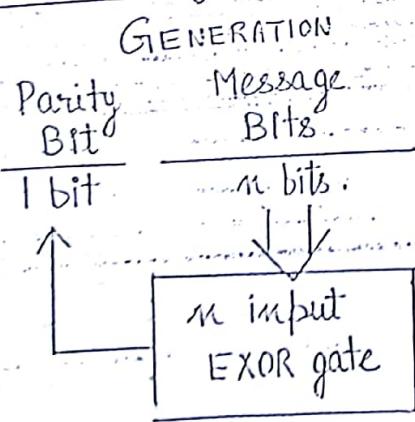
$$\text{eg: } 1101 \rightarrow \underline{0} \quad 1101$$

$$1001 \rightarrow \underline{1} \quad 1001$$

↓
Parity Bit

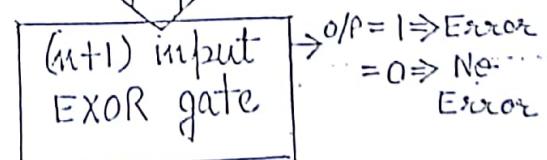
Note: Parity Bit (both even and odd parity) can detect only odd no. of errors.

Even Parity: Generation and Detection



DETECTION

Parity Bit	Message Bits
(n+1) bits	



ASCII (American Standard Code for Information Interchange)

→ Alphanumeric Code.

→ 7 bit code \Rightarrow 128 characters.

 | \rightarrow Graphical characters. (94)

 | \rightarrow Control characters. (34)

(Arrow, Caps Lock, etc.)

Extended ASCII

→ 8 bit code

X

46

→ Memory Size = $2^k \times n$ bits.

→ the n data lines transfer data to/from one complete word at a time.

→ the binary address is applied on the k address lines. An internal $k \times 2^k$ decoder selects one of the 2^k words.

→ Read / Write = 0 \Rightarrow Data Write operation.
 = 1 \Rightarrow Data Read operation.

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Types of RAM

SRAM (Static RAM)

→ Data is stored in flip flops.

→ Periodic Recharging is not required.

∴ faster in operation (Advantage)

→ Implemented using both BJT and MOS transistors.

DRAM (Dynamic RAM)

→ Data is stored in the form of charge on capacitor.

→ Periodic Recharging (Refreshing) is required

to restore the charge on capacitor

∴ slower in operation (Disadv)

→ Implemented using only MOS transistors.