Graph Data Structures

Mayank Pratap Singh

August 29, 2024

1 Introduction to Graphs

A graph is a collection of nodes (vertices) connected by edges. The nodes can represent any object, and the edges represent the connections or relationships between these objects.

2 Trees vs. Graphs

2.1 Tree

- A special type of graph with no cycles.
- Hierarchical and connected, with a root node and child nodes.
- Traversal Methods: Preorder, Inorder, Postorder (all Depth-First Traversals).

2.2 Graph

- Can have cycles and multiple connections between nodes.
- No root node, and the connections can be in any form.
- Traversal Methods: Depth-First Search (DFS), Breadth-First Search (BFS).

3 Key Differences

Feature	Tree	Graph
Structure	Hierarchical	Network-like
Root Node	Has a root node	No root node
Cycles	No cycles (acyclic)	Can have cycles
Edges	Represents parent-child	Represents any connection
Connectedness	Always connected	Can be connected or disconnected
Applications	Hierarchical data (e.g., files)	Networks, social connections, etc.
Traversal	Preorder, Inorder, Postorder	DFS, BFS

4 Applications of Graphs

- **Social Networks:** Users are represented as nodes and connections as edges.
- **Web Page Link Structure:** Web pages are nodes, hyperlinks are edges.
- **Road Networks:** Intersections are nodes, roads are edges.
- **Recommendation Systems:** Users and items are nodes, interactions are edges.
- **Computer Networks:** Devices are nodes, connections are edges.

5 Simple Graph vs. Multigraph

Feature	Simple Graph	Multigraph
Self Loops	Not allowed	Allowed
Parallel Edges	Not allowed	Allowed
Example Application	Social Networks (no self-loops)	Special cases like certain algorithms

6 Graph Terminology

- **Degree of a Vertex:** Number of edges connected to a vertex.
- **Maximum Degree:** $n - 1$
- **Minimum Degree:** 0
- **Complete Graph:** A graph where every node is connected to every other node.
- **Degree Calculation:** $n(n-1)/2$

7 Graph Properties

- **Sum of Degrees:** $2 \times$ number of edges.
- **Edge Count:** $E = \frac{n(n-1)}{2}$
- **Logarithmic Relationship:** $\log E \approx \log V$

8 Traversal Algorithms

8.1 Depth-First Traversal (DFT)

Uses a stack (or recursion) to explore as far as possible along each branch before backtracking.

8.2 Breadth-First Traversal (BFT)

Uses a queue to explore all neighbors of a node before moving to the next level.

9 Practical Applications

- Google Maps uses graphs to represent roads and intersections.
- Social Networks use graphs to represent connections between people.
- Recommendation Systems use bipartite graphs to connect users and items.

10 Recursion in Trees

Recursion is used in tree traversal methods like Inorder, Preorder, and Postorder.

11 Advanced Graph Concepts

- **Null Graph:** A graph with no edges.
- **Complete Graph:** A graph in which each pair of vertices is connected by an edge.