K-Map

Karnaugh Map (K-Map)

- a tool for simplification of Boolean algebra
- is made up of squares
- is a graphical representation of SOP (Minterm)
- extensively reduces the calculation and provides best minimized solution
- solves the expression with grouping of neighbor cells

In choosing adjacent squares in a map, we must ensure that:

(1) all the minterms of the function are covered when we combine the squares,

(2) the number of terms in the expression is minimized, and

(3) there are no redundant terms (i.e., minterms already covered by other terms).

Karnaugh Map (K-Map)

- 2 variable
- 3 variable
- 4 variable

K-Map Simplification Rule

1) Construct K-Map and place 1s in the squares according to the truth table.
2) Groupings can contain only 1s.
3) Groups can be formed only at right angles; diagonal groups are not allowed.
4) The number of 1's in a group must be a power of 2.
5) The groups must **be made as large** as possible.
6) Groups can overlap and wrap around the sides of the Kmap.
7) Every group puts a term in the solution.

Optimized Solution
Minimum number of group
Each group covers maximum possible squares

KMAP

A\B	0	1
0	m_0	m_1
1	m_2	m_3

2 variable

A\BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

3 variable

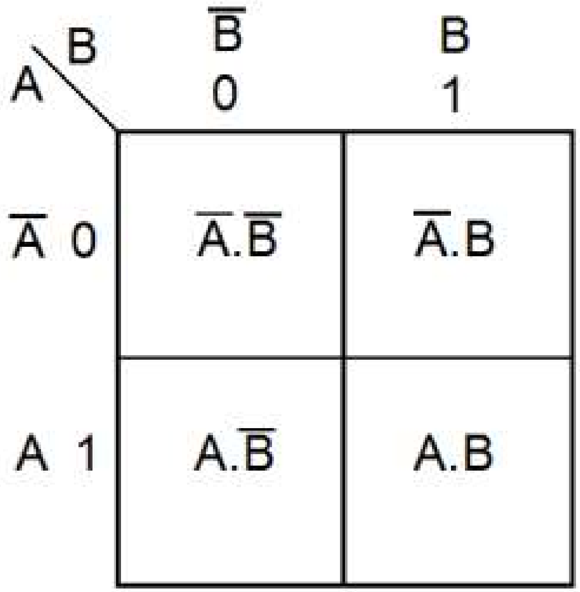
AB\CD	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m10

4 variable

2 Variable K-map

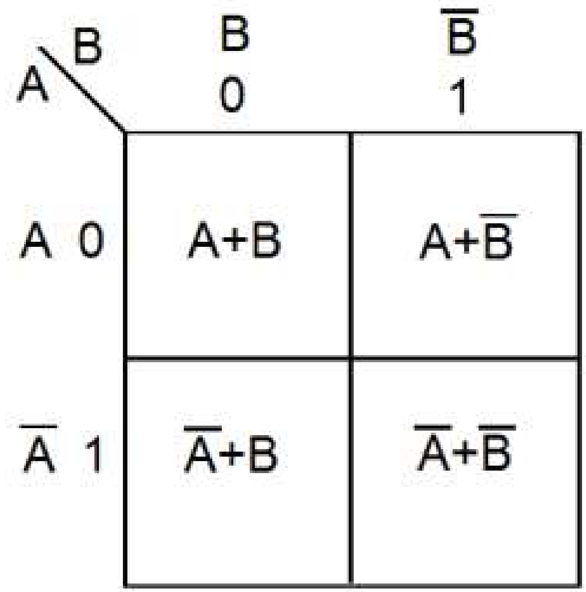
- There are four minterms for two variables; hence, the map consists of four squares, one for each minterm.
- The map (b) shows the relationship between the squares and the two variables x and y.
- The 0 and 1 marked in each row and column designate the values of variables.
- Variable x appears primed in row 0 and unprimed in row 1. Similarly, y appears primed in column 0 and unprimed in column 1.

A. SOP: -

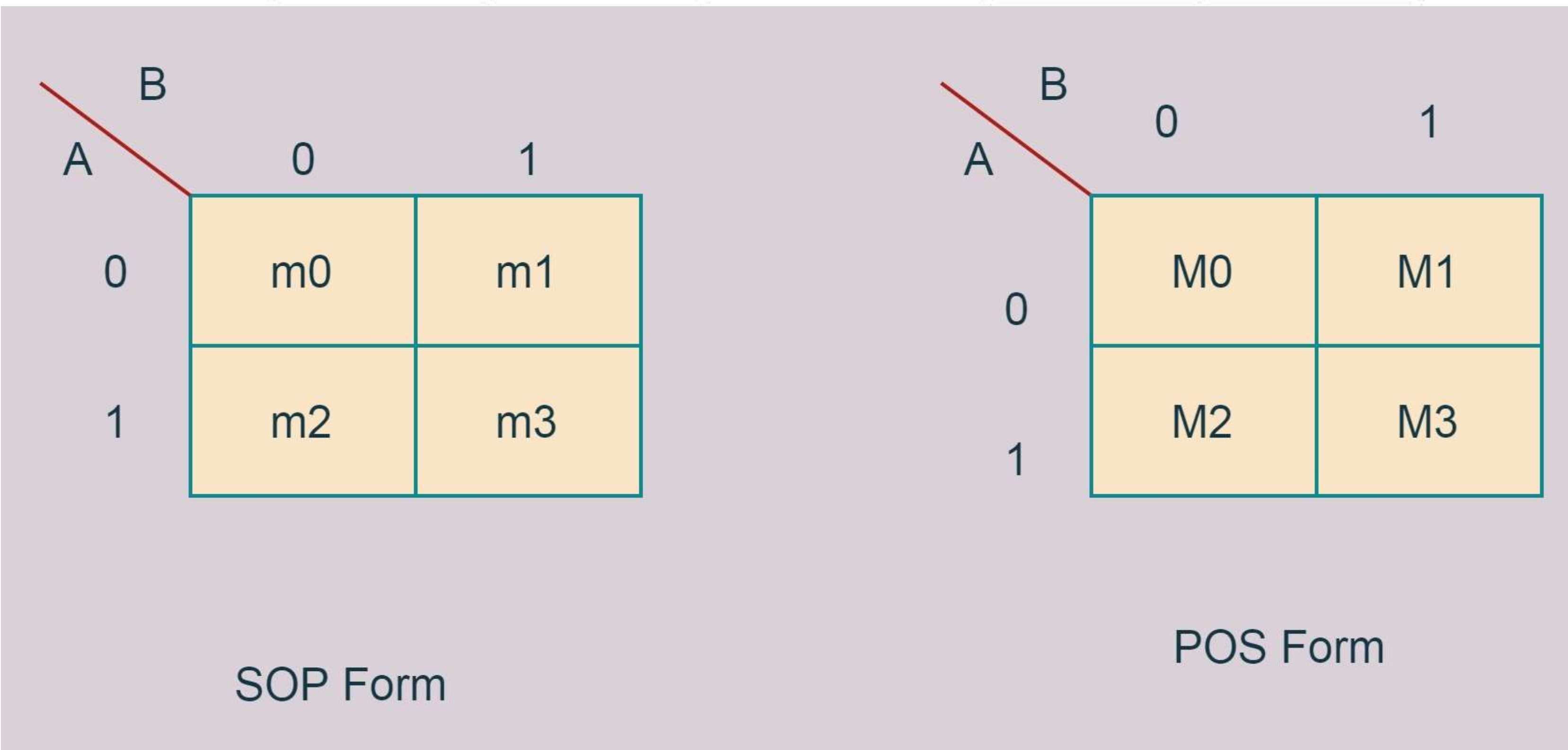


A
B
$\overline{B}$ 0
B 1
$\overline{A}$ 0
A 1
$\overline{A}.\overline{B}$
$\overline{A}.B$
$A.\overline{B}$
A.B

B. POS: -

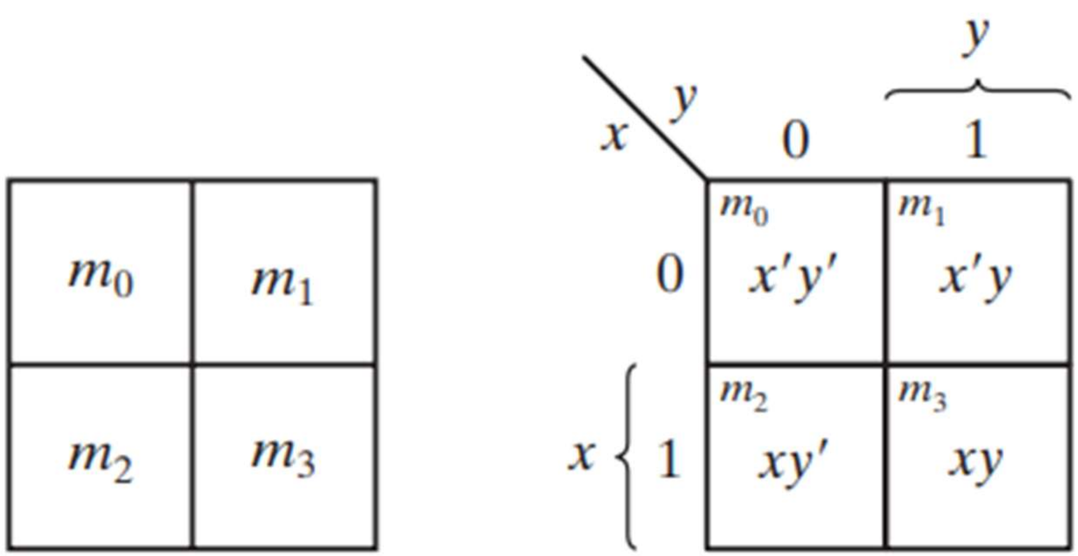


A
B
B 0
$\overline{B}$ 1
A 0
$\overline{A}$ 1
A+B
$A+\overline{B}$
$\overline{A}+B$
$\overline{A}+\overline{B}$

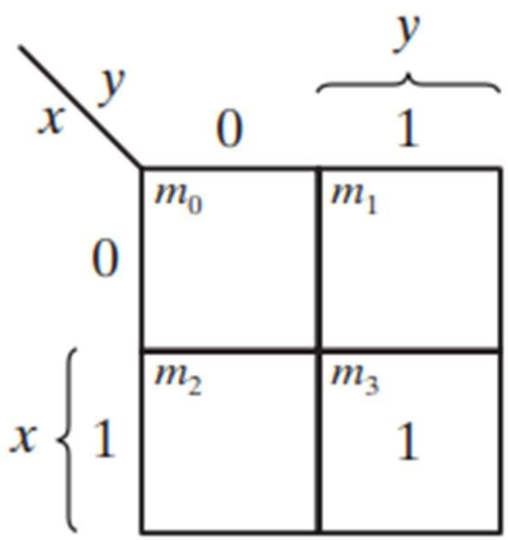


A
B
0
1
m0
m1
m2
m3
SOP Form
A
B
0
1
M0
M1
M2
M3
POS Form

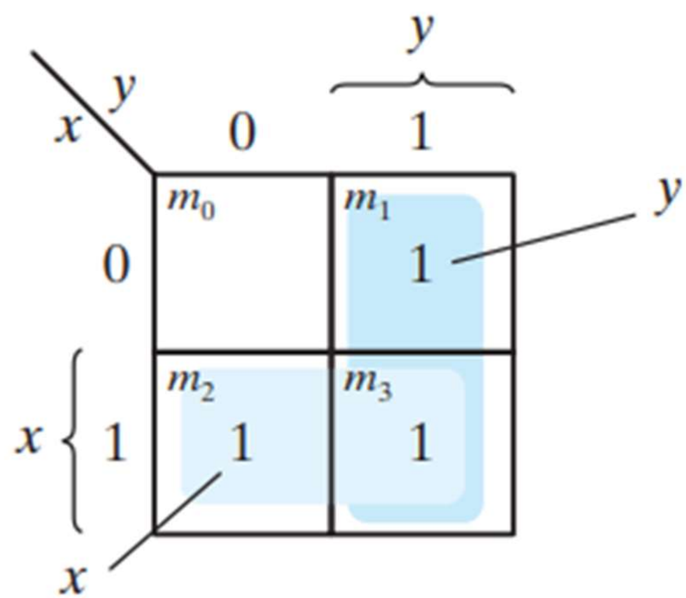
2 Variable K-map



(a) (b)



(a) xy

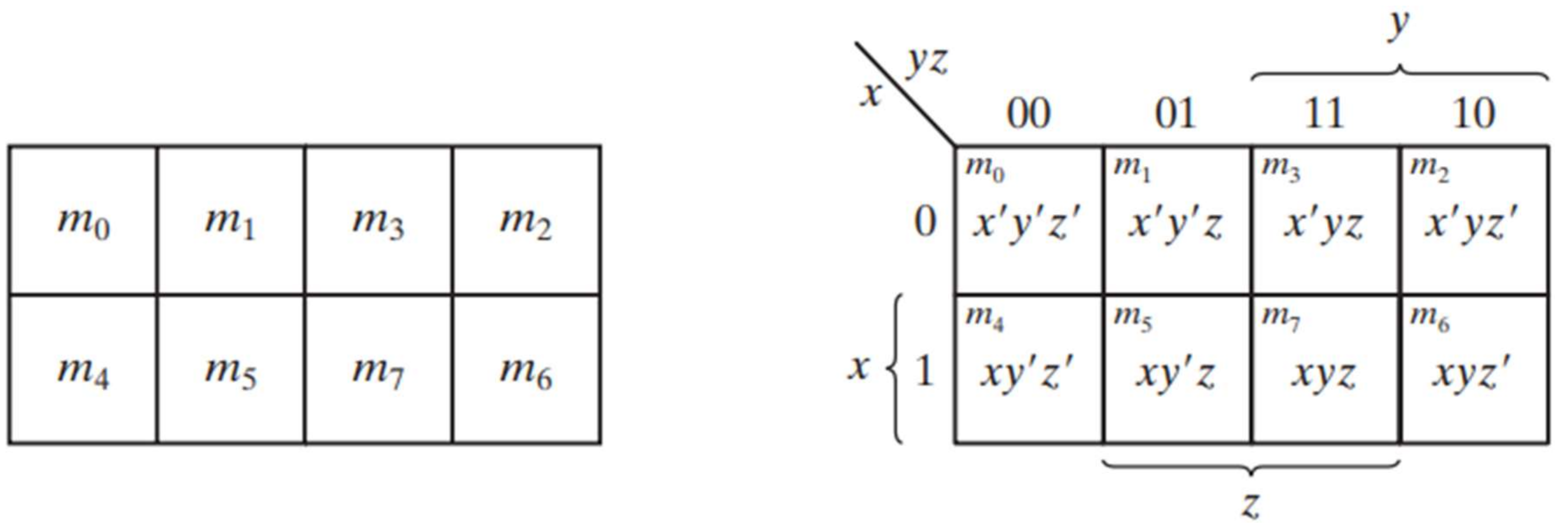


(b) $x + y$

$$m_1 + m_2 + m_3 = x'y + xy' + xy = x + y$$

3 Variable K-map

- There are eight minterms for three binary variables; therefore, the map consists of eight squares.
- Note that the minterms are arranged, not in a binary sequence, but in a sequence similar to the Gray code.
- The characteristic of this sequence is that only one bit changes in value from one adjacent column to the next.
- Any two adjacent squares in the map differ by only one variable.



m_0 m_1 m_3 m_2
m_4 m_5 m_7 m_6
yz
x
y
00 01 11 10
0
m_0 $x'y'z'$
m_1 $x'y'z$
m_3 $x'yz$
m_2 $x'yz'$
x 1
m_4 $xy'z'$
m_5 $xy'z$
m_7 xyz
m_6 xyz'
z

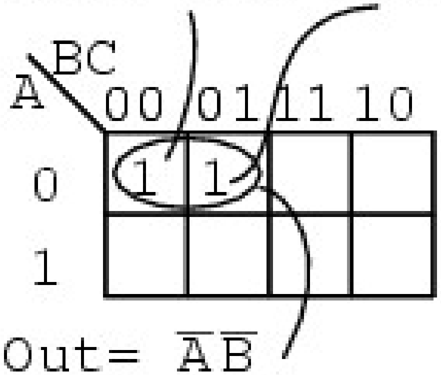
(a) (b)

Pairing of m5 and m7

$$m_5 + m_7 = xy'z + xyz = xz(y' + y) = xz$$

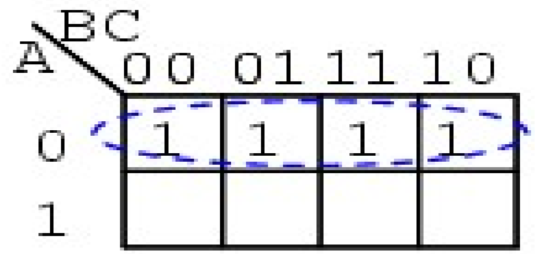
One square represents one minterm, giving a term with three literals.
Two adjacent squares represent a term with two literals.
Four adjacent squares represent a term with one literal.
Eight adjacent squares encompass the entire map and produce a function that is always equal to 1.

$$\text{Out} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C$$

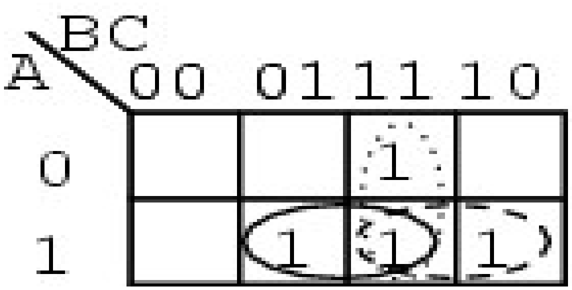


$$\text{Out} = \overline{A}\,\overline{B}$$

$$\text{Out} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$$

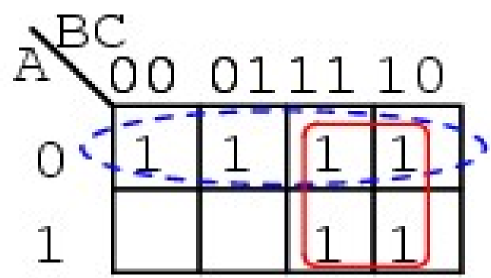


$$\text{Out} = \overline{A}$$



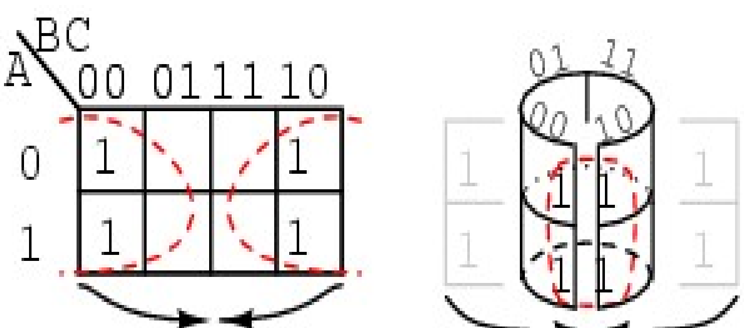
Output = AB + BC + AC

$$\text{Out} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + ABC + A\,B\,\overline{C}$$



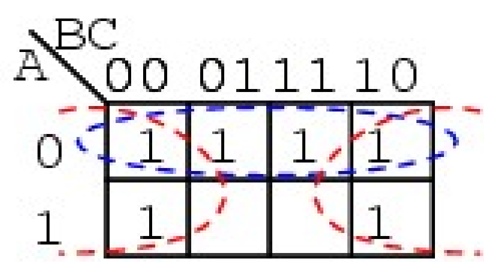
$$\text{Out} = \overline{A} + B$$

$$\text{Out} = \overline{A}\,\overline{B}\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\,B\,\overline{C}$$



$$\text{Out} = \overline{C}$$

$$\text{Out} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$



$$\text{Out} = \overline{A} + \overline{C}$$

Simplify the Boolean function

$$F(x, y, z) = \Sigma(2, 3, 4, 5)$$

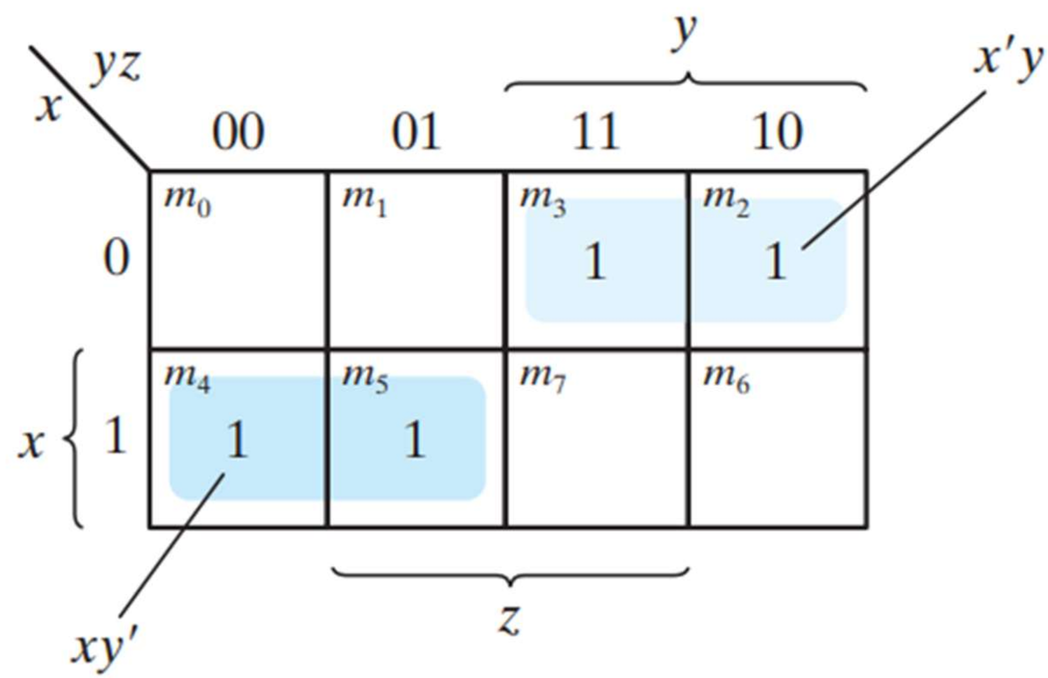


FIGURE 3.4
Map for Example 3.1, $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

$$m_0 + m_2 = x'y'z' + x'yz' = x'z'(y' + y) = x'z'$$

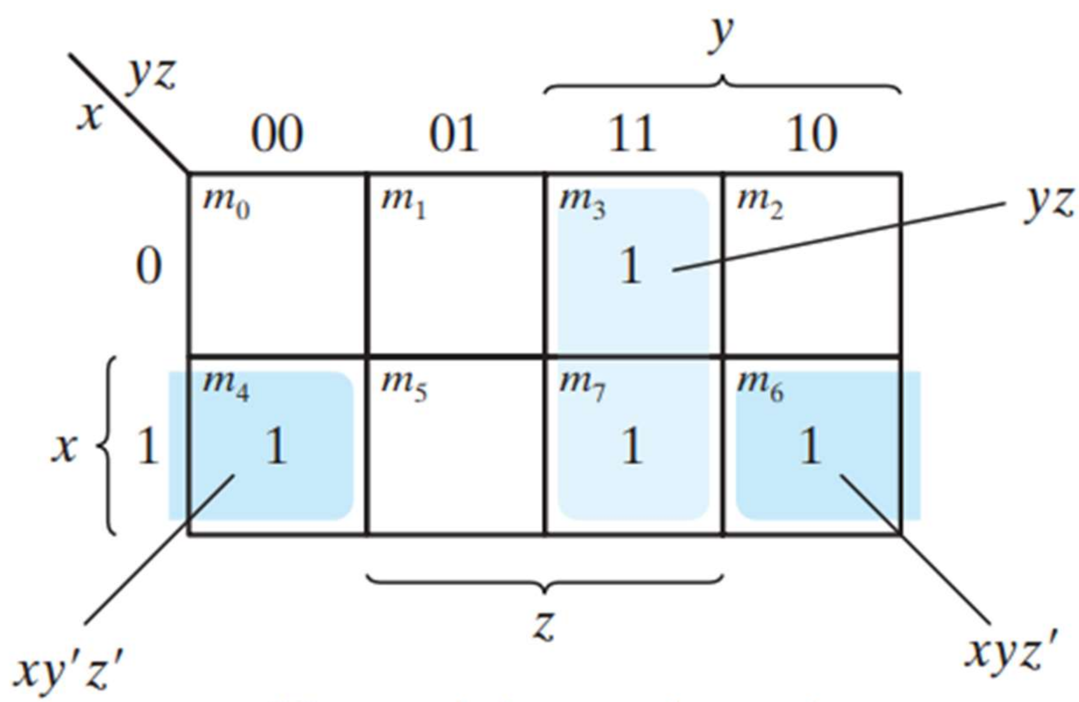
$$m_4 + m_6 = xy'z' + xyz' = xz' + (y' + y) = xz'$$

$$F = x'y + xy'$$

Simplify the Boolean function

$$F(x, y, z) = \Sigma(3, 4, 6, 7)$$

$$F = yz + xz'$$



Note: $xy'z' + xyz' = xz'$

FIGURE 3.5
Map for Example 3.2, $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

Consider now any combination of four adjacent squares in the three-variable map. Any such combination represents the logical sum of four minterms and results in an expression with only one literal. As an example, the logical sum of the four adjacent minterms 0, 2, 4, and 6 reduces to the single literal term z':

$$
\begin{aligned}
m_0 + m_2 + m_4 + m_6 &= x'y'z' + x'yz' + xy'z' + xyz' \\
&= x'z'(y' + y) + xz'(y' + y) \\
&= x'z' + xz' = z'(x' + x) = z'
\end{aligned}
$$

The number of adjacent squares that may be combined must always represent a number that is a power of two, such as 1, 2, 4, and 8. As more adjacent squares are combined, we obtain a product term with fewer literals.

Simplify the Boolean function

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

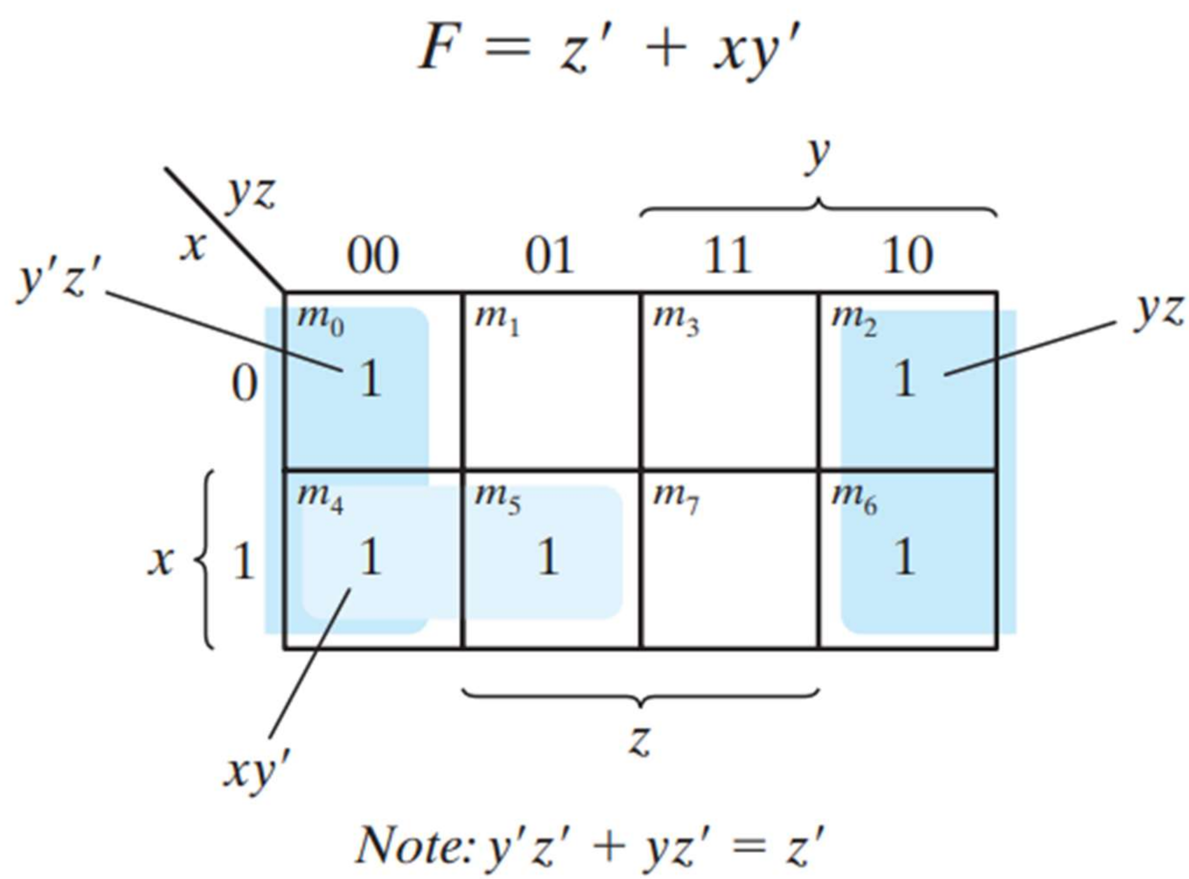


FIGURE 3.6
Map for Example 3.3, $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$

For the Boolean function

$$F = A'C + A'B + AB'C + BC$$

(a) Express this function as a sum of minterms.

(b) Find the minimal sum-of-products expression.

$$F(A, B, C) = \Sigma(1, 2, 3, 5, 7)$$

The sum-of-products expression, as originally given, has too many terms. It can be simplified, as shown in the map, to an expression with only two terms:

$$F = C + A'B$$

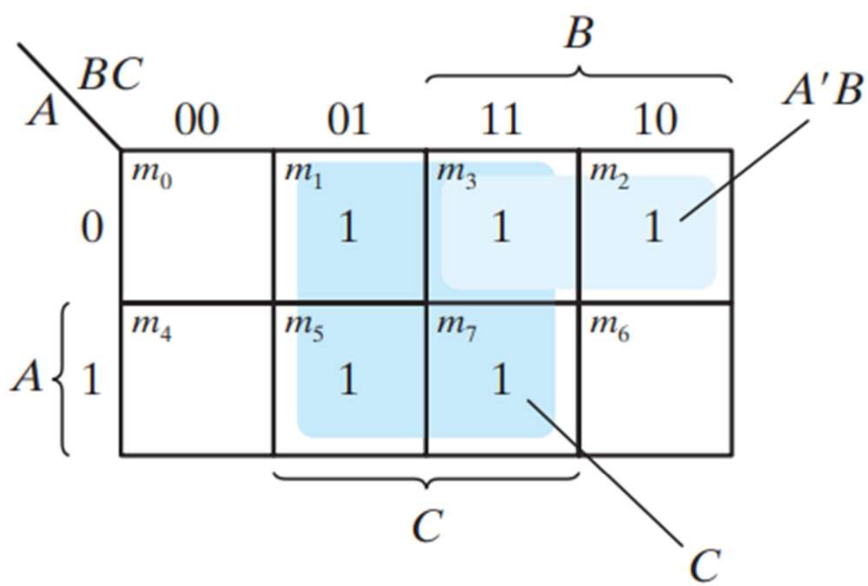


FIGURE 3.7
Map of Example 3.4, $A'C + A'B + AB'C + BC = C + A'B$

4 Variable K-map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

$wx \backslash yz$	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$

y (columns 11, 10); x (rows 01, 11); w (rows 11, 10); z (columns 01, 11)

(b)

FOUR VARIABLE K-MAP

AB \ CD		C'D' 00	C'D 01	CD 11	CD' 10
A'B'	00	A'B'C'D' 0	A'B'C'D 1	A'B'CD 3	A'B'CD' 2
A'B	01	A'BC'D' 4	A'BC'D 5	A'BCD 7	A'BCD' 6
AB	11	ABC'D' 12	ABC'D 13	ABCD 15	ABCD' 14
AB'	10	AB'C'D' 8	AB'C'D 9	AB'CD 11	AB'CD' 10

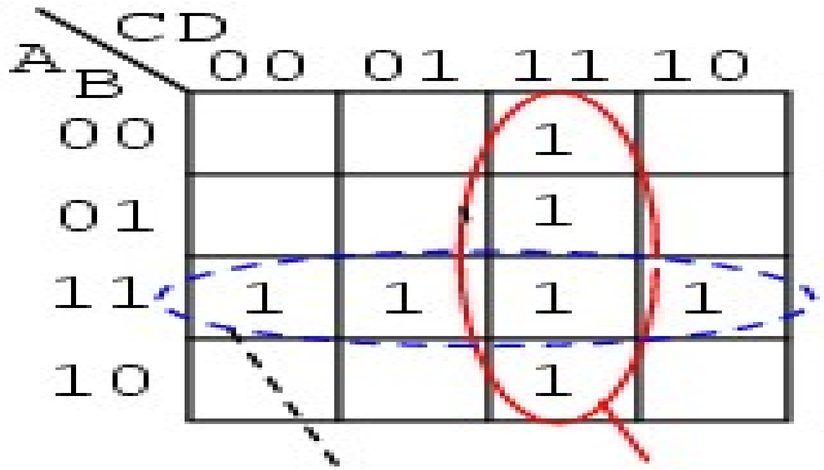
SOP(MINTERMS)

AB \ CD		C+D 00	C+D' 01	C'+D' 11	C'+D 10
A + B	00	A+B+C+D 0	A+B+C+D' 1	A+B+C'+D' 3	A+B+C'+D 2
A + B'	01	A+B'+C+D 4	A+B'+C+D' 5	A+B'+C'+D' 7	A+B'+C'+D 6
A'+B'	11	A'+B'+C+D' 12	A'+B'+C+D' 13	A'+B'+C'+D' 15	A'+B'+C'+D 14
A'+B	10	A'+B+C+D 8	A'+B+C+D' 9	A'+B+C'+D' 11	A'+B+C'+D 10

POS(MAXTERMS)

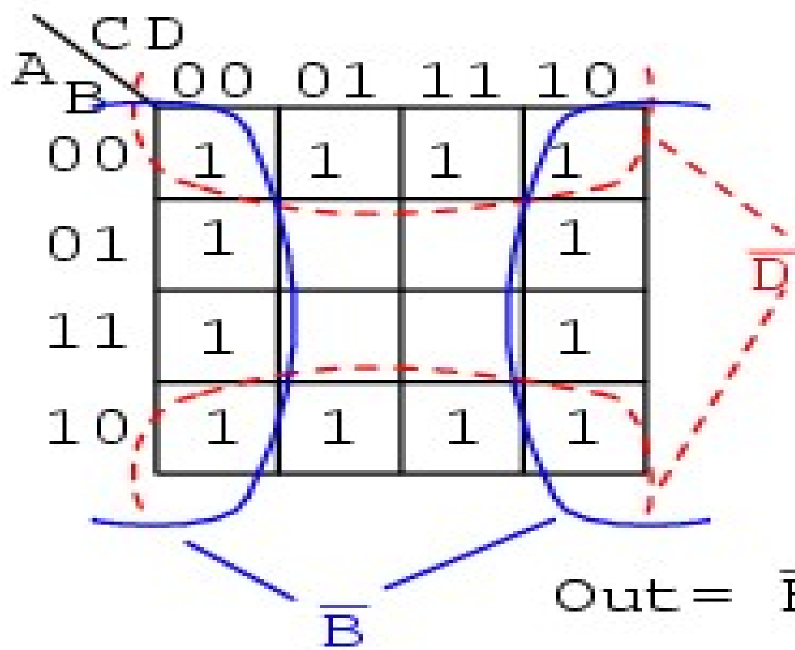
One square represents one minterm, giving a term with four literals.
Two adjacent squares represent a term with three literals.
Four adjacent squares represent a term with two literals.
Eight adjacent squares represent a term with one literal.
Sixteen adjacent squares produce a function that is always equal to 1.

$$Out = \bar{A}\bar{B}CD + \bar{A}BCD + ABCD + A\bar{B}CD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D}$$



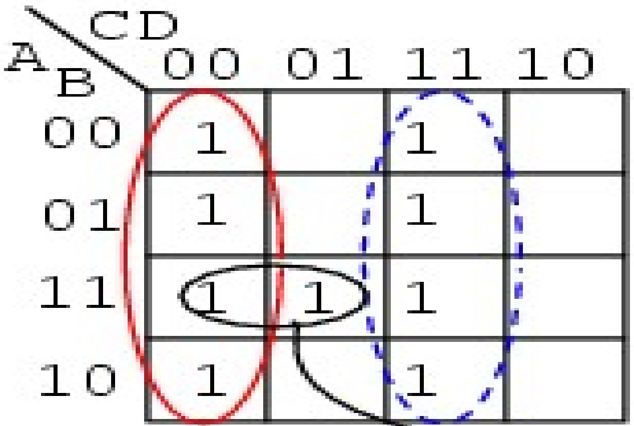
$$Out = AB + CD$$

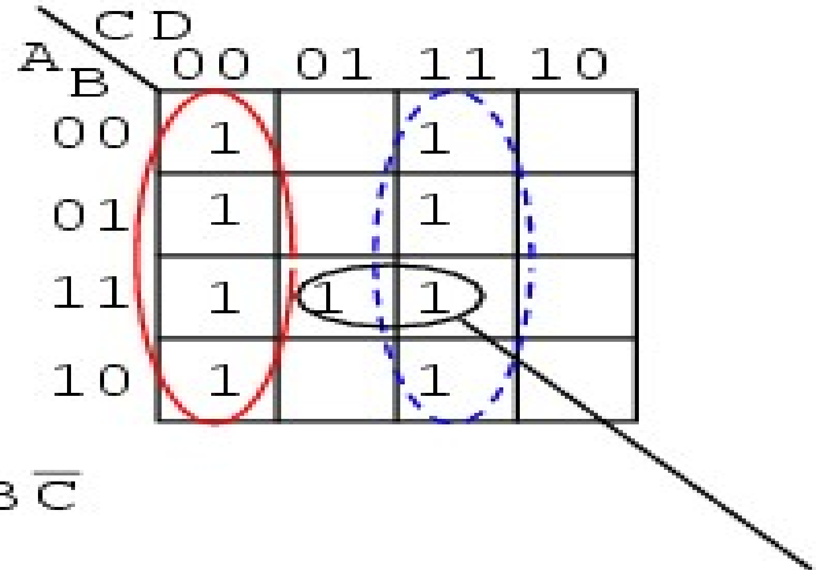
$$Out = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$$
$$+ B\bar{C}\bar{D} + BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}D + A\bar{B}C\bar{D}$$



$$Out = \bar{B} + \bar{D}$$

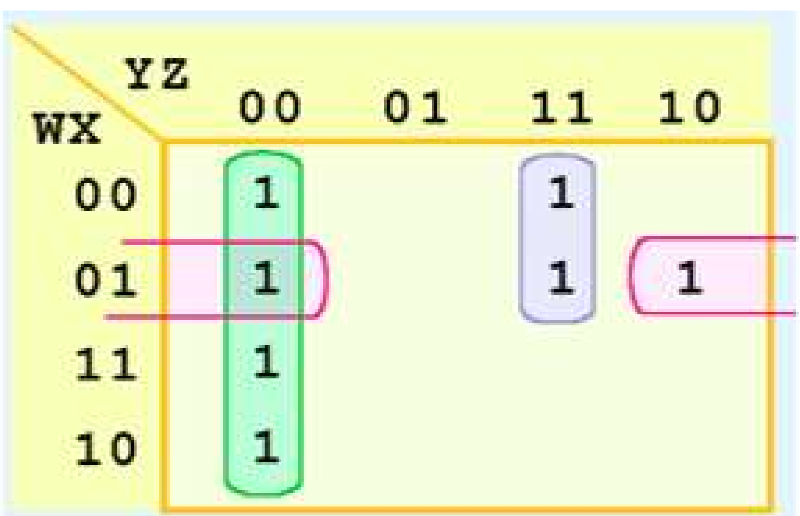
$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + \overline{A}BCD + AB\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD$$

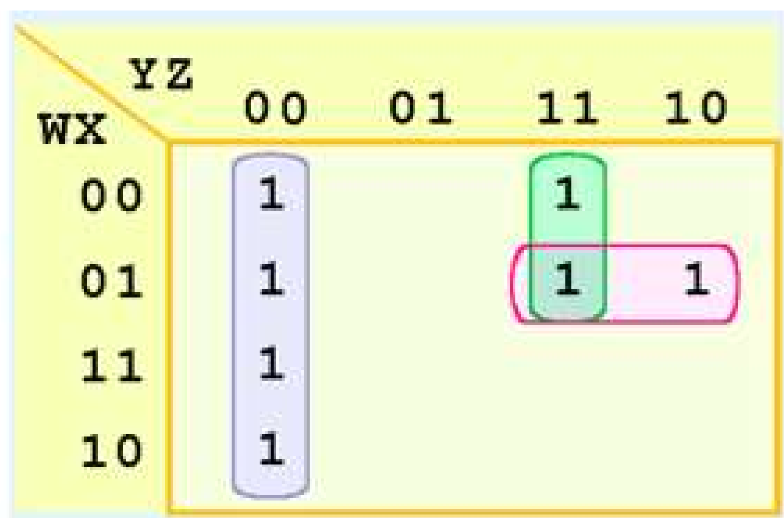




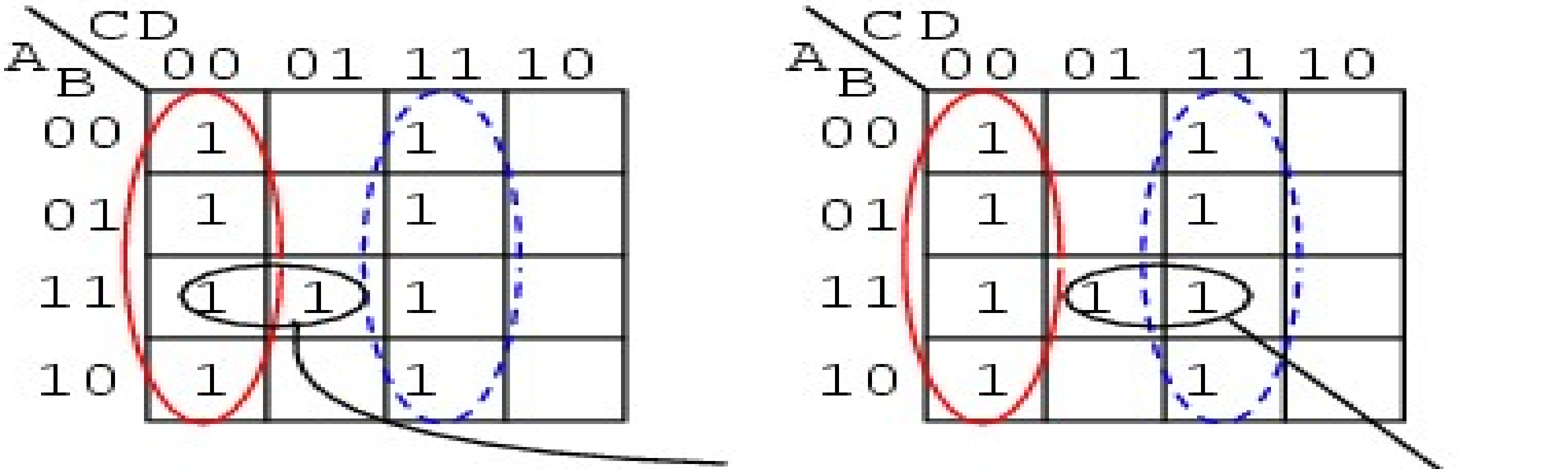
$$Out = \overline{C}\,\overline{D} + CD + AB\overline{C}$$

$$Out = \overline{C}\,\overline{D} + CD + ABD$$



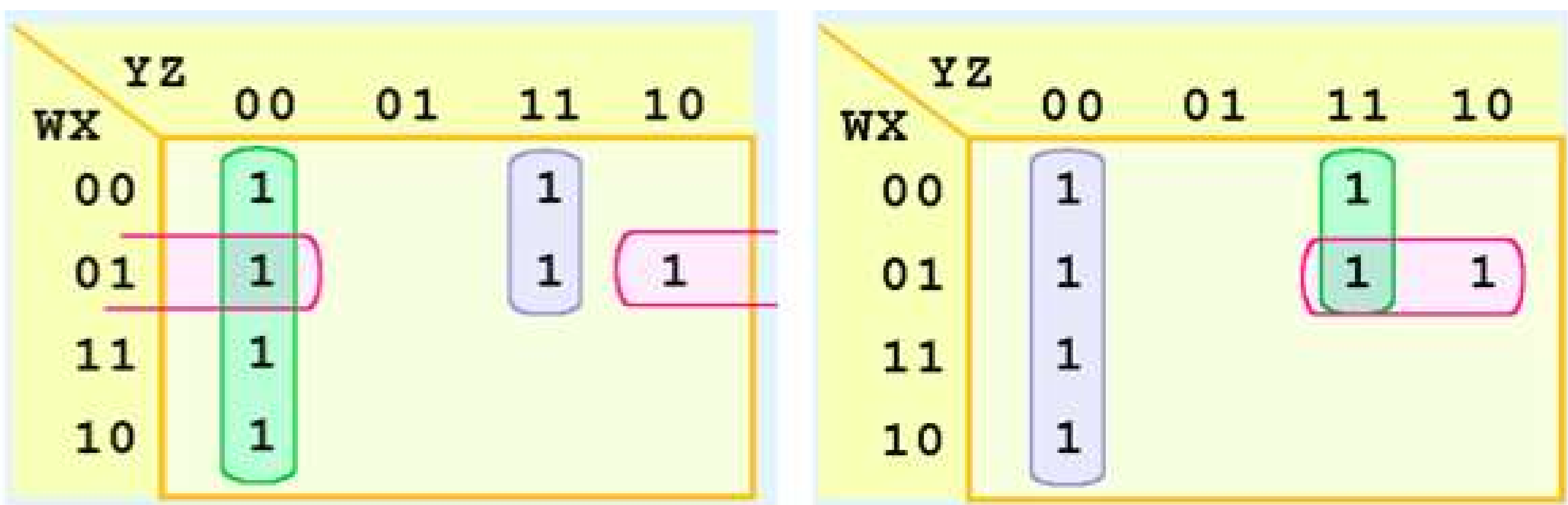


$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,C\,D + A\,B\,\overline{C}\,\overline{D}$$
$$+ A\,B\,\overline{C}\,D + A\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,D$$



$$Out = \overline{C}\,\overline{D} + CD + AB\overline{C}$$

$$Out = \overline{C}\,\overline{D} + CD + ABD$$



$$\text{Out} = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D$$
$$+ \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + \overline{A}\,B\,C\,D$$
$$+ A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,C\,D$$

$$f(A,B,C,D) = \sum m(0,1,3,4,5,7,12,13,15)$$

$$f(A,B,C,D) = \prod M(2,6,8,9,10,11,14)$$

AB \ CD	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	1	1	1	0
10	0	0	0	0

$$\overline{A}\,\overline{C} + \overline{A}D + B\overline{C} + BD$$

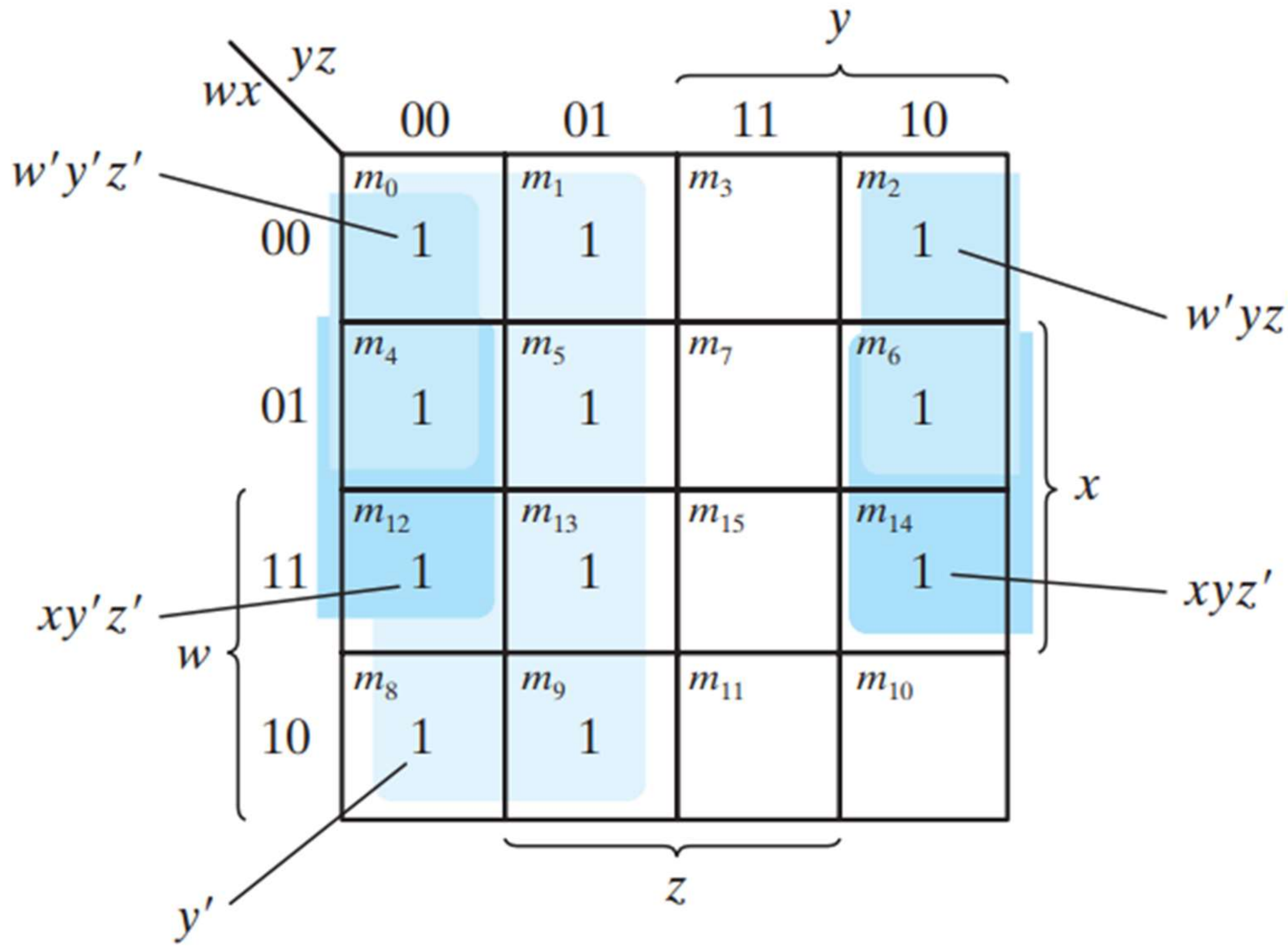
AB \ CD	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	1	1	1	0
10	0	0	0	0

$$f(A,B,C,D) = (\overline{A}+B)(\overline{C}+D)$$

Simplify the Boolean function

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

$$F = y' + w'z' + xz'$$



Note: $w'y'z' + w'yz' = w'z'$

$xy'z' + xyz' = xz'$

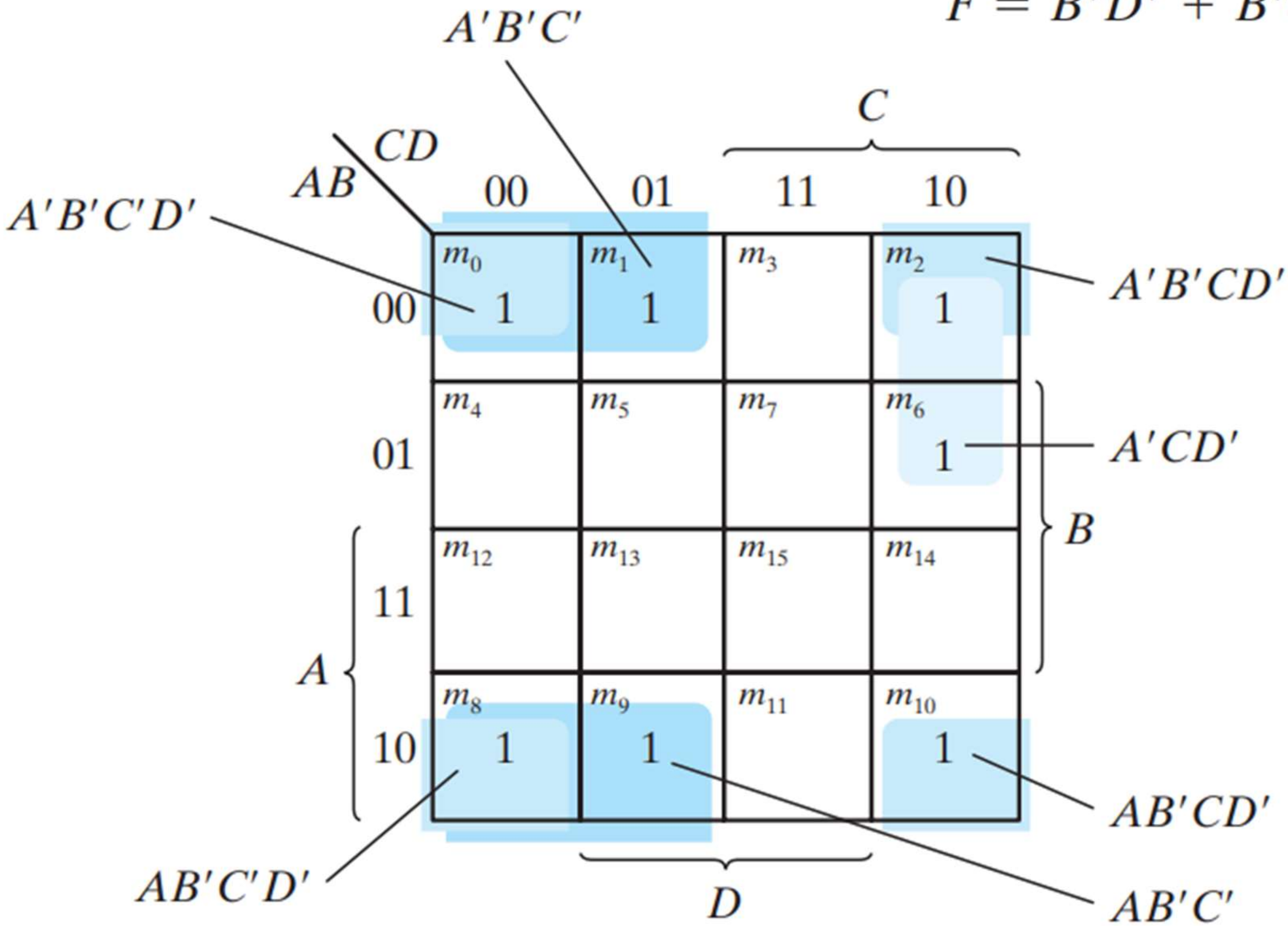
FIGURE 3.9

Map for Example 3.5, $F(w, x, y, z) = \Sigma(0,1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'$

Simplify the Boolean function

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

$$F = B'D' + B'C' + A'CD'$$



$$
\begin{aligned}
\textit{Note: } A'B'C'D' + A'B'CD' &= A'B'D' \\
AB'C'D' + AB'CD' &= AB'D' \\
A'B'D' + AB'D' &= B'D' \\
A'B'C' + AB'C' &= B'C'
\end{aligned}
$$

FIGURE 3.10

Map for Example 3.6, $A'B'C' + B'CD' + A'BCD' + AB'C' = B'D' + B'C' + A'CD'$

Quiz Time

How many cells are present in KMAP of 3 variable function?

A. 3
B. 4
C. 8
D. 16

Quiz Time

How many cells are present in KMAP of 3 variable function?

A. 3

B. 4

C. 8 ✓

D. 16

K-Map with Don’t Care

- In a K-Map, a don’t care condition is identified by an *X* in the cell of the minterm(s).
- In simplification, we are free to include or ignore the *X*’s when creating our groups.

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

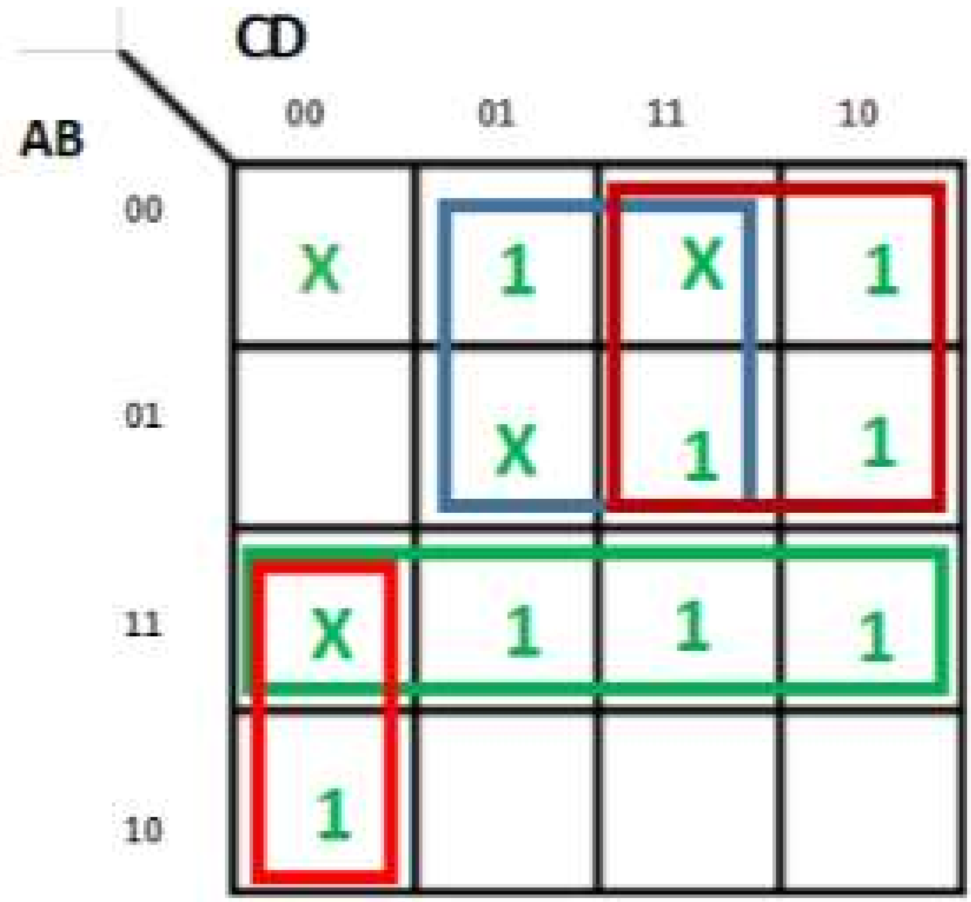
$$F(W,X,Y,Z) = \overline{W}\,\overline{Y} + YZ$$

K-Map with Don’t Care

$$F(A, B, C, D) = m(1, 2, 6, 7, 8, 13, 14, 15) + d(0, 3, 5, 12)$$

Kmap with Don't Care

$$F(A, B, C, D) = m(1, 2, 6, 7, 8, 13, 14, 15) + d(0, 3, 5, 12)$$



Prime implicants

- Sometimes there may be two or more expressions that satisfy the simplification criteria. The procedure for combining squares in the map may be made more systematic if we understand the meaning of two special types of terms.
- A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map.
- If a minterm in a square is covered by only one prime implicant, that prime implicant is said to be essential.

- The prime implicants of a function can be obtained from the map by combining all possible maximum numbers of squares.
- This means that a single 1 on a map represents a prime implicant if it is not adjacent to any other 1's.

- Two adjacent 1’s form a prime implicant, provided that they are not within a group of four adjacent squares.
- Four adjacent 1’s form a prime implicant if they are not within a group of eight adjacent squares, and so on.
- The essential prime implicants are found by looking at each square marked with a 1 and checking the number of prime implicants that cover it.
- The prime implicant is essential if it is the only prime implicant that covers the minterm.