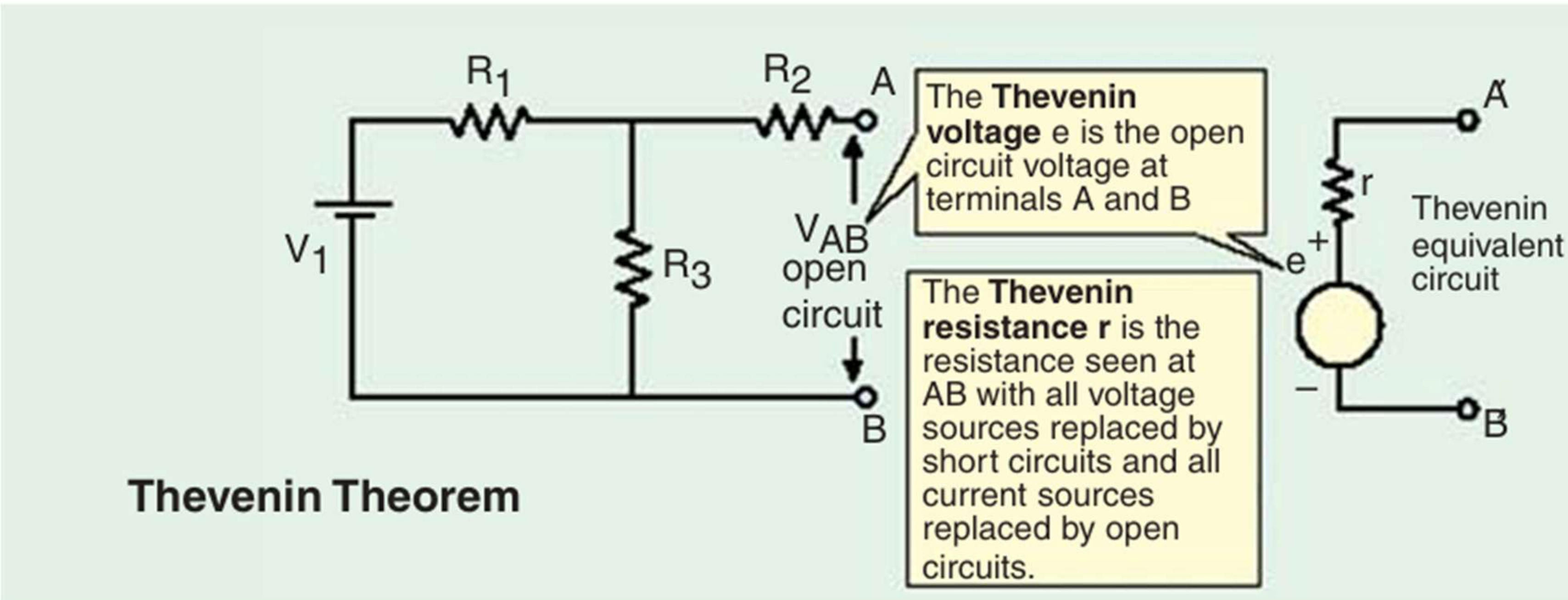
Thevenin's Theorem

- Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thevenin Theorem



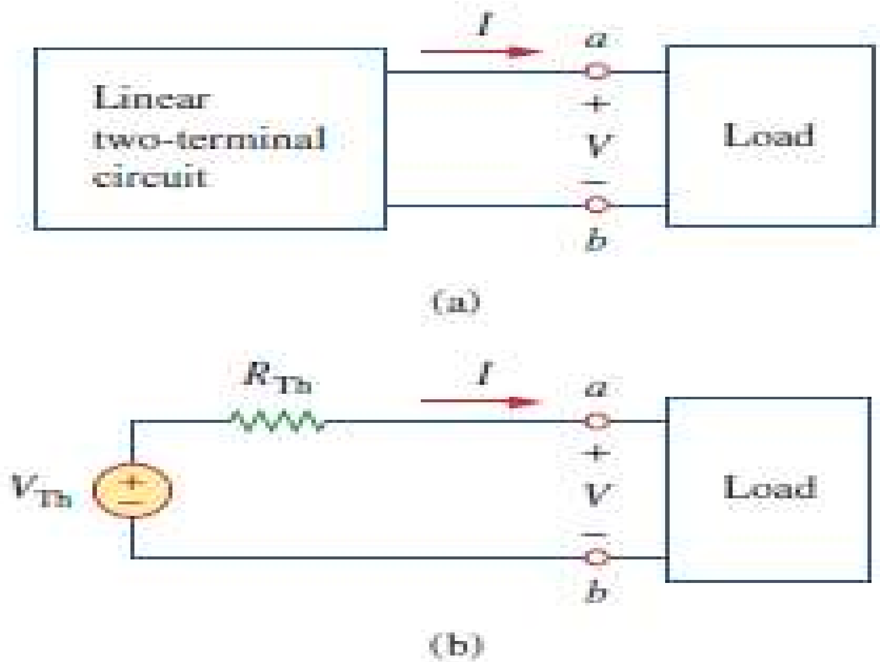
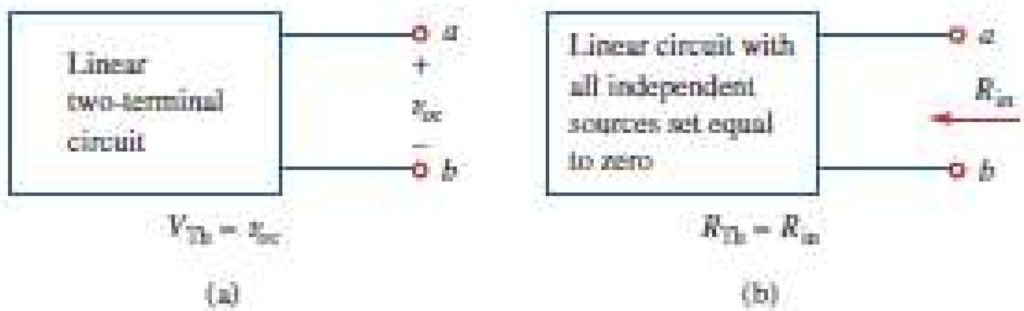


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

$$V_{Th} = v_{oc} \quad \textbf{(4.6)}$$

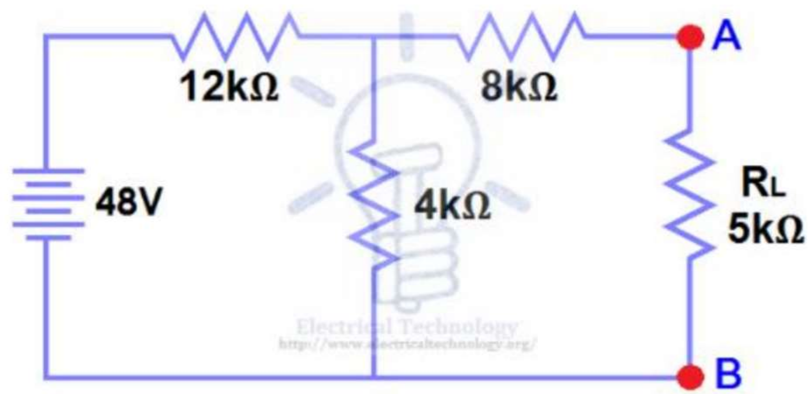


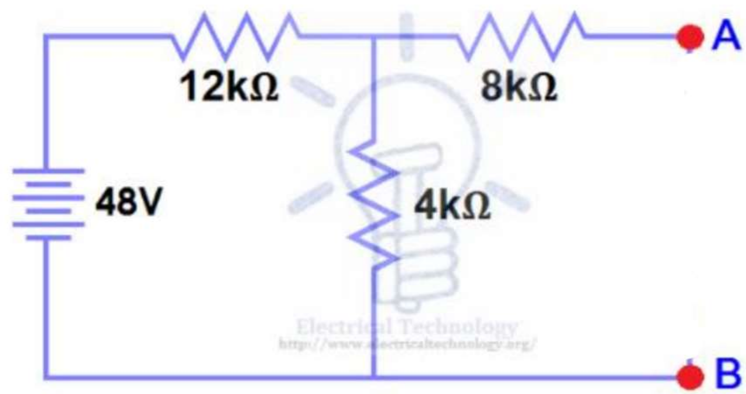
How to Thevenize a Given Circuit ?

1. Temporarily remove the resistance (called load resistance R_L) whose current is required.
2. Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th}.
3. Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance R_{th} or T_i.
4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i.
5. Connect R_L back to its terminals from where it was previously removed.
6. Finally, calculate the current flowing through R_L by using the equation,

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

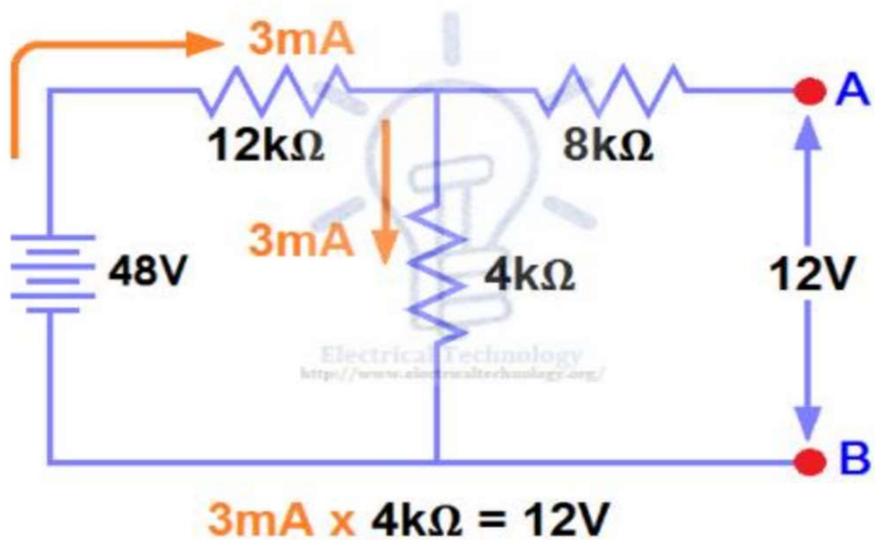
Q1. **Find V_{TH}, R_{TH}** and the load current I_L flowing through and load voltage across the load resistor in fig (1) *by using Thevenin's Theorem.*



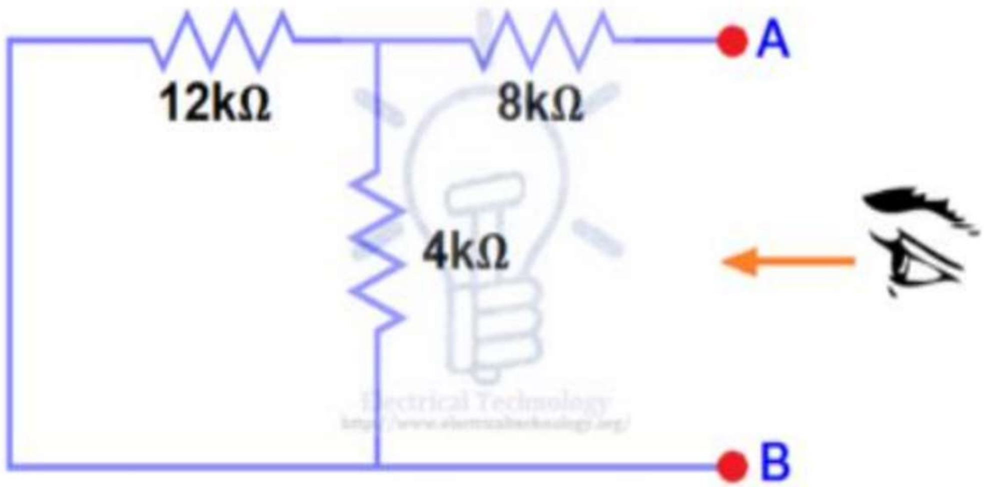


12kΩ
8kΩ
A
48V
4kΩ
B
Electrical Technology
http://www.electricaltechnology.org/

$V_{TH} = 12V$

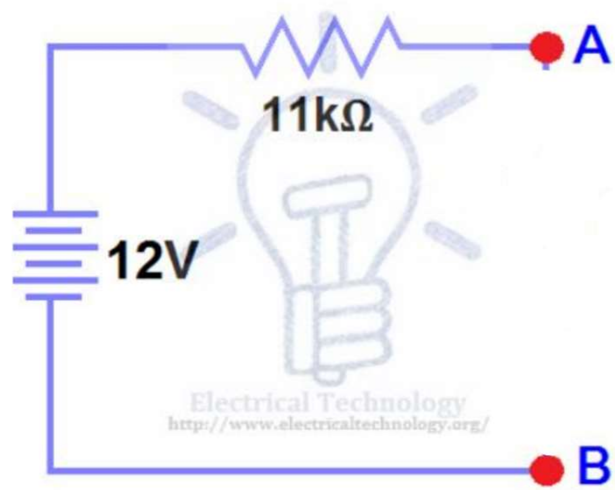


3mA
12kΩ
8kΩ
A
3mA
48V
4kΩ
12V
B
Electrical Technology
3mA x 4kΩ = 12V

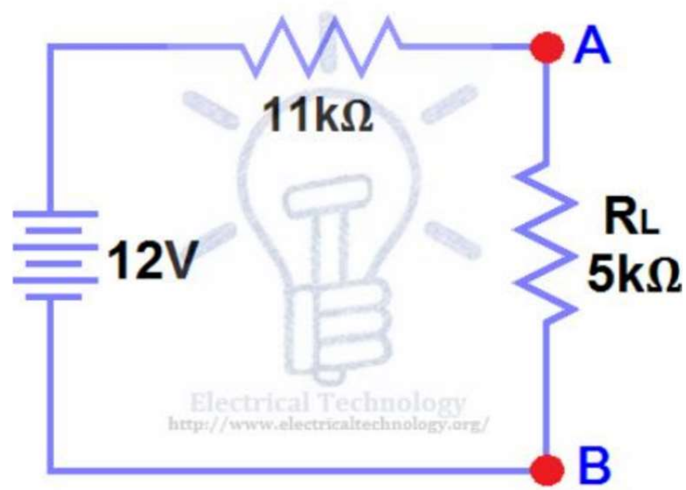


$$= 8k\Omega + (4k\,\Omega \parallel 12k\Omega) \rightarrow = 8k\Omega + 3k\Omega$$

$$\boxed{R_{TH} = 11k\Omega}$$



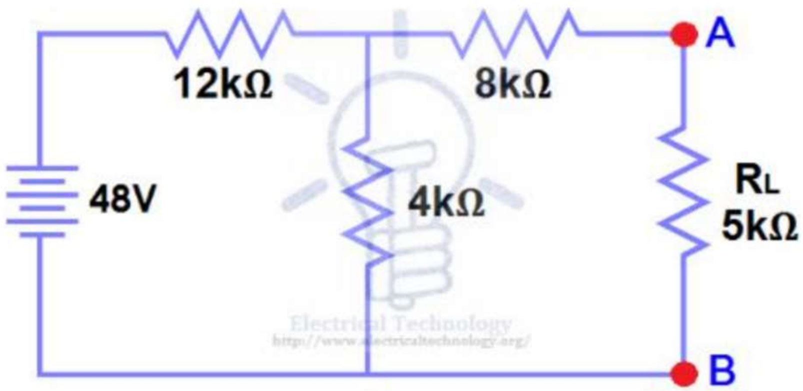
Thevenin's equivalent circuit

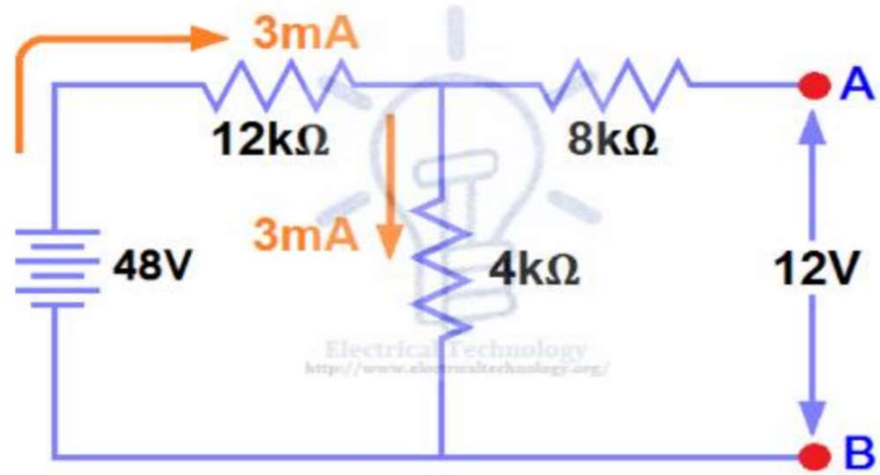


Thevenin's equivalent circuit

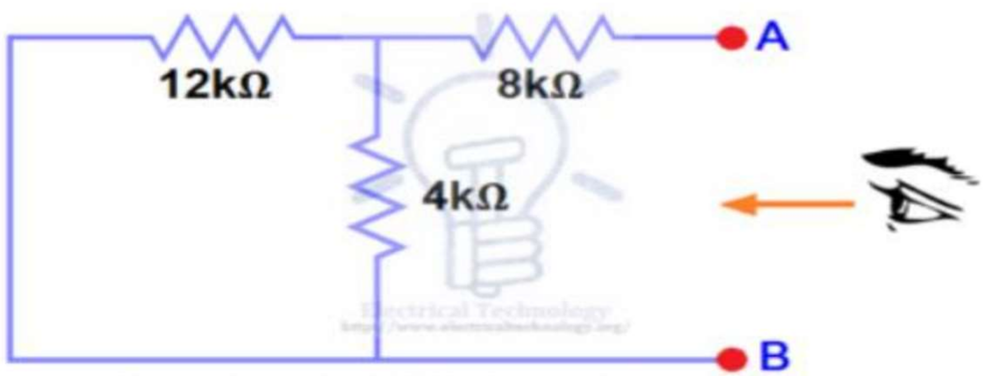
Find V_{TH}, R_{TH} and the load current $\mathbf{I_L}$ flowing through and load voltage across the load resistor in fig (1) *by using Thevenin's Theorem.*

V_{TH} = 12V



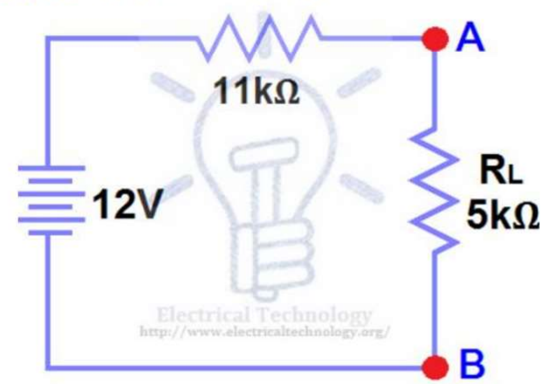


3mA x 4kΩ = 12V



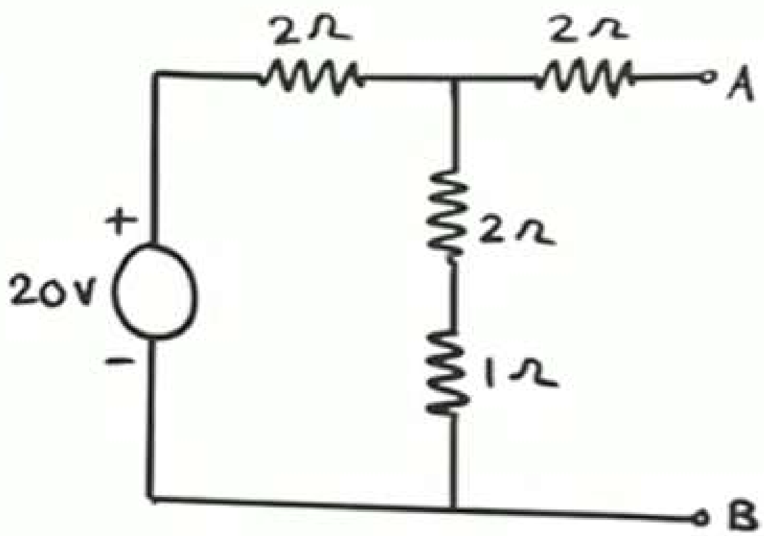
= 8kΩ + (4k Ω || 12kΩ) → = 8kΩ + 3kΩ

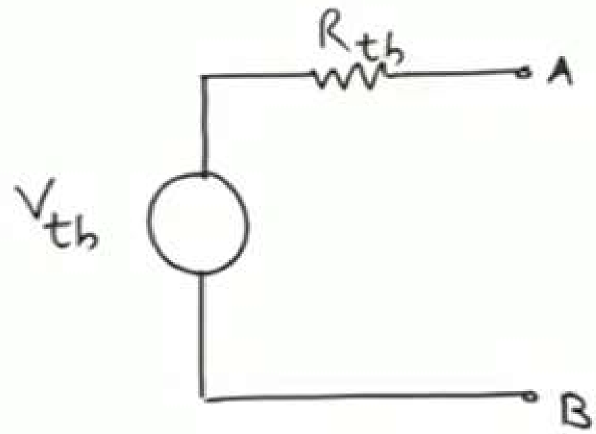
$$\boxed{R_{TH} = 11k\Omega}$$



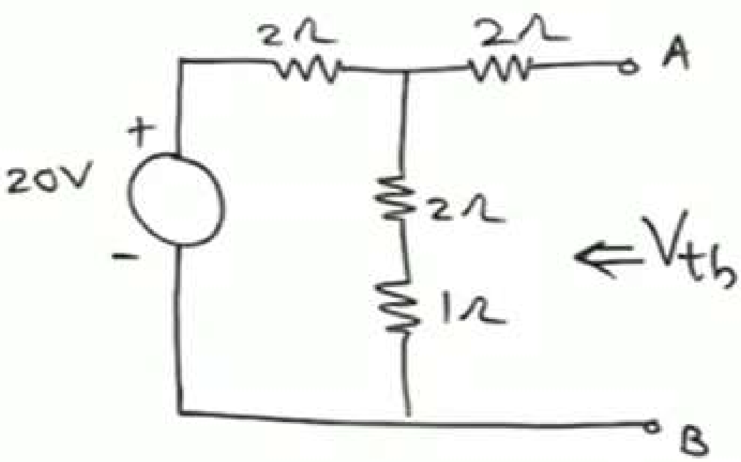
Thevenin's equivalent circuit

Q2. Find its Thevenin's equivalent circuit.





step 1> Calculation of V_{th}



Apply KVL in loop 1

$$+20 - 2I_1 - 2I_1 - 1I_1 = 0$$

$$+20 - 5I_1 = 0$$

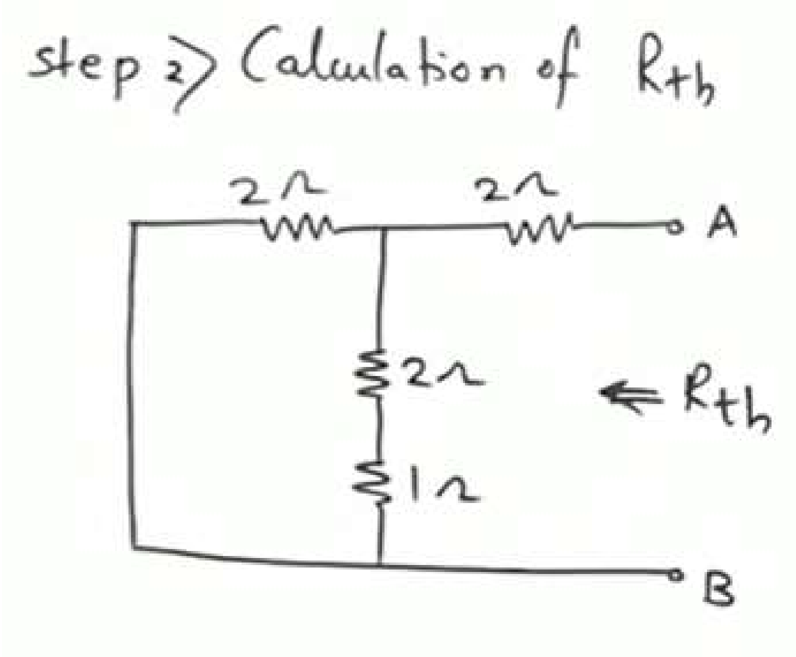
$$5I_1 = 20$$

$$\boxed{I_1 = 4A}$$

$$\therefore V_{th} = I_1 \times (2+1)$$

$$= 4 \times 3$$

$$\boxed{V_{th} = 12\,V}$$



$$R_{th} = (2 \parallel 3) + 2$$

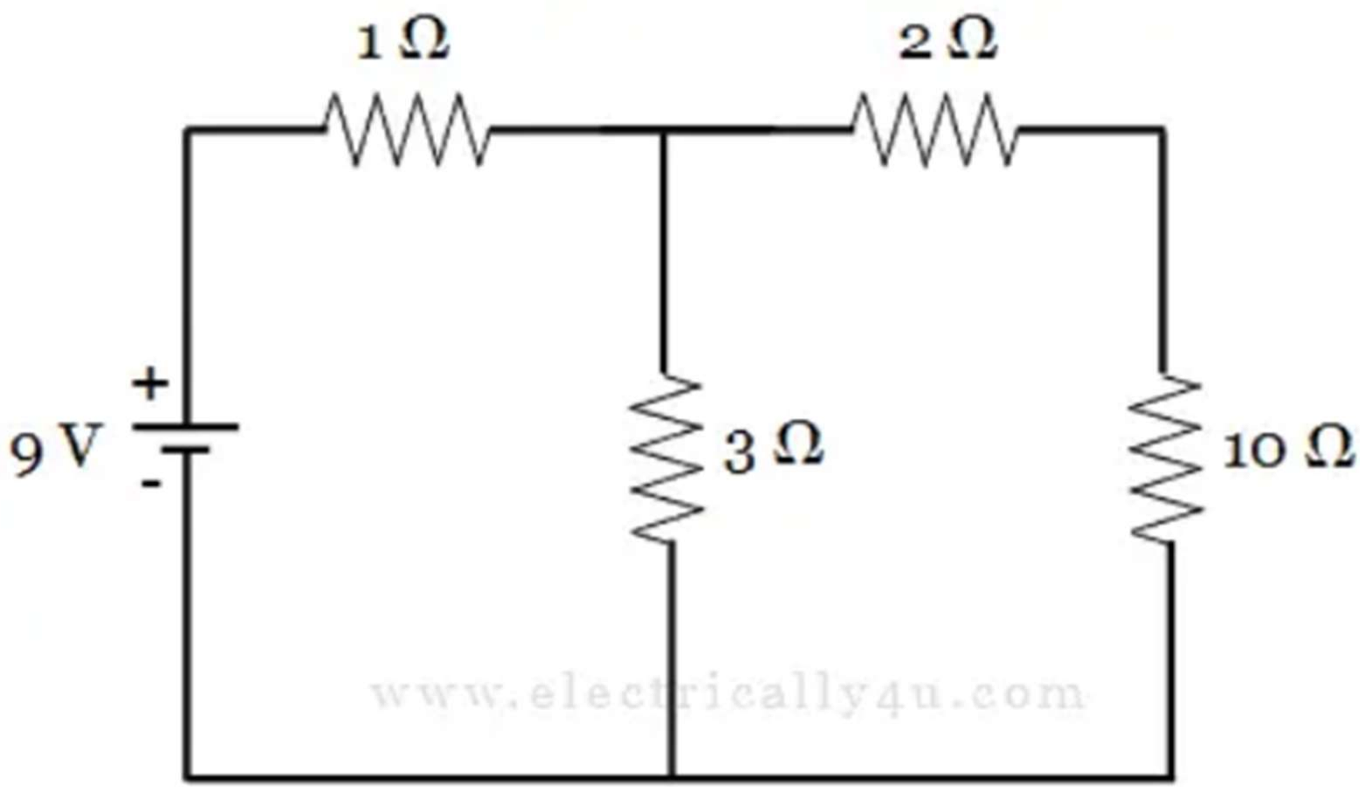
$$= \frac{2 \times 3}{2+3} + 2$$

$$\boxed{R_{th} = 3.2\,\Omega}$$

For finding Thevenin resistance or equivalence resistance:

- Voltage source is replaced by short circuit.
- Current source is replaced by open circuit.

Q3. ***Solve the given circuit to find the current through 10 Ω using Thevenin's Theorem***



By Ohm's law,

$$I = \frac{V}{R} = \frac{9}{4} = 2.25A$$

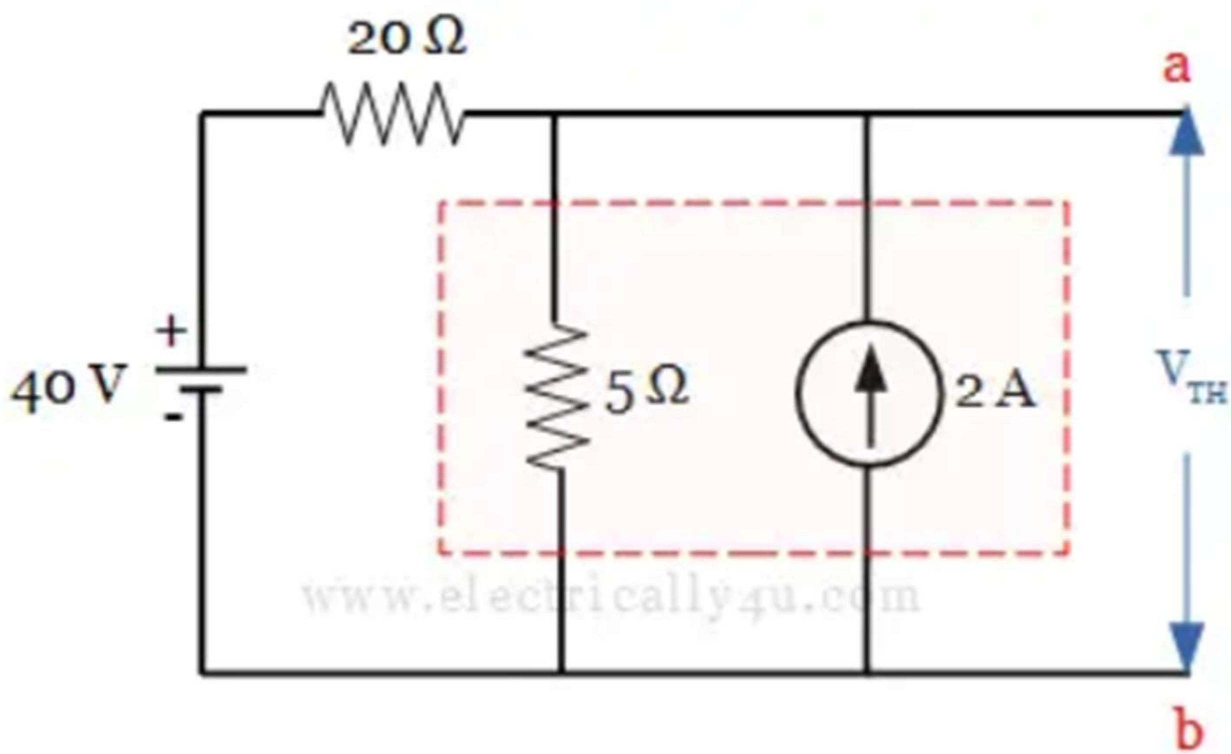
Thus, the voltage across 3 Ω resistor(or Thevenin's voltage V_{TH}) is given by,

$$V_{ab} = V_{TH} = I * R = 2.25 * 3 = 6.75V$$

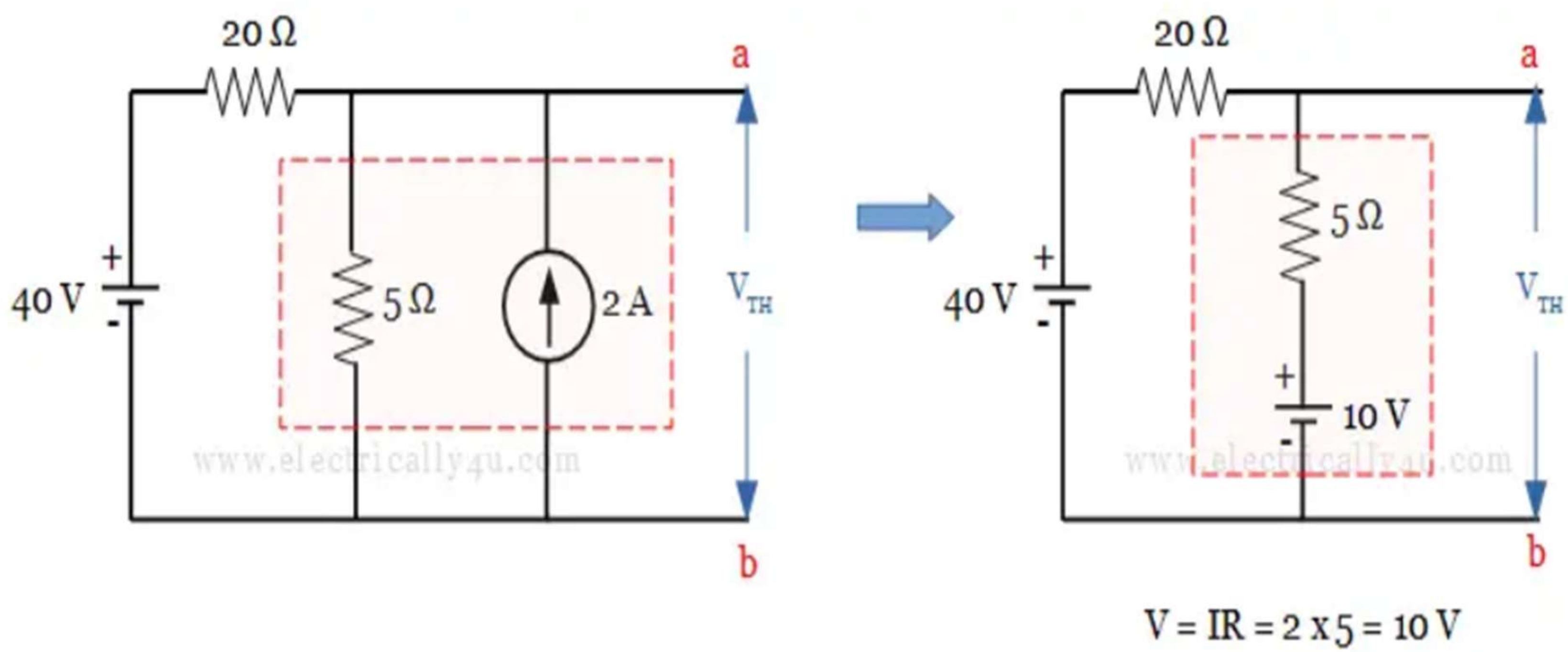
$$R_{TH} = 2 + \frac{1 * 3}{1 + 3} = 2.75\Omega$$

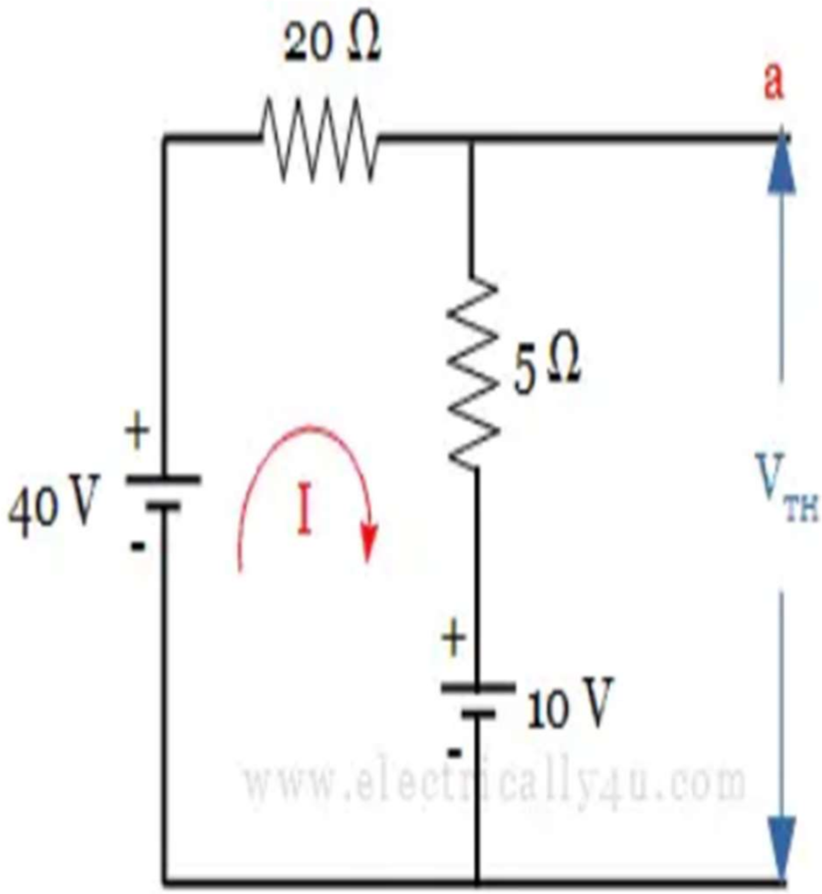
$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{6.75}{2.75 + 10} = 0.529A$$

Q4. ***Solve the given circuit to find the load current using Thevenin's Theorem.***



By source transformation, let us convert this current source into its equivalent voltage source in series with 5 Ω resistor.





$$R_{TH} = \frac{5 * 20}{5 + 20} = 4\Omega$$

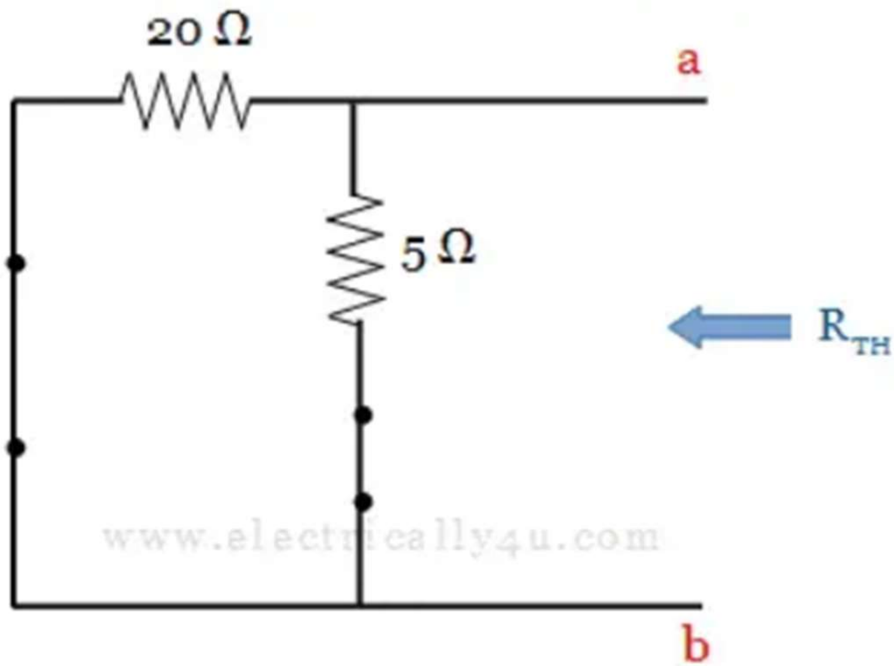
Let us apply Kirchoff's Voltage Law to this loop and find the value of loop current.

Applying KVL, $20I + 5I + 10 - 40 = 0$

$$25I = 30$$

$$I = 1.2\ A$$

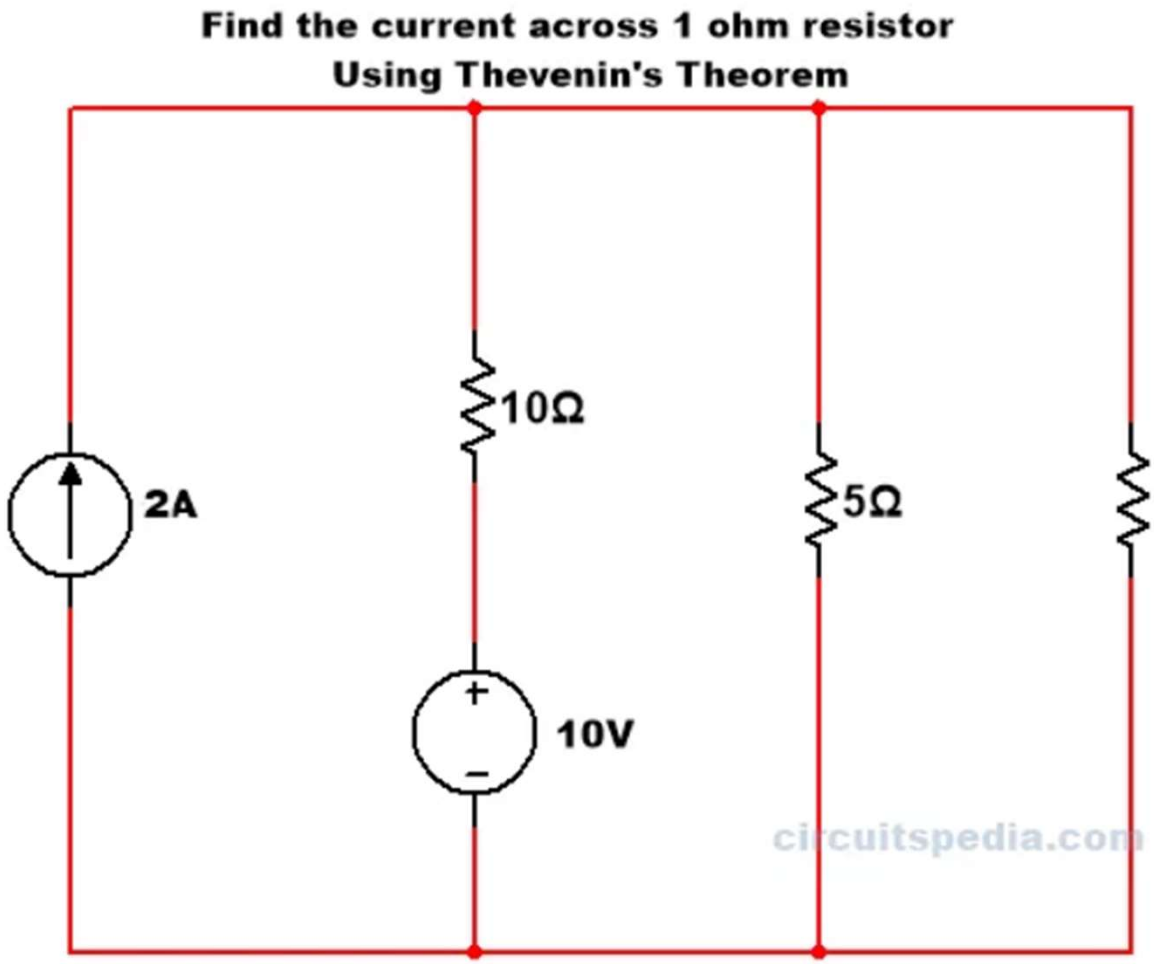
$$V_{TH} = 10 + (5 * 1.2) = 16\ v$$

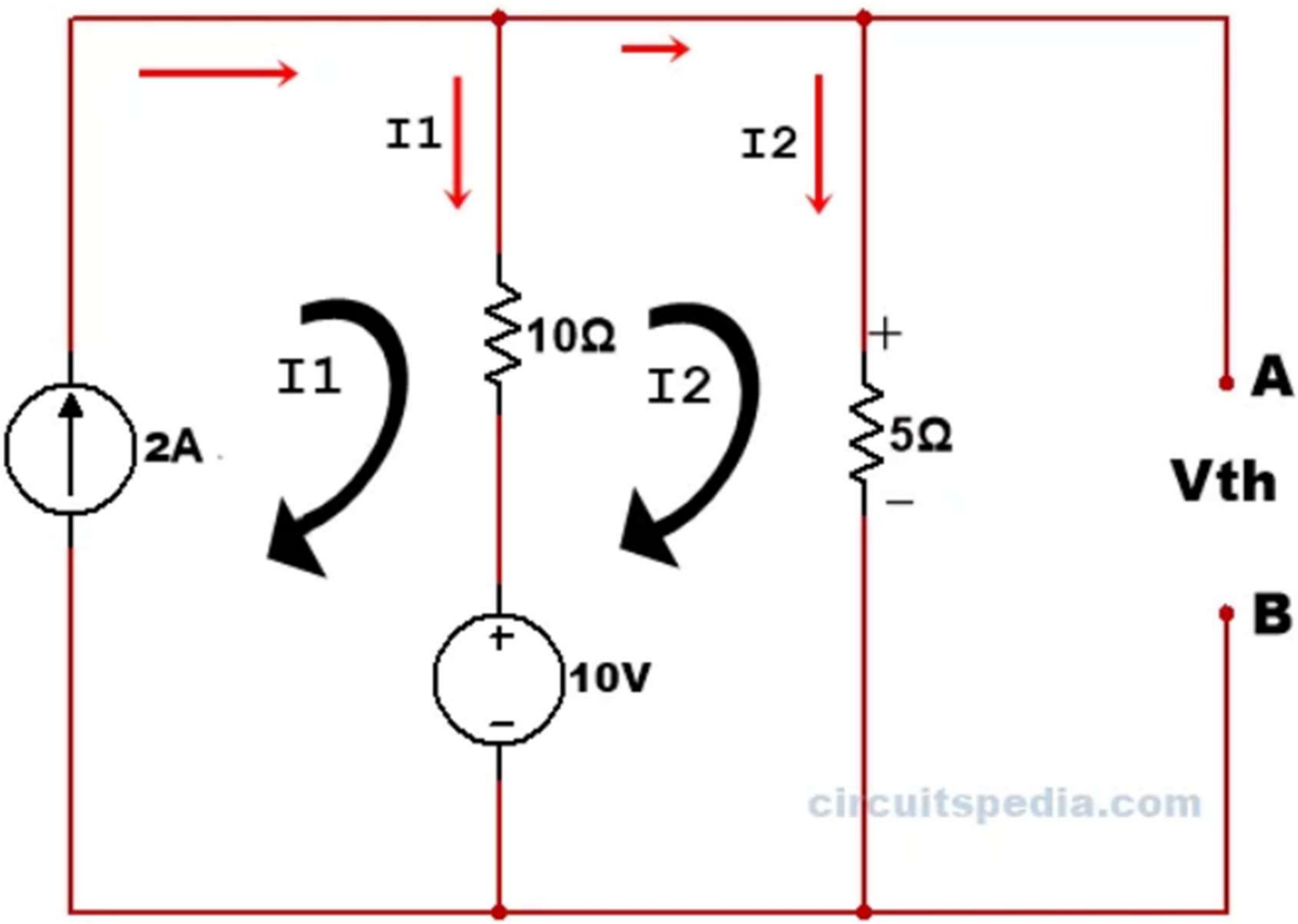


$$R_{TH} = \frac{5 * 20}{5 + 20} = 4\Omega$$

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{16}{4 + 15} = 0.842A$$

Q. Find the value of current through 1Ω Resistor in the given circuit using Thevenin's theorem.




I1
I2
I1
10Ω
I2
+
5Ω
−
2A
A
Vth
B
+
10V
−
circuitspedia.com

Applying KVL in All meshes.

$I_1=2A$

$-5.I_2 + 10 + 10(I_1-I_2) = 0$

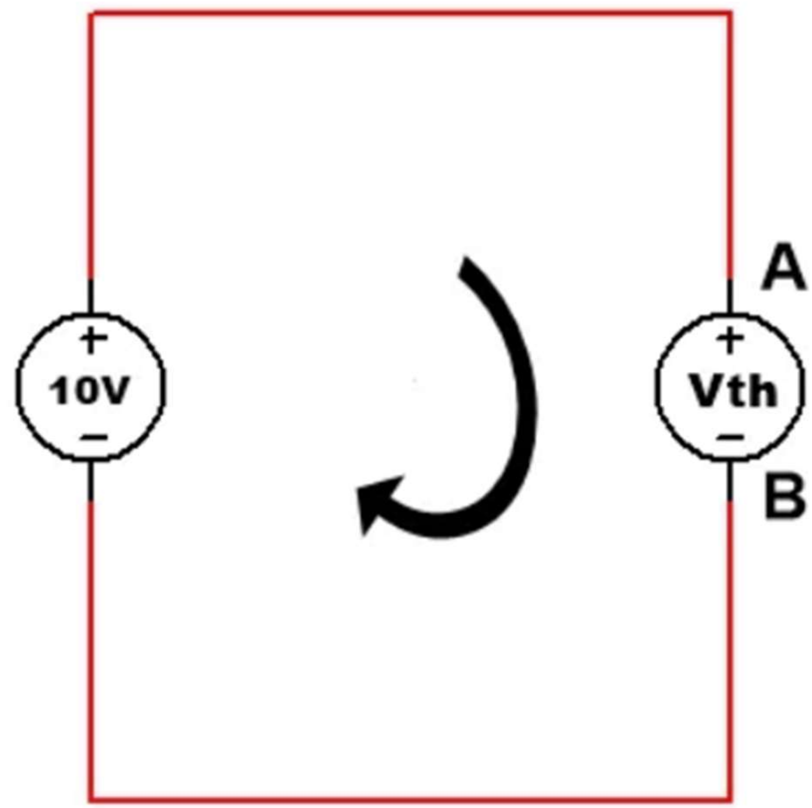
$10-10(I_2-I_1)-5I_2=0$

$-15I_2+10I_1=-10$

$-15I_2=-10-20=-30$

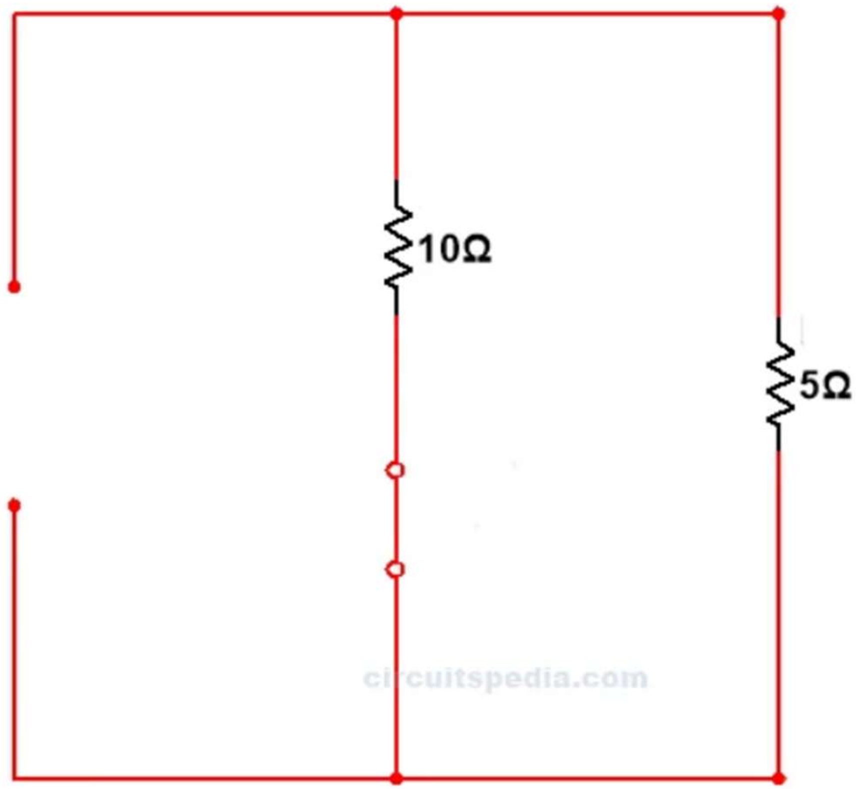
$-15I_2=-30$

$I_2=30/15=2A$

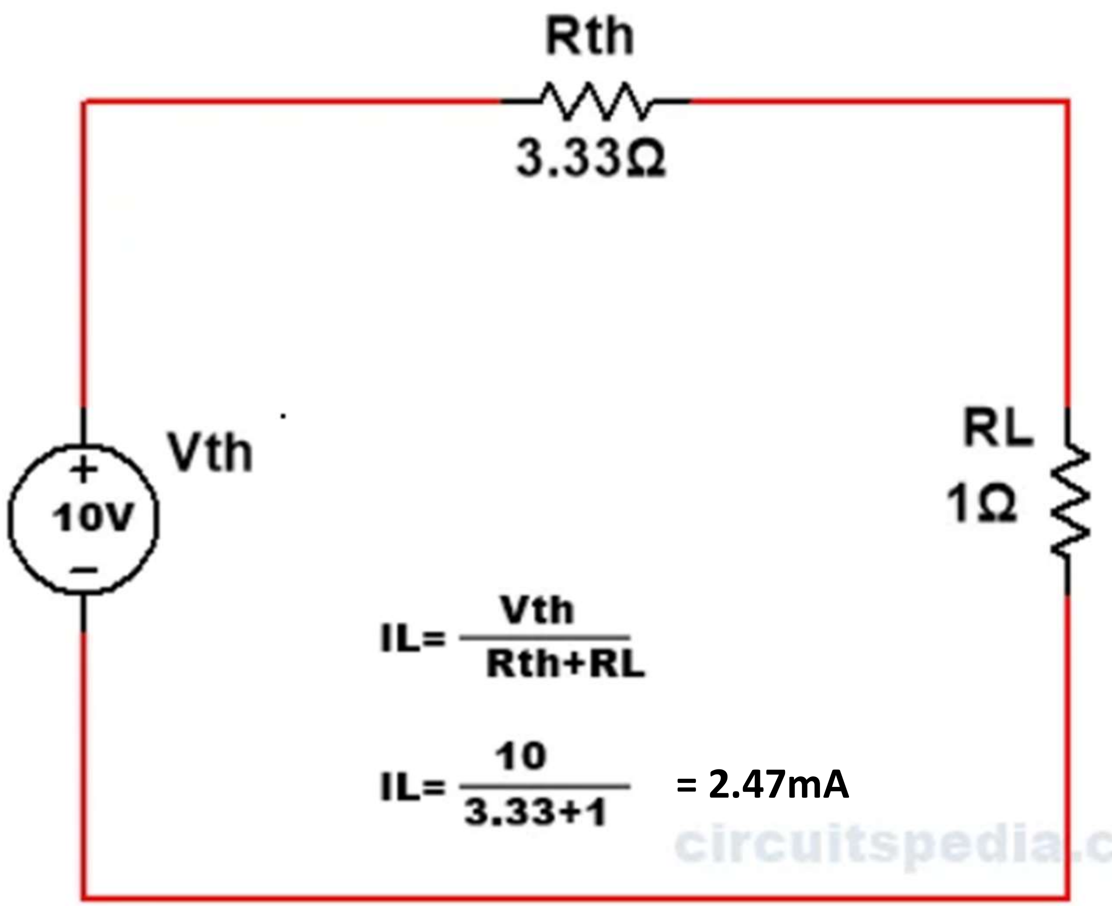


The voltage across the 5Ω resistor is **V=I*R=2*5=10Ω**

Now we replace all active voltage and current sources with their internal resistance

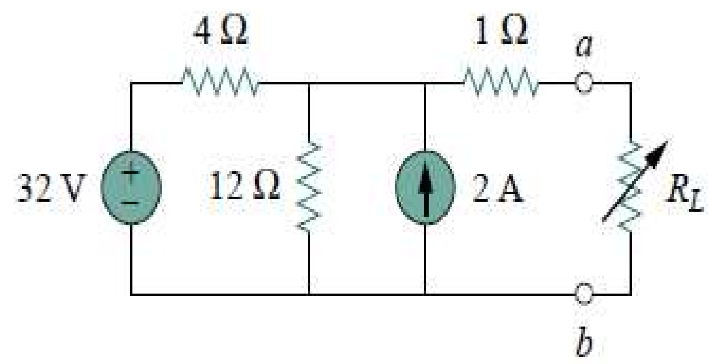


Rth= 10//5=3.33 ohm



Rth
3.33Ω
Vth
+
10V
−
RL
1Ω
IL= Vth / Rth+RL
IL= 10 / 3.33+1
= 2.47mA

MCQ



Find the Thevenin equivalent circuit, for the circuit shown in Fig. to the left of the terminals *a*-*b*, if RL=6 Ω.

(a) Rth= 4 ohm , Vth= 30 V

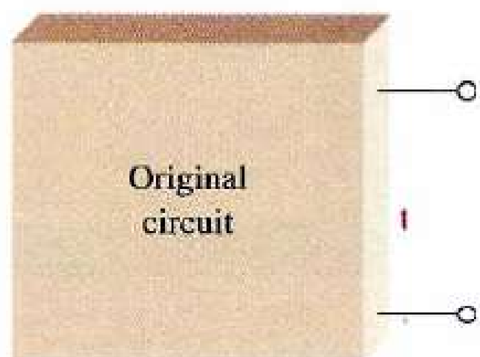
(b) Rth= 16 ohm , Vth= 30 V

(c) Rth= 17 ohm , Vth= 20 V

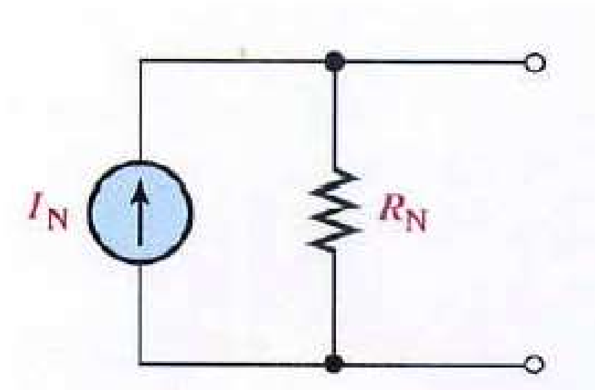
(d) Rth= 17 ohm , Vth= 30 V

Norton's Theorem

Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler equivalent form.



$\equiv$



Norton's Theorem

This theorem is an alternative to the Thevenin's theorem. In fact, it is the dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

How To Nortonize a Given Circuit ?

This procedure is based on the first statement of the theorem given above.

1. Remove the resistance (if any) across the two given terminals and put a short-circuit across them.
2. Compute the short-circuit current I_{SC}.
3. Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open-circuits *i.e.* by infinite resistance.
4. Next, find the resistance R_1 (also called R_N) of the network as looked into from the given terminals. It is exactly the same as R_{th}
5. The current source (I_{SC}) joined in parallel across R_i between the two terminals gives Norton's equivalent circuit.

Norton's Theorem

Summary of Norton's Theorem

Step 1. Short the two terminals between which you want to find the Norton equivalent circuit.

Step 2. Determine the current (I_N) through the shorted terminals.

Step 3. Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.

Step 4. Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

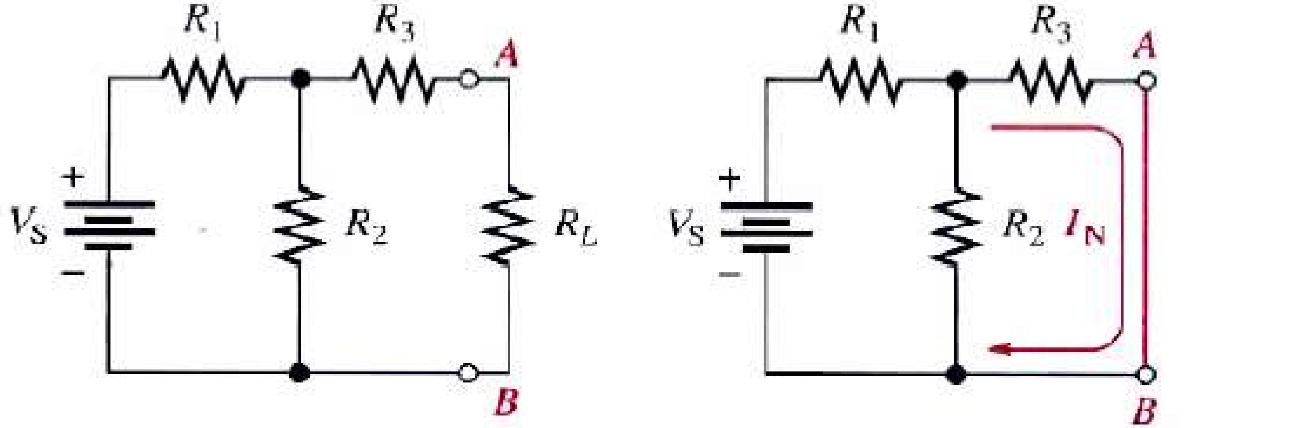
Steps to solve NORTON'S theorem

- **Step 1: Remove the Load Resistor**
- **Step 2: Calculate the Norton Current :**To find the Norton current, short circuit the connection between the load points and determine the resultant current
- **Step 3: Calculate Norton Resistance** (shorting all voltage sources or by open circuiting all the current sources)
- **Step 4: Draw the Norton Equivalent Circuit**

Norton's Theorem

Norton's Equivalent Current (I_N)

Norton's equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.



(a) Original circuit

(b) Short the terminals to get I_N.

Find equivalent Norton Circuit across R?

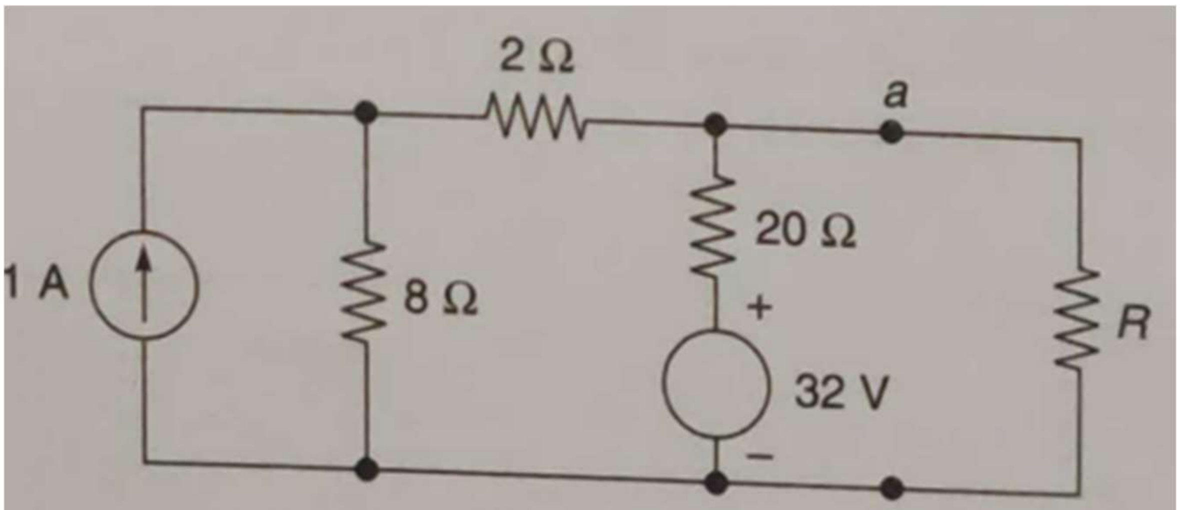


Fig. 2.38

Find equivalent Norton Circuit across R?

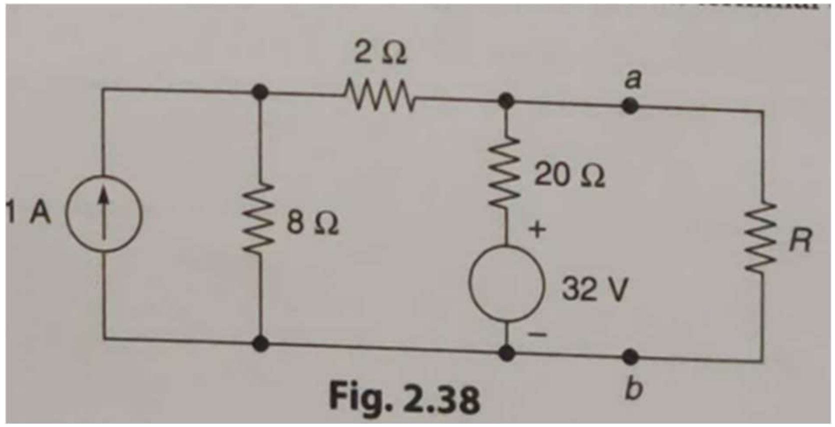
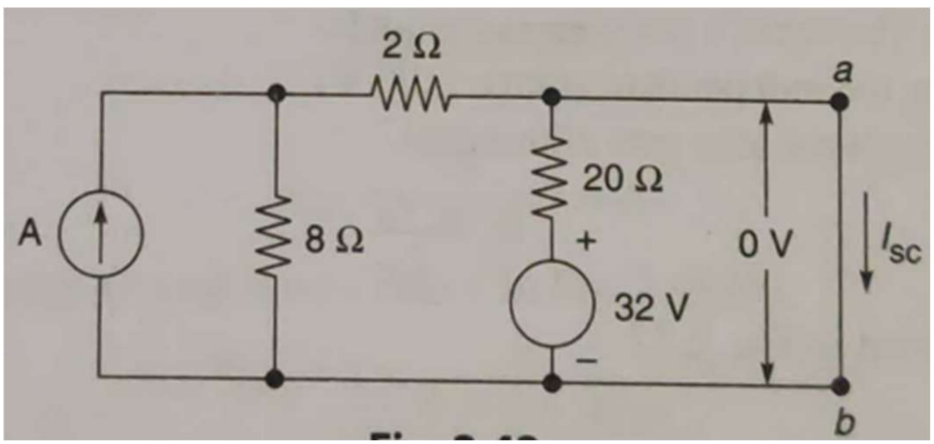
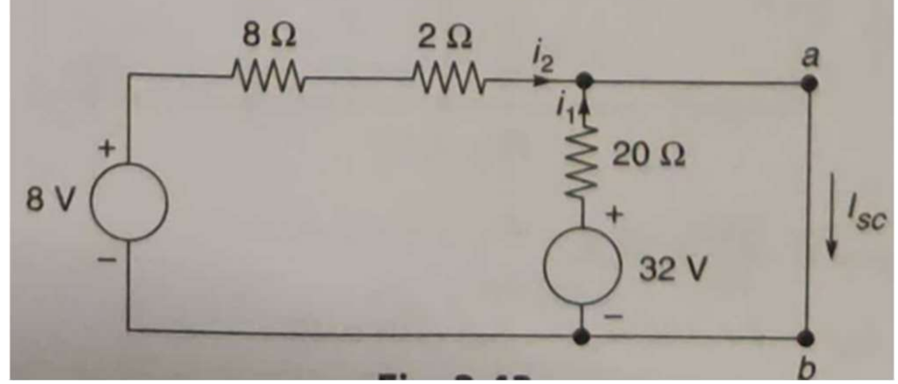


Fig. 2.38



$$I_{SC} = I_1 + I_2 = \frac{32}{20} + \frac{8}{10} = \frac{48}{20} = 2.4 \text{ A}$$



Open-circuit 1 A source and short-circuit 32 V source.

$$R_0 = \frac{20 \times 10}{20 + 10} = \frac{20}{3}\ \Omega$$

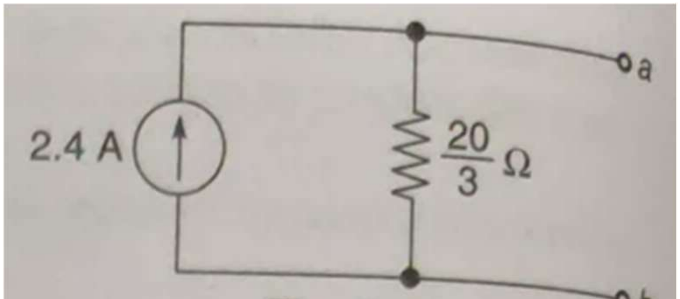
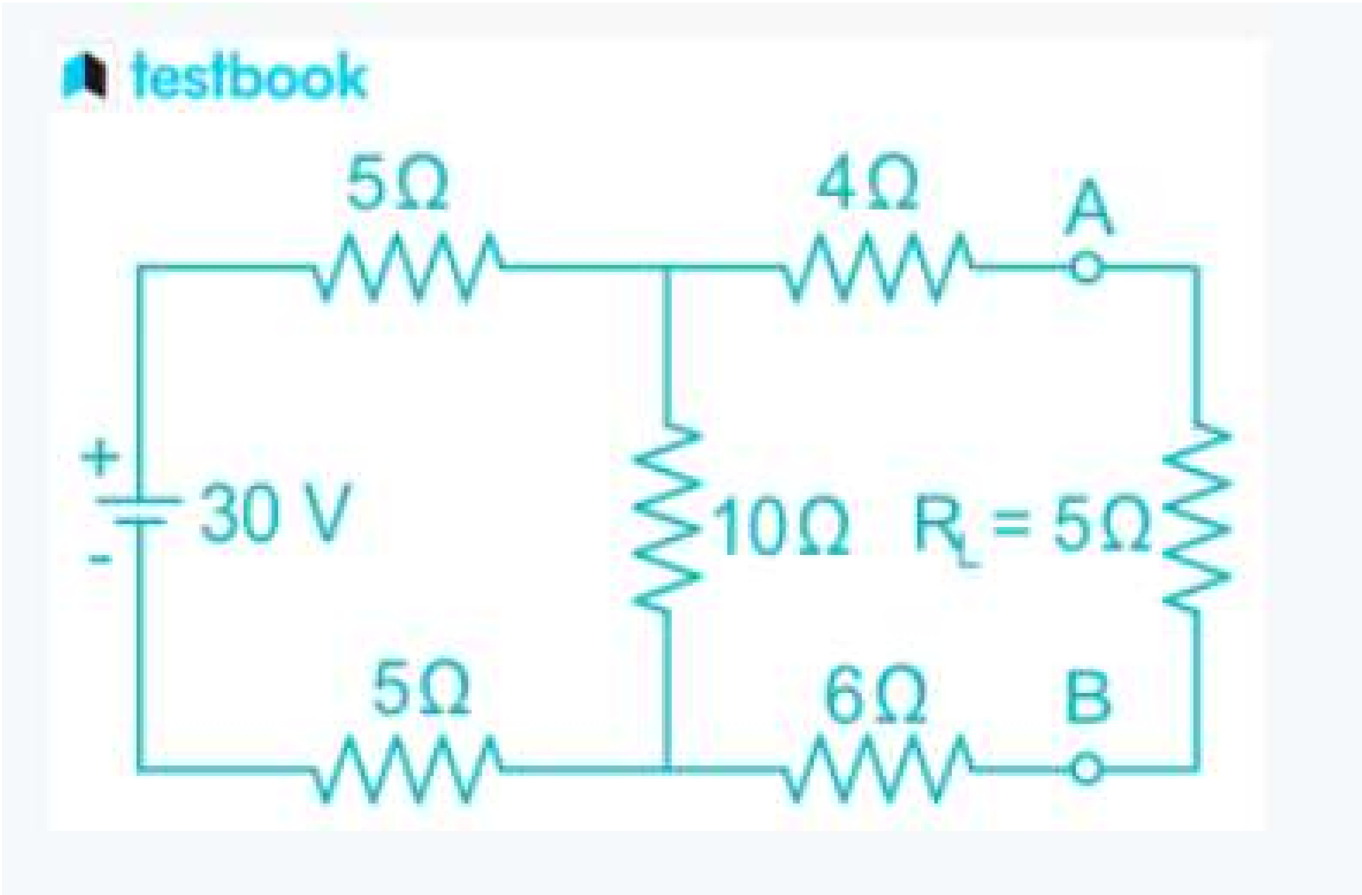


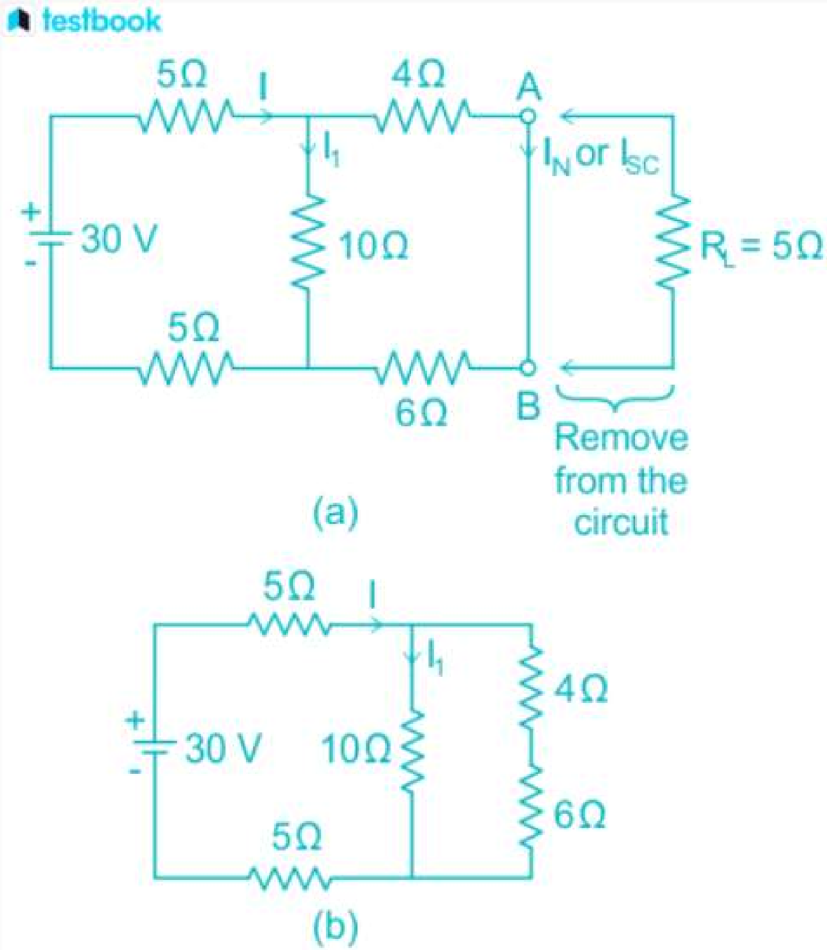
Fig. 2.44

HW: Find Current flowing in R and voltage across R if R=1000ohm

Example 1: In the network shown in figure, calculate the current through the load resistor *RL* by using Norton's Theorem.



Step 1 : Remove the load resistance $R_L = 5\Omega$ and short circuit the terminals A and B



Step 2 : Find the current I_N flowing through A and B

Total resistance R = $(5+5)\Omega + (10\Omega||(4+6)\Omega)$

$= 10 + \frac{10\times10}{10+10} = 10 + 5 = 15\Omega$

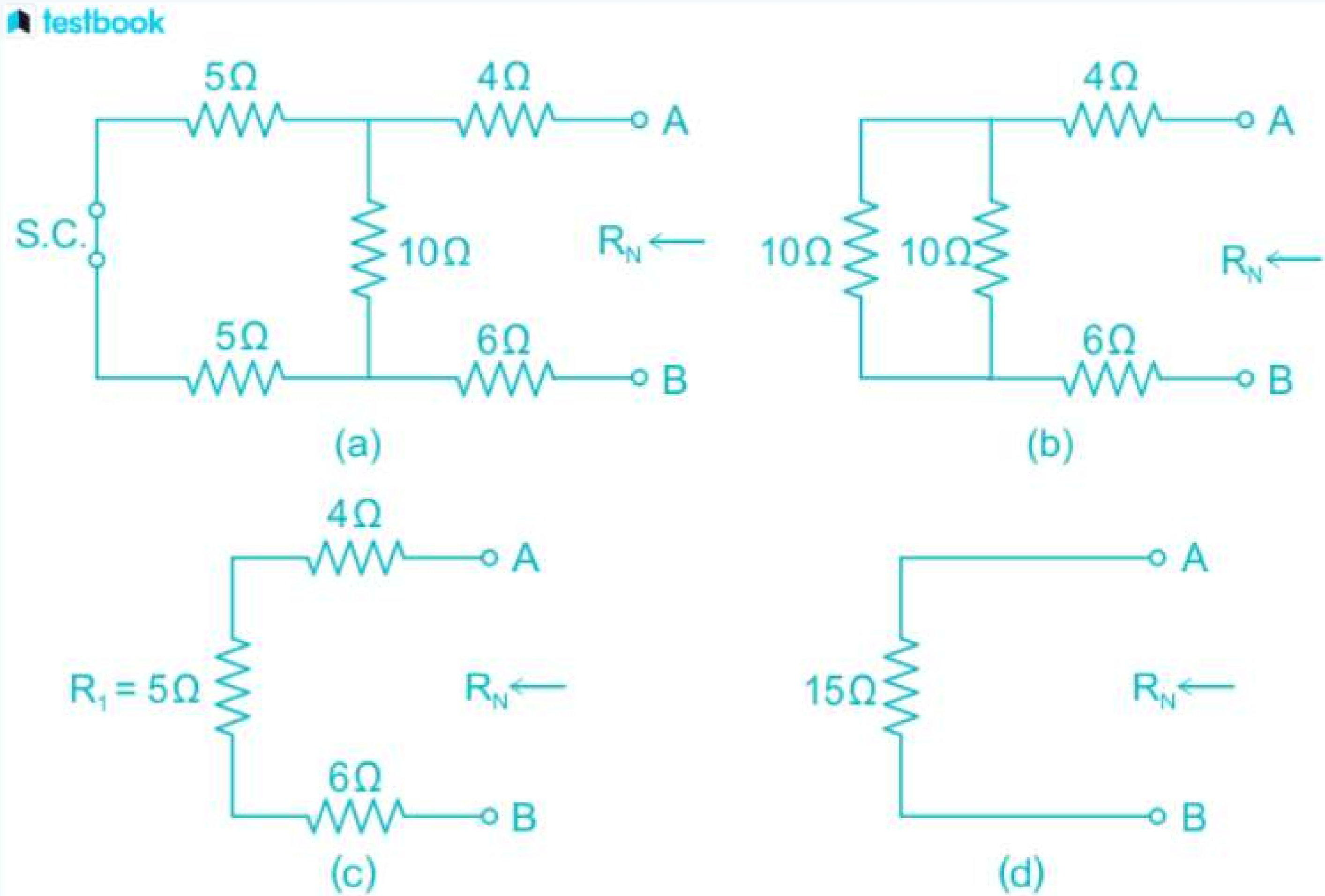
Current $I = \frac{V}{R}$

$I = \frac{30}{15} = 2A$

Short circuit current,I_{SC} or I_N = $I \times \frac{10}{10+4+6}$

$2 \times \frac{10}{20} = 1A$

Step 3: Short circuit the voltage source and find the Norton equivalent resistance R_N as seen from the terminal A and B.

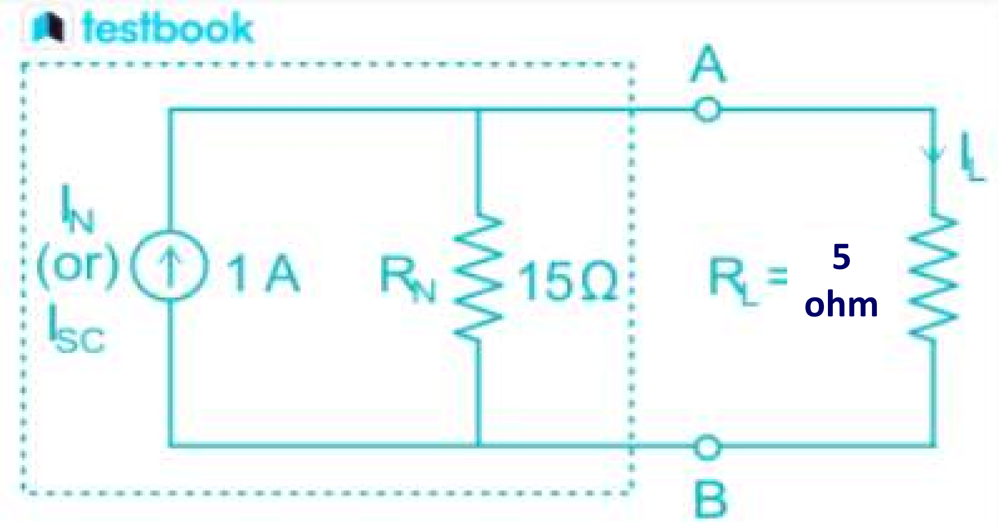


$R_1 = 10\Omega || 10\Omega = \frac{10 \times 10}{10+10} = 5\Omega$

$R_N = R_{AB} = 4 + R_1 + 6$

$4+5+6 = 15\Omega$

Step 4: Draw Norton's equivalent circuit by replacing the entire network with a single current course, I_N in parallel with R_N



Current through load resistance,

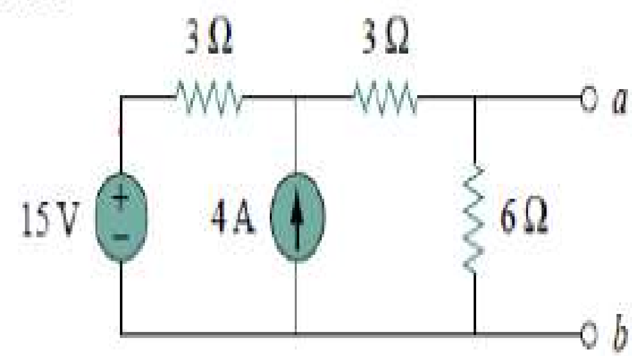
$$R_L = 5\Omega$$

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

$$I_L = 1 \times \frac{15}{15+5} = \frac{15}{20} = 0.75$$

MCQ

Find the Norton equivalent circuit for the circuit in Fig

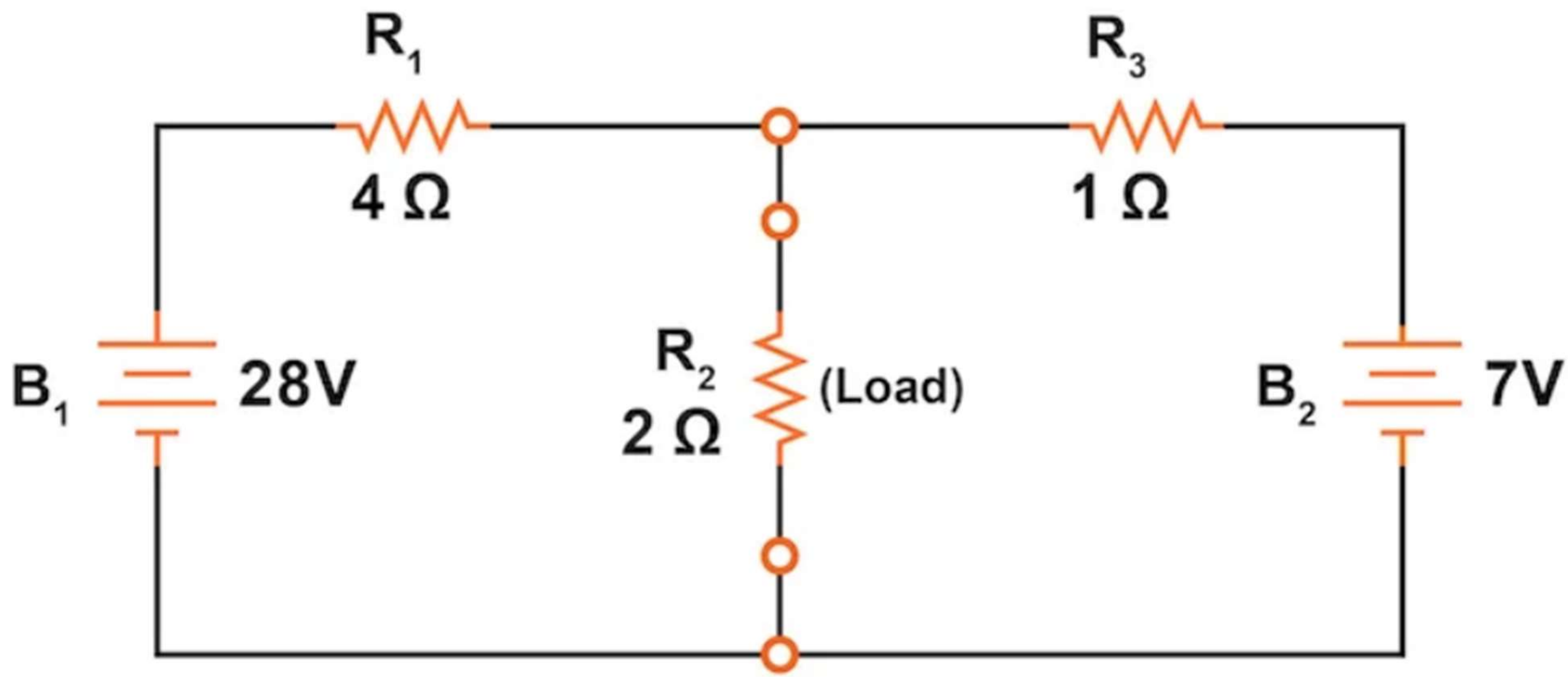


(a) $R_N = 3\ \Omega$, $I_N = 4.5$ A

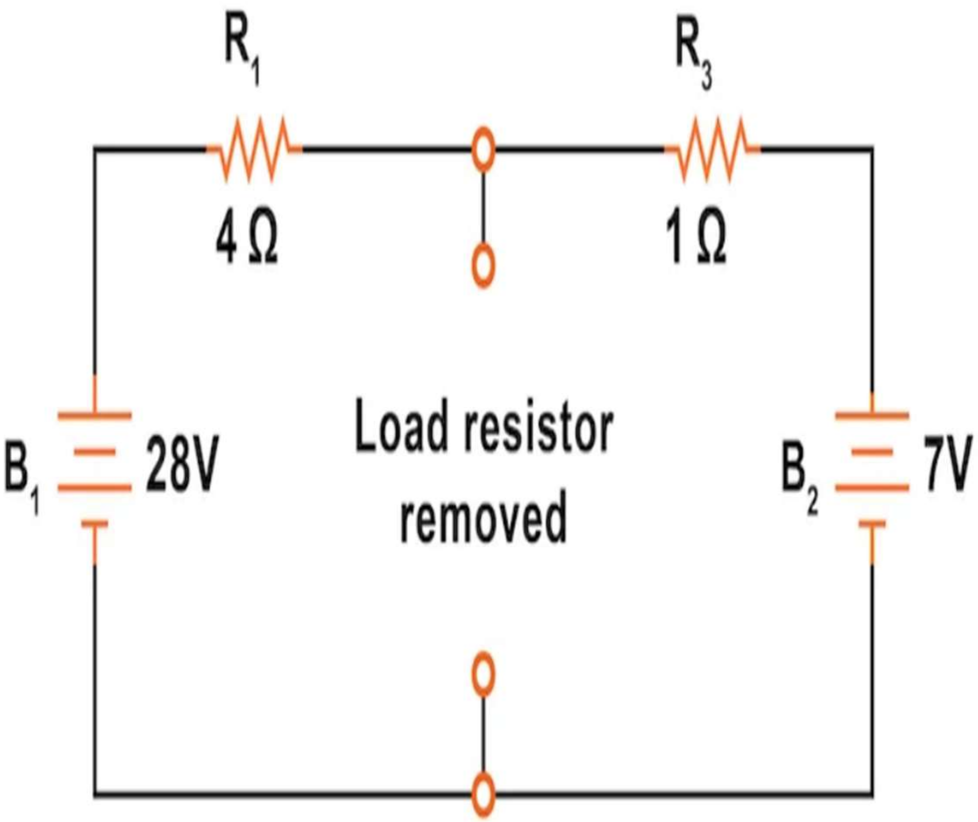
(b) $R_N = 6\ \Omega$, $I_N = 4.5$ A

(c) $R_N = 3\ \Omega$, $I_N = 1.5$ A

(d) $R_N = 3\ \Omega$, $I_N = 2.5$ A

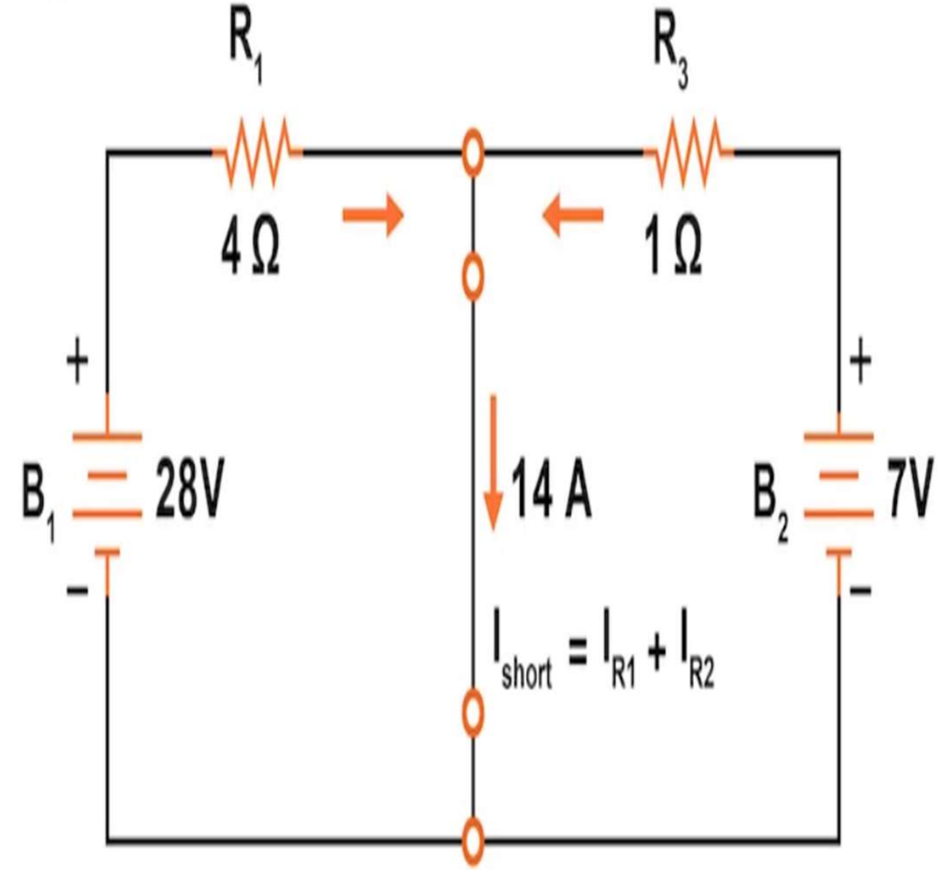

R1
4 Ω
R3
1 Ω
B1
28V
R2
2 Ω
(Load)
B2
7V

Step 1: Remove the Load Resistor

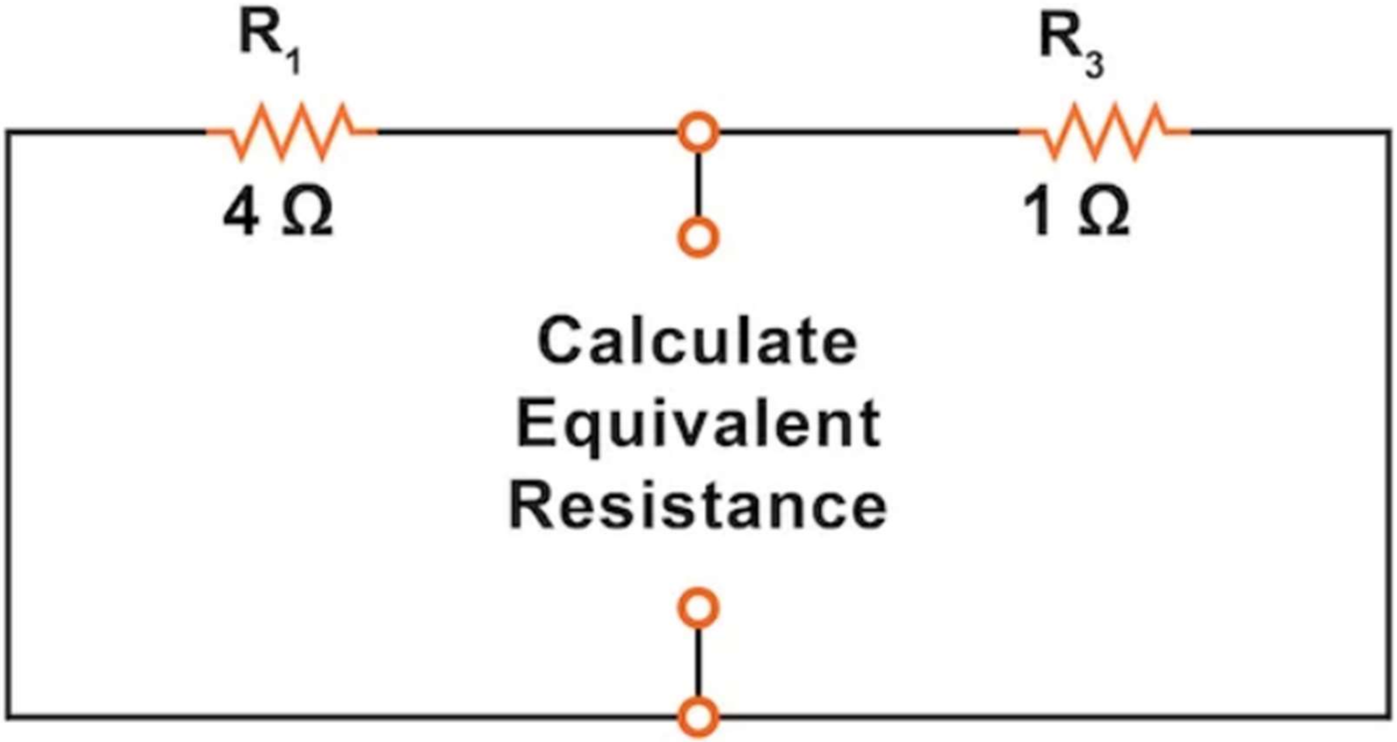


$$I_{short} = I_{R1} + I_{R2} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

Step 2: Calculate the Norton Current



$$I_{Norton} = I_{short} = \frac{28}{4} + \frac{7}{1} = 7 + 7 = 14\ \text{A}$$



The Norton equivalent resistance is calculated as:

$$R_{Norton} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3}} = \frac{1}{\frac{1}{4} + \frac{1}{1}} = 0.8\ \Omega$$

Solution Short-circuiting ab and converting the current source to a voltage source as in Fig. 2.56(a), we get

$$I_{sc} = (45 - 9)/6 = 6\text{A}$$

Short-circuiting the voltage source and open-circuiting the current source as in Fig. 2.56(b), we obtain $R_o = (5 + 1) \parallel 3 = 2\Omega$.

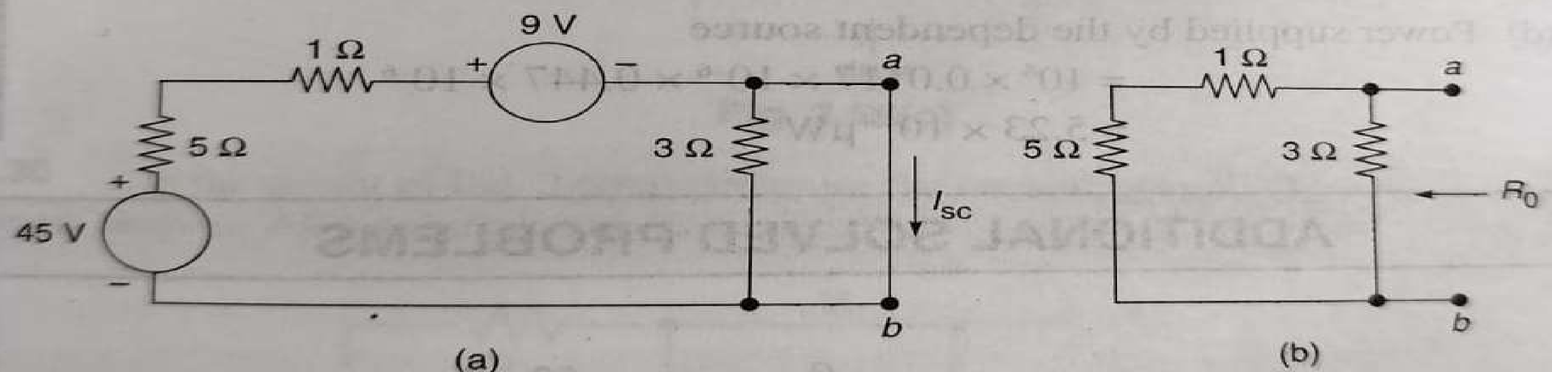


Fig. 2.56

The Norton equivalent is drawn in Fig. 2.57. Notice that the direction of the current in the current source in the equivalent is such as to cause the short-circuit current at ab in the same direction as shown in Fig. 2.56(a).

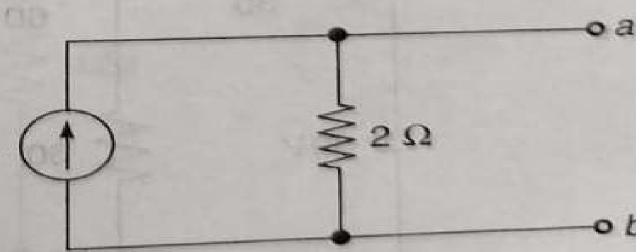


Fig. 2.57

Find Norton’s Equivalent circuit of the following circuit: