UNIT 1: DC CIRCUITS

Lecture 3

Kirchhoff's Law

- Ohm's law by itself is not sufficient to analyze circuits.
- However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.
- These laws are:

1. Kirchhoff's Current Law (KCL)
2. Kirchhoff's Voltage Law (KVL)

Kirchhoff's Current Law (KCL)

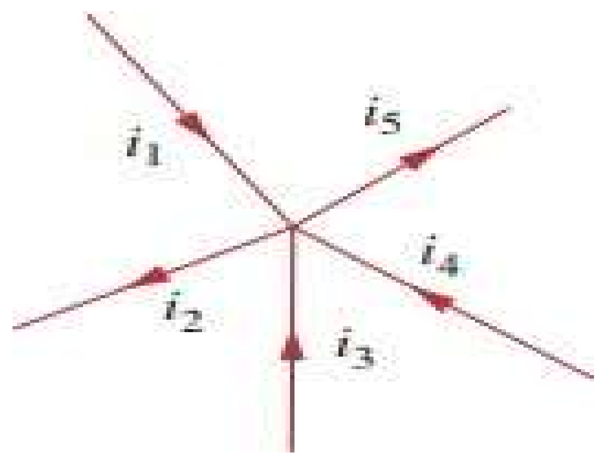
- It states that:

"the algebraic sum of currents entering a node is zero".

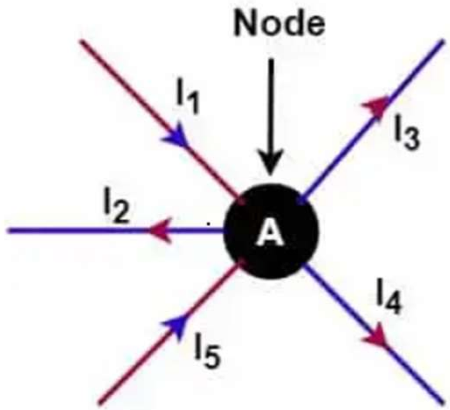
OR

" Sum of currents entering a node = Sum of currents leaving a node "

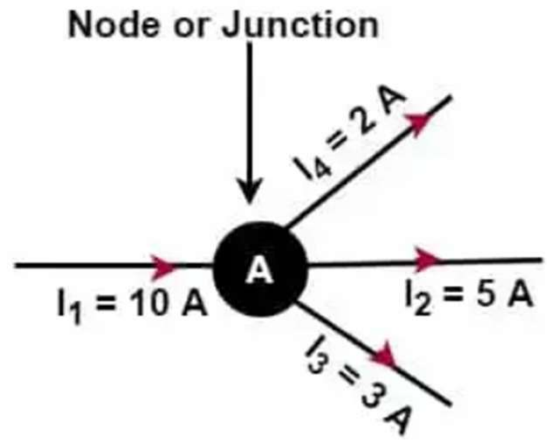
- Based on Law of Conservation of Charge.
- Mathematically, $\sum I = 0$



$$I_1 + I_5 = I_2 + I_3 + I_4$$



Fig(a)



Fig(b)

Current Entering the node is equal to current leaving the node

QUICK QUIZ (Poll 1)

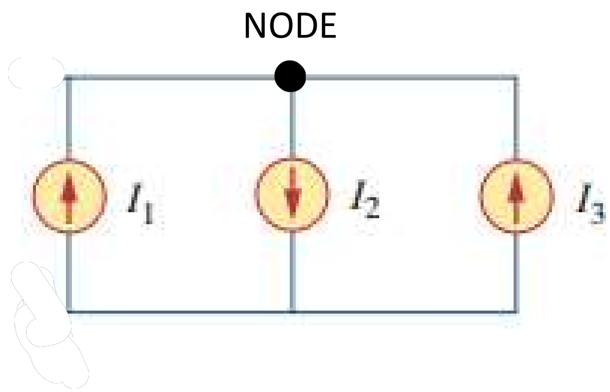
KCL equation for the given network is:

A. $I_1 + I_2 + I_3$

B. $I_1 + I_2 - I_3$

C. $I_1 - I_2 + I_3$

D. $-I_1 - I_2 + I_3$



Kirchhoff's Voltage Law (KVL)

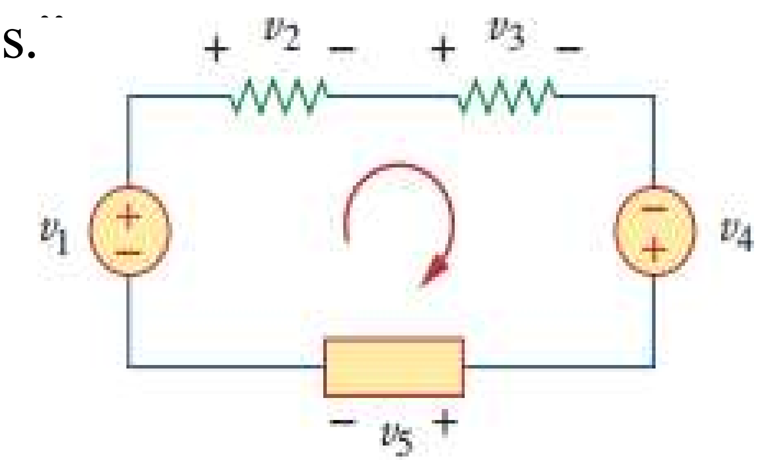
- It states that:

"algebraic sum of all voltages around a closed path (or loop) is zero."

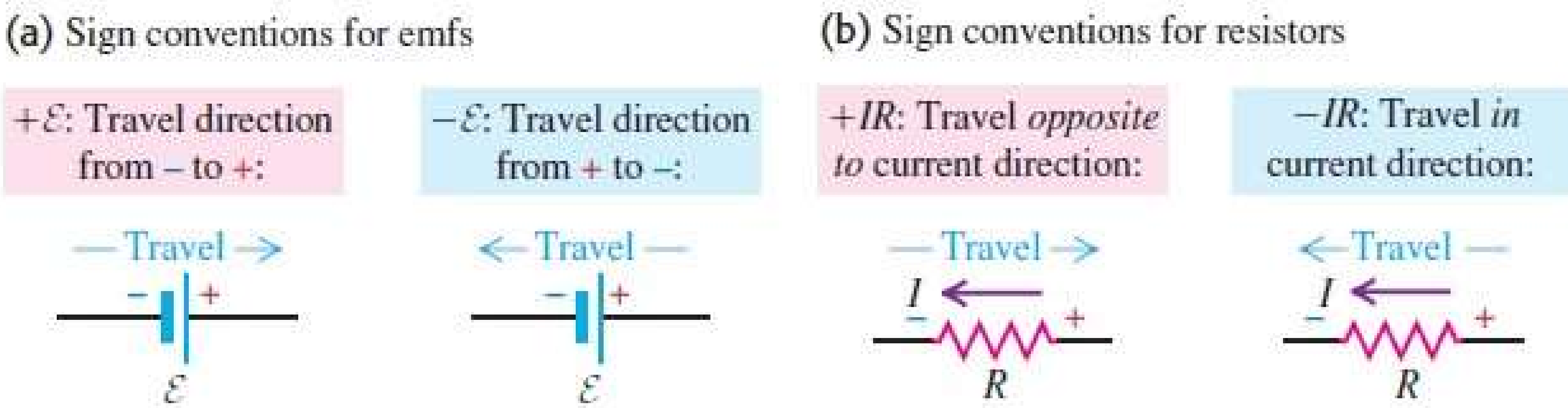
OR

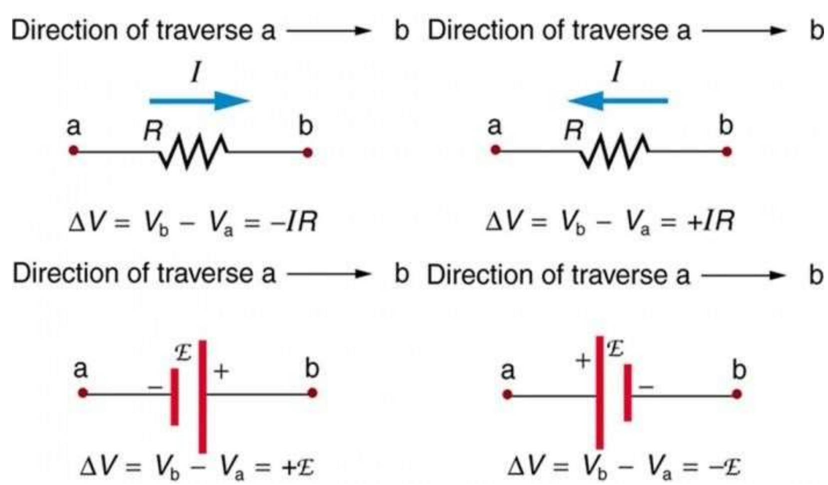
" Sum of voltage drops = Sum of voltage rises."

- Based on Law of Conservation of Energy
- Mathematically, $\sum V = 0$



Sign Convention for KVL





Let us Recall!

- Taking Clockwise direction (Def. 1):

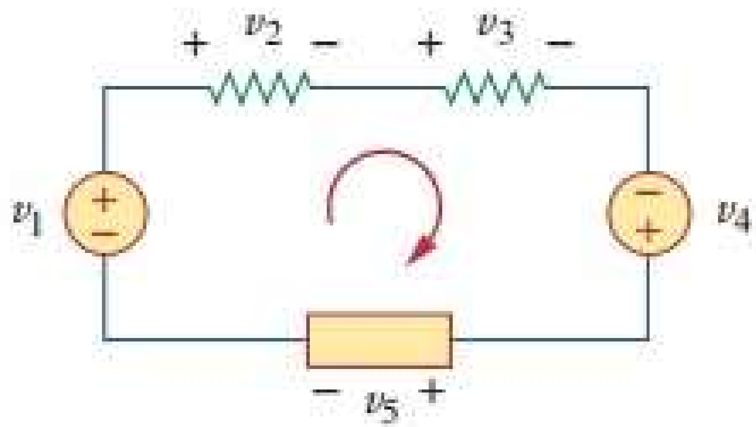
$$+V_1 - V_2 - V_3 + V_4 - V_5 = 0$$

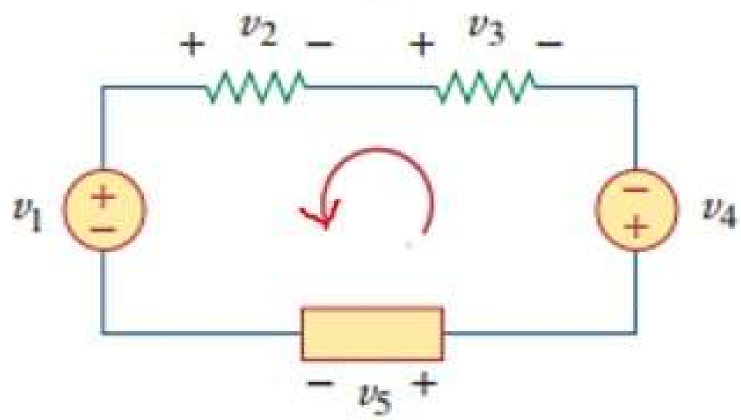
- Taking Anti-clockwise direction(Def. 1):

$$-V_4 + V_3 + V_2 - V_1 + V_5 = 0$$

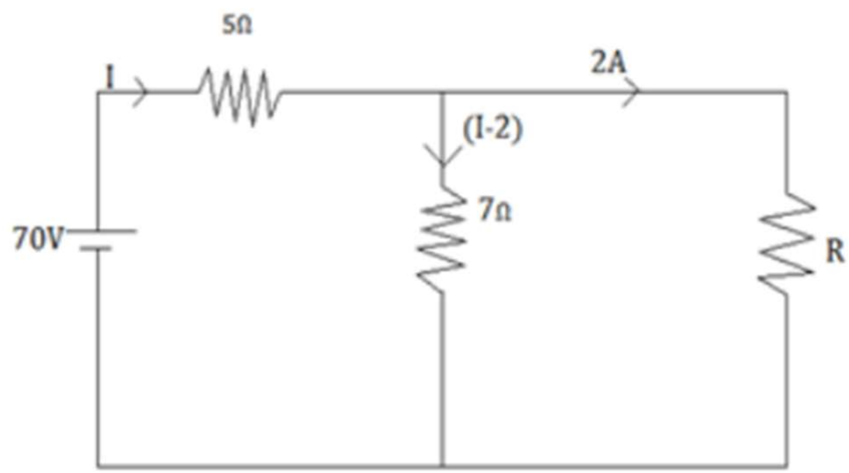
- Voltage rise = Voltage drop

$$+V_1 + V_4 = V_2 + V_3 + V_5$$





Q. Find R-value from the below circuit using KVL.



KVL: $70 - 5I - 7(I - 2) = 0$

$I = 7A$

KVL to 2nd loop: $7(I - 2) - 2R = 0$

$R = 17.5\Omega$

QUICK QUIZ (Poll 2)

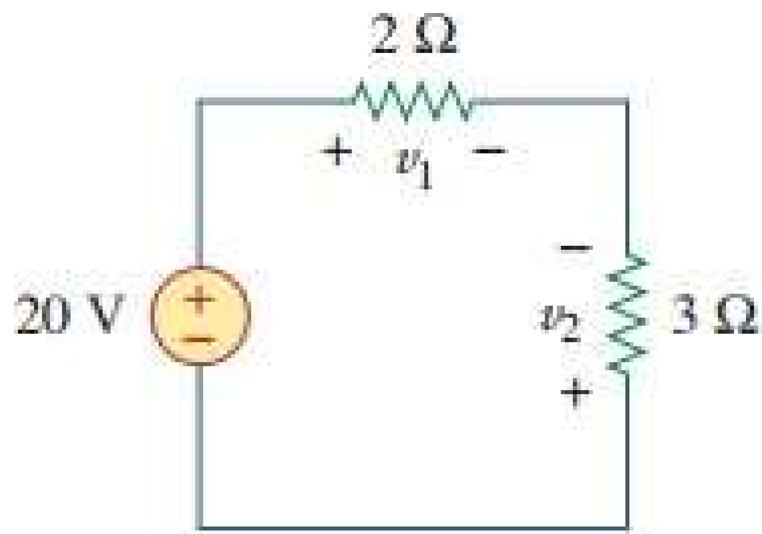
Find voltages V_1 and V_2 in the given circuit:

A. $V_1 = 16\ V\ and\ V_2 = 12\ V$

B. $V_1 = 16\ V\ and\ V_2 = -8\ V$

C. $V_1 = 8\ V\ and\ V_2 = -12\ V$

D. $V_1 = -12\ V\ and\ V_2 = 8\ V$



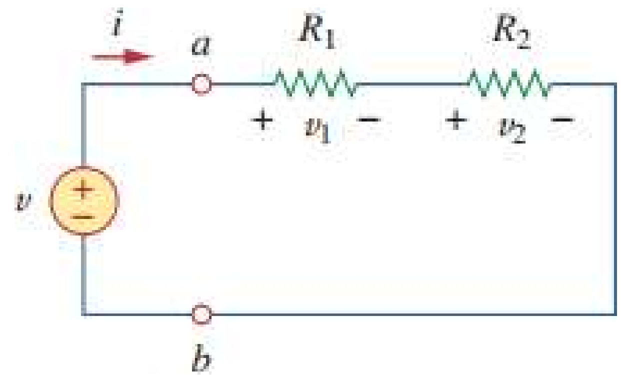
$$20V - V1 + V2 = 0$$
$$20V - (V1 - V2) = 0$$

Voltage Division Rule

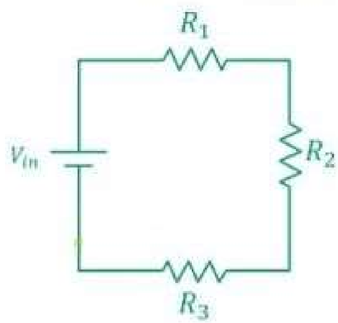
The voltage division rule states that the voltage across any of the series components in a series circuit is equal to the product of value of that resistance and the total supply voltage, divided by the total resistance of the series circuit.

- The important relations are:

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$



VOLTAGE DIVISION RULE FOR 3- RESISTORS



$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} * V_{in}$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} * V_{in}$$

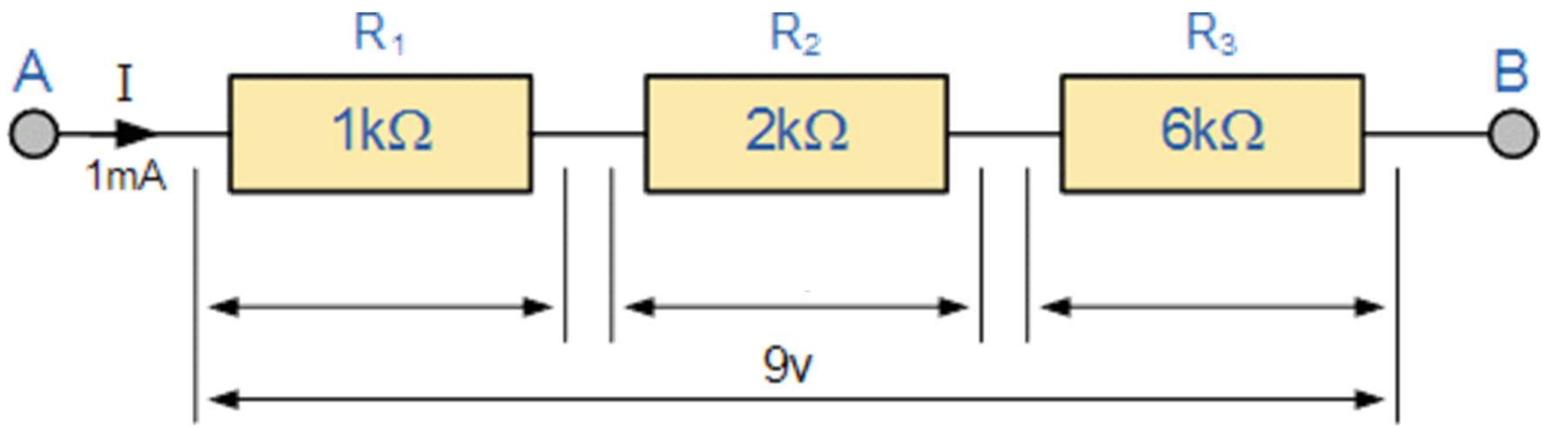
$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} * V_{in}$$

Voltage Division Rule for N-Resistors

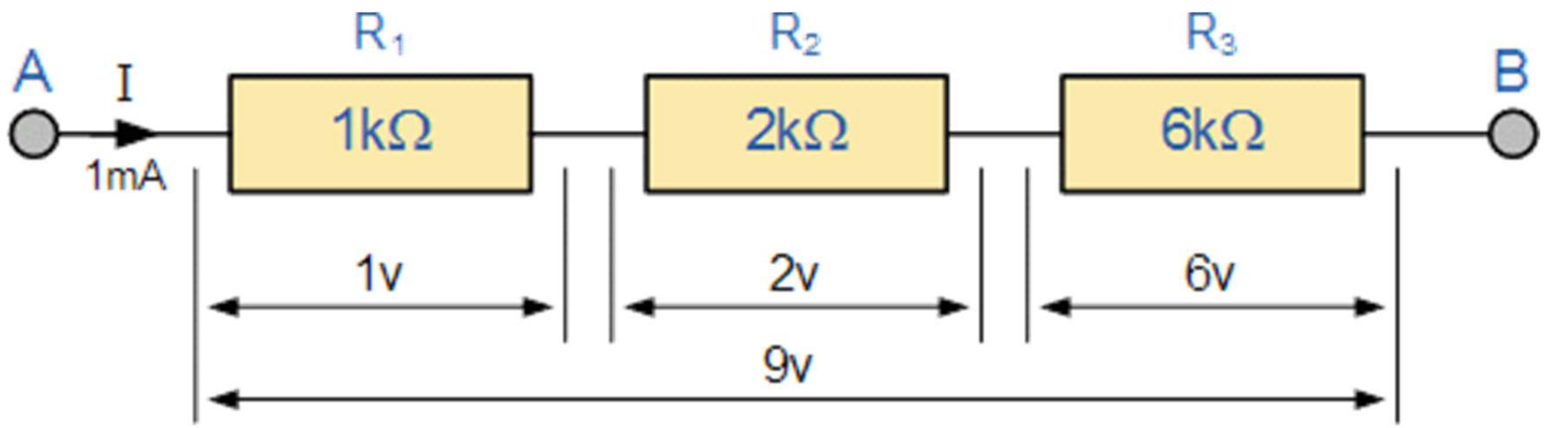
$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Example for Voltage Division Rule

Find V1, V2 and V3.



Example for Voltage Division Rule



$$V1 = [R1/(R1 + R2 + R3)] \times V$$
$$= [1/(1+2+6)] \;.\; 9$$
$$= 1 \text{ V}$$

$$V2 = [R2/(R1 + R2 + R3)] \times V$$
$$= [2/(1+2+6)] \;.\; 9$$
$$= 2 \text{ V}$$

$$V3 = [R3/(R1 + R2 + R3)] \times V$$
$$= [6/(1+2+6)] \;.\; 9$$
$$= 6 \text{ V}$$

Find the voltage across resistors R_1 and R_2 in the circuit shown in Figure-4.

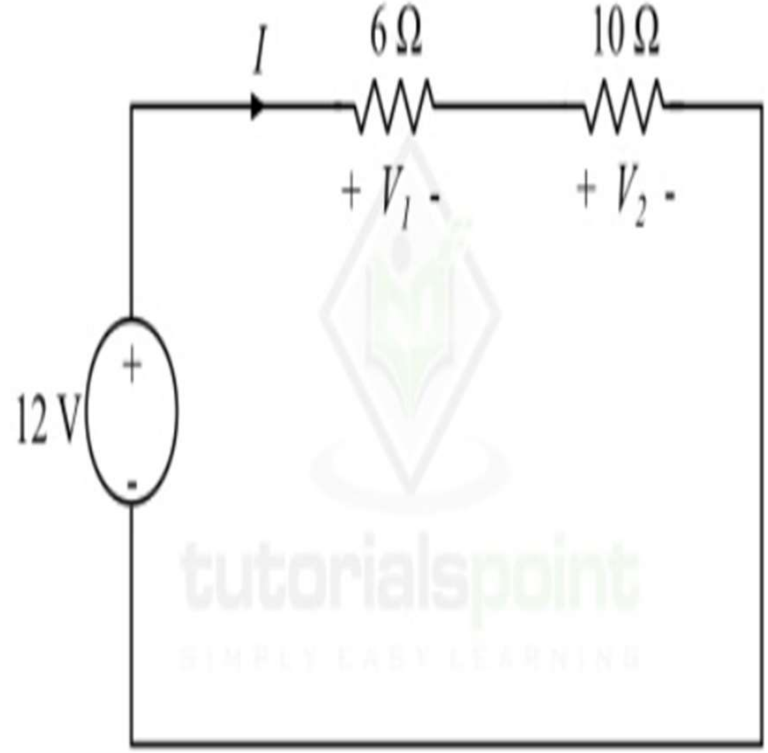


Figure 4

Solution for Figure 4:

$$V_1 = \frac{VR_1}{R_1 + R_2} = \frac{12 \times 6}{6 + 10} = 4.5V$$

The voltage across the resistor R_2 will be,

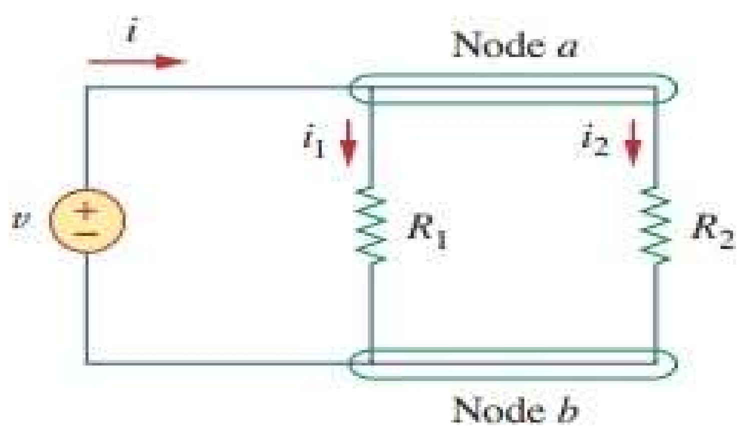
$$V_2 = \frac{VR_2}{R_1 + R_2} = \frac{12 \times 10}{6 + 10} = 7.5V$$

Current Division Rule

The current division rule states that the current in any of the parallel branches of a parallel circuit is equal to the ratio of opposite branch resistance to the sum of all parallel resistances, multiplied by the total current.

- The important relations are:

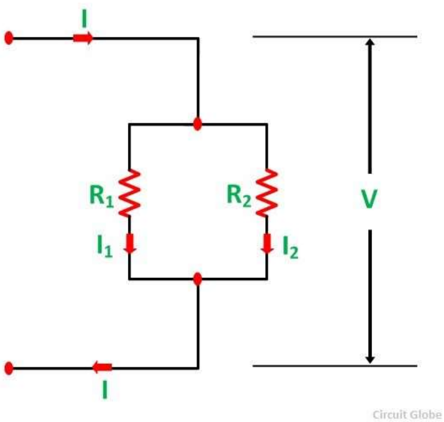
$$\boxed{i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}}$$

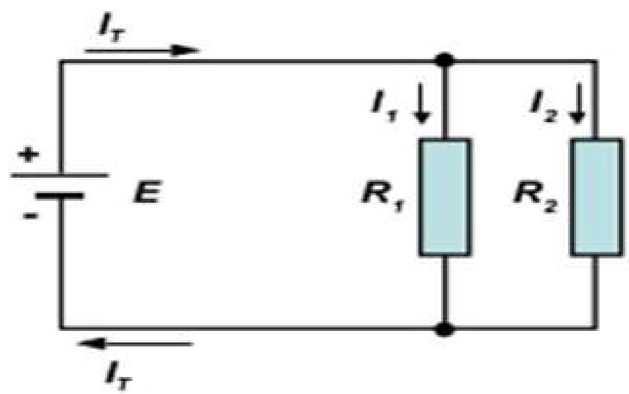


CURRENT DIVISION RULE

In parallel circuits the current I_T divides up through the various branch networks, I_1, I_2.

The ratio between any two branch currents is the inverse ratio of the branch resistances.





$$I_1 = I_T \frac{R_2}{R_1 + R_2}$$

$$I_2 = I_T \frac{R_1}{R_1 + R_2}$$

This procedure is only suitable where there are two parallel branches.

QUICK QUIZ (Poll 3)

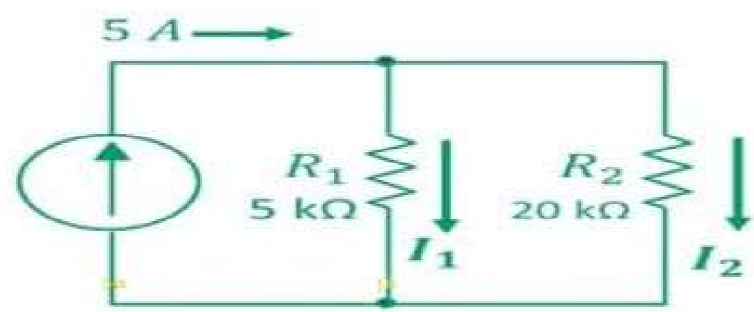
Find current across two resistors?

A. $I_1 = 4\ A\ and\ I_2 = 16\ A$

B. $I_1 = -2\ A\ and\ I_2 = 1\ A$

C. $I_1 = 4\ A\ and\ I_2 = 1\ A$

D. $I_1 = 1\ A\ and\ I_2 = 4\ A$



Find the currents I_1 and I_2 in the parallel circuit shown in Figure3

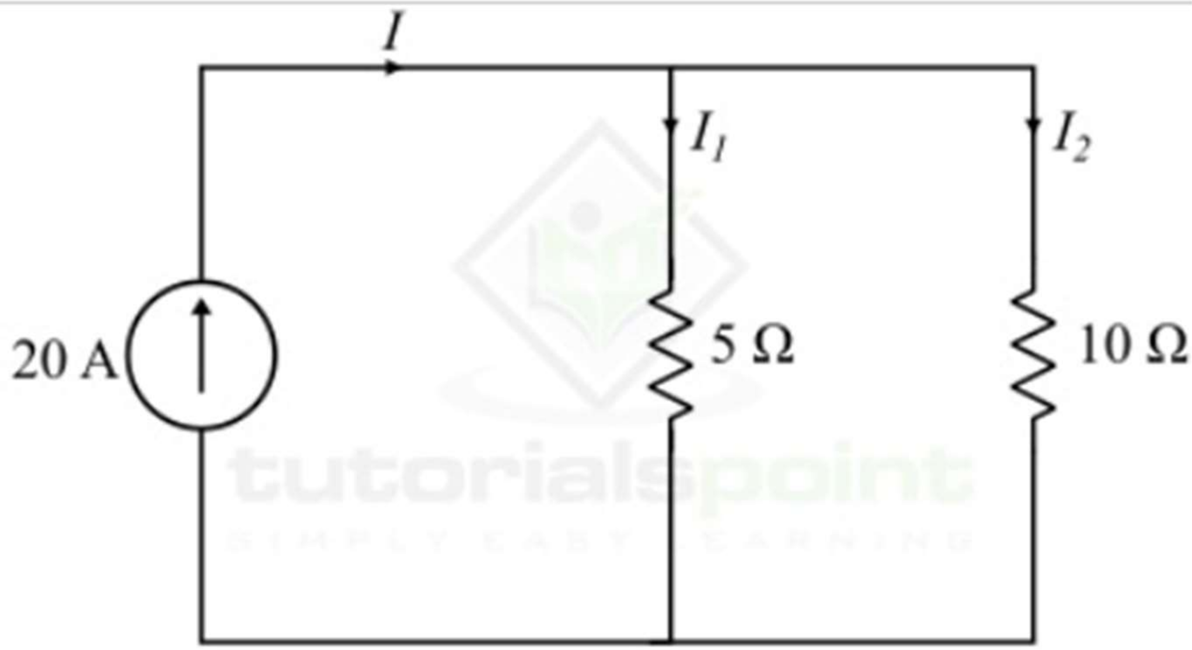


Figure 3

Using the current division rule, the current through resistor R_1 is,

$$I_1 = I \times \frac{R_2}{R_1 + R_2} = 20 \times \frac{10}{5 + 10}$$

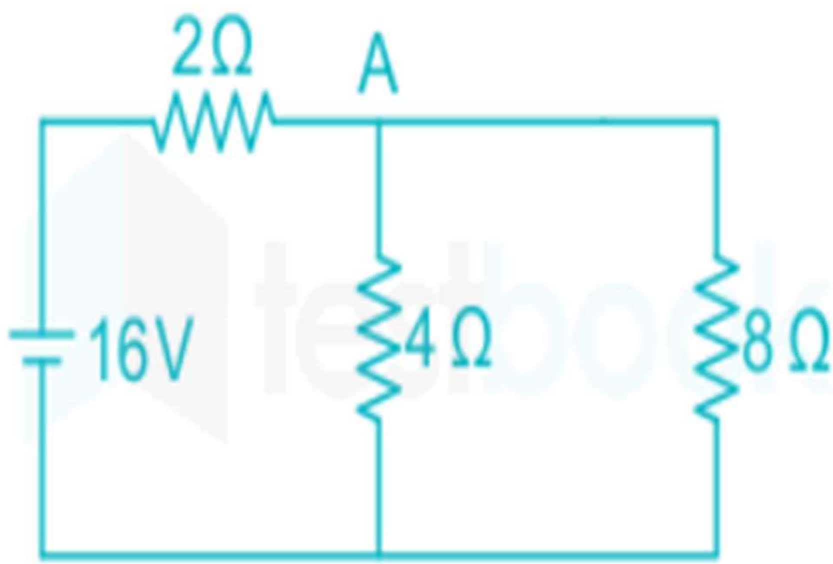
$$\therefore I_1 = 13.33 \text{ A}$$

The current through resistor R_2 will be,

$$I_2 = I \times \frac{R_1}{R_1 + R_2} = 20 \times \frac{5}{5 + 10}$$

$$\therefore I_2 = 6.67 \text{ A}$$

Find current across 8 ohm resistor in below Figure.



$R_T = (\frac{1}{4} + \frac{1}{8})^{-1} + 2$

$= \frac{8}{3} + 2$

$= \frac{14}{3} \Omega$

The total current

$I = \frac{V}{R}$

$I = \frac{16}{14} \times 3$

$= \frac{24}{7}$ A

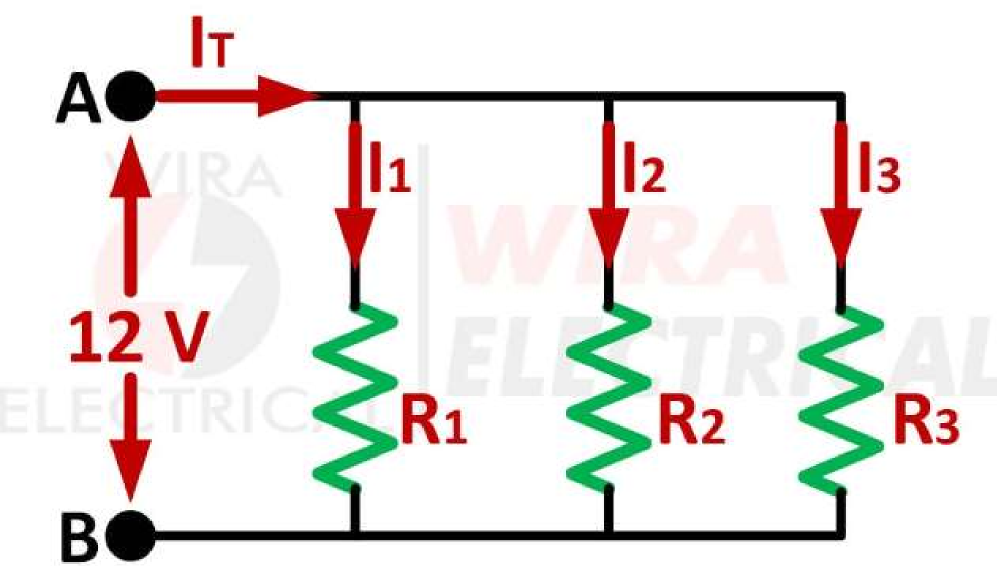
By the current divider rule

The current through 8 Ω resistor is

$$I_2 = I \times \frac{R_1}{R_1+R_2}$$

$= \frac{24}{7} \times \frac{4}{(8+4)}$

$= \frac{8}{7}$ A



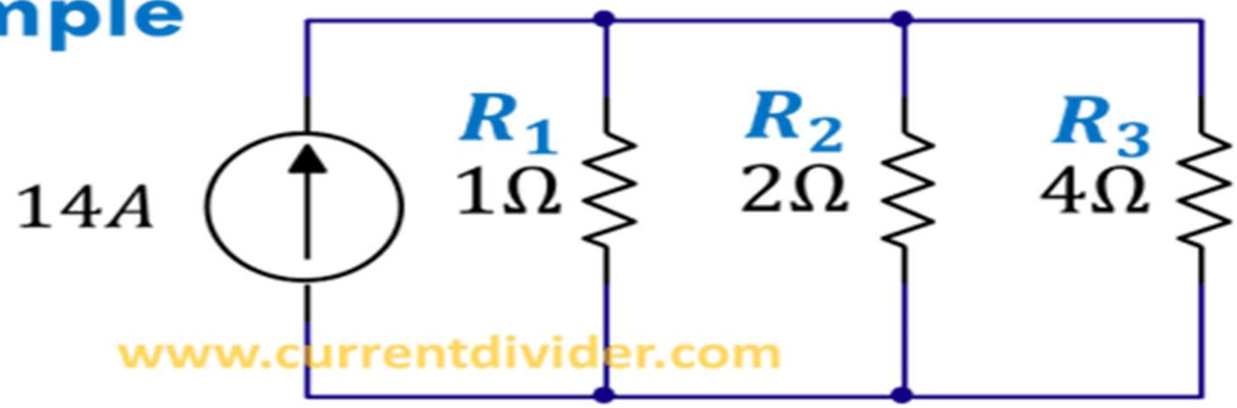
Current division for any number of parallel resistors:

$$I_X = I_T \times \frac{R_T}{R_X}$$

where:

- I_X - branch current.
- I_T - current entering branches.
- R_X - branch resistance.
- R_T - equivalent resistance of parallel circuit.

Example



$$Where\ R_t = R_1 || R_2 || R_3 = 0.5714\ \Omega$$

$$I_{R_1} = \frac{0.5714\ \Omega}{1\ \Omega} * 14\ A$$

$$I_{R_1} = 7.99\ \text{A}$$

$$I_{R_2} = \frac{0.5714\ \Omega}{2\ \Omega} * 14\ A$$

$$I_{R_2} = 3.99\ \text{A}$$

$$I_{R_3} = \frac{0.5714\ \Omega}{4\ \Omega} * 14A$$

$$I_{R_3} = 2\ \text{A}$$

Applications of Kirchhoff's Laws

- They can be used to analyze any electrical circuit.
- Computation of current and voltage of complex circuits.

Limitations of Kirchhoff's Laws

- The limitation of Kirchhoff's both laws is that it works under the assumption that there is no fluctuating magnetic field in the closed loop and the current flows only through conductors and wires.

$$\frac{\partial \phi_B}{\partial t} = 0$$

Outside elements

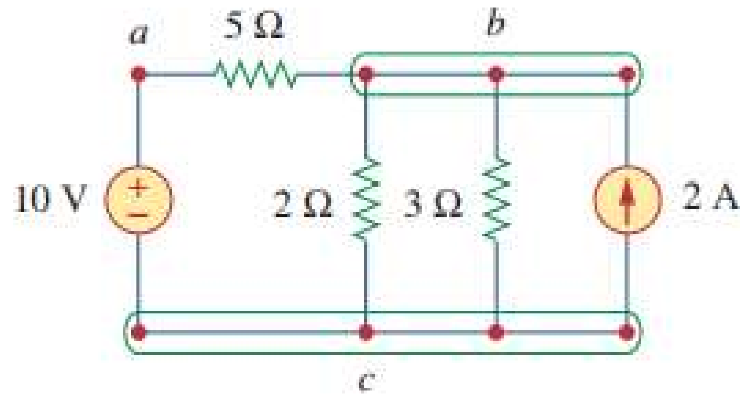
$$\frac{\partial q}{\partial t} = 0$$

Inside elements

wires resistors sources

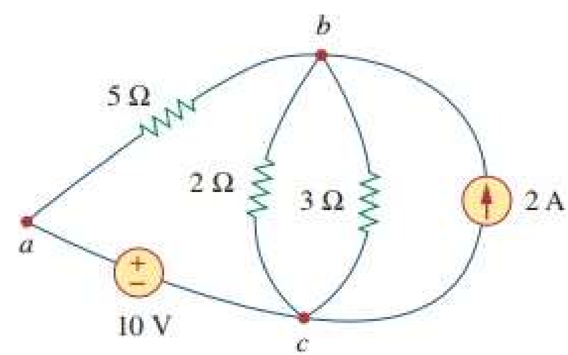
Nodes, Branches, and Loops

- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit



NOTE:

- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.



QUICK QUIZ (Poll 4)

How many branches, nodes and independent loops are present in the given circuit?

A. b=3, n=5, l=6

B. b=5, n=3, l=6

C. b=5, n=3, l=3

D. b=3, n=5, l=3