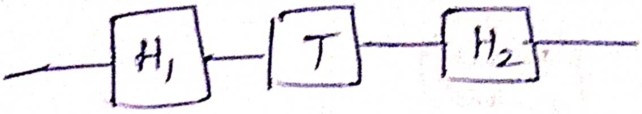
13/02/23



$$T(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$ (puts a phase infront of 1)

$$T(\theta) \Rightarrow \underset{\text{input}}{\frac{|0\rangle + |1\rangle}{\sqrt{2}}} \Rightarrow \underset{\text{output}}{\frac{|0\rangle + e^{i\theta}|1\rangle}{\sqrt{2}}}$$

Hadamard

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\theta = \frac{\pi}{4} \Rightarrow e^{i\theta} = \frac{1+i}{\sqrt{2}} = \omega$$

We know matrices for individual gates

Matrix for whole system:

$$H_2 T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{bmatrix}$$

$$H_2 T H_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2} \begin{bmatrix} 1+e^{i\theta} & 1-e^{i\theta} \\ 1-e^{i\theta} & 1+e^{i\theta} \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1+\omega & 1-\omega \\ 1-\omega & 1+\omega \end{bmatrix}$$

$H_2 T H_1 \,|\text{input}\rangle$ $\qquad$ $H_2 T H_1 = \frac{1}{2}\begin{pmatrix} 1+\omega & 1-\omega \\ 1\mp\omega & 1+\omega \end{pmatrix}$

If $|\text{input}\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$$H_2 T H_1 \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1+\omega\\1-\omega\end{pmatrix}$$

Is it normalised?

Hadamard is unitary

T is unitary

Product of unitary matrices is also a unitary matrix

$(AB)^\dagger = B^\dagger A^\dagger$

$A, B \to$ unitary.

$$(H_1 T H_2)^\dagger = H_2^\dagger T^\dagger H_1^\dagger = H_2 T H_1$$

$$(H_1 T H_2)^\dagger (H_1 T H_2) = H_2 T^\dagger H_1^\dagger H_1 T H_2$$

$$= I$$

So the norm of the output will be 1 for this circuit.

Make measurements such that

$\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are the post measurement states

What is the probability that we will get $\begin{pmatrix}1\\0\end{pmatrix}$ as the post measurement state?

$$\frac{1}{2}\begin{pmatrix}1+\omega\\1-\omega\end{pmatrix} = \frac{(1+\omega)}{2}\begin{pmatrix}1\\0\end{pmatrix} + \frac{(1\mp\omega)}{2}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\text{Probability} = \left(\frac{1+\omega}{2}\right)^2 \text{ for } \begin{pmatrix}1\\0\end{pmatrix}$$

ω

$\text{prob}\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = |\alpha|^2$ where

$|\text{output}\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$

α = Inner product of $\begin{pmatrix}1\\0\end{pmatrix}$ with output

$$= \left\langle \begin{pmatrix}1\\0\end{pmatrix}, |\text{output}\rangle \right\rangle$$

$$= \begin{pmatrix}1 & 0\end{pmatrix} \frac{1}{2}\begin{pmatrix}1+\omega\\1-\omega\end{pmatrix}$$

$$= \frac{1}{2}(1+\omega)$$

$$|\alpha|^2 = \left(\frac{1+\omega}{2}\right)^2 \rightarrow \text{irrational}$$

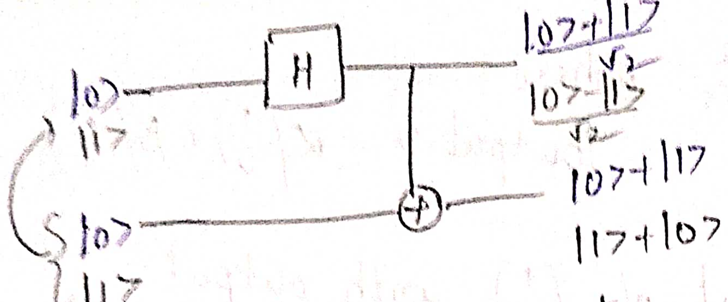
Bell states

$$|B\,00\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|B\,01\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|B\,10\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|B\,11\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

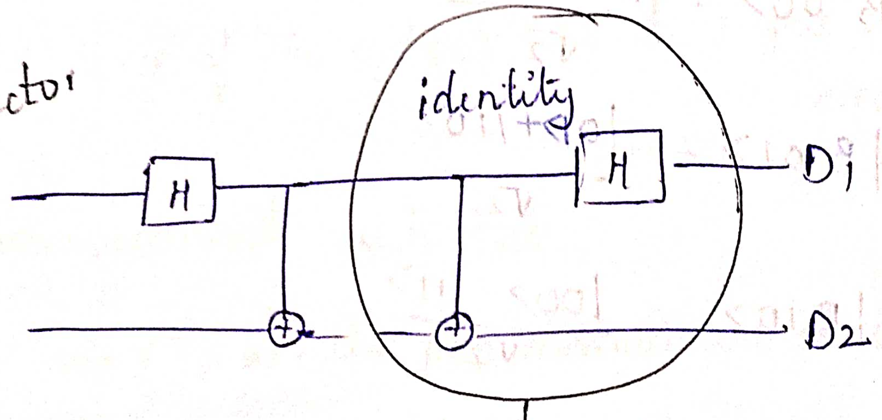


Input	Output
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2}$
$\|01\rangle$	$(\|01\rangle + \|10\rangle)/\sqrt{2}$
$\|10\rangle$	$(\|00\rangle - \|11\rangle)/\sqrt{2}$
$\|11\rangle$	$(\|01\rangle - \|10\rangle)/\sqrt{2}$

Bellstate measurement

2 Qubit system - Bipartite system.

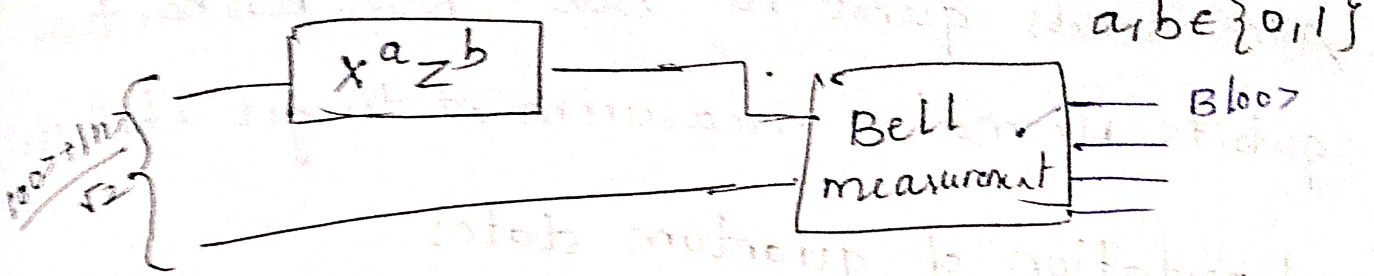
Bell detector



Bell state measurement device

D_1 detects $|0\rangle$ or $|1\rangle$ first Qubit
D_2 detects $|0\rangle$ or $|1\rangle$ second qubit.

You cannot write bellstate as a tensor product of 2 states.



If $a = b = 0$

$X^0 \rightarrow$ identity
$Z^0 \rightarrow$ identity
Output: $|B00\rangle$

Z - flips sign of $|1\rangle$
X - swaps $|0\rangle$ and $|1\rangle$

If $a = 1, b = 0$ $\quad X^1 Z^0$

a	b	Output
0	0	$\|B00\rangle$
1	0	$\|B01\rangle$
0	1	$\|B10\rangle$
1	1	$\|B11\rangle$

$\frac{|10\rangle + |01\rangle}{\sqrt{2}}$

$\underbrace{X^a Z^b}_{2\times 2} \otimes \underbrace{I}_{2\times 2} \rightarrow$ 4x4 matrix will act on input.

Quantum
Dense coding

Transfer ~~this~~ one qubit, to get 4 qubits of information.

Entanglement allows you to do dense coding.

Alice and Bob share info through qubits.

Alice has access to first qubit

Alice sends qubit to Bob. Now bob has two qubits He makes measurement to get a bell state.

Teleportation of quantum states

Transportation of quantum state.

We do not physically transfer $e^{\ominus}$ but we transport states.

We have unknown state of system we have another system whose state is known.

We need this 2nd system to be oriented in the 1st system. This is called teleportation.

Without entanglement, we cannot do this -

Let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (to be teleported)

Resource: a pair of qubits in $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

3 Qubit system

$$|\psi\rangle\left(\frac{|00\rangle+|11\rangle}{\sqrt{2}}\right)$$

($\alpha, \beta \rightarrow$ unknown)

$$= \underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{\text{state to be teleported}} \otimes \underbrace{\left(\frac{|00\rangle+|11\rangle}{\sqrt{2}}\right)}_{\text{state of shared entangled resource}}$$

$$= \frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\beta}{\sqrt{2}} |100\rangle + \frac{\beta}{\sqrt{2}} |111\rangle$$

$$= \frac{1}{2} |00\rangle \frac{\alpha|0\rangle + \beta|1\rangle}{\sqrt{2}} + \frac{1}{2} |00\rangle \frac{\alpha|0\rangle - \beta|1\rangle}{\sqrt{2}}$$

$$+ \frac{1}{2} |01\rangle \frac{\alpha|1\rangle + \beta|0\rangle}{\sqrt{2}} + \frac{1}{2} |01\rangle \frac{\alpha|1\rangle - \beta|0\rangle}{\sqrt{2}}$$

$$+ \frac{1}{2} \quad \cdots$$

$$+ \frac{1}{2} \quad \cdots$$

$$= \frac{1}{2} \frac{|00\rangle + |11\rangle}{\sqrt{2}} (\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2} \frac{(|01\rangle + |10\rangle)}{\sqrt{2}} (\alpha|1\rangle + \beta|0\rangle)$$

$$+ \frac{1}{2} \frac{|00\rangle - |11\rangle}{\sqrt{2}} (\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2} \frac{|01\rangle - |10\rangle}{\sqrt{2}} \alpha|1\rangle - \beta|0\rangle$$

We take last two qubits to be teleported to first two qubits

Remaining will be bell states

α, β are unknown, but the state of 3rd qubit at the output is same as the first.

Here teleportation occured.

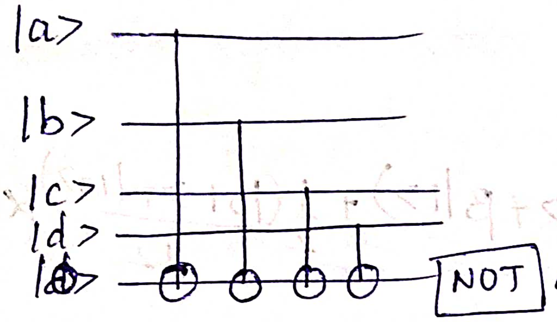
By transmitting 1 qubit,

Locality

20/2/23:

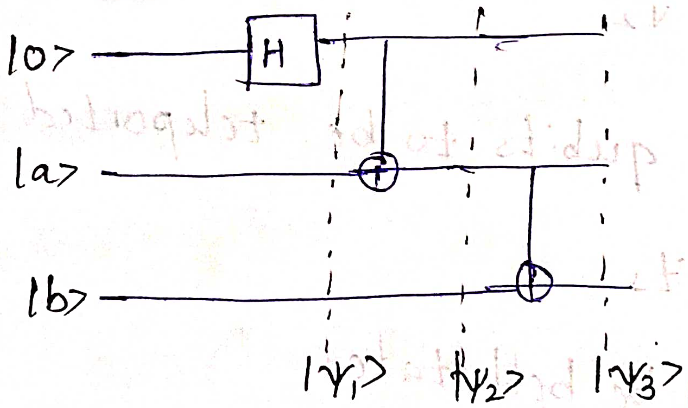
Quiz 3

(1) For parity checking



← Measure this qubit

(2)



$$|\psi_1\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|a\rangle|b\rangle = \frac{|0\rangle|a\rangle|b\rangle + |1\rangle|a\rangle|b\rangle}{\sqrt{2}}$$

$$|\psi_2\rangle = \frac{|0\rangle|0\rangle\oplus|a\rangle|b\rangle + |1\rangle|1\rangle\oplus|a\rangle|b\rangle}{\sqrt{2}}$$

$$= \frac{|0\rangle|a\rangle|b\rangle + |1\rangle|1\oplus a\rangle|b\rangle}{\sqrt{2}}$$

$$|\psi_3\rangle = \frac{|0\rangle|a\rangle|a\oplus b\rangle + |1\rangle|1\oplus a\rangle|1\oplus a\oplus b\rangle}{\sqrt{2}}$$

a	b	$|\psi_3\rangle$
0	0	$\frac{|000\rangle + |111\rangle}{\sqrt{2}}$
0	1	$\frac{|001\rangle + |110\rangle}{\sqrt{2}}$
1	0	$\frac{|011\rangle + |100\rangle}{\sqrt{2}}$
1	1	$\frac{|010\rangle + |101\rangle}{\sqrt{2}}$

Function

Maps elements of one set to another set.

e^x → takes input of all +ve and -ve
gives output as non negative values.

Single input

input A: $\{0,1\}$

output B: $\{0,1\}$

Ex:1
$f_1(0) = 1$
$f_1(1) = 0$

Ex 2
$f_1(0) = 0$
$f_1(1) = 1$

Balanced function

Ex 3.
$f_1(0) = 1$
$f_1(1) = 1$

Exa Consta

$f_1(0) = 0$
$f_2(1) = 0$

constant function

$f_1 : A \rightarrow B$ (Boolean function)

George Bool.

2 inputs

A_1: $\{0,1\}$

A_2: $\{0,1\}$

B_1: $\{0,1\}$

$f_2(x, y) = g(z)$

$x \in A_1$
$y \in A_2$

x	y	$f_2(x,y)$
0	0	0
0	1	0
1	0	0
1	1	0
0	0	1
0	1	1
1	0	1
1	1	1

constant function

0	0
0	1
1	0
1	1

$$f_2 : \underbrace{A_1 \times A_2}_{\text{Cartesian product}} \rightarrow B_1$$

$$A_1 \times A_2 = \{(0,0),(0,1),(1,0),(1,1)\}$$

$$f_3 : \underbrace{A_1 \times A_2}_{\text{Domain}} \rightarrow \underbrace{A_1 \times A_2}_{\text{Range}}$$

$$f_3(x,y) = (y, x) \qquad x \in A_1$$

$$f_3(x,y) = (x \oplus 1, y \oplus 1) \quad y \in A_2$$

$f_3(x,y) = (x, x-y)$

Generalisation:

$$f: A_1 \otimes A_2 \otimes A_3 \ldots \times A_p \Rightarrow A_1 \times A_2 \times A_3 \ldots A_m$$

$$\{0,1\} = Z_2$$

$$\underbrace{Z_2 \times Z_2 \times Z_2 \cdots Z_2}_{m \text{ times}} = Z_2^m$$

Cartesian product of Z_2 with itself 'm' times.

contains 2^m elements

$$f: Z_2^m \to Z_2^p$$

Given a fun^c how to decide it is a constant fun^c?

$|x\rangle \; |y\rangle$

$|0\rangle \; |0\rangle = |00\rangle \equiv |0\rangle_2$ $\quad x \in Z_2$

$|0\rangle \; |1\rangle = |01\rangle \equiv |1\rangle_2$ $\quad y \in Z_2$

$|1\rangle \; |0\rangle = |10\rangle \equiv |2\rangle_2$

$|1\rangle \; |1\rangle = |11\rangle \equiv |3\rangle_2$

Tensor product of two qubits.

$|x\rangle \; |y\rangle \; |z\rangle$

$|0\rangle \; |0\rangle \; |0\rangle \quad |0\rangle_3$

$|0\rangle \; |0\rangle \; |1\rangle \quad |1\rangle_3$

$|0\rangle \; |1\rangle \; |0\rangle \quad |2\rangle_3$

$\vdots \quad \vdots \quad \vdots$

$|1\rangle \; |1\rangle \; |1\rangle \quad |7\rangle_3$

$X \otimes Z |0\rangle|0\rangle = X|0\rangle \otimes Z|0\rangle$

$X \otimes X |0\rangle|0\rangle = X|0\rangle \otimes X|0\rangle = X^{\otimes 2}|0\rangle_2$

~~$X|0\rangle$~~

$X \otimes X \otimes Z \otimes Z \otimes Z |3\rangle_5 = X^{\otimes 2} \otimes Z^{\otimes 3} |3\rangle_5$

$f : \{0,1\} \rightarrow \{0,1\}$

Given : f is either constant or balanced

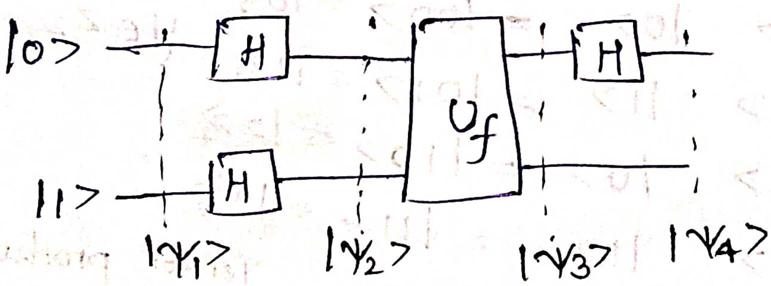
Qn : Is f constant or balanced ?

Classically : i) Calculate $f(0)$

ii) Calculate $f(1)$

iii) Compare.

Quantum version



$U_f |x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$

$|\psi_1\rangle = |0\rangle|1\rangle$

$|\psi_2\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$

$= \frac{|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle}{2}$

$$\cancel{U_f|0\rangle|0\rangle = |0\rangle}$$

$$U_f|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{|0\rangle|0\oplus f(0)\rangle - |0\rangle|1\oplus f(0)\rangle}{\sqrt{2}}$$

if $f(0)=0$

$$= \frac{|0\rangle|0\rangle - |0\rangle|1\rangle}{\sqrt{2}}$$

$$= |0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$U_f|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} = |0\rangle\frac{0-|1\rangle}{\sqrt{2}}$$

if $f(0)=1$

$$U_f|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{|0\rangle|1\rangle-|0\rangle|0\rangle}{\sqrt{2}}$$

$$= |0\rangle\frac{|1\rangle-|0\rangle}{\sqrt{2}}$$

$$U_f|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} = -|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$U_f|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{|1\rangle|0\oplus f(1)\rangle - |1\rangle|1\oplus f(1)\rangle}{\sqrt{2}}$$

if $f(1)=0$

$$= \frac{|1\rangle|0\rangle - |1\rangle|1\rangle}{\sqrt{2}}$$

$$U_f|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} = |1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

if $f(1) = 1$

$$U_f |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{|1\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

$$= |1\rangle \frac{|1\rangle - |0\rangle}{\sqrt{2}}$$

$$U_f |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} = -|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$U_f (|x\rangle|y\rangle) = \begin{cases} |x\rangle|y\rangle & f(0) = 0, f(1) = 0 \\ -|x\rangle|y\rangle & f(0) = 1, f(1) = 1 \end{cases}$$

$$U_f |x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} = (-1)^{f(x)} |x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$U_f \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$= \frac{(-1)^{f(x)}|0\rangle + (-1)^{f(x)}|1\rangle}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

If f is constant

$f(0) = f(1) = 1$

$$|\psi_3\rangle = -\frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$f(0) = f(1) = 0$

$$|\psi_3\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

If the function is constant

$$|\psi_3\rangle = \pm \frac{|0\rangle+|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

If f is balanced

$f(0) = 0$

$f(1) = 1$

$$|\psi_3\rangle = \frac{|0\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}} - \frac{|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$= \frac{|0\rangle-|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

if $f(0) = 1$

$f(1) = 0$

$$|\psi_3\rangle = -\frac{|0\rangle+|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$= -\frac{|0\rangle-|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$|\psi_3\rangle = \pm \frac{|0\rangle-|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$|\psi_4\rangle = H \otimes I \, |\psi_3\rangle$$

$$= \pm |1\rangle \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$ if f is balanced

$$= \pm |0\rangle \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$ if f is constant

By seeing the 1st state we can say if f is balanced or constant function

22/2/23

Deutsch Josza

$f : Z_2^m \rightarrow Z_2$

$\{0,1\}^m \rightarrow \{0,1\}$

Given f is either balanced or constant

Ex: $\{0,1\}^2$

$Z_2^2 = \{0,1\} \times \{0,1\}$

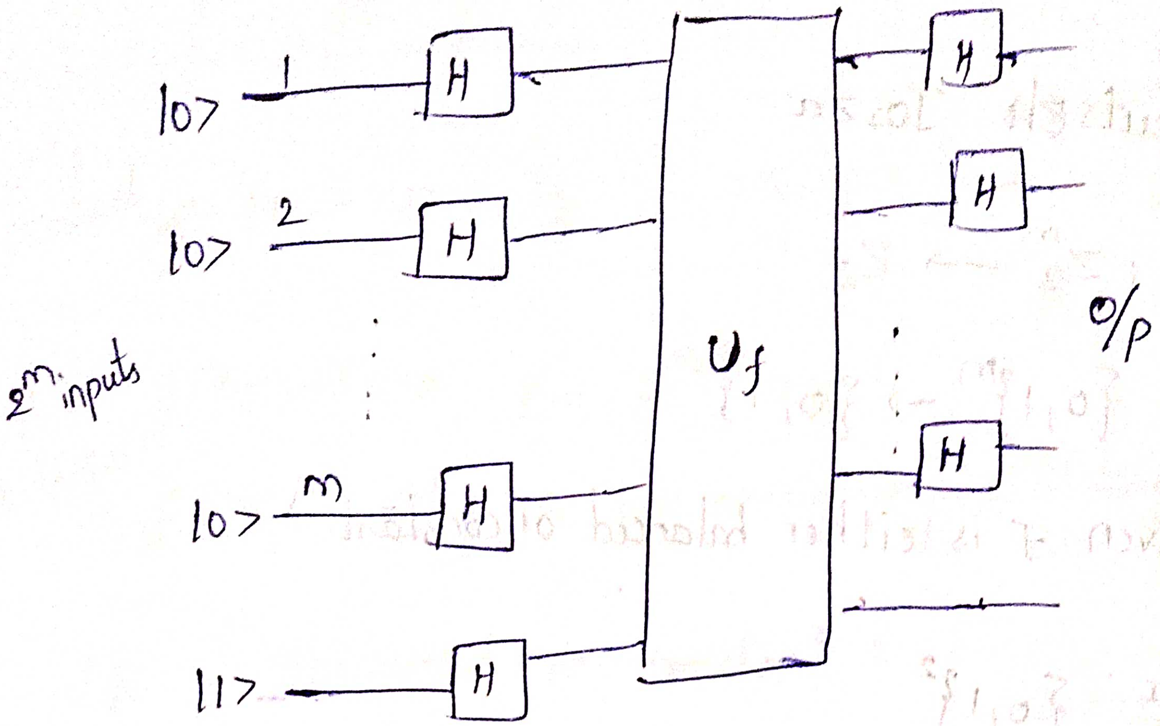
$= \{00, 01, 10, 11\}$

Ex: $f : Z_2^2 \rightarrow Z_2$

$$\left.\begin{aligned} f(00) &= 0 \\ f(01) &= 0 \\ f(10) &= 1 \\ f(11) &= 1 \end{aligned}\right\} \text{Balanced}$$

Ex

$$\left.\begin{aligned} f(00) &= 0 \\ f(01) &= 0 \\ f(10) &= 0 \\ f(11) &= 0 \end{aligned}\right\} \text{constant}$$



$$\underbrace{|0\rangle|0\rangle \cdots |0\rangle}_{m \text{ times}} = |000\cdots0\rangle = |0\rangle\rangle$$

$$H|0\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$$

$$H|1\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$H|x\rangle = \frac{|0\rangle+(-1)^x|1\rangle}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\sum_{z=0}^{1}(-1)^{xz}|z\rangle$$

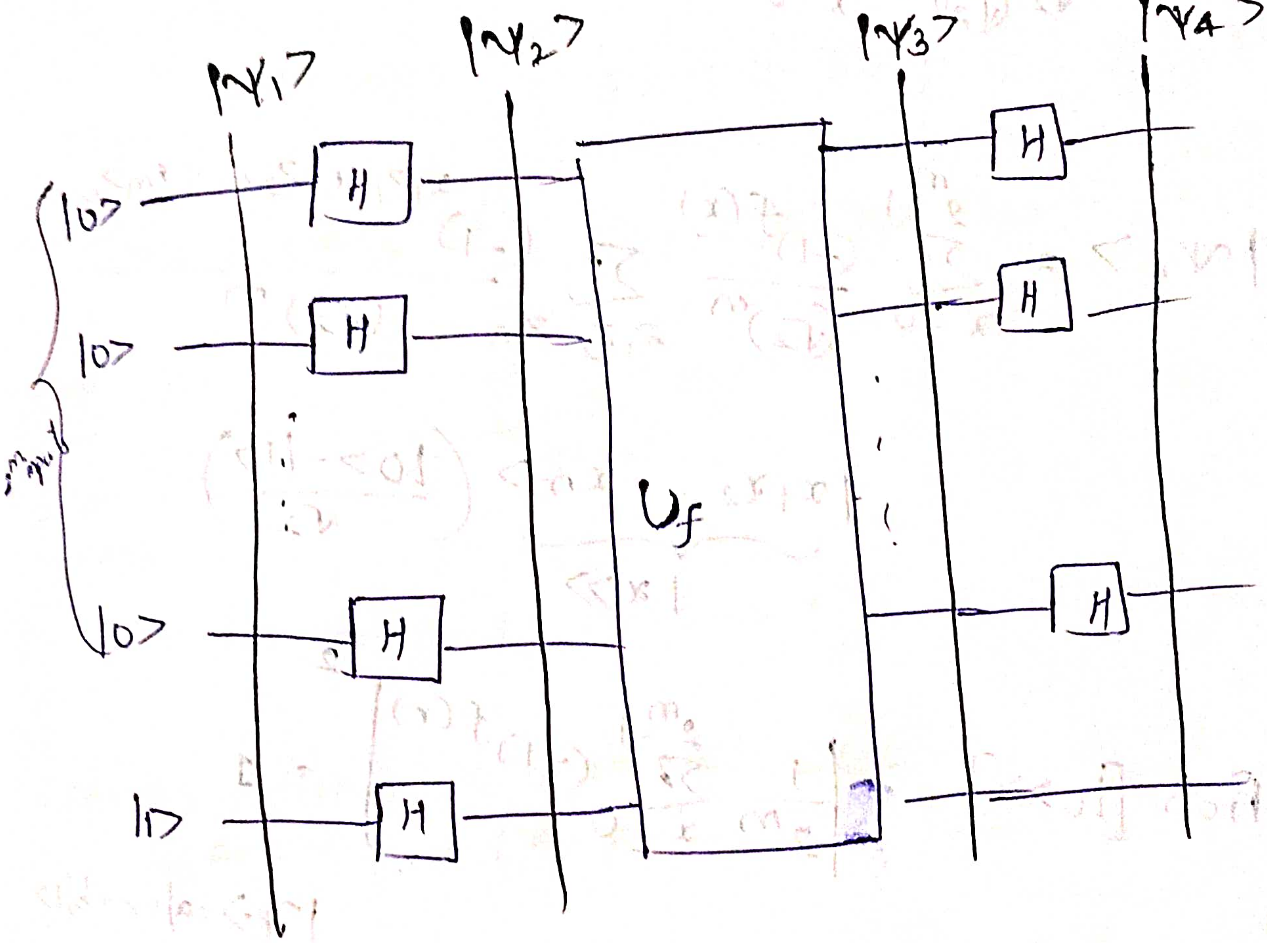
$$H\otimes H\,|x_1\rangle|x_2\rangle = H|x_1\rangle \otimes H|x_2\rangle$$

$$= \left(\frac{1}{\sqrt{2}}\sum_{z_1=0}^{1}(-1)^{x_1z_1}|z_1\rangle\right)\otimes\left(\frac{1}{\sqrt{2}}\sum_{z_2=0}^{1}(-1)^{x_2z_2}|z_2\rangle\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \sum_{z_1}\sum_{z_2} (-1)^{x_1z_1+x_2z_2} |z_1\rangle|z_2\rangle$$

$$H^{\otimes m}|x_1x_2\ldots x_m\rangle$$

$$= \frac{1}{(\sqrt{2})^m}\sum_{z_1}\sum_{z_2}\cdots\sum_{z_m}(-1)^{x_1z_1+x_2z_2+\cdots+x_mz_m}|z_1\rangle|z_2\rangle\ldots|z_m\rangle$$



$$|\psi_1\rangle = |0\rangle\rangle|1\rangle$$

$$|\psi_2\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\cdots\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{(\sqrt{2})^m}\left[|0\rangle\rangle+|1\rangle\rangle+\cdots+|2^m-1\rangle\rangle\right]\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{(\sqrt{2})^m}\left(\sum_{x=0}^{2^m-1}|x\rangle\rangle\right)\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

$$U_f|\psi_2\rangle = \frac{1}{(\sqrt{2})^m}\sum_{x=0}^{2^m-1} U_f\left[|x\rangle\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}}\frac{1}{(\sqrt{2})^m}\sum_{x=0}^{2^m-1}\left(U_f|x\rangle\rangle|0\rangle - U_f|x\rangle\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\frac{1}{(\sqrt{2})^m}\sum_{x=0}^{2^m-1}\left(|x\rangle\rangle|f(x)\rangle - |x\rangle\rangle|1\oplus f(x)\rangle\right)$$

$$|\psi_4\rangle = \sum_{x=0}^{2^m-1}\frac{(-1)^{f(x)}}{(\sqrt{2})^m}\sum_{z_1 z_2 \, z_m}\frac{(-1)^{x_1z_1+x_2z_2+\dots+x_mz_m}}{(\sqrt{2})^m}$$

$$\underbrace{|x_1x_2\dots x_m\rangle}_{|x\rangle\rangle}\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$\text{Prob}\,[|0\rangle\rangle] = \left|\frac{1}{2^m}\sum_{x=0}^{2^m-1}(-1)^{f(x)}\right|^2 = 1$$

$|\psi\rangle = a|0\rangle + b|1\rangle$

$P(|0\rangle) = |a|^2$

$P(|1\rangle) = |b|^2$

01/03/23

$$\hat{H}|x\rangle = \frac{1}{\sqrt{2}}\overbrace{\sum_{y=0}^{1} \underbrace{(-1)^{xy}}_{\text{phase of } |y\rangle} |y\rangle}^{|\psi\rangle} \qquad |x\rangle = \{|0\rangle, |1\rangle\}$$

What is x, given $|\psi\rangle$?

$H \otimes H = I$

$H|\psi\rangle = |x\rangle$

Only works if $|\psi\rangle$ is of the form $\frac{1}{\sqrt{2}}\sum_{y=0}^{1}(-1)^{xy}|y\rangle$

General phase in a superposition

state of 'n' qubit system $\quad |\psi\rangle = \sum_{y=0}^{2^n-1} \underbrace{e^{2\pi i w y}}_{\text{phase}} |y\rangle$

Can we design a circuit to find w given the state $|\psi\rangle$?

Fourier expansion of aperiodic function

$\sum a_n \sin(nx) + \sum b_n \cos(nx)$

n qubit system

$$|\psi\rangle_n = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} (-1)^{x \cdot y} |y\rangle$$

↳ Hadamard can only identify this form

$H^{\otimes n} |\psi\rangle_n = |x\rangle_n$

w can be any number only an integer.

$$w = b_n b_{n-1} b_{n-2} b_{n-3} \ldots b_0 \cdot a_1 a_2 a_3 \ldots$$

$$= b_n \times 2^n + \ldots + b_1 \times 2^1 + b_0 \times 2^0 + \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots$$

$$e^{2\pi i w y} = \underbrace{e^{2\pi i (b_n b_{n-1} \ldots b_0) y}}_{1} \; e^{2\pi i (0 \cdot a_1 a_2 \ldots) y}$$

integer integer

$$w \in (0,1)$$

$$e^{2\pi i (0 \cdot a_1 a_2 \ldots a_m) y}$$

$$(0 \cdot a_1 a_2 \ldots) = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots$$

nonnegative integer

$$\underbrace{2^k}_{} \cdot w = a_1 a_2 a_3 \ldots a_k \cdot a_{k+1} a_{k+2} a_{k+3} \ldots$$

shifts the ~~decir~~ decimal point by k bits

2 Qubit System

$$|\psi_2\rangle = \frac{1}{(\sqrt{2})^2} \sum_{y=0}^{2^2-1=3} e^{2\pi i (0 \cdot x_1 x_2) y} |y\rangle$$

$$= \frac{1}{(\sqrt{2})^2} \Big(\overset{|0\rangle|0\rangle}{|0\rangle\rangle} + e^{2\pi i (0 \cdot x_1 x_2)} \overset{|0\rangle|1\rangle}{|1\rangle\rangle} + e^{2\pi i (0 \cdot x_1 x_2) 2} \overset{|1\rangle|0\rangle}{|2\rangle\rangle} + e^{2\pi i (0 \cdot x_1 x_2) 3} \overset{|1\rangle|1\rangle}{|3\rangle\rangle} \Big)$$

(2^1+1)

Multi Qubit System (Double angular bracket)

$$= \frac{1}{(\sqrt{2})^2}\Big[|0\rangle\rangle + e^{2\pi i(0.x_1x_2)}|1\rangle\rangle + e^{2\pi i(x_1.x_2)}|2\rangle\rangle + e^{2\pi i(0.x_1x_2)2}\, e^{2\pi i(0.x_1x_2)1}|3\rangle\rangle\Big]$$

$$= \frac{1}{(\sqrt{2})^2}\Big[|0\rangle\rangle + e^{2\pi i(0.x_1x_2)}|1\rangle\rangle + e^{2\pi i(x_1.x_2)}|2\rangle\rangle + e^{2\pi i(x_1.x_2)}\, e^{2\pi i(0.x_1x_2)}|3\rangle\rangle\Big]$$

Neglecting x_1 bcoz it has no effect on phase

$$= \left(\frac{|0\rangle + e^{2\pi i(0.\cancel{x_1}x_2)}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle + e^{2\pi i(0.x_1x_2)}|1\rangle}{\sqrt{2}}\right)$$

Phase gate

$$R(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i\theta} \end{bmatrix}$$

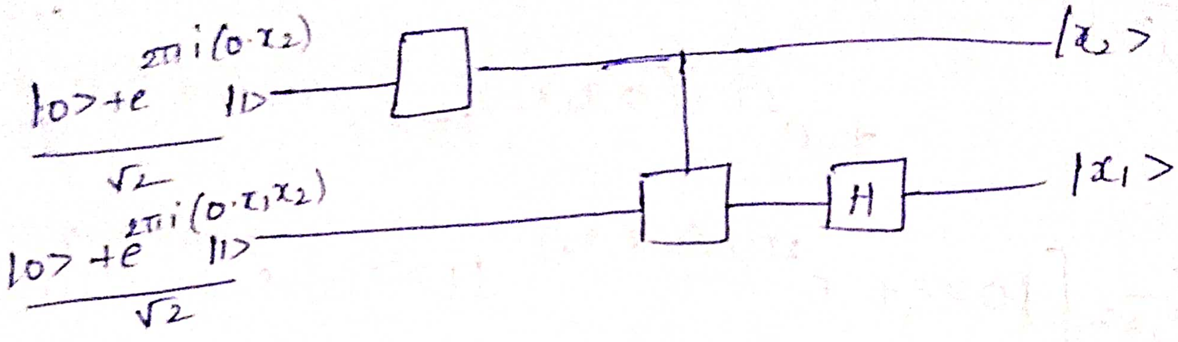
$$R_2 = R\left(\theta = \frac{1}{2^2}\right) = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^2}} \end{bmatrix}$$

$$R_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{-2\pi i}{2^2}} \end{bmatrix}$$

$$e^{-2\pi i(0.01)}$$

R_2^{-1} on the 2nd qubit

$$R_2^{-1}\left[\frac{|0\rangle + e^{2\pi i(0.x_1x_2)}|1\rangle}{\sqrt{2}}\right] = \frac{|0\rangle + e^{2\pi i(0.x_1)}}{\sqrt{2}} \quad \text{if } x_2 = 1$$



These helps in factorization

- ATM pins (how are they safe)
- Discrete logarithm problem

3 qubit system

$$|\psi_3\rangle_3 = \left(\frac{|0\rangle + e^{2\pi i(0.x_3)}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle + e^{2\pi i(0.x_2x_3)}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle + e^{2\pi i(0.x_1x_2x_3)}|1\rangle}{\sqrt{2}}\right)$$