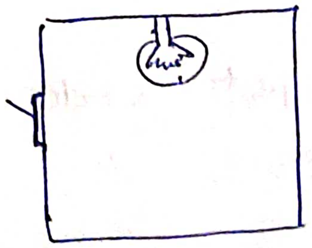
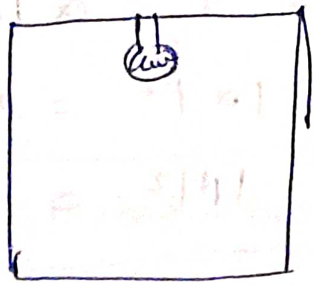
Classical Switch



Quantum Switch

Quantum switch



state of room (in Quantum switch) $= \frac{1}{\sqrt{2}} |light\rangle + \frac{1}{\sqrt{2}} |dark\rangle$ → When we touch at middle.

state of room $= \sqrt{0.9}\,|l_1\rangle + \sqrt{0.1}\,|dark\rangle$ → When we touch near the north pole or south pole

$\sqrt{0.9}$ → 90%, $\sqrt{0.1}$ → 10%

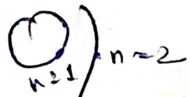
* Qubit → (Quantum Bit)

↳ basic unit of information in QC. (quantum computing)

→ It is 2-level system.

→ Spin ↑ up ↓ down

→ Photon → (polarization) → (horizontal or vertical)



spin $|0\rangle$ ↑ $|1\rangle$ ↓

$\frac{1}{4} + \frac{9}{4} = \frac{10}{4}$

State $\psi = \alpha|0\rangle + \beta|1\rangle$

$|\alpha|^2$ = Probability to get state $|0\rangle$

$|\beta|^2$ = " " " $|1\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

(a) $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ = $\cancel{\frac{1}{2}+\frac{1}{2}=1}$

~~0.64 + 0.3~~

(b) $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

(c) $|\psi\rangle = 0.8|0\rangle + 0.6|1\rangle$ ~~0.64 + 0.36~~

x (d) $\psi = \frac{1}{2}|0\rangle + \frac{3}{2}|1\rangle$ $\boxed{\frac{1}{4} + \frac{3}{4} = \frac{10}{4} \neq 1}$

3-Level function

$\rightarrow |\psi\rangle = \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle + \left(\frac{1}{2} + \frac{i}{2}\right)|2\rangle$

$\frac{1}{4} + \frac{1}{4} + \left(\frac{1}{4} + \frac{1}{4}\right)$ $\frac{4}{4} = 1$

Outcome	P	new state
if 0	1/4	$\|0\rangle$
if 1	1/4	$\|1\rangle$
if 2	1/2	$\|2\rangle$

~~If $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$~~

~~$\langle 0|\psi\rangle$ =~~

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$= 1 \cdot 1 \cos\theta$

$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1 i}{\sqrt{2}}|1\rangle$

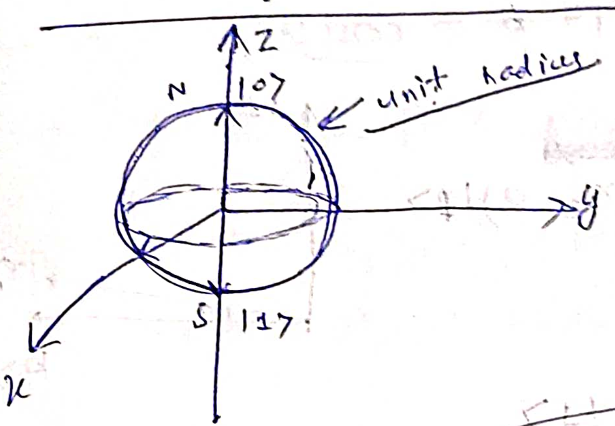
$\cos 45|0\rangle + \sin 45|1\rangle$

$\boxed{\text{Probability} = \cos\theta}$

Now, $\langle 0|\psi\rangle = \frac{1}{\sqrt{2}}$

Visualize Bloch sphere

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

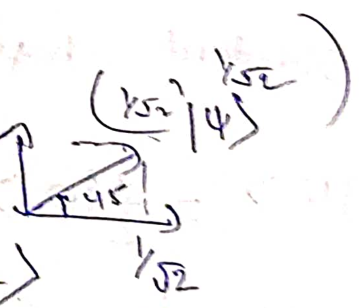


state of $X = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\mathbb{C}^2$ $|v\rangle$ Lin. $|0\rangle$ and $|1\rangle$ → standard basis

$\mathbb{C}^2$ $|v\rangle$ Lin. $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ → sign basis

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = c_1(\ ) + c_2(\ )$ and $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$



$|\psi\rangle = \underbrace{\cos\theta}|0\rangle + \underbrace{\sin\theta}|1\rangle$

$\cos^2\theta$

$\sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\langle 0|\psi\rangle = \cos\theta$

$|0\rangle$

Measure in

$|1\rangle$

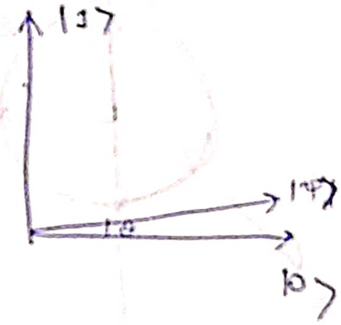
$\mathbb{C}^2$ prob. at stage $|0\rangle$ $\leftarrow |\alpha|^2$ $|\beta|^2$ — probability at the stage $|1\rangle$.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\langle 0|\psi\rangle = \alpha \underbrace{\langle 0|0\rangle}_{\|0\|^2 = 1} + \beta \underbrace{\langle 0|1\rangle}_{0}$$

→ Now, $|\langle 0|\psi\rangle|^2 = |\alpha|^2 = \underline{\cos^2\theta}$.

$$|\psi\rangle = \cos\theta\,|0\rangle + \cos(\pi/2 - \theta)|1\rangle$$



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Sign basis or <u>Hadamard Basis</u>

Standard basis $|0\rangle$ & $|1\rangle$

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle \quad \underline{\text{Std. basis}}$$

Measure in sign basis $|+\rangle$ and $|-\rangle$

$$|\psi\rangle = \beta_0|+\rangle + \beta_1|-\rangle$$
$$= \frac{\beta_0}{\sqrt{2}}$$

$|\langle 0|\psi\rangle|^2 = 1/2 = 1/4$ → standard basis

$|\langle +|\psi\rangle|^2 = |\beta_0|^2$ sign basis

$$|\langle - | \psi \rangle|^2 = |\beta_1|^2$$

⊛ Summary → In 1 Qubit

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$

Two Qubits

→ $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$

→ $|\phi\rangle = \beta_0|0\rangle + \beta_1|1\rangle$

$\Rightarrow (\alpha_0|0\rangle + \alpha_1|1\rangle)(\beta_0|0\rangle + \beta_1|1\rangle)$

$\Rightarrow \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle$

→ It is basically tensor product b/w

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}_{2\times 1} \otimes \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}_{2\times 1} = \begin{bmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_0 \\ \alpha_1\beta_1 \end{bmatrix}_{4\times 1}$$

⊛ If $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$|\phi\rangle = \frac{1}{2}|0\rangle - \sqrt{3}/2\,|1\rangle$

$$\begin{bmatrix} 1 \\ -\frac{\sqrt{3}}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ -\frac{\sqrt{3}}{4} \end{bmatrix}_{4\times 1}$$

⊛ Bell state or Entangled state :- When we cannot factorize qubits or we cannot write tensor product of two states.

* In $\mathbb{C}^4$ $|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle +$
$\alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

* $|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$.

* $|\psi\rangle = \begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}$

* Ex:- $\underbrace{\frac{1}{\sqrt{2}}|00\rangle + \frac{i}{2}|01\rangle}_{\text{I.e. is not normalized}} + \frac{1}{2}|11\rangle$

* You measure only first qubit

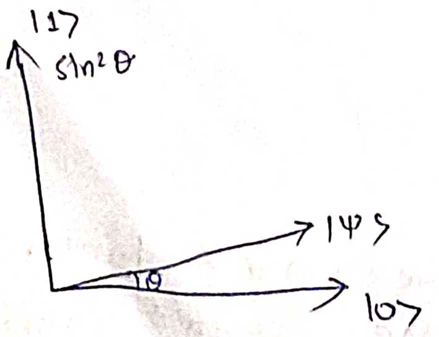
$P[0,\ ] = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

So, $\dfrac{\frac{1}{\sqrt{2}}|00\rangle + \frac{i}{2}|01\rangle}{\sqrt{\frac{1}{2}+\frac{1}{4}} = \sqrt{\frac{2+1}{4}} = \frac{\sqrt{3}}{2}}$

* $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \neq \underbrace{|\psi\rangle}_{\alpha_0|0\rangle + \alpha_1|1\rangle} \otimes |\phi\rangle$

$\beta_0|0\rangle + \beta_1|1\rangle$

* Recap ~~Revision~~



if $\theta \approx 0$, Then probability of $\cos^2\theta = 1$ be 100%.

(*) 2 Qubits

$$|\psi\rangle = (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$\begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}, \quad |00\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftarrow |00\rangle$$

Ex: $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle + 0 \cdot |01\rangle + 0|10\rangle$

$\neq |\psi\rangle \otimes |\phi\rangle$

Bell state or entangled state.

(*) Chapter

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, Is it Hermitian

$\downarrow$

Pauli matrix

For Hermitian $X = X^{+}$

For unitary $X^{+} = X^{-1}$

$\boxed{\text{GATE}}$ $\rightarrow$ Bit-Flip Gate or not gate

$X(\alpha|0\rangle + \beta|1\rangle) = \alpha X|0\rangle + \beta X|1\rangle$

$\downarrow$

not gate

$= \alpha|1\rangle + \beta|0\rangle \quad = \beta|0\rangle + \alpha|1\rangle$

Q: $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|0\rangle = 1|0\rangle + 0 \cdot |1\rangle$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|1\rangle = 0|0\rangle$

$\left.\begin{array}{l} X|0\rangle = |1\rangle \\ X|1\rangle = |0\rangle \end{array}\right\}$

$\overset{a}{0}|0\rangle + \overset{c}{1}|1\rangle + 1|1\rangle$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\overset{b}{1}|0\rangle + \overset{d}{0}|1\rangle$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = |-\rangle$$

$H|0\rangle = |+\rangle \longrightarrow \boxed{H} \longrightarrow |0\rangle$

$H|1\rangle = |-\rangle \longrightarrow \boxed{H} \longrightarrow |1\rangle$

$H|+\rangle = |0\rangle$ ✓
& $H|-\rangle = |1\rangle$ ✓

* Z gate $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ ✓

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$$

* $\alpha|0\rangle + \beta|1\rangle$
  $\alpha|0\rangle - \beta|1\rangle$

$\boxed{\text{Phase - Flip gate}}$

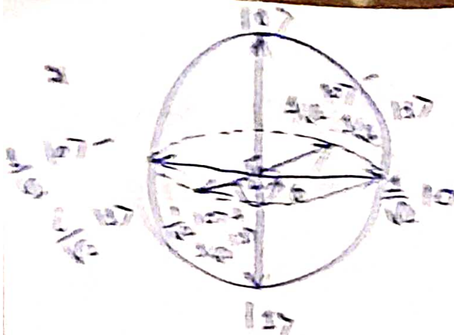
* Gates
  - X   Y
  - H   S
  - Z   T ($\pi/4$ rotation)

Ex $|\psi\rangle = (\alpha\cdot|0\rangle + \beta|1\rangle)$

If we multiply by $e^{i\theta}$, then no changes will occur,

$|\psi\rangle' = (\alpha|0\rangle + \beta|1\rangle)\, e^{i\theta}$ ⟶ global Phase

$$\boxed{|\psi\rangle = |\psi\rangle'}$$



Bloch Sphere

r, θ, ϕ

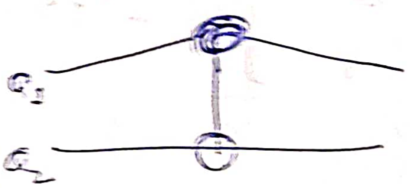
$$|0\rangle \cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

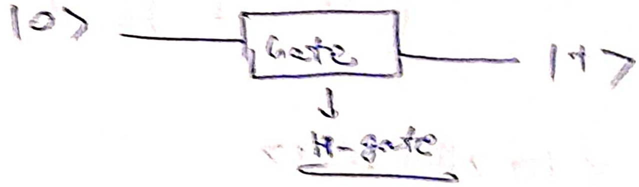
at $\theta, \phi = 0$

$\theta, \phi = \pi$

④ Two Qubit Gate

CNOT → (Controlled NOT)





$\alpha_{00} = |00\rangle \rightarrow |00\rangle \longrightarrow 1|00\rangle + 0|01\rangle + 0|10\rangle + 0|11\rangle$

$\alpha_{01} = |01\rangle \rightarrow |01\rangle$

$\alpha_{10} = |10\rangle \rightarrow |11\rangle \longrightarrow 0|00\rangle + 0|01\rangle + 0|10\rangle + 1|11\rangle$

If first is 1 then it will change second

$\alpha_{11} = |11\rangle \longrightarrow |10\rangle$

CNOT GATE

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

$|00\rangle \Rightarrow |00\rangle$

$|01\rangle$

$|10\rangle \Rightarrow |11\rangle$

$|11\rangle$

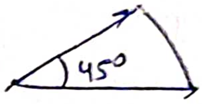
Problem 1.1

ⓐ $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ Let $\theta = 45°$

$R_\theta |0\rangle =$

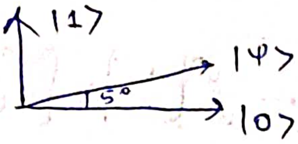
Unitary

$R_\theta^{+} R_\theta = I$



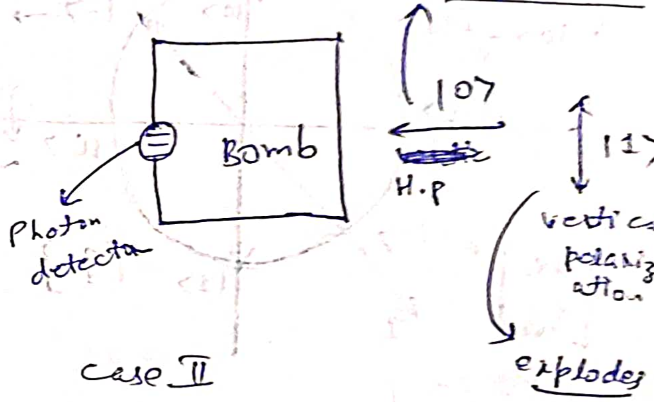
$|0\rangle$

ⓑ $|\psi\rangle = \cos\theta |0\rangle + \sin\theta |1\rangle$



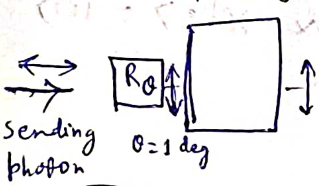
ⓒ Elitzur – Vaidman bombs

10 bomb — Faulty / Live



Now, case I

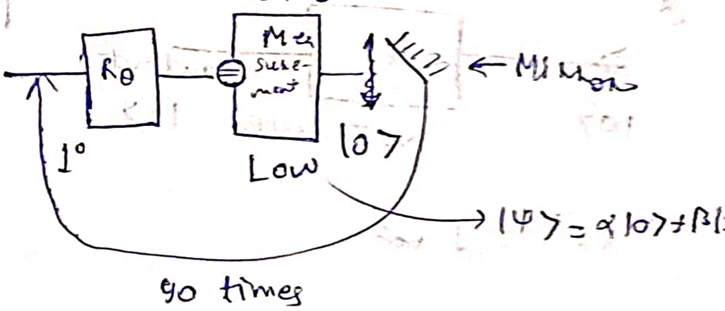
Faulty



90 times then it will vertical ↕

case II

Live



90 times

→ If explosion doesn't happen then dir. of photon will be horizontal ⟷, and state will $|0\rangle$.

ⓓ

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1&1\\1&-1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$$

2×1 $\quad$ 2×2

$$\begin{bmatrix}1\\0\end{bmatrix} - i\begin{bmatrix}0\\1\end{bmatrix} \qquad \begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$$

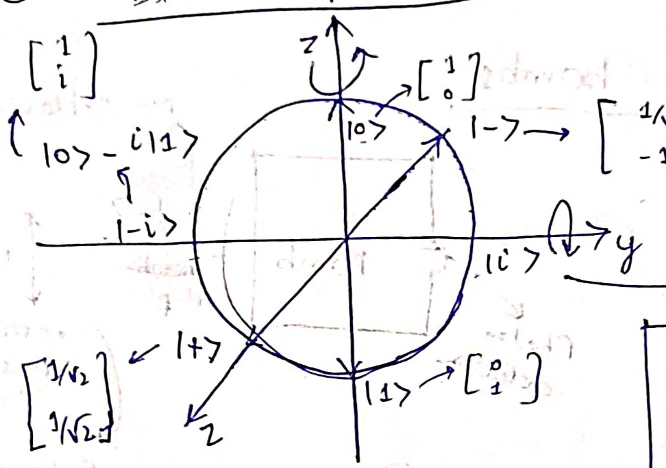
2×2 $\quad$ 2×1

$e^{i\theta}$

$$\begin{bmatrix}1\\-1\end{bmatrix} \quad \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}$$

$$\begin{bmatrix}0&-i\\i&0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\i\end{bmatrix} = i\begin{bmatrix}0\\1\end{bmatrix}$$

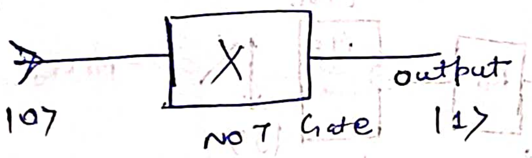
Bloch Sphere



$\to |0\rangle + i|1\rangle \to \begin{bmatrix}1\\i\end{bmatrix}$

$$Y = \begin{bmatrix}0&-i\\i&0\end{bmatrix}, \quad Y|i\rangle = |i\rangle, \quad Y|0\rangle = |1\rangle$$

$X|0\rangle = |1\rangle$ & $X|1\rangle = |0\rangle$



$XX^{+} = I$

$$X|i\rangle = \begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1\\i\end{bmatrix} = \begin{bmatrix}i\\1\end{bmatrix} = |-i\rangle.$$

$$\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix} = |+\rangle$$

$$Z \equiv \begin{bmatrix}1&0\\0&-1\end{bmatrix}, \quad Z|+\rangle = |-\rangle, \quad Z|-\rangle = |+\rangle$$

$$Z|i\rangle = |-i\rangle, \quad Z|-i\rangle = |i\rangle$$

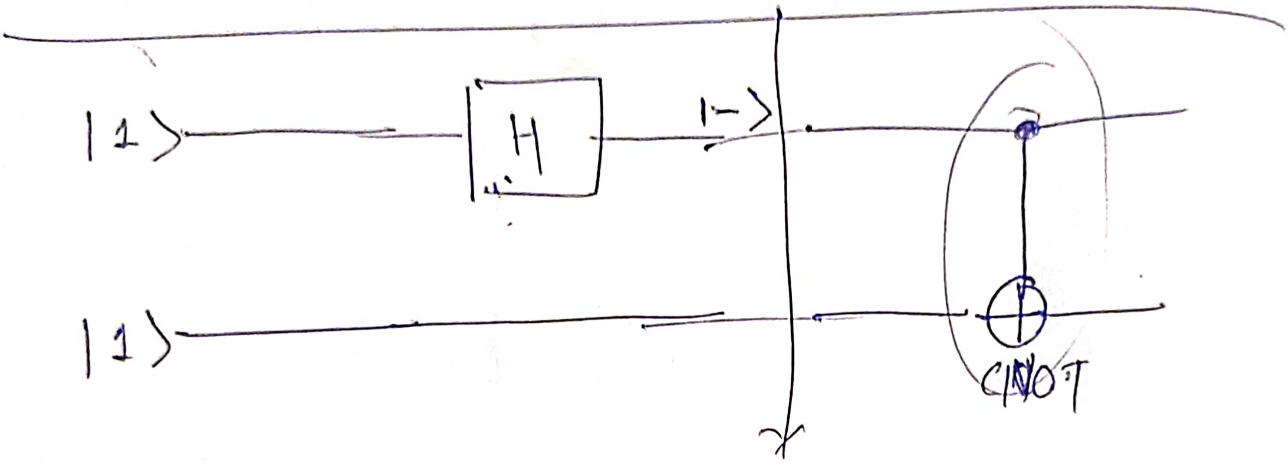
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$\uparrow$ $\downarrow$

$|0\rangle$ $|1\rangle$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

New st $|0\rangle$ $|1\rangle$



$|11\rangle$ $|-\ 1\rangle$

CNOT

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |1\rangle$$

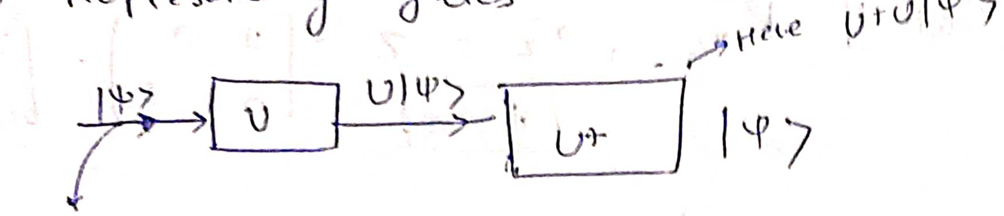
$$= \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$$

$$= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$ Bell State

* Pure state to super position state → Hadamard Gate

* All these gates are reversible.

* Representing gates



$\alpha|0\rangle + \beta|1\rangle$

$\alpha|0\rangle + \beta|1\rangle$

* <u>Bell state or Entangled state</u>

$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\,|00\rangle + |11\rangle\,\right] \neq (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)$

Probability that 1st qubit is in $|0\rangle = 1/2$

" " 2nd " " ,

doubt

first $|0\rangle$

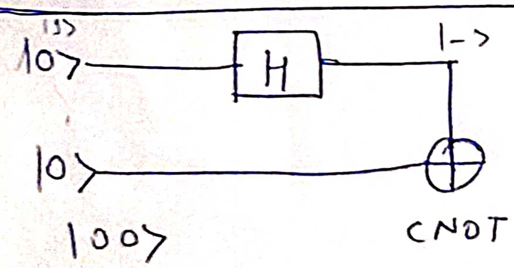
Total st $1.\,|00\rangle$

0 1 0 1

00 01 10 11

$|11\rangle$

(4) $1/\sqrt{2}|0\rangle - 1/\sqrt{2}|11\rangle$



$1/\sqrt{2}\,(\,|00\rangle + |10\rangle\,)$ (11)

* If $|1\rangle$ $|1\rangle$ $= 1/\sqrt{2}\,(\,|01\rangle - |10\rangle\,)$

$|0\rangle$ $|1\rangle$ $= 1/\sqrt{2}\,(\,|01\rangle + |10\rangle\,)$

$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\underset{\uparrow P}{0}1\rangle + |10\rangle\right)$

$\boxed{\text{S Gate:}}$ $R_{\pi/2}$, $\sigma_z \equiv Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\pi i} \end{bmatrix}$ $\boxed{\phi = \pi}$

$$R_{\pi/2} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$S|i\rangle = |-\rangle$

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \xrightarrow{\text{normalized}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = |-\rangle$$

$\boxed{S^{+} \text{ gate}}$

$$R_{3\pi/2} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$
$\downarrow$
$R_{-\pi/2}$

T gate

$$R_{\pi/4} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$\sqrt{\text{NOT}}$ GATE

$$\frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$ i.e $\sqrt{\text{NOT}} \times \sqrt{\text{NOT}} = X$

Home Work

Bell states $\leftarrow$ EPR Paradox 1935

Einstein Podolstay Rosen

$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

$|u\rangle = \begin{bmatrix} 3 \\ -i \end{bmatrix}$ $\quad |w\rangle = \begin{bmatrix} 2+i \\ 4 \end{bmatrix}$

$\langle u | w \rangle =$

$\begin{bmatrix} 3 & i \end{bmatrix} \begin{bmatrix} 2+i \\ 4 \end{bmatrix} = 6+3i + 4i$

$= 6+7i$

$\begin{bmatrix} 3 \\ -i \end{bmatrix}_{2\times 2} \cancel{\otimes \begin{bmatrix} (2-i) & 4 \end{bmatrix}}_{1\times 2}$

$\begin{bmatrix} 2+i \\ 4 \end{bmatrix} \begin{bmatrix} 3 & +i \end{bmatrix}$

$= \begin{bmatrix} 6+3i & 2i-1 \\ 12 & 4i \end{bmatrix}$

$\cancel{|w\rangle\langle u|}$

$M_0 = |0\rangle\langle 0|$

$M_0 \; (\alpha|0\rangle + \beta|1\rangle)$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\cancel{\langle \alpha|0\rangle|0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle)}$

$|v\rangle = [\;]\begin{bmatrix} 3 \\ -i \end{bmatrix}$ $\quad |w\rangle = \begin{bmatrix} 2+i \\ 4 \end{bmatrix}$

$\langle v|w\rangle =$

$\begin{bmatrix} 3 & i \end{bmatrix}\begin{bmatrix} 2+i \\ 4 \end{bmatrix} = 6 + 3i + 4i$

$= 6 + 7i$

$\begin{bmatrix} 3 \\ -i \end{bmatrix} \otimes \begin{bmatrix} (2-i) & 4 \end{bmatrix}$

$2\times 2 \qquad 1\times 2$

$\begin{bmatrix} 6-3i & 12 \\ 6 & \end{bmatrix}$ $\quad \begin{bmatrix} 3\times(2-i) \\ \end{bmatrix}$

2×4

$\begin{bmatrix} 2+i \\ 4 \end{bmatrix}\begin{bmatrix} 3 \end{bmatrix}$

$= \begin{bmatrix} 2+i \\ 4 \end{bmatrix}\begin{bmatrix} 3 & +i \end{bmatrix}$

$= \begin{bmatrix} 6+3i & 2i-1 \\ 12 & 4i \end{bmatrix}$

~~$|w\rangle\langle v|$~~

$M_0 = |0\rangle\langle 0|$

$M_0\ (\alpha|0\rangle + \beta|1\rangle)$

$\psi = \alpha|0\rangle + \beta|1\rangle$

$\langle$ ~~$\alpha|0\rangle|0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle)$~~

~~$\langle\psi|$~~

$\langle\psi| = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} = [\alpha^* \ \beta^*]$

$[\alpha^* \ \beta^*]|0\rangle \ \alpha$ $\boxed{\langle\psi| = \alpha^*\langle 0| + \beta^*\langle 1|}$

$(\langle 0|\alpha^* + \beta^*\langle 1|)(|0\rangle\langle 0|)(\alpha|0\rangle + \beta|1\rangle)$

$= (\langle 0|\alpha^* + \beta^*\langle 1|)(|0\rangle\alpha)$

$= |\alpha|^2$

$\langle\psi|M_1|\psi\rangle = |\beta|^2$, $M_1 = |1\rangle\langle 1|$

$= (\alpha^*\langle 0| + \beta^*\langle 1|)(|1\rangle\langle 1|)(\alpha|0\rangle + \beta|1\rangle)$

$\beta\beta^* = |\beta|^2$

---

(7) **2-Qubit**

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$\langle 00|00\rangle = 1$

$\langle 00|01\rangle = 0$

$[\ \ ]_{4\times 4}$

3-Qubits

$|000\rangle \quad |100\rangle$

$|001\rangle \quad |101\rangle$

$|010\rangle \quad |110\rangle$

$|011\rangle \quad |111\rangle$

$[\ \ ]_{8\times 8}$

---

(8) **1 Qubit H-gate**

$$H = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}_{2\times 2} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Quantum SWAP Gate

2- Qubits

$|\psi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$

$|\psi_2\rangle = \alpha_2 |0\rangle + \beta_2 |1\rangle$

(i) $|\psi_1\rangle \otimes |\psi_2\rangle$.

$$= (\alpha_1 |0\rangle + \beta_1 |1\rangle) \otimes (\alpha_2 |0\rangle + \beta_2 |1\rangle)$$

$$= \begin{bmatrix} \alpha_1 \alpha_2 \\ \alpha_1 \beta_2 \\ \beta_1 \alpha_2 \\ \beta_1 \beta_2 \end{bmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

(ii) $|\psi_2\rangle \otimes |\psi_1\rangle$

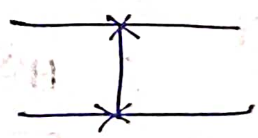
$$(\alpha_2 |0\rangle + \beta_2 |1\rangle) \otimes (\alpha_1 |0\rangle + \beta_1 |1\rangle)$$

$$= \begin{bmatrix} \alpha_2 \alpha_1 \\ \alpha_2 \beta_1 \\ \beta_2 \alpha_1 \\ \beta_2 \beta_1 \end{bmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

$\circledast$ $\boxed{\alpha_2 \alpha_1 = \alpha_1 \alpha_2}$

$$A \left(|\psi_1\rangle \otimes |\psi_2\rangle \right) = |\psi_2\rangle \otimes |\psi_1\rangle$$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



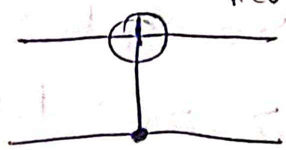
2 Qubit → 2 wire

~~Homework~~ Homework

(A) ~~KP*~~ $H^{\otimes 2}$ CNOT $H^{\otimes 2}$ doubt

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ & \end{bmatrix}$$

Reverse CNOT



00 → 00

01 ⟷ 11

10 → 10

11 → 01

(4) Circuit

q_0 i $|0\rangle$ —[X]— $|1\rangle$

Initialized

q_0 i $|0\rangle$ —[measure]— $|m_0\rangle = |0\rangle$

If q_0 i $|0\rangle$ —[H]—[measure]— $|m_0\rangle = |0\rangle$ or $|1\rangle$

Probability is same

(1) q_0 i $|0\rangle$ —[H]—[H]—[measure]— $|m_0\rangle = |0\rangle$

(2) q_0 i $|0\rangle$ —[H]—[measure]—[H]—[measure]— $|m_0\rangle = |0\rangle$ or $|1\rangle$

$|0\rangle \to |+\rangle$ $\left(1/\sqrt{2}|0\rangle + 1/\sqrt{2}|1\rangle\right)$

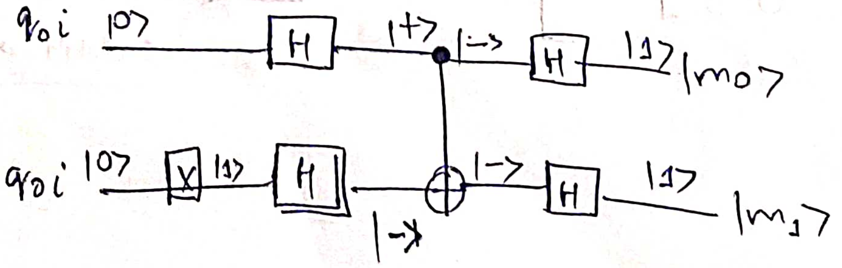
$|1\rangle \to |-\rangle$ $\left(1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle\right)$

(8) Two Qubit

Reverse CNOT

If 2nd Qubit is 1, flip the state of the 1st qubit

$H^{\otimes 2}$ CNOT $H^{\otimes 2}$



$|11\rangle$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

so, applying CNOT

$= \frac{1}{2}(|00\rangle - |01\rangle + |11\rangle - |10\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \cdot \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= |-\rangle \cdot |-\rangle$

Q $|+\rangle, |+\rangle$

$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

$= \frac{1}{2}(|00\rangle + |01\rangle + |11\rangle + |10\rangle)$

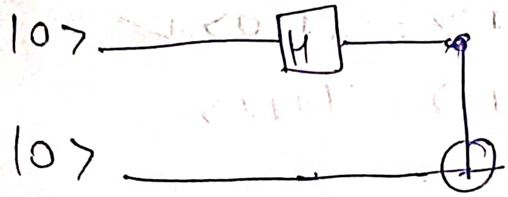
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|+\rangle \quad |+\rangle$

Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$|0\rangle$ —[H]— $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|0\rangle$ ——— $|0\rangle$ $\equiv \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$



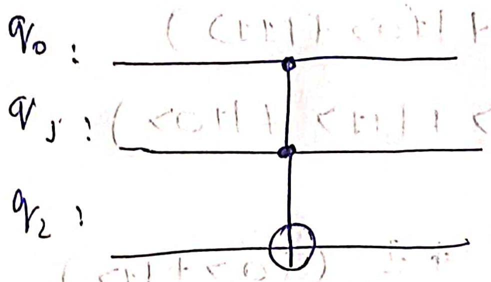
$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

For 10 Qubits $[\quad]_{2^{10} \times 2^{10}}$

(1)

$$\begin{array}{c} |1000\rangle \\ |1001\rangle \\ |1010\rangle \\ |1011\rangle \\ |1100\rangle \\ |1101\rangle \\ |1110\rangle \\ |1111\rangle \end{array} \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 0 & 1 \\ & & & & & & 1 & 0 \end{bmatrix}$$

Quantum Toffoli CC NOT Gate CCX



If state of first two qubit is $|1\rangle$ $|1\rangle$, flip state of the 3rd Qubit

$|110\rangle \Rightarrow |111\rangle$, $|111\rangle \Rightarrow |110\rangle$

Quantum Fredkin C SWAP GATE

If the state of the first qubit is $|1\rangle$ then states of the 2nd and 3rd qubits are swaped.

$|100\rangle = |100\rangle$, $|101\rangle = |110\rangle$

$|110\rangle = |101\rangle$, $|111\rangle = |111\rangle$