



MDOrbital

Interplanetary Mission Optimization



Prepared By:
Elamin, Mayar | Pagan, Sebastian | Weber, Jacob

Project Pivot: Ramjet Optimization → Interplanetary Mission Optimization

- ◆ **Initial Focus:** Ramjet optimization constrained by speed, altitude, combustion limits, and geometry.
- ◆ **Optimization Goal:** Intake pressure, fuel input, heat addition, geometry, and thrust ratios.
- ◆ **Challenges Identified:** Project scope exceeded available resources, time, and current expertise.
- ◆ **New Focus:** Optimization of Δv and staging masses in interplanetary mission planning.

Introduction

- ◇ Deep space exploration has technical & economic challenges.
- ◇ Growing interest in Europa & Mars demands efficient mission planning.
- ◇ **Study:** Earth → Europa using Mars & Jupiter gravity assists to save fuel.
- ◇ **Focus:** Optimize flight path & staging within mission constraints.
- ◇ **Key factors:**
 - Delta-V budgets
 - Planetary alignment windows
 - Gravitational assist strategies
- ◇ **Goal:** A framework for efficient, cost-effective deep space missions.

Literature Review

Historic Missions:

- ◇ Pioneer 10 & 11 (1970s) - first close-up data.
- ◇ Voyager 1 & 2 - revealed active, young surface.
- ◇ Galileo - confirmed likely subsurface salty ocean.

Europa Clipper (2024-present):

- ◇ Dozens of flybys, studying habitability & surface composition.
- ◇ Avoids direct orbit to limit radiation exposure.

Gravity Assists:

- ◇ Used in past missions to save fuel (Galileo, Clipper).

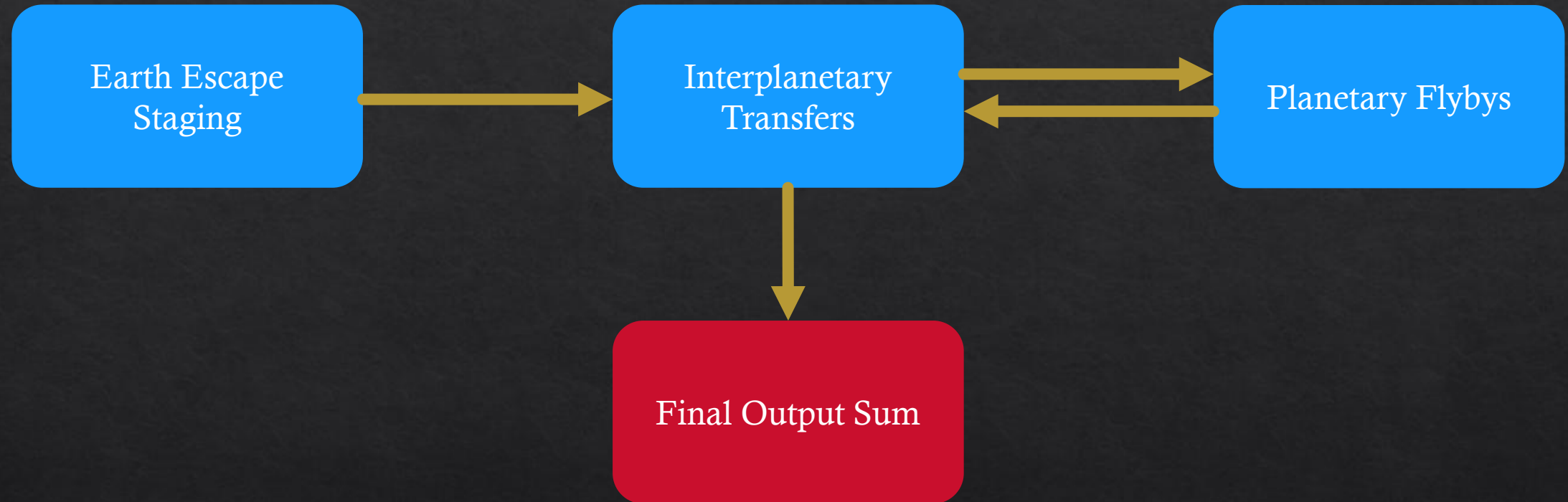
This Study:

- ◇ Builds on proven techniques using Mars & Jupiter flybys.
- ◇ Applies MDO for optimal trajectory and staging design.

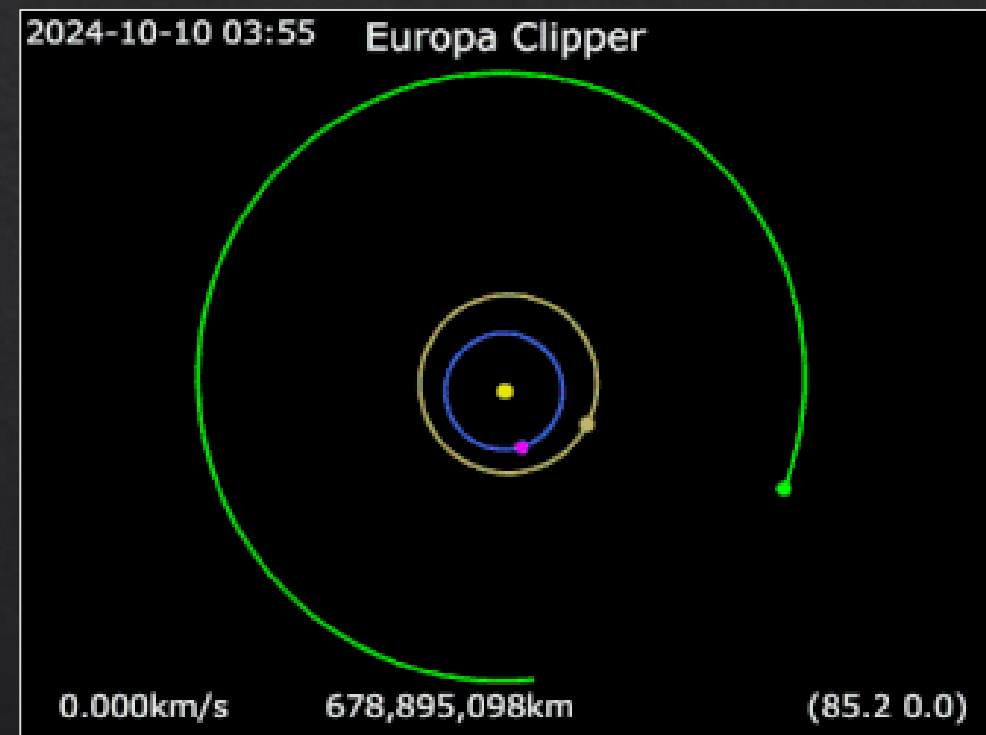
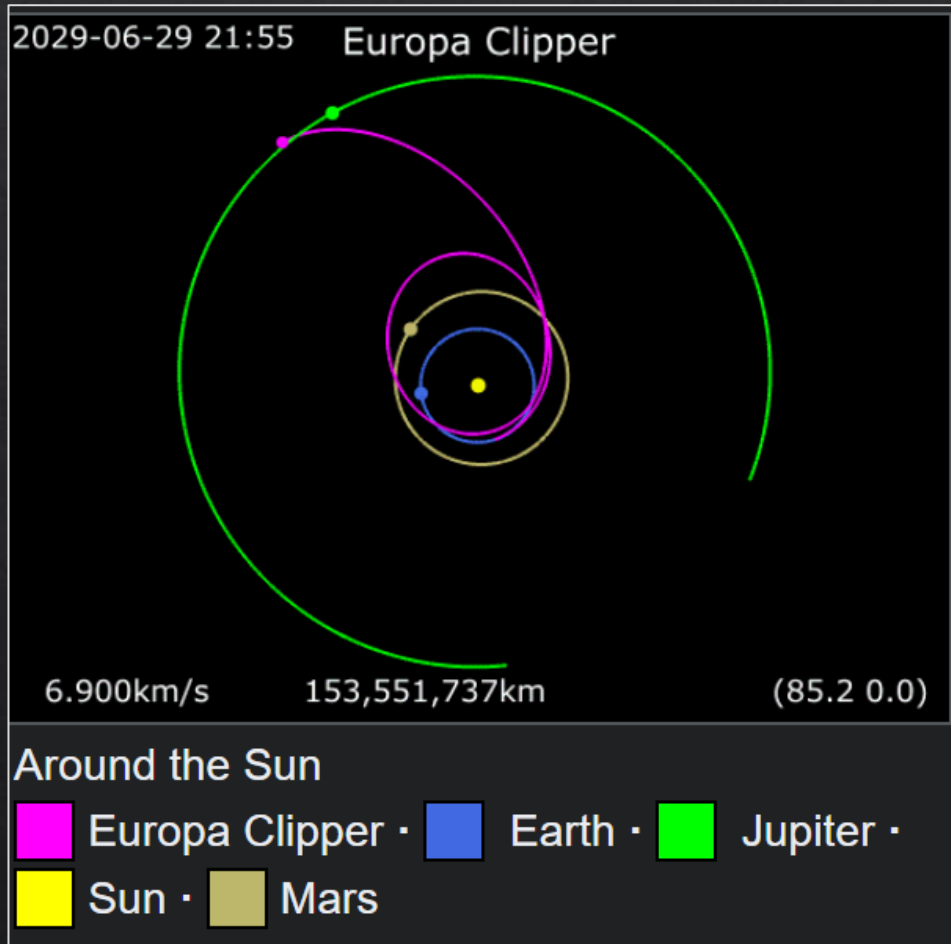


Europa Clipper launched on October 14, 2025

Technical Approach



Our Journey



Video Depicting Europa Clipper Mission's Trajectory

Technical Approach – Rocket Staging Optimization

Purpose:

- ◆ Optimize multi-stage rocket design to minimize total launch mass while meeting a delta-V requirement.

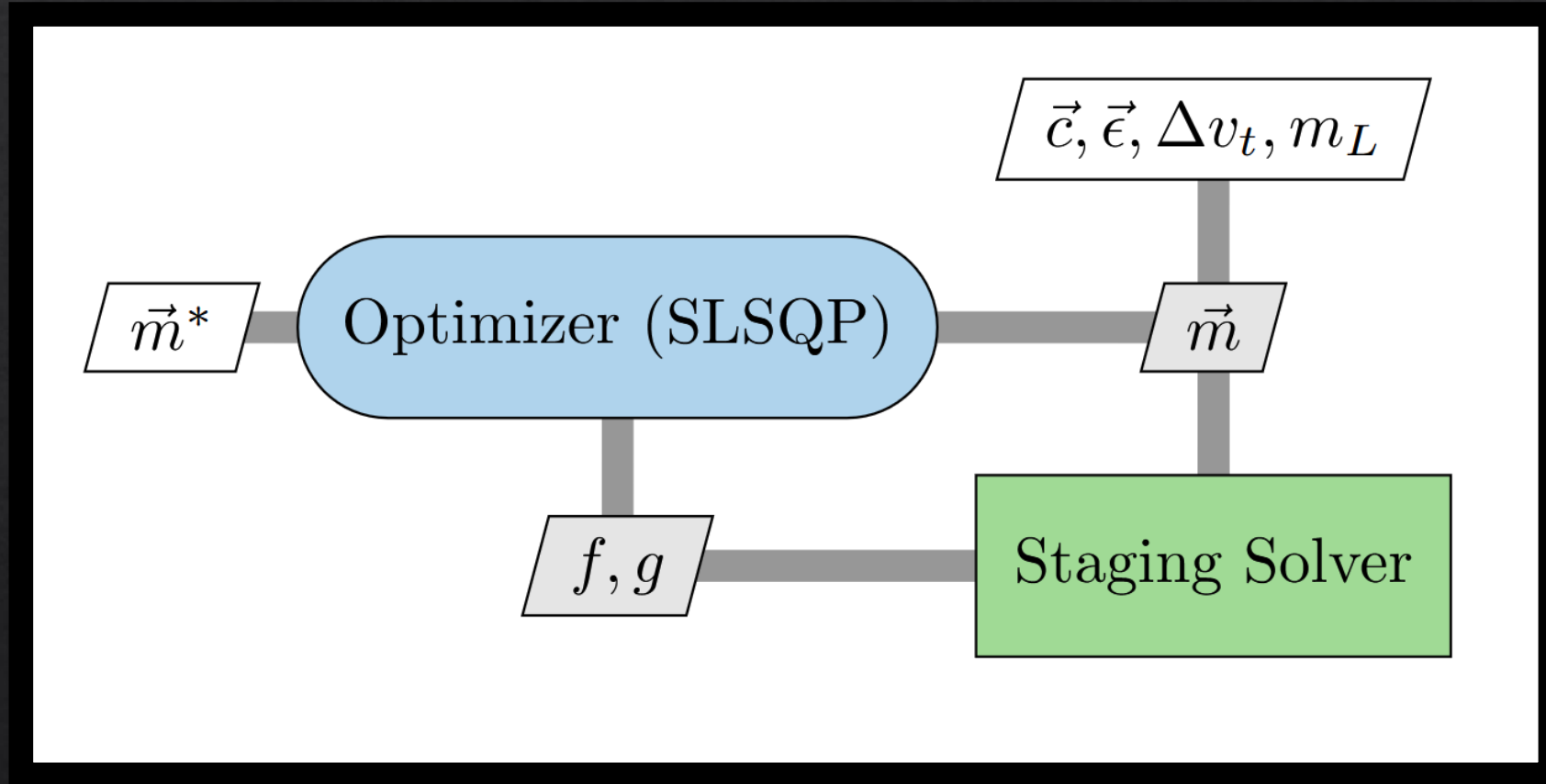
Approach:

- ◆ Uses the Optimal Staging Rocket Equation to calculate achievable delta-V for a staged system.

$$\sum_{i=1}^n c_i \ln \frac{m_i + m_{i+1} + \dots + m_n}{\varepsilon_i m_i + m_{i+1} + \dots + m_n} - \Delta V_T$$

- ◆ Applies constraint-based optimization to balance fuel mass, structural mass, and payload capacity.

XDSM Diagram - Staging



Technical Approach – Rocket Staging Optimization

◇ Design Variables:

Rocket Stage Masses (\vec{m}): Represents the combined structural and fuel masses of each rocket stage arranged in a vectorial form for ease of computation.

◇ Objective: Minimize stage mass components m_i

◇ Constraints: $m_1, m_2 > m_L$,

Initial Values

Full Stage 1 Mass = 433,180.7 kg

Full Stage 2 Mass = 111,583.7 kg

Total Mass = 544,764.4 kg

Technical Approach – Interplanetary Transfers

Purpose:

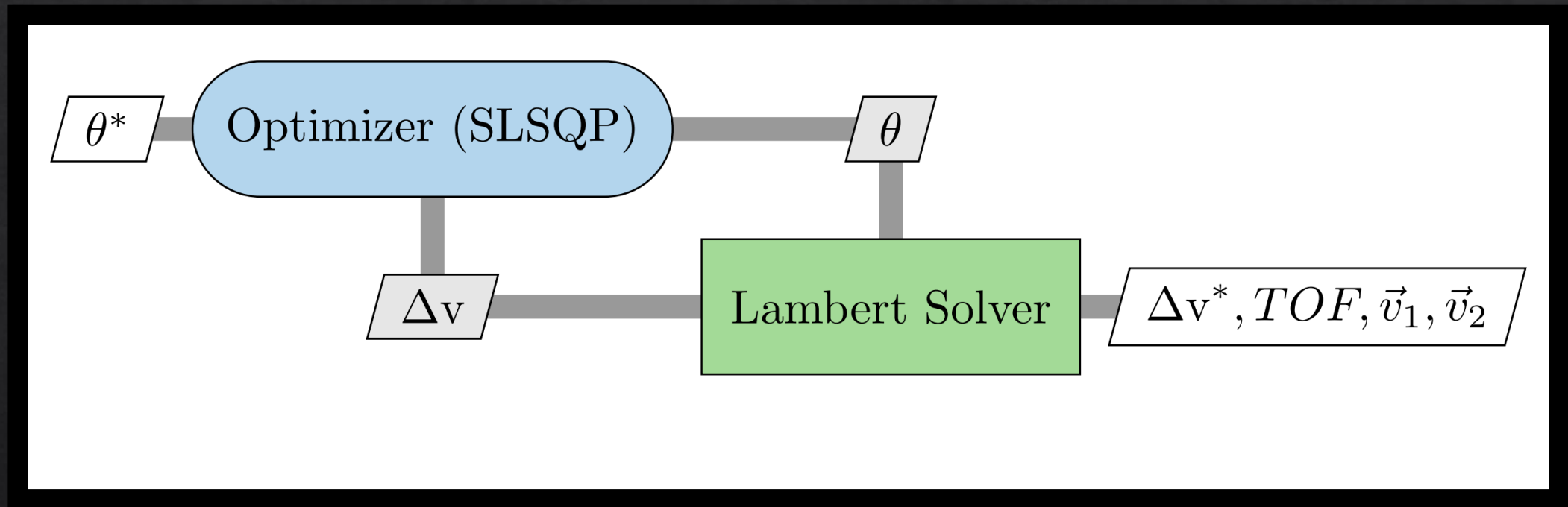
- ◆ Optimize Δv cost of a transfer orbit injection for both transfers.

Approach:

- ◆ Uses a specialized formulation of Lambert's problem to optimize the minimum energy transfer orbit.

$$\sqrt{\mu}t_m = \left[\frac{s^3}{8} \right]^{1/2} (\pi - \beta_m + \sin(\beta_m))$$
$$\Delta v_1 = |v_1 - v_p|, \Delta v_2 = |v_0 - v_2|$$

XDSM Diagram - Interplanetary Transfers



Technical Approach – Interplanetary Transfers

◆ Design Variables:

Transfer Angle (θ): Represents the total angle spanned by the transfer between Earth-Mars and Earth-Jupiter in heliocentric frame.

◆ **Objective:** Minimize Δv required to inject the transfer orbit

◆ **Constraints:** $0 \leq \theta \leq 2\pi$, $TOF \leq 365 \text{ days}$

Initial Values

$$\theta = 75^\circ$$

$$TOF = 182 \text{ days}$$

$$\Delta v = 17.4 \text{ km/s}$$

Technical Approach – Mars Flyby Into Ellipse

Purpose:

- ◊ Optimize Δv for flyby and establish heliocentric ellipse

Approach:

- ◊ Perform optimal gravity-assist at Mars that leads into an ellipse to meet back up with Earth.

$$v_0^2 = v_{Earth}^2 + v_{\infty Earth}^2 - 2v_{Earth}v_{\infty Earth} \cos(\pi - \delta)$$

$$\beta = \sin^{-1} \left(\frac{v_{\infty Earth}}{v_0} \right)$$

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}, T = 2\pi \sqrt{\frac{a^3}{\mu}}$$

Technical Approach – Mars Flyby Into Ellipse

◆ Design Variables:

Semimajor Axis (a): Find semimajor axis to create a feasible ellipse post-Martian flyby

◆ Objective: Δv post-flyby

◆ Constraints: $2/\pi \leq \delta \leq \pi$, $Altitude > r_{Earth}$, $r_{peri_{ellipse}} < 1$, $0 < e < 1$

Initial Values

$$a = 1.2 \text{ AU}$$

$$e = 0.5$$

$$TOF = 422 \text{ days}$$

$$r_p = 1 \text{ AU}$$

$$alt = 4,000 \text{ km}$$

$$\Delta v = 75.44 \text{ km/s}$$

Technical Approach – Orbit Raise to Europa

Purpose:

- ◆ Optimize Δv for an orbit raise from established rotation around Jupiter into Europa's orbital radius

Approach:

- ◆ By optimizing trajectory of the orbit with respect to transfer time, the magnitude of Δv will decrease given a large TOF considering a 2D plane coordination system

Results and Discussion

| Staging | | | | Mars FlyBy Into Ellipse | | | |
|--------------------------|---------------|-----------------|------------|-------------------------------|---------------|-----------------|------------|
| Parameter | Initial Value | Optimized Value | % improved | Parameter | Initial Value | Optimized Value | % improved |
| Stage 1 Mass (kg) | 433,180.7 | 96,618.0 | 70 | a | 1.2 AU | 1.1 AU | 92 |
| Stage 2 Mass (kg) | 111,583.7 | 58,117.2 | | e | 0.5 | 0.14 | |
| Total Mass (kg) | 544,764.4 | 154,735.27 | | TOF | 1,000 days | 372.3 days | |
| Earth to Mars Transfer | | | | r_p | 1 AU | 0.96 AU | |
| θ (deg) | 75° | 180° | 83 | alt | 4,000 km | 47,158 km | |
| TOF (days) | 182 | 260 | | Δv | 75.44 km/s | 6.24 km/s | |
| Δv (km/s) | 17.4 | 2.91 | | Jupiter to Europa Orbit Raise | | | |
| Mars to Jupiter Transfer | | | | Parameter | Initial Value | Optimized Value | % improved |
| θ (deg) | 190° | 79° | 92 | TOF (min) | 1250.0 | 1933.49 | 33 |
| TOF (days) | 1000 | 861 | | Total Orbit Raise (km) | 320900 | 370900 | |
| Δv (km/s) | 75.44 | 6.24 | | Δv (km/s) | 18.8 | 12.52 | |

References

- [1] NASA. “Pioneer 10 - NASA Science.” *Science.nasa.gov*, 6 Nov. 2024, science.nasa.gov/mission/pioneer-10/.
- [2] NASA. “Voyager 1 - NASA Science.” *Science.nasa.gov*, 2 Nov. 2024, science.nasa.gov/mission/voyager/voyager-1/.
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- [4] NASA. “About | Mission – NASA’s Europa Clipper.” *NASA’s Europa Clipper*, Oct. 2019, europa.nasa.gov/mission/about/.
- [5] Prussing, John E Prussing, and Bruce A Conway. *Orbital Mechanics*. New York, Oxford University Press, 2013.

Team Tasks

◆ Jacob Weber:

- Reviewed Orbital Mechanics textbook and other astrodynamics literature for equations used in coding.
- Contributed in staging optimization and transfer optimization scripts.

◆ Mayar Elamin:

- Worked on developing XDSM diagrams.
- Contributed in staging optimization script.
- Commented staging optimization script in detail.

◆ Sebastian Pagan:

- Worked on developing XDSM diagrams.
- Contributed in staging optimization script.
- Worked on flyby scripts.