



# MDOrbit

## Interplanetary Mission Optimization

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# Project Pivot: Ramjet Optimization → Interplanetary Mission Optimization

- ❖ **Initial Focus:** Ramjet optimization constrained by speed, altitude, combustion limits, and geometry.
- ❖ **Optimization Goal:** Intake pressure, fuel input, heat addition, geometry, and thrust ratios.
- ❖ **Challenges Identified:** Project scope exceeded available resources, time, and current expertise.
- ❖ **New Focus:** Optimization of  $\Delta v$  and staging masses in interplanetary mission planning.

# Introduction

- ❖ Deep space exploration has technical & economic challenges.
- ❖ Growing interest in Europa & Mars demands efficient mission planning.
- ❖ Study: Earth → Europa using Mars & Jupiter gravity assists to save fuel.
- ❖ Focus: Optimize flight path & staging within mission constraints.
- ❖ Key factors:
  - Delta-V budgets
  - Planetary alignment windows
  - Gravitational assist strategies
- ❖ Goal: A framework for efficient, cost-effective deep space missions.

# Literature Review

## Historic Missions:

- ❖ Pioneer 10 & 11 (1970s) - first close-up data.
- ❖ Voyager 1 & 2 - revealed active, young surface.
- ❖ Galileo - confirmed likely subsurface salty ocean.

## Europa Clipper (2024-present):

- ❖ Dozens of flybys, studying habitability & surface composition.
- ❖ Avoids direct orbit to limit radiation exposure.

## Gravity Assists:

- ❖ Used in past missions to save fuel (Galileo, Clipper).

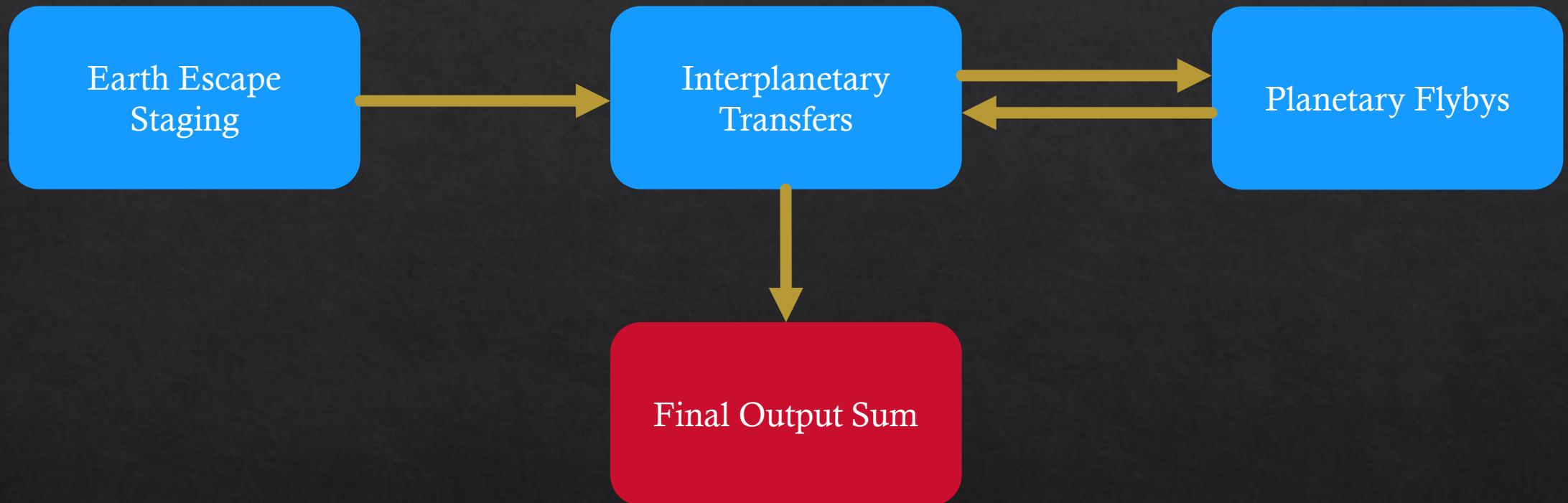
## This Study:

- ❖ Builds on proven techniques using Mars & Jupiter flybys.
- ❖ Applies MDO for optimal trajectory and staging design.

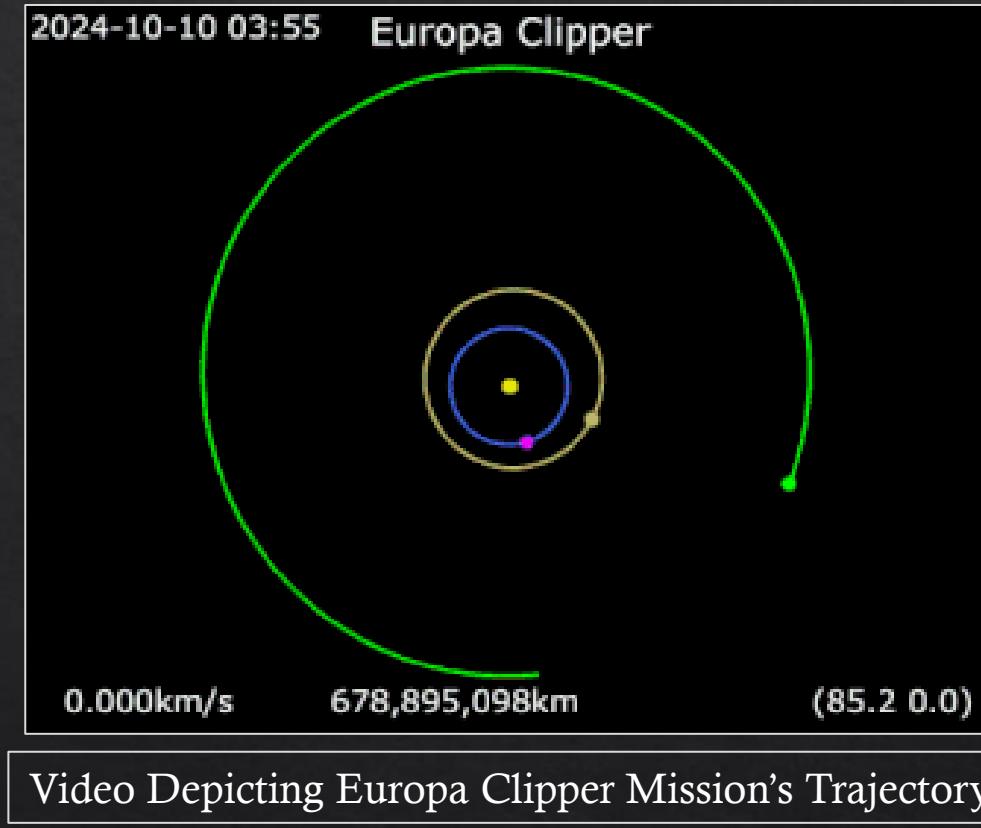
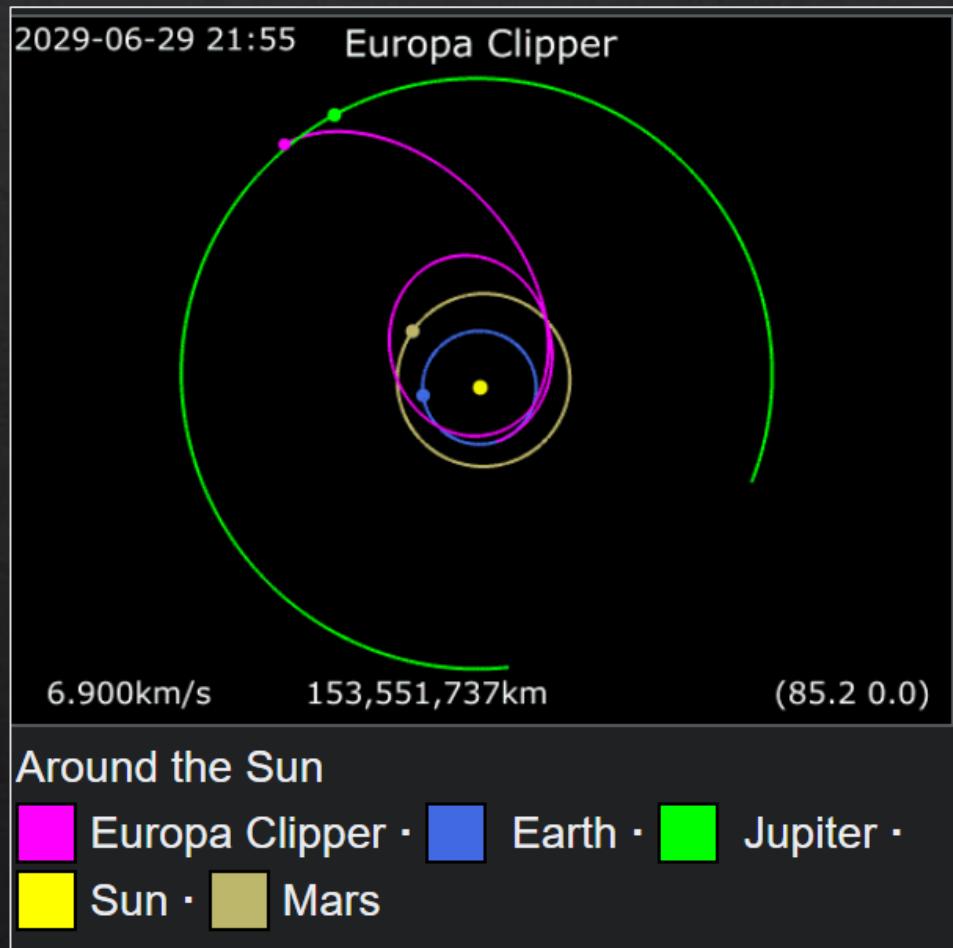


Europa Clipper launched on October 14, 2025

# Technical Approach



# Our Journey



# Technical Approach – Rocket Staging Optimization

## Purpose:

- ❖ Optimize multi-stage rocket design to minimize total launch mass while meeting a delta-V requirement.

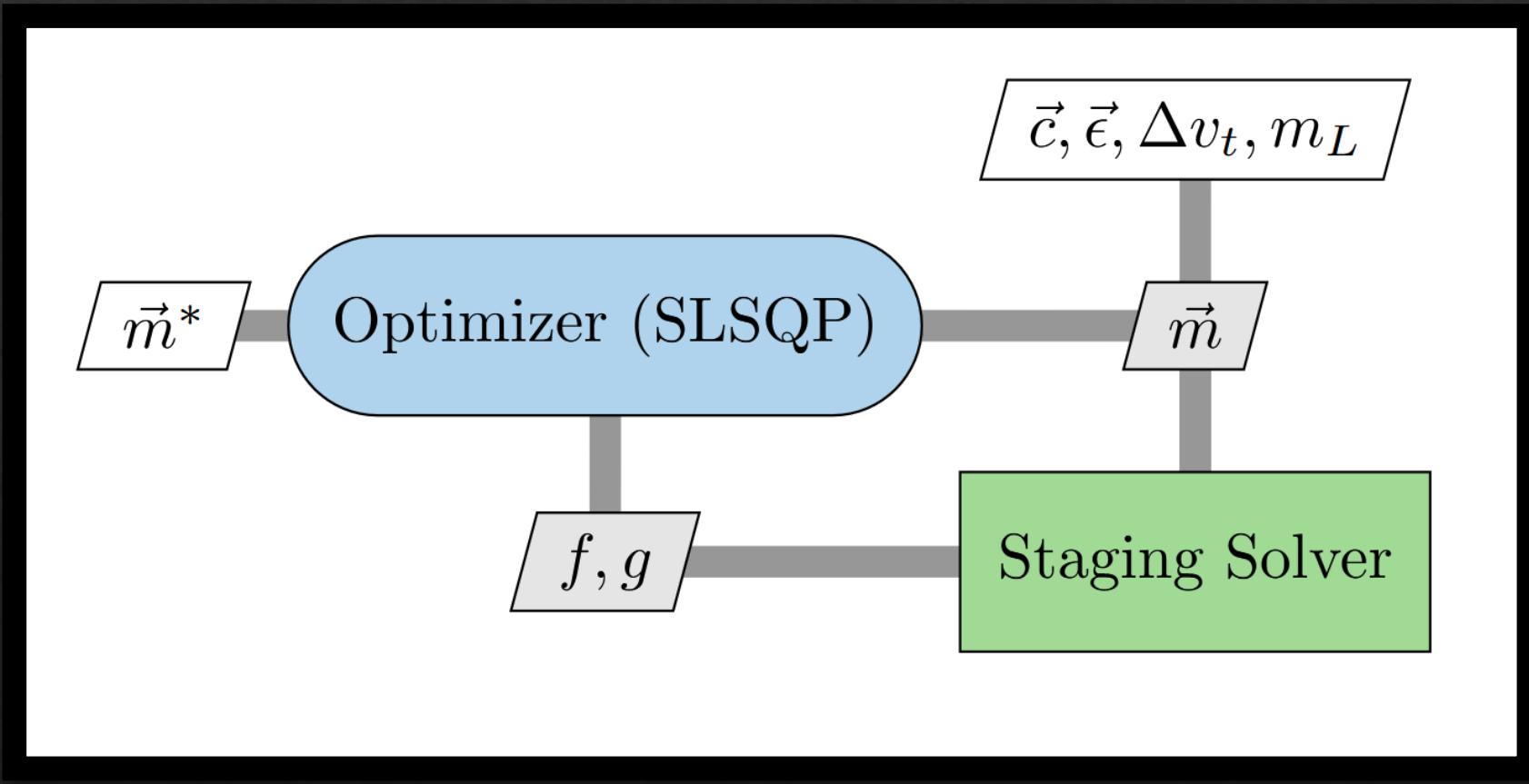
## Approach:

- ❖ Uses the Optimal Staging Rocket Equation to calculate achievable delta-V for a staged system.

$$\sum_{i=1}^n c_i \ln \frac{m_i + m_{i+1} + \dots + m_n}{\varepsilon_i m_i + m_{i+1} + \dots + m_n} - \Delta V_T$$

- ❖ Applies constraint-based optimization to balance fuel mass, structural mass, and payload capacity.

# XDSM Diagram - Staging



# Technical Approach – Rocket Staging Optimization

- ❖ Design Variables:

Rocket Stage Masses ( $\vec{m}$ ): Represents the combined structural and fuel masses of each rocket stage arranged in a vectorial form for ease of computation.

- ❖ Objective: Minimize stage mass components  $m_i$
- ❖ Constraints:  $m_1, m_2 > m_L$ ,

Initial Values

Full Stage 1 Mass = 433,180.7 kg

Full Stage 2 Mass = 111,583.7 kg

Total Mass = 544,764.4 kg

# Technical Approach – Interplanetary Transfers

## Purpose:

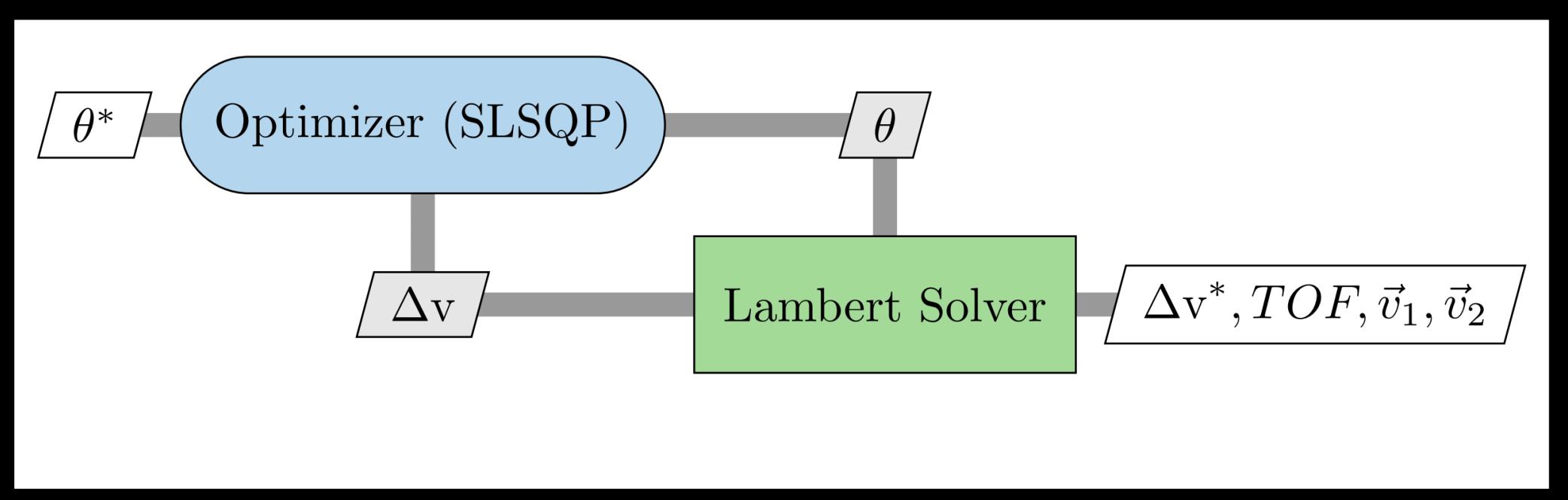
- ❖ Optimize  $\Delta v$  cost of a transfer orbit injection for both transfers.

## Approach:

- ❖ Uses a specialized formulation of Lambert's problem to optimize the minimum energy transfer orbit.

$$\sqrt{\mu}t_m = \left[ \frac{s^3}{8} \right]^{1/2} (\pi - \beta_m + \sin(\beta_m))$$
$$\Delta v_1 = |v_1 - v_p|, \Delta v_2 = |v_0 - v_2|$$

# XDSM Diagram - Interplanetary Transfers



# Technical Approach – Interplanetary Transfers

- ❖ Design Variables:

Transfer Angle ( $\theta$ ): Represents the total angle spanned by the transfer between Earth-Mars and Earth-Jupiter in heliocentric frame.

- ❖ Objective: Minimize  $\Delta v$  required to inject the transfer orbit
- ❖ Constraints:  $0 \leq \theta \leq 2\pi$ ,  $TOF \leq 365 \text{ days}$

Initial Values

$$\theta = 75^\circ$$

$$TOF = 182 \text{ days}$$

$$\Delta v = 17.4 \text{ km/s}$$

# Technical Approach – Mars Flyby Into Ellipse

Purpose:

- ❖ Optimize  $\Delta v$  for flyby and establish heliocentric ellipse

Approach:

- ❖ Perform optimal gravity-assist at Mars that leads into an ellipse to meet back up with Earth.

$$v_0^2 = v_{Earth}^2 + v_{\infty Earth}^2 - 2v_{Earth}v_{\infty Earth} \cos(\pi - \delta)$$

$$\beta = \sin^{-1} \left( \frac{v_{\infty Earth}}{v_0} \right)$$

$$v = \sqrt{\mu \left( \frac{2}{r} - \frac{1}{a} \right)}, T = 2\pi \sqrt{\frac{a^3}{\mu}}$$

# Technical Approach – Mars Flyby Into Ellipse

- ❖ Design Variables:

Semimajor Axis ( $a$ ): Find semimajor axis to create a feasible ellipse post-Martian flyby

- ❖ Objective:  $\Delta v$  post-flyby

- ❖ Constraints:  $2/\pi \leq \delta \leq \pi$ , Altitude >  $r_{Earth}$ ,  $r_{peri\ ellipse} < 1$ ,  $0 < e < 1$

Initial Values

$a = 1.2$  AU

$e = 0.5$

TOF = 422 days

$r_p = 1$  AU

alt = 4,000 km

$\Delta v = 75.44$  km/s

# Technical Approach – Orbit Raise to Europa

## Purpose:

- ❖ Optimize  $\Delta v$  for an orbit raise from established rotation around Jupiter into Europa's orbital radius

## Approach:

- ❖ By optimizing trajectory of the orbit with respect to transfer time, the magnitude of  $\Delta v$  will decrease given a large TOF considering a 2D plane coordination system

# Results and Discussion

Staging				Mars FlyBy Into Ellipse			
Parameter	Initial Value	Optimized Value	% improved	Parameter	Initial Value	Optimized Value	% improved
Stage 1 Mass (kg)	433,180.7	96,618.0	70	a	1.2 AU	1.1 AU	92
Stage 2 Mass (kg)	111,583.7	58,117.2		e	0.5	0.14	
Total Mass (kg)	544,764.4	154,735.27		TOF	1,000 days	372.3 days	
Earth to Mars Transfer				$r_p$	1 AU	0.96 AU	
$\theta$ (deg)	75°	180°	83	alt	4,000 km	47,158 km	33
TOF (days)	182	260		$\Delta v$	75.44 km/s	6.24 km/s	
$\Delta v$ (km/s)	17.4	2.91		Jupiter to Europa Orbit Raise			
Mars to Jupiter Transfer				Parameter	Initial Value	Optimized Value	% improved
$\theta$ (deg)	190°	79°	92	TOF (min)	1250.0	1933.49	33
TOF (days)	1000	861		Total Orbit Raise (km)	320900	370900	
$\Delta v$ (km/s)	75.44	6.24		$\Delta v$ (km/s)	18.8	12.52	

# References

- [1] NASA. “Pioneer 10 - NASA Science.” *Science.nasa.gov*, 6 Nov. 2024, [science.nasa.gov/mission/pioneer-10/](https://science.nasa.gov/mission/pioneer-10/).
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- [3] NASA. “Galileo - NASA Science.” *Science.nasa.gov*, 2024, [science.nasa.gov/mission/galileo/](https://science.nasa.gov/mission/galileo/).
- [4] NASA. “About | Mission – NASA’s Europa Clipper.” *NASA’s Europa Clipper*, Oct. 2019, [europa.nasa.gov/mission/about/](https://europa.nasa.gov/mission/about/).
- [5] Prussing, John E Prussing, and Bruce A Conway. *Orbital Mechanics*. New York, Oxford University Press, 2013.

# Team Tasks

## ◊ Jacob Weber:

- Reviewed Orbital Mechanics textbook and other astrodynamics literature for equations used in coding.
- Contributed in staging optimization and transfer optimization scripts.

## ◊ Mayar Elamin:

- Worked on developing XDSM diagrams.
- Contributed in staging optimization script.
- Commented staging optimization script in detail.

## ◊ Sebastian Pagan:

- Worked on developing XDSM diagrams.
- Contributed in staging optimization script.
- Worked on flyby scripts.