Denoising and Dimensionality Reduction Using Multilinear Tools for Hyperspectral Images

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Denoising and Dimensionality Reduction Using Multilinear Tools for Hyperspectral Images

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***Abstract*— In hyperspectral image (HSI) analysis, classification requires spectral dimensionality reduction. While common dimensionality reduction methods use linear algebra, we propose a multilinear algebra method to jointly achieve denoising and dimensionality reduction. Multilinear tools consider HSI data as a whole by processing jointly spatial and spectral ways. The lower rank-(K_1, K_2, K_3) tensor approximation ($LRTA$-(K_1, K_2, K_3)) was successfully applied to denoise multi-way data such as color images. Firstly, we demonstrate that the $LRTA$-(K_1, K_2, K_3) performs well as a denoising preprocessing to improve classification results. Then we propose a novel method, referred to as $LRTA_{dr}$-(K_1, K_2, D_3) which performs both spatial lower rank approximation and spectral dimensionality reduction. Classification algorithm SAM is applied to the output of three dimensionality and noise reduction methods to compare their efficiency : the proposed $LRTA_{dr}$-(K_1, K_2, D_3), PCA_{dr}, and PCA_{dr} associated with Wiener filtering or soft shrinkage of wavelet transform coefficients.**

***Index Terms*— Classification, dimensionality reduction, multilinear algebra.**

I. INTRODUCTION

THE emergence of hyperspectral images (HSI) implies the exploration and the collection of a huge amount of data. Hyperspectral imaging sensors provide a huge number of spectral bands, typically up to several hundreds. This unreasonably large dimension of HSI not only increases computational complexity but also degrades classification accuracy [?]. Because HSI contains a large amount of spectral redundancy and lacks training data, reduction of spectral dimensionality has proven necessary to apply classification algorithms. A previous work [?] has shown that high-dimensional data spaces are mostly empty, indicating that the involved data structure exists primarily in a subspace.

Due to its simplicity and ease of use, the most popular dimensionality reduction (DR) approach is the PCA [?], referred to as PCA_{dr}, which maximizes data variance by orthogonal projection. A refinement of PCA_{dr} is the independent component analysis (ICA) [?], referred to as ICA_{dr} which uses higher-order statistics. But those two linear DR methods require a preliminary data arrangement. Indeed, when dealing with three-way data a first step consists in vectorizing all images yielding two-way data, permitting the use of signal processing but neglecting spatial rearrangement. To overcome it, [?] proposes a multichannel mathematical morphology operator-based DR method which incorporates the image representation. In this paper, a multilinear algebra-based DR method is proposed to extract spectral principal components by taking into account spatial information.

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Moreover some radiometric noise degrades classification results. This noise is due to sensor, photon effects and calibration error [?], [?]. Then, in addition to DR, a noise reduction (NR) processing is also required to decrease the spectral variability, which is useful for further classification algorithms. In this context, maximum noise fraction [?], Wiener and wavelet thresholding, shrinkage [?] are useful denoising techniques. Commonly, the hybrid scheme [?], [?] consists in spectral band decorrelation by PCA, Fourier or discrete cosine transform to apply 2D-filtering on each decorrelated band. But as was pointed out in [?] the intuitive representation of a collection of images is a three-way array, or third-order tensor. Hence, instead of adapting data to classical matrix-based techniques, the 3D representation has proven its usefulness for compression applications [?], [?].

In this paper, our multilinear algebra method simultaneously reduces the spectral dimension and jointly denoise all ways. Based on the Tucker3 decomposition, we show the ability of the lower rank-(K_1, K_2, K_3) tensor approximation [?] as a denoising tool. We propose a novel method, referred to as $LRTA_{dr}$-(K_1, K_2, D_3) which performs both spatial lower rank approximation and spectral dimensionality reduction. The main purpose is to introduce those two methods and then to show how they work in classification experiments [?].

The remainder of the paper is organized as follows: Section II presents the multi-way representation. Section III introduces the $LRTA$-(K_1, K_2, K_3) and its formulation of the classical noise-removing problem. Section IV introduces the proposed spectral dimensionality reduction method, the $LRTA_{dr}$-(K_1, K_2, D_3). Section V contains some comparative results concerning the performance of spectral angle mapper (SAM) classifier [?] when it is applied after either denoising and/or dimensionality reduction of HSI. Section VI concludes the paper.

II. MULTI-WAY REPRESENTATION AND PROPERTIES

In this paper we consider a three-way array as a third order tensor, the entries of which are accessed via three indices. It is denoted by $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, with elements arranged as $x_{i_1 i_2 i_3}$, $i_1 = 1, \ldots, I_1;\ i_2 = 1, \ldots, I_2;\ i_3 = 1, \ldots, I_3$ and $\mathbb{R}$ is the real manifold. Each index is called way or mode and the number of levels on one mode is called dimension of that mode. The n-mode vectors are the I_n- dimensional vectors obtained from a tensor by varying index i_n while keeping the other indices fixed. The so-called n-mode flattened matrix $\mathbf{X}_n$ of $\mathcal{X}$ is such that its columns are the n-mode vectors.

The analysis of HSI which is a set of I_3 images with size $I_1 \times I_2$ resulting from the confluence of multiple modes is a problem in multilinear algebra. Within this mathematical framework, the HSI data can be represented as a three-way array or as a third-order tensor: two ways for rows and columns and one way for the spectral band.

Foremost, let us give a brief review of tensor rank definitions which can be found in [?]. The n-mode rank of multi-way data $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, denoted by $Rank_n(\mathcal{X})$, is defined as the dimension of the vector space generated by the I_n-dimensional vectors obtained from $\mathcal{X}$ by varying index i_n while keeping the other indices fixed. $\mathcal{X}$ is called a rank-(K_1, K_2, K_3) tensor if $Rank_n(\mathcal{X}) = K_n$ for all $n = 1, 2, 3$.

This multi-way representation considers HSI data as a whole data set which involves a joint processing along each mode without separability assumption, rather than splitting data or processing only the vectorized images. This representation intuitively implies processing technics based on multilinear algebra [**?**], [?]. The Tucker3 decomposition [**?**] permits to achieve $LRTA$-(K_1, K_2, K_3).

III. Denoising tool : $LRTA$-(K_1, K_2, K_3)

Several tensor decomposition models have been developed [?], [**?**], [**?**]. The Tucker3 tensor decomposition [?] generalizes the SVD. Following the Tucker3 model, the decomposition of any three-way data $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ can be expressed as

$$\mathcal{X} = \mathcal{C} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)} \tag{1}$$

where $\mathbf{U}^{(n)}$ is the orthogonal n-mode matrix holding the K_n eigenvectors associated with the K_n largest eigenvalues of the flattened matrix $\mathbf{X}_n$, $\mathcal{C} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ is the core tensor analogous to the diagonal singular value matrix in conventional matrix SVD, and $\times_n$ is the n-mode product.

Given a real-valued three-way data $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, the purpose of $LRTA$-(K_1, K_2, K_3) is to find the lower rank-(K_1, K_2, K_3) multi-way data $\widehat{\mathcal{X}}$, with $K_n < I_n$, for all $n = 1$ to 3, which minimizes the following quadratic Frobenius norm: $\left\| \mathcal{X} - \widehat{\mathcal{X}} \right\|_F^2$. Thus the best lower rank-(K_1, K_2, K_3) multi-way approximation of $\mathcal{X}$ is expressed as [?]

$$\widehat{\mathcal{X}} = \mathcal{X} \times_1 \mathbf{P}^{(1)} \times_2 \mathbf{P}^{(2)} \times_3 \mathbf{P}^{(3)}, \tag{2}$$

where $\mathbf{P}^{(n)} = \mathbf{U}^{(n)} \mathbf{U}^{(n)^T}$, n = 1 to 3, $\mathbf{P}^{(n)}$ is achieved after an alternating least squares (ALS) algorithm convergence [?]. Thanks to this procedure any projector along a given mode depends on the projectors along all other modes.

In HSI context, we assume that each spectral band is impaired by additive noise [?], [?] due to radiometric effects. According to the data representation adopted in this paper, this additive noise can, as well, be represented as a three-way array, $\mathcal{N}$. So, the observed HSI data can be represented as $\mathcal{R} = \mathcal{X} + \mathcal{N}$, where $\mathcal{R}$, $\mathcal{X}$ and $\mathcal{N}$ are three-way array data of $\mathbb{R}^{I_1 \times I_2 \times I_3}$. We aim at estimating tensor $\mathcal{X}$, using subspace-based methods. In 2-D signal processing these methods use the signal subspace spanned by the eigenvectors associated with the largest eigenvalues of the covariance matrix of the set of observation vectors. In the same way when noise $\mathcal{N}$ is not correlated with signal tensor $\mathcal{X}$, the classical subspace-based approach can be extended to multi-way data by assuming that whatever the n-mode, $E^{(n)}$ is the direct sum of a signal subspace $E_{ss}^{(n)}$ of dimension K_n and a noise orthogonal subspace $E_{ns}^{(n)}$ of dimension $I_n - K_n$.

In the tensor formulation, the projectors on the n-mode vector spaces are determined by computing the $LRTA$-(K_1, K_2, K_3): one way to estimate signal tensor $\mathcal{X}$ from the noised tensor $\mathcal{R}$ is to estimate $E_{ss}^{(n)}$ in every n-mode of $\mathcal{R}$ and to orthogonally project every n-mode vector of $\mathcal{R}$ on the n-mode signal subspace $E_{ss}^{(n)}$, for all $n = 1, 2, 3$. The $LRTA$-(K_1, K_2, K_3) is the appropriate tool since it permits, for each n-mode, to keep only the K_n eigenvectors associated with the K_n largest eigenvalues of the n-mode flattened matrix $\mathbf{R}_n$. This statement can be written as

$$\widehat{\mathcal{X}} = \mathcal{R} \times_1 \mathbf{P}^{(1)} \times_2 \mathbf{P}^{(2)} \times_3 \mathbf{P}^{(3)} \tag{3}$$

where $\mathbf{P}^{(n)} = \mathbf{U}^{(n)} \mathbf{U}^{(n)^T}$, with $\mathbf{U}^{(n)} = [\mathbf{u}_1, \cdots, \mathbf{u}_{K_n}]$, is the projector upon the n-mode signal subspace. To estimate the signal subspace dimension K_n the well-known Akaike information criterion (AIC) is extended in [**?**]. The main issue of this multi-way data analysis is to remove noise using jointly spatial and spectral information thanks to ALS algorithm.

IV. Dimensionality reduction tool : $LRTA_{dr}$-(K_1, K_2, D_3)

In HSI context, we are interested in reducing the number of spectral bands by selecting more significant spectral features in order to improve classification.

The principles of PCA_{dr}-(D_3) are the following: I_3 images of full size $I_1 \cdot I_2$ are considered. Each image is transformed into a vector by row concatenation. Data tensor $\mathcal{R} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ composed of all I_3 images as slice matrices becomes a matrix $\mathbf{R} \in \mathbb{R}^{I_3 \times p}$ where $p = I_1 \times I_2$. The aim of DR is to extract a small number $D_3 < I_3$ of features, called principal components (PCs). Therefore the D_3 PCs generate a reducing matrix $\mathbf{Z} \in \mathbb{R}^{D_3 \times p}$

$$\mathbf{Z} = \mathbf{R} \times \Lambda^{-1/2} \mathbf{U}^T. \tag{4}$$

The data can be reshaped as a three-way array $\mathcal{Z} \in \mathbb{R}^{I_1 \times I_2 \times D_3}$.

In tensor formulation [?], [?], [?], [?], the previously obtained matrix $\mathbf{R}$ is equivalent to the 3-mode flattened matrix of $\mathcal{R}$ noted $\mathbf{R}_3$. Then equation (4) can be written

$$\mathcal{Z} = \mathcal{R} \times_3 \mathbf{\Lambda}^{-1/2} \mathbf{U}^{(3)^T}. \tag{5}$$

In the same way, we can turn the $LRTA$-(K_1, K_2, K_3) into a spectral dimensionality reduction tool. This tool is referred to $LRTA_{dr}$-(K_1, K_2, D_3) in this paper, where D_3 represents the number of spectral principal components. $LRTA_{dr}$-(K_1, K_2, D_3) extracts D_3 spectral PCs in order to obtain the three-way array $\mathcal{Z} \in \mathbb{R}^{I_1 \times I_2 \times D_3}$. The challenge is carried out thanks to the $LRTA_{dr}$-(K_1, K_2, D_3) is to jointly reduce the dimensionality of the spectral mode and to project the information along the spatial modes onto lower (K_1, K_2)-dimensional subspaces. The latter processing permits to compress and to spatially denoise the data. Different parameter values -(K_1, K_2, D_3)- can be retained for each mode. The $LRTA_{dr}$-(K_1, K_2, D_3) model reads

$$\mathcal{Z} = \mathcal{R} \times_1 \mathbf{P}^{(1)} \times_2 \mathbf{P}^{(2)} \times_3 \mathbf{\Lambda}^{-1/2} \mathbf{U}^{(3)^T}, \tag{6}$$

Where $\mathbf{U}^{(3)}$ is a matrix holding D_3 selected eigenvectors, $\mathbf{\Lambda}$ is the diagonal eigenvalue matrix holding the D_3 largest eigenvalues and $\mathbf{P}^{(n)}$ is the projector for the n-mode, defined in the above section III.

The main $LRTA_{dr}$-(K_1, K_2, D_3) attributes in relation to the PCA_{dr}-(D_3) is dual. First, $LRTA_{dr}$-(K_1, K_2, D_3) joint uses spatial and spectral information to extract the spectral principal components. Secondly, $LRTA_{dr}$-(K_1, K_2, D_3) denoises the extracted spectral principal components thanks to the estimated spatial projectors, $\mathbf{P}^{(n)}$ $n = 1, 2$. The iterative ALS algorithm permits to take into account the cross-dependence between NR and DR. One ALS iteration approximatively requires 6 sec. on a 3.2 GHz PC running Windows and using Matlab® 7. Five iterations yield satisfactory results.

V. EXPERIMENTS

The data used in the following experiments are real-world data collected by HYDICE imaging, with a 1.5 m spatial and 10 nm spectral resolution and including 148 spectral bands (from 435 to 2326 nm), 310 rows and 220 columns. This HSI can be represented as a multi-way array data, denoted by $\mathcal{R} \in \mathbb{R}^{310\times220\times148}$. A preprocessing step removes the mean of each vector pixel of the initial multi-way data $\mathcal{R}$.

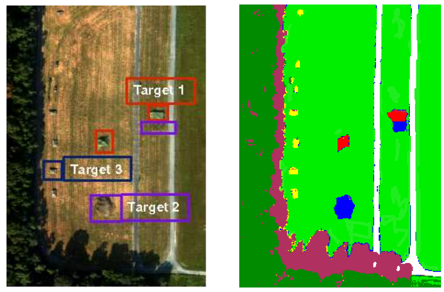
In this paper, we focus on the classification result obtained after each preprocessing method. Figs. 1 a) and b) show the entire scene used for experiments. The land cover classes are : field, trees, road, shadow and 3 different targets. The resulting number of training and testing pixels for the seven classes are given on Table I. Classification is performed thanks to the spectral angle mapper (SAM) algorithm [?] which is very largely applied to HSI data. To appreciate quantifiable comparisons, we determine the overall accuracy (OA) in percentage exhibited by SAM classifier. For P classes C_i, $i = 1, \ldots, P$, if a_{ij} is the number of test samples that actually belong to class C_i and are classified into C_j for $i, j = 1, \ldots, P$, then OA is defined as follows

$$OA = \frac{1}{M} \sum_{i=1}^{i=P} a_{ii}, \tag{7}$$

where M is the total number of samples, P is the number of classes C_i for $i = 1, \ldots, P$ and a_{ii} is equal to a_{ij} for $i = j$.

Three experiments are presented. The first one shows the ability of the $LRTA$-(K_1, K_2, K_3) to reduce the system imaging noise; the second experiment shows the ability of the $LRTA_{dr}$-(K_1, K_2, D_3) as a dimensionality reduction tool; and the third one shows the robustness of the proposed $LRTA_{dr}$-(K_1, K_2, D_3) algorithm in presence of additive noise.

For all those experiments algebraic-based methods are applied. Firstly, to evaluate the interest of dealing with spatial information to extract the spectral components, experiments compare the OA results obtained after $LRTA_{dr}$-(K_1, K_2, D_3) and after PCA_{dr}. Secondly, to evaluate the denoising efficiency of the $LRTA$-(K_1, K_2, K_3), experiments compare the OA results after applying two hybrid filters [?], [?], which consist in a 2D-Wiener filtering or soft wavelet thresholding (SWT) on the spectral bands reconstructed and decorrelated by PCA. Those methods are referred to as PCA-$Wiener$ and



a) b)

Fig. 1: a) Classes in the HYDICE image RGB, b) ground truth.

TABLE I: Information classes and samples

Classes	Training samples	Test samples	Color
field	1 002	40 811	green 1
trees	1 367	5 537	green 2
road	139	3 226	white
shadow	372	5 036	pink
target 1	128	519	red
target 2	78	285	blue
target 3	37	223	yellow

PCA-SWT respectively. For the wavelet transform, we use the Daubechies order-2 wavelet (db2) and a 4-level wavelet decomposition. In dimensionality reduction context, they are referred to as PCA_{dr}-(D_3)-$Wiener$ and PCA_{dr}-(D_3)-SWT, 2D filtering is applied to the D_3 retained spectral components.

A. Classification without dimensionality reduction

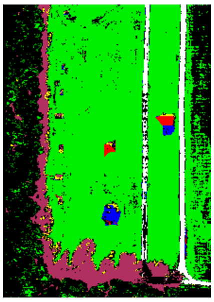
The $LRTA$-(K_1, K_2, K_3) is tested on real-world HSI which is impaired because of some properties [?], [?] of imaging system. This experiment evaluates the necessity of denoising real-world data to improve the classification results. The (K_1, K_2, K_3)-values of the $LRTA$-(K_1, K_2, K_3) are estimated by AIC criterion [?]. For this experiment, the (K_1, K_2, K_3)-values are estimated to (100,100,80).

Fig. 2 shows classification results obtained from the real-world multi-way array HSI $\mathcal{R} \in \mathbb{R}^{310\times220\times148}$ and from the three multi-way arrays $\widehat{\mathcal{X}} \in \mathbb{R}^{310\times220\times148}$ estimated by each NR method. The black pixels present in the classification result represent the unclassified pixels. We notice from the visual results and OA values that denoising is a necessary preprocessing step. For this real-world data the $LRTA$-(K_1, K_2, K_3) outperforms comparative methods. The next experiment shows the additional improvement when DR is performed.

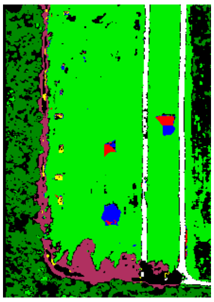
B. Classification after dimensionality reduction

This experiment highlights the advantage of applying DR method on real-world data before classification. We compare the classification result after applying the $LRTA_{dr}$-(K_1, K_2, D_3), the PCA_{dr}-(D_3) and the two hybrid filters : PCA_{dr}-(D_3)-$Wiener$ and PCA_{dr}-(D_3)-SWT.

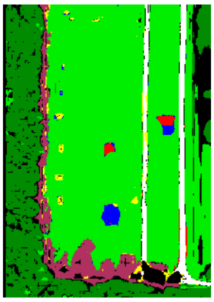
It is conceded that the number of retained spectral features has an impact on the classification efficiency. Then, we first evaluate the influence of spatial ranks, (K_1, K_2) values, on



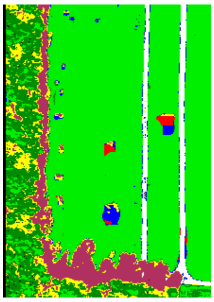
a) SAM result from real-world HSI, OA = 78.98



b) SAM result after PCA-Wiener, OA = 84.36



c) SAM result after PCA-SWT, OA = 89.28



d) SAM result after LRTA-(100, 100, 80), OA = 93.42

Fig. 2: Imaging system noise reduction outcome for classification applied on all 148 spectral bands.

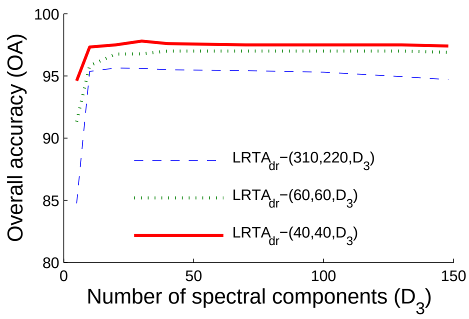


Fig. 3: Impact of spatial ranks, (K_1, K_2) values, on the overall accuracy as a function of the spectral dimension.

classification efficiency for various numbers of retained spectral features.

Fig. 3 presents OA results obtained with the $LRTA_{dr}$-(K_1, K_2, D_3) method, and shows that the (K_1, K_2) values of the spatial ranks also have much impact. For each value of D_3, the lower (K_1, K_2) values the better the classification results. This optimal interplay between parameters (K_1, K_2) and D_3 is not permitted when PCA_{dr}-(D_3) is used.

Fig. 4 draws the OA as a function of the number of retained spectral components obtained with the comparative DR methods. Knowing that from the initial tensor $\mathcal{R} \in \mathbb{R}^{310\times220\times148}$, OA is equal to 78.98, Fig. 4 highlights the DR interest when the aim is the classification. Indeed, for PCA_{dr} DR method, we note that there is an optimal spectral dimension : using too few or too much components decreases the classification efficiency. We notice also that the $LRTA_{dr}$-(K_1, K_2, D_3) leads to better OA than PCA_{dr}-(D_3) for all D_3-values. Instead of decreasing when

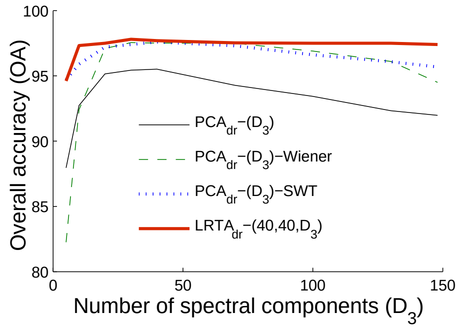


Fig. 4: Dimensionality reduction outcome for SAM classification. OA as a function of the number of retained spectral components. (From the initial tensor $\mathcal{R}$, OA=78.98.)

the D_3-value increases, the OA obtained after $LRTA_{dr}$-$(40, 40, D_3)$ is constant.

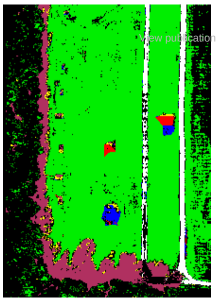
Fig. 5 shows a visual classification result obtained from the original tensor $\mathcal{R}$ and after the DR methods which select D_3= 30 spectral features and where (K_1, K_2)-dimensions of the spatial subspaces have been fixed to 40 for the $LRTA_{dr}$-$(40, 40, 30)$. Fig. 5 a) permits visually to appreciate the DR usefulness. Fig. 5 shows that including a denoising step is successful. Indeed this processing permits to have classes which are more homogeneous and the mean area corresponding to the background and the target are more identifiable with less unclassified pixels. The $LRTA_{dr}$-(K_1, K_2, D_3) permits to reduce simultaneously the spectral dimension and the dimensions of the spatial subspaces which is of great interest for SAM classifier. Furthermore, multilinear-based DR method $LRTA_{dr}$-(K_1, K_2, D_3) leads to better classification results than PCA_{dr} and than PCA_{dr} associated with 2D-spatial filtering considered in this paper.

C. Classification in noisy environment

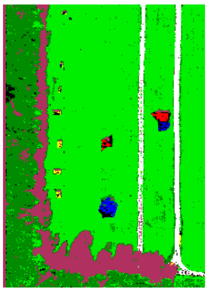
In this experiment, we test the noise robustness of the $LRTA_{dr}$-(K_1, K_2, D_3) method. For this issue, Gaussian stationary and zero mean noise is added to real-world HSI data with standard deviation varying from 5 to 25. For convenience, OA results are only obtained from $D_3 = 30$ spectral components retained thanks to the DR methods. Table II shows that the OA values obtained from PCA_{dr} result decrease when standard deviation increases. The 2D-filtering ($Wiener$ or SWT) applied on spectral components permits to improve the OA-value. The $LRTA_{dr}$-(40,40,30) reaches the same effect by yielding classification results which are robust to noise impairment and sensibly better than the PCA_{dr}-$Wiener$ and PCA_{dr}-SWT.

VI. Conclusion

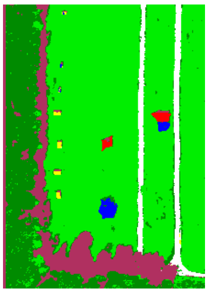
A novel multilinear algebra-based algorithm, named $LRTA_{dr}$-(K_1, K_2, D_3), is proposed for joint noise and dimensionality reduction. This joint spatial-spectral processing is cross-dependent thanks to the ALS algorithm. We focused on the ability of $LRTA_{dr}$-(K_1, K_2, D_3) as a preprocessing



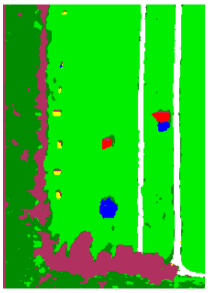
a) SAM result from real-world data, OA = 78.98



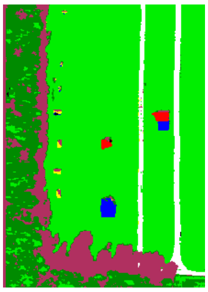
b) SAM result after PCA_{dr}-(30), OA = 92.73



c) SAM result PCA_{dr}-(30)-Wiener, OA = 97.58



d) SAM result after PCA_{dr}-(30)-SWT, OA = 97.6



d) SAM result after $LRTA_{dr}$-$(40, 40, 30)$, OA = 97.67

Fig. 5: Dimensionality reduction outcome for classification applied on 30 spectral components.

TABLE II: Overall accuracy from SAM in noisy environment. Results obtained from 30 retained spectral components.

standard deviation	PCA_{dr}	PCA_{dr}-$Wiener$	PCA_{dr}-SWT	$LRTA_{dr}$-(40,40,30)
5	94,9	97,6	97,6	97,55
15	90,7	96,78	97,14	97,58
20	80	96,9	97,12	97,5
25	75.5	96,6	97,09	97.5

algorithm that improves SAM classification result applied to real-world Hydice data. Quantitative results based on OA criterion evaluate the impact on the spatial ranks, (K_1, K_2) values, and compare the performance with selected dimensionality reduction methods. Indeed in comparison with PCA_{dr}-(D_3), the $LRTA_{dr}$-(K_1, K_2, D_3) permits to extract spectral components by taking into account spatial information by simultaneously estimating spatial projectors to denoise them. The comparison with selected hybrid filters, which perform 2D-spatial filtering of the retained spectral components, permits to appreciate the denoising efficiency of our method, in the application of target classification in noisy Hydice data.

These promising results encourage us to integrate a multilinear approach in ICA_{dr} method with the same proposed strategy. This further work could overcome a major issue for PCA_{dr}-(D_3) [**?**] which is that many subtle materials or rare targets require higher-order statistics to be characterized.

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