Applications of Non-Markovian Hybrid Petri-Nets in Power Engineering

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***Abstract*—This paper is focused on simulation and graphical stochastic modeling of electrical power systems. The main goal of this paper is to develop a tool for modeling and simulation of very large systems with probabilistic incidents. For this purpose, a new extension of Petri nets is introduced in this paper. Using this extension which is suitable for power engineering applications, a large system with stochastic transitions can be studied and analyzed. After a preliminary introduction of Petri nets, the proposed extension is introduced. In order to clarify the modeling process, a case study regarding modeling and analyzing long-term behavior of a Plug-in Hybrid Electric Vehicle (PHEV) is studied and simulated. The simulation results demonstrate the long-term stochastic dynamics of a charging station without any requirement for rigorous analytical studies of the overall stochastic process.**

***Index Terms*—Reliability, Petri nets, Monte-Carlo simulation, charging station.**

I. Introduction

Integration of Distributed Energy Resources (DER) with power systems has led to new requirements in managing and monitoring power systems. Traditionally, a power system has to cope with random failure incidents and stochastic variations of loads. For this purpose, large safety margins in reserve capacity are incorporated. In addition, load shedding is a common procedure in maintaining stable operation of the system. In recent years, dependency of the society to electrical energy has increased the equivalent cost of power outages. Therefore, electricity cooperatives are not interested in practicing load shedding procedures. Furthermore, increasing demand for electric energy as well as the high cost of generation has reduced the feasible reserved capacity. Also, integration of renewable energy resources has introduced another factor of randomness to power systems. These resources are not dispatchable and often observe sudden variations of generated power which can be as much as 100% of the generation capacity. Also, decision making systems such as smartgrid controllers can disconnect an electrical region during voltage and frequency anomalies. Although islanding will prevent a local microgrid from observing power outages, on the overall grid, disconnection of an electrical region can significantly impact the stability of the utility grid. In fact, massive integration of solar generation has already affected several countries such as Germany (i.e. *Germany's 50.2Hz problem*). These added complexities have rendered conventional off-line deterministic studies insufficient in capturing stochastic dynamics of modern power systems.

Unlike deterministic methods, stochastic modeling provide more accurate results in studying dynamics of power systems. Due to the extremely large number of components in power systems, an accurate analytical model of the system is hard to derive (Unless the system is simplified to a Markovian representation). Monte Carlo simulation methods have been widely used in simulating stochastic processes. However, a complete framework for modeling, simulating, and analyzing a power system is of interest.

Petri Nets (PN) are graphical and mathematical tools for modeling and analysis of very large systems. These tools provide a framework that is robust to largeness and further expansions of a system. PN was first introduced by C. Petri. PNs are bipartite directional graphs that contain three basic primitives; i. places which contain the information or state of the system, ii. transitions which manage the evolution of the states, iii. and arcs which connect places and transitions. Petri nets have been widely used in modeling complex algorithms and software, processes, and manufacturing systems [1], [2]. Coloured (colored) PNs are one of the early extensions of PNs where each token has a colour (i.e. color) property. With this extension, similar PNs can be integrated into a single PN where each token is associated with a different color to distinguish between individual systems. This extension of PNs has been widely used in computer and process engineering [3]. In this paper, dynamic colored tokens are introduced to integrate a portion of the model within the token for reducing the size of the overall PN.

Stochastic Petri nets were introduced to integrate probabilistic transitions into the standard PN. However, in order to maintain closed-form analytical solutions, the stochastic transitions are restricted to exponential distributions [4], [5]. These PNs are very similar in concept to continuous time Markov chains while the transitions provide more flexibility in modeling complex systems. Since majority of reliability studies are using exponential distributions for analytical simplicity, stochastic PNs have been widely used in system reliability analysis [6]–[9]. In order to relax restrictions on using exponential distributions in stochastic Petri nets, queueing Petri nets were introduced. In these PNs, a new set of *queueing* places were introduced that were capable of releasing tokens with a general probability distribution. However, the arrivals were still bounded to transitions with exponential distributions [10],

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[11]. Non-Markovian PNs are capable of modeling probabilistic incidents with general distribution functions [12], [13]. However, the conventional methods for generating closed-form analytical solutions for PNs are no longer applicable to these nets. Various simulation methods have been studied for fast simulation of non-Markovian PNs [14]. Fluid or hybrid PNs are an strong extension to PNs. These nets allow for integration of continuous places and hence, can form a hybrid discrete-continuous system. A continuous system can be modeled by either hybrid stochastic Petri nets or stochastic hybrid systems (SHS)(which is a similar modeling tool mainly used in control engineering). However, non-Markovian hybrid PNs can model a wider range of systems in comparison with SHS. A very small number of studies have utilized PNs for analysis of power engineering systems [15]. Protection systems are an ideal case study for Petri nets. These nets can explore every possible state of a large protection system [16]. Reliability in connection of a distribution network can be studied using PNs [17]. Fluid (hybrid) stochastic Petri nets have been incorporated to study the stochastic power injection from renewable energy resources [18].

In this paper, a non-Markovian hybrid PN is introduced that is suitable for applications in power engineering. After the introduction of this modeling framework, a case study of a hybrid vehicle charging station is modeled and simulated using this PN.

II. Non-Markovian Hybrid Petri Nets with Dynamic Tokens

In this paper, a more general form of Petri nets is introduced. This net will be able to incorporate the dynamics and probability functions required to model a real-life power system. In this formalism, a Non-Markovian Hybrid Petri Net (NHPN) with dynamic tokens is defined by a multi-tuple

$$\mathrm{PN} \; : \; \langle \mathcal{P}, \mathcal{T}, \mathcal{A}, \mathcal{W}, \mathcal{G}, \mathcal{S}_0, \tau, \mathcal{D}, \mathcal{D}_{\mathcal{T}}, \mathfrak{p} \rangle \tag{1}$$

where $\mathcal{P}$ is the set of places. This set is divided into two subsets of $\mathcal{P}_D$ which is the set of discrete places in conventional Petri nets, and $\mathcal{P}_C$ which is the set of fluid (i.e. continuous) places and is denoted by two concentric circles. For simplicity, the notation of Marking, $\mathcal{M}(\mathcal{P}_i)$, is used for both discrete and continuous places. Furthermore, the state of the system is the vector of all markings in time (i.e. $\mathcal{S} := \{\mathcal{M}(\mathcal{P}), \tau\}$.

$\mathcal{T}$ is the set of all transitions. For non-Markovian nets, this set has the two subset of immediate transitions (i.e. $\mathcal{T}_I$) and timed transitions (i.e. $\mathcal{T}_G$). Timed transitions, after enabling, have a firing time with a cumulative distribution function $G(\tau)$. Unlike stochastic Petri nets, non-Markovian nets are not restricted to exponential firing distributions. In addition, this expansion eliminates the usage of queueing Petri nets since these places can be integrated within the non-Markovian Petri nets, directly. Immediate transitions are divided into deterministic transitions (standard immediate transitions) and probabilistic transitions (semi-immediate).

$\mathcal{A}$ is the set of arcs which connect places and transitions. In Petri nets, no arc can exist between two transitions or two places. The set of arcs is divided into four subsets; i. standard arcs with labelings which define the amount each arc carries over, $\mathcal{A}_n : ((\mathcal{P}\times\mathcal{T})\cup(\mathcal{T}\times\mathcal{P})) \to \mathbb{R}$. In this definition, discrete and continuous arcs are modeled simultaneously. Therefore, if this arc is connected to a continuous place, the labeling will act as a Dirac impulse. ii. Inhibitor arcs which act with an inverse logic, $\mathcal{A}_h : (\mathcal{P} \times \mathcal{T}) \to \mathbb{R}$. Inhibitor arcs do not remove any token or mass from places. iii. Test arcs, $\mathcal{A}_t : (\mathcal{P} \times \mathcal{T}) \to \mathbb{R}$, which are similar to the standard arcs. However, these arcs do not remove any mass or token from places. These arcs are shown with a dashed line. iv. fluid or continuous arcs with labelings which define the rate of mass/fluid each arc carries over, $\mathcal{A}_c : ((\mathcal{P}_C \times \mathcal{T}) \cup (\mathcal{T} \times \mathcal{P}_C)) \to \mathbb{R}$. This are defines the rate of variations of mass/fluid in a continuous place and can be used to model differential equations. Traditionally, these arcs are shown with a thick line.

$\mathcal{W}$ is the set of priority of firing for immediate transitions. Without this definition, conflicts can occur if two transitions are enabled simultaneously. For semi-immediate transitions, this set acts as the probability of the transition. In this extension, a semi-immediate transition is an immediate transition that contains a probability of firing relative to the other simultaneously enabled transitions. In this case, the probability of transition $\mathcal{T}_i$ firing is $\Pr\{Ti\} = w_i/\sum_j w_j$ where j is the set of all enabled transitions connected to a same place, for which, firing of one transition disables the others. This extension enables for modeling probabilistic chains such as Markov chain.

$\mathcal{S}_0$ if the equivalent to initial marking in conventional definitions. In the proposed Petri net, time and initial ages are crucial to accurate modeling of the system, therefore, the initial marking is replaced with the initial state which consists of markings of discrete and continuous variables as well as age of each token. $\tau \in \mathbb{R}_+^{N_\tau}$ tracks the age of each token or timed transition. $\mathcal{D} \in \mathbb{R}_+^{N_D}$ is the colour (i.e. color) associated with each token. However, in this extension, tokens are dynamic and demonstrate the state of each token. Based on the age of each token and history of the states and transitions, the token can change color. More accurately, tokens contain information and dynamics. Therefore, in this formalism, the notation for color is changed to data set, $\mathcal{D}$ and the set of all functions modeling the dynamics of each token is $\mathcal{D}_{\mathcal{T}}$.

$\mathcal{G}$ is the set of all arc labels, firing distributions, and firing functions (guards). $\mathfrak{p}$ defines the policy of transitions in the Petri net. Traditionally, policy has been an important factor in defining Petri nets. Three major policies are being widely used which are i. Preemptive Repeat Difference (PRD), ii. Preemptive Resume (PRS), and iii. Preemptive Repeat Identical (PRI). In this paper, the default policy is PRD unless noted otherwise. On the other hand, this set defines the resampling policy of each transition when other transitions are fired. The default policy is PRS. In the case of PRD, the history will be erased after firing. However, in the case of PRS, the history will resume to evolve.

At any given time, t, for the set of enabled transitions in state $\mathcal{S}_t$, the variations of the markings are given by

$$\forall p \in \mathcal{P}_D,\ \mathcal{M}(p^{t+}) = \mathcal{M}(p^t) + A_n^{\mathcal{T}_-\cdot p}(\mathcal{S}) - A_n^{p\cdot\mathcal{T}_+}(\mathcal{S}) \tag{2}$$

$$\begin{aligned}\forall p \in \mathcal{P}_C,\ \frac{d\mathcal{M}(p)}{dt} &= \left(A_n^{\mathcal{T}_-\cdot p}(\mathcal{S}) - A_n^{p\cdot\mathcal{T}_+}(\mathcal{S})\right)\delta(t) \\ &\quad + A_c^{\mathcal{T}_-\cdot p}(\mathcal{S}) - A_c^{p\cdot\mathcal{T}_+}(\mathcal{S})\end{aligned} \tag{3}$$

where $\mathcal{T}_-$ and $\mathcal{T}_+$ refer to prior and subsequent enabled transitions to p, respectively. In this extension, the mass/level of the fluid is not limited to $\mathbb{R}^+$. The proposed formalism in this paper is derived and extended from several of the extensions of PNs especially hybrid PNs. It should be noted that the design and formalism introduced in this paper is best fit for discrete time simulation of the Petri net. Continuous states, stochastic processes, and dynamics can be approximated as discrete states for short periods of time. In fact, the applications of PNs in power systems is best fitted for long-term uncertainty analysis. Hence, a fairly small simulation time-step can provide a good approximation of the original process.

III. Modeling with NHPN

In this section, modeling electrical power systems using Petri nets is introduced. Modeling is a system dependent process. For this reason, this process is introduced using a case study. A Plug-in Electric Hybrid Vehicle (PHEV) charging station is studied in this section. This charging station has two charging ports which are connected to the utility grid. In general, a PHEV charging station is associated with various probabilistic properties:

Availability:

i. Time of arrival of each PHEV.
ii. Time to departure of each PHEV.
iii. The willingness to participate in energy conversion.

Electrical:

iv. Capacity of the battery-bank of each PHEV.
v. State-Of-Charge (SOC) in each PHEV.
vi. The requested SOC at the time of departure.
vii. Reliability of the charging circuit.

Economic:

viii. Limits on the price of electricity.

In order to study this system, besides the definition of the NHPN which was introduced in (1), a probability space is required. Let Ω be a sample space and let $\mathcal{F} = \mathfrak{B}(\Omega)$ be the σ-algebra of all the Borel sets on the sample space. Let $\Pr : \mathcal{F} \to [0, 1]$ be a measure on $\mathcal{F}$. Then $(\Omega, \mathcal{F}, \Pr)$ is a probability space with a natural filtration of $\{\mathcal{F}_t\}_{t\in I}$, where $I \in \mathbb{R}^+$. This probability space will be shared among all stochastic processes in the model. In this paper, survival of a random variable T is defined as

$$S(t) = \Pr\{T > t\} = 1 - F(t) \tag{4}$$

where $F(t)$ is the cumulative distribution function. Furthermore, for stochastic processes, it is of interest to define the survival function as a function of the incident rate. Therefore, a rate can be defined as

$$\lambda(t) = \Pr\{t < T \leq t + \delta | t < T\}_{\delta\to 0} \tag{5}$$

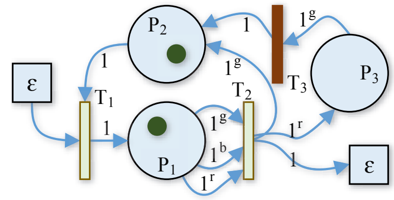

Fig. 1. Petri net model of the charging station.

to calculate the survival function as

$$S(t) = e^{-\int_0^t \lambda(u)du} = e^{-\Lambda(t)} \tag{6}$$

It is assumed that the charging station has two chargers. Therefore, after arrival of two cars, the capacity of the station is full until the departure of a vehicle. This process is shown in Figure 1. $\mathcal{P}_1$ represents the number of vehicles under charge while $\mathcal{P}_2$ is the remaining capacity of the station. In the system under study, time to the next arrival of a vehicle has a general cumulative probability function of $W_a(t)$. Since the purpose of this modeling is to simulate the behavior of the system, restrictions of the off-line methods are not applicable to this modeling. Hence, $W_a(t)$ can be as simple as a homogeneous Poisson process, or a more detailed doubly stochastic Poisson process [19]. Unlike conventional methods that require rigorous mathematical relations, in PN modeling and simulation, analytical relations are more relaxed. This is due to the time-based simulation process which can eliminate the requirements for complete closed-form relations. For instance, in a Renewal process, in order to develop a closed form equation for the process as a whole, the solution to the fundamental renewal equation is required. However, in general, this equation cannot be simplified any further. Also, if the time to the next arrival follows a new probability distribution function for each inte-roccurrence time, the Renewal process cannot be represented as a single equation. Unlike off-line mathematical methods, PN provides a temporal relaxation for these processes. After each arrival, the time to the next arrival can be sampled based on the probability function of interest. Further details on the concept of age and relative time in stochastic processes, are available in [19]. The sampling process in a simulator is a Pseudo-random number sampling technique such as Inverse Transform Sampling (ITS) or with some approximations, sampling on the discretized distribution using discrete search methods.

In this system, departure is also a Poisson process with a cumulative function $W_d(t)$. For simplicity, time to next arrival is considered to be an exponential random variable with a time dependent rate of $\alpha(t)$. Hence, the residual remaining time to arrival (or departure) can be calculated as

$$\Pr\{x_a | s_a\} = e^{-(\Lambda(x_a+s_a)-\Lambda(s_a))} \tag{7}$$

where $x_a \in \mathbb{R}^+$ is the time in the future and $s_a \in \tau$ is the age of the arrival (or departure) transitions (i.e. “the moment that the process was observed”). The benefit of maintaining the local age (i.e. time) is to treat each sampling of the remaining time, individually. Failure of the internal circuity of a PHEV can be modeled with a random variable T_p. In the case of failure, the charging process will stop and the vehicle will leave the station when the driver arrives. On the other hand, failure of the internal circuitry of each charging stations can be modeled by random variables T_i which stops a charger from operating. Therefore, after the vehicle is moved by the driver, the repair process starts. Repair will be finished after a random time of T_r. These processes are shown in Figure 1. Transition $\mathcal{T}_1$ emulates the arrival of a PHEV while $\mathcal{T}_2$ emulates the departure of a vehicle. In this extension of the Petri nets, dynamic token are defined as tokens that can evolve in time. After the arrival of each vehicle, a token will be reduced from $\mathcal{P}_2$ and a token will be added to $\mathcal{P}_1$ to demonstrated the occupancy of a charger. The firing rate of $\mathcal{T}_1$ is based on the process $W_a(t)$. Tokens in place $\mathcal{P}_1$ represent a charger-PHEV combination during normal operation. However, if a charger or a vehicle fails to operate, the corresponding token will change color. In this study, the token will acquire the color (i.e. property) "red" for failure of the charger and "black" for failure of the PHEV. The internal dynamics associated with this change of color can be expressed as

$$S_c^r = S_i^{(k+1)_i} \tag{8}$$

$$S_c^b = S_p^{(k+1)_p} \tag{9}$$

where S_c^r and S_c^b are the calendar time of change of color ($S \in \mathbb{R}^+$ is the arrival time) while $S_i^{k_i}$ and $S_p^{k_p}$ are the calendar times of the k_i-th and k_p-th failures of the chargers and PEHVs, respectively. Hence, the survival function of the token is (the token can acquire both properties simultaneously)

$$S_c^r(t)_{t>S_i^{k_i}} = S_i^{(k+1)_i}(t_i) = e^{-\Lambda_i(t-S_i^{k_i})} \tag{10}$$

$$S_c^b(t)_{t>S_p^{k_p}} = S_p^{(k+1)_p}(t_p) = e^{-\Lambda_p(t-S_p^{k_p})} \tag{11}$$

while $S_i^{(k+1)_i}(t)$ and $S_p^{(k+1)_p}(t)$ are the survival functions of the $(k+1)_i$-th and $(k+1)_p$-th time to next incident of the Poisson process of the chargers and PEHVs, respectively. Therefore, a token can change color based on the survival function $S_c^{r\ or\ b}(t)$. Also, t_i and t_p are the local times of the survival functions of the $(k+1)$-th incidents. In order to clarify this concept, the following example can be studied. Consider a charger that has operated for 5 years since it was repaired and a vehicle that has operated for 3 years since it was purchased. Depending on the "average time to failure" for each of these components, distribution of the time that the charger-PHEV combination will fail, can vary. The history, $\mathcal{F}$, of each process contains the time of the last incident. Hence, the survival function $S_i^{(k+1)_i}(t_i)$ is in fact the distribution of the $(k+1)$-th interoccurrence time of the process $W_i(t)$ which follows the failures of the charger. Similarly, the local time $t_i = t - S_i^{k_i}$. Therefore, by knowing the history $\mathcal{F}$, which is a benefit of time-domain simulation, the survival function of the next incidents are derived.

If a failure occurs, the vehicle will wait at the station until the driver arrives. Therefore, the rate of departure will not change. This is also illustrated in Figure 1. After the departure, if the failure was within the vehicle, the station will go to normal mode of operation. However, if a charger has failed, the station will start the repair procedure. The repair procedure is usually modeled with a log-normal distribution. Hence, the survival function of the token in changing from "red" to "green" is

$$\begin{aligned} S_c^g(t) &= 1 - \Phi((\ln(t - S_d) - \mu)/\sigma) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\mu - \ln(t-S_d))/\sigma} e^{-t^2/2} dt \end{aligned} \tag{12}$$

where S_d is the calendar time of the departure, and $\Phi(.)$ is the standard normal distribution. In the model, after transition $\mathcal{T}_2$ which has the rate equivalent to the departure rate, the vehicle will depart even if its internal charging circuitry has failed. However, if the charging station has failed, the token will go to $\mathcal{P}_3$ until it is repaired (i.e. transition to "green"). Afterwards, it is instantly transfered to $\mathcal{P}_2$ and the charger becomes available for new arrivals of PHEVs.

Meanwhile, when a vehicle arrives, the State-Of-Charge in the battery is a random variable SOC_k with cumulative distribution function of $F_{S_k}(c)$ while the capacity of the battery is a random variable C_k with distribution of F_{C_k}. Beta distribution is a good model for these random variables. Majority of the vehicles who arrive at the charging station have a low SOC. Therefore, by tuning a beta distribution function to the statistical data collected from the charging station, the model can be derived. For each PHEV, the charger will charge the vehicle until the vehicle is full or is departed. In order to complete the model introduced in Figure 1, an index is added to each token to clearly distinguish the charging port. Although the rate of charge varies with time, for simplicity, a fixed rate of 10kW is selected. The complete model is shown in Figure 2. In this model, $\mathcal{P}_5$ and $\mathcal{P}_6$ represent the added charge to the battery of each PHEV. The charge is at a rate of 10kW and is activated through $\mathcal{T}_4$ and $\mathcal{T}_5$. However, these transitions will be active iff the proper token is available in $\mathcal{P}_1$. Tokens "1g" and "2g" correspond to fully operational charger-PHEV at station 1 and 2, respectively. Also, after the departure of each vehicle, transition $\mathcal{T}_7$ will empty the respective $\mathcal{P}_5$ or $\mathcal{P}_6$.

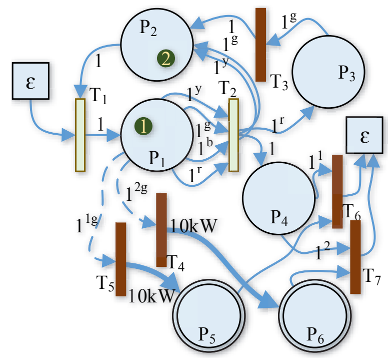

Fig. 2. Petri net model of the charging station.

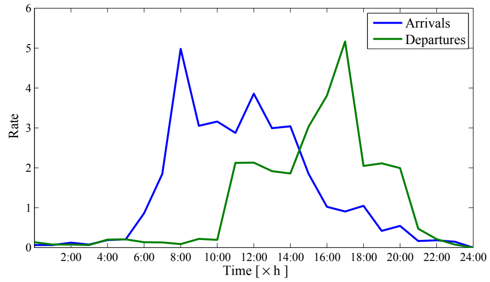

Fig. 3. Average hourly rates of arrivals and departures during the simulation.

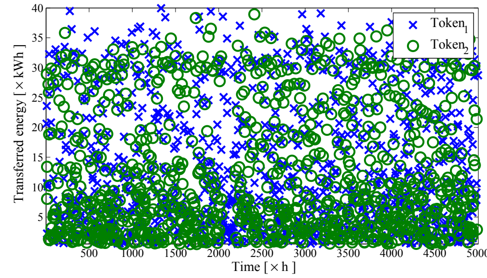

Fig. 6. Total energy transfered to each vehicle at the time of departure.

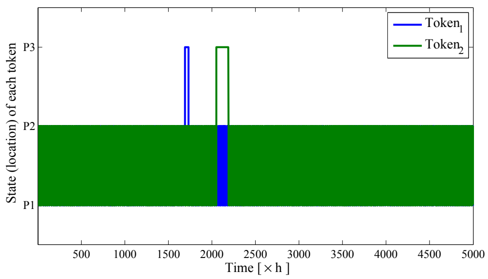

Fig. 4. Location of each token during simulation.

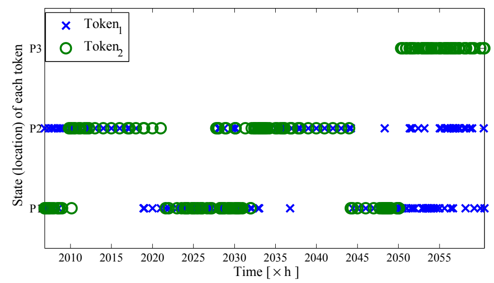

Fig. 7. Location of each token during the period of 2000h to 2060h.

IV. Simulation Results

In order to simulated the Petri net illustrated in Figure 2, hourly rates for arrival and departures of vehicles have been selected. These rates are for a charging station located at a business district. Therefore, majority of arrivals occur during the morning hours while departures are dense in early evening. The model was simulated for half a year (i.e. 5000 hours). Average hourly number of arrivals and departures during this simulation is shown in Figure 3. Furthermore, in order to achieve some failure incidents during the simulation, the failure rate of the PHEVs and chargers are reduced from an average of 50000h to 3000h (otherwise, with a 50000h long simulation period (instead of 5000h), the details will not be visible in a published paper). The repair process is modeled with a log-normal distribution with a mean repair period of 5 days.

The location of each token for non-vanishing places (i.e. $\mathcal{P}_4$) during the simulation is shown in Figure 4. Also, the color of each token is shown in Figure 5. It can be observed that after 1700h charger 1 and after 2000h charger 2 has failed. In addition, in 700h, there was an incident of a PHEV failing at the station 1. Based on Figure 5, it can be observed that majority of the vehicles leave the charging station in a fully charged state. Total energy transfered to each vehicle is shown in Figure 6. Although the average capacity of the vehicles is 30kWh, there is a large number of vehicles leaving after receiving below 10kWh. This shows one of the strengths of numerical simulations of a large stochastic system. Analytical calculation of the statistical data in Figure 6 is a complicated process while these data are very important in designing a charging station and energy planning for these systems.

In order to observe details of the simulation, the time period between 2000h and 2060h is studied in details. Location of each token during this period is shown in Figure 7 and their color is illustrated in Figure 8. Based on Figure 8, the charger 2 has failed to operate at 2034h. It can be seen that there is an arrival at charger 2 at 2044h not knowing the charger is not operating and token 2 is transfered to location $\mathcal{P}_1$. The vehicle has left the charging station after 6 hours at 2050h.

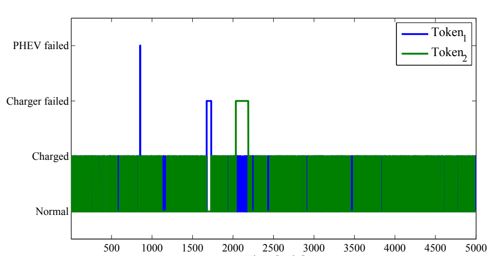

Fig. 5. Color (i.e. property, status) of each token during simulation.

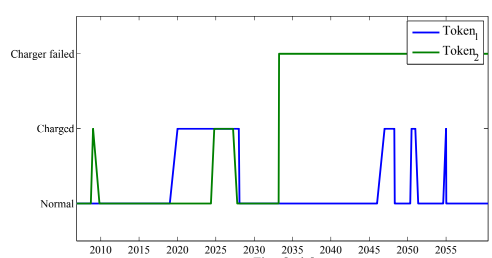

Fig. 8. Color (i.e. property, status) of each token during the period of 2000h to 2060h.

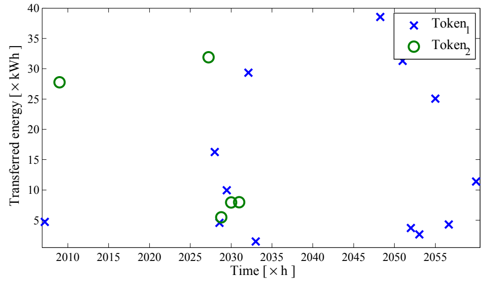

Fig. 9. Total energy transfered to each vehicle at the time of departure.

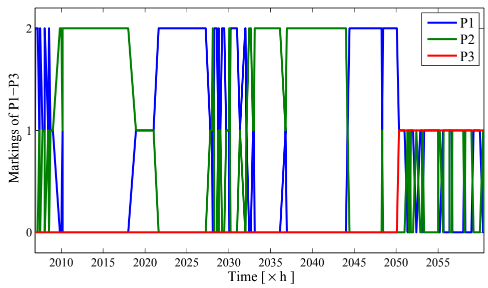

Fig. 10. Markings of non-vanishing discrete places during the period of 2000h to 2060h.

Afterwards, the repair procedure of the charger is started and the location of token 2 is transfered to $\mathcal{P}_3$. Based on Figure 5, the repair process was finished at 2188h (138 hours of repair). Meanwhile, charger 1 was operating normally and has accepted a customer at 2037h, 2044h, 2048h, 2052h, and so on. In fact, the vehicle that has arrived at 2044h was fully charged at 2047h and has left this charger at 2048h. The charge delivered to each vehicle in this period is shown in Figure 9. Therefore, the fully charged car that has left the charger 1 at 2048h has carried 38.5kWh of charge.

Marking of each place is illustrated in Figure 10. The sum of the markings of all non-vanishing places at any time is 2. This is due to the fact that there are only 2 chargers in the system. Furthermore, at 2050h, the second charger has started the repair process which can be seen from this figure.

V. Conclusion

In this paper, applications of Petri nets for analysis of complex stochastic power system was studied. For this purpose, a new extension of hybrid Petri nets with dynamic tokens was introduced. In this extension, each token can have dynamic properties which increases the flexibility of the Petri net while reducing the size of the overall net. In order to explore the modeling process, a plug-in vehicle charging station was modeled and simulated using the proposed Petri net. This model was benefiting from non-Markovian transitions. Each token and transition has a local time relative to the global calendar time. Simulation results from this model were studied and analyzed.

In conclusion, Petri nets are strong frameworks that can be incorporated for designing, energy planning, reliability analysis, and studies of dynamic behaviors in a large power system.

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