东南大学

Matlab 仿真实验报告

课程名称：　　　　　生物系统建模分析

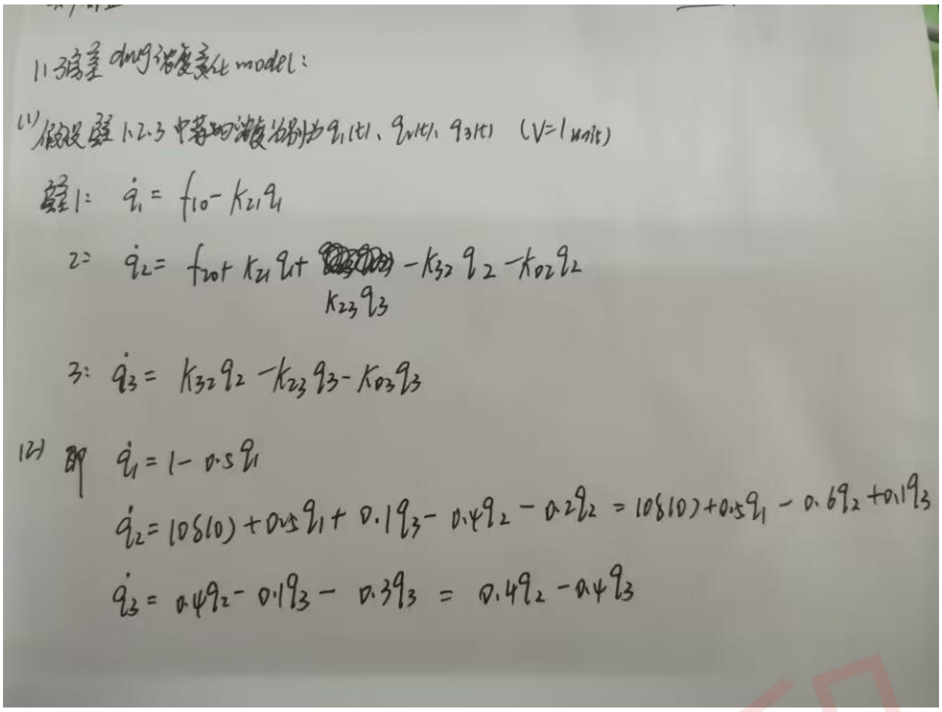
作业周次：　第 6 周

姓　　　名：

学　　　号：

1，3 房室模型：

（1）数学模型：



1) 确立 drug 浓度变化 model:

(1) 假设室 1.2.3 中药物浓度分别为 $q_1(t)$、$q_2(t)$、$q_3(t)$ (V=1 unit)

室1: $\dot{q}_1 = f_{10} - k_{21} q_1$

2: $\dot{q}_2 = f_{20} + k_{21} q_1 + k_{23} q_3 - k_{32} q_2 - k_{02} q_2$

3: $\dot{q}_3 = k_{32} q_2 - k_{23} q_3 - k_{03} q_3$

(2) 即 $\dot{q}_1 = 1 - 0.5 q_1$

$\dot{q}_2 = 10\delta(0) + 0.5 q_1 + 0.1 q_3 - 0.4 q_2 - 0.2 q_2 = 10\delta(0) + 0.5 q_1 - 0.6 q_2 + 0.1 q_3$

$\dot{q}_3 = 0.4 q_2 - 0.1 q_3 - 0.3 q_3 = 0.4 q_2 - 0.4 q_3$

（2）Matlab ode 求解器代码

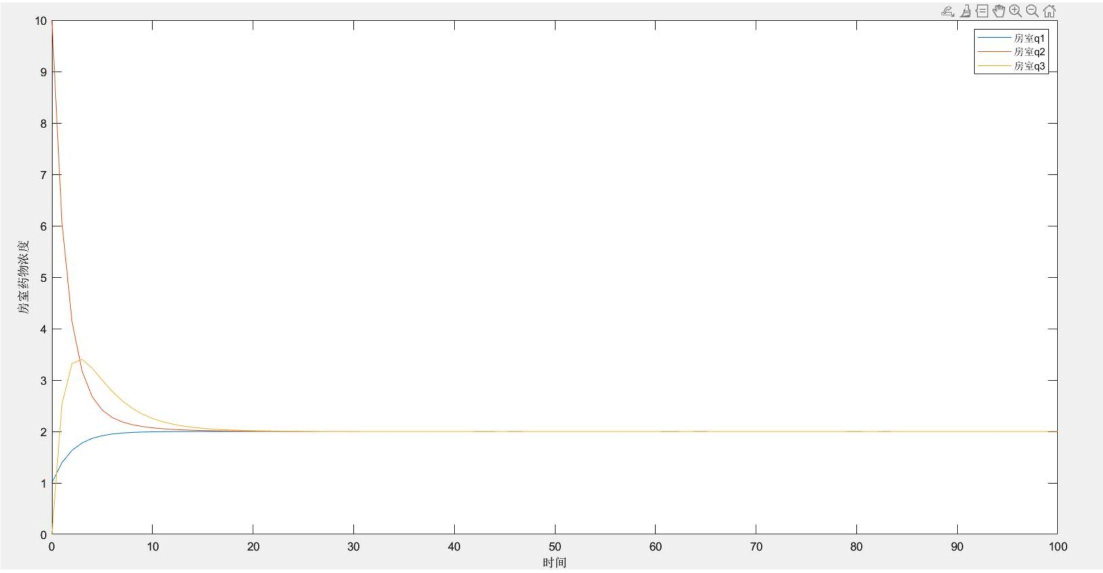
ex1.m

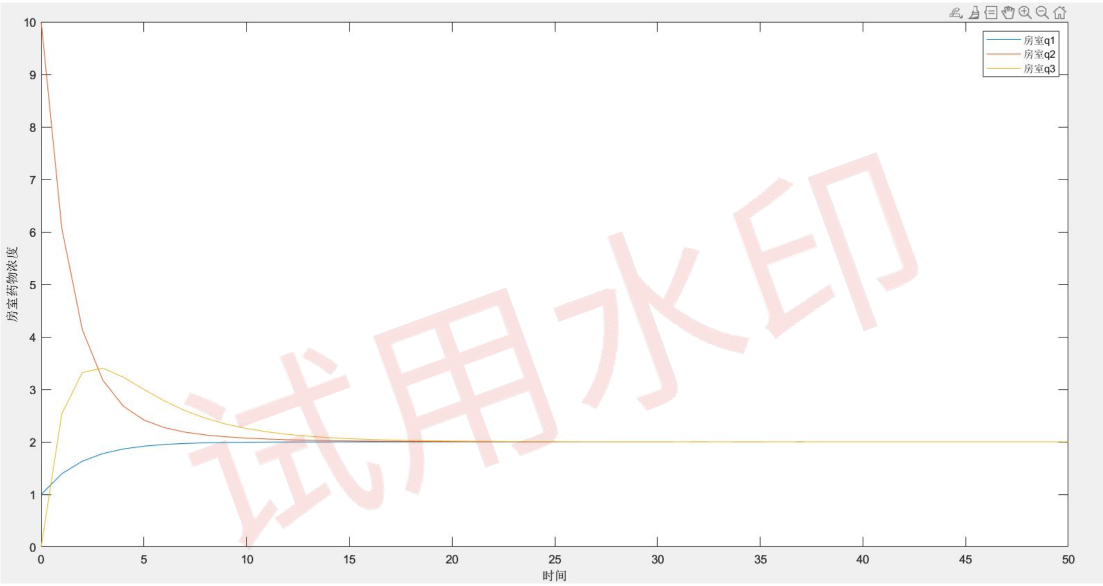
```
f10=1;k21=0.5;k32=0.4;k23=0.1;k02=0.2;k03=0.3;
f=@(t,q)[f10-k21*q(1);k21*q(1)+k23*q(3)-k32*q(2)-k02*q(2);k32*q(2)-k23*q(3)-k03*q(3)];
tspan=0:100;
q0=[1,10,0];  %f20=10*dirac(0)这一块还是体现在初值上算了


[t,q]=ode45(f,tspan,q0);
plot(t,q(:,1),t,q(:,2),t,q(:,3))
legend('房室q1', '房室q2','房室q3');
xlabel('时间');
ylabel('房室药物浓度');
```

此处直接使用 ode45（5 阶 R-K），然后需要注意的地方就是 q0（其实就是 3 个房室的初始药物浓度的地方要注意一下），这里 1+f10 系数一直保持，10 因为是冲激函数所以 f20 就没有保持了

但是在初值里体现了

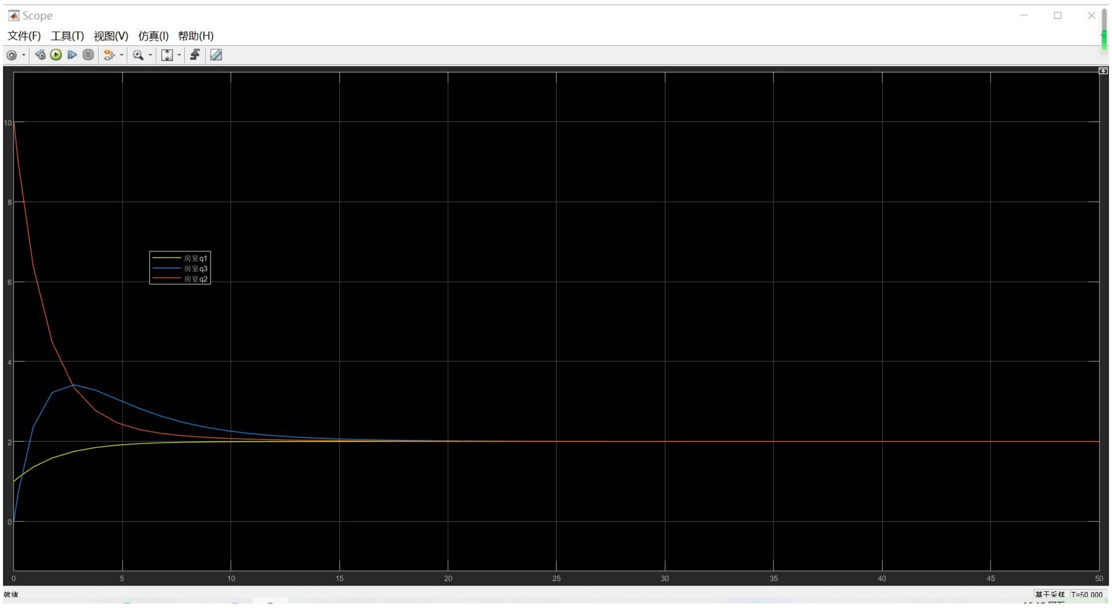
见附件 ex1.m





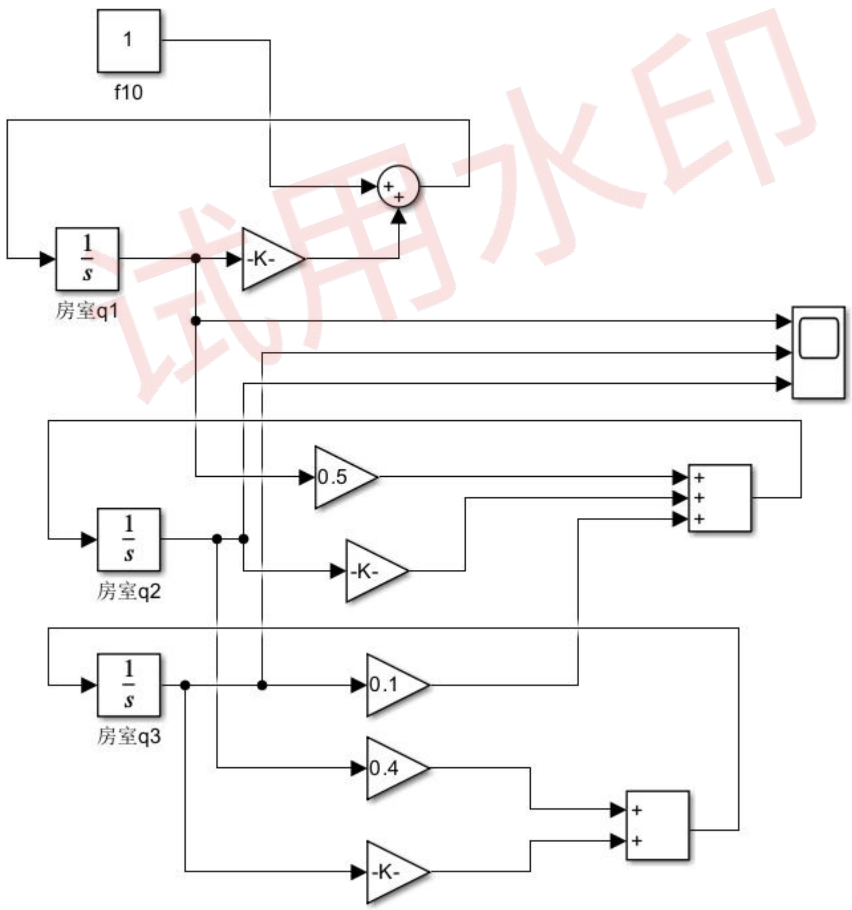
（3）Simulink

没找到 dirac 或者是 impulse 模块，所以处理方法同上 ode45



基本相符

仿真设计如下：见附件 ex01. slx



2,

E4.27 ***Two-Compartment Model Time-Domain Solutions***. Analytically generate time-domain solutions the two-compartment model of Eqs. (4.9), with $u_2 = 0$ and u_1 arbitrary, two ways. First, convert the two ODEs to a single second-order ODE and find the free and forced responses. Second, write the general solution for vector q(t) in terms of the zero-input and zero state (convolution integral) responses (see Chapter 2).

Model Dynamics with Mass State Variables $q_i(t)$

$$\begin{aligned}\dot{q}_1 &= -(k_{01}+k_{21})q_1 + k_{12}q_2 + u_1 \\ \dot{q}_2 &= k_{21}q_1 - (k_{02}+k_{12})q_2 + u_2\end{aligned} \tag{4.9}$$

or

$$\dot{\boldsymbol{q}} = \begin{bmatrix} -(k_{01}+k_{21}) & k_{12} \\ k_{21} & -(k_{02}+k_{12}) \end{bmatrix}\boldsymbol{q} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\boldsymbol{u} \equiv K\boldsymbol{q} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4.10}$$

or, more compactly,

$$\dot{\boldsymbol{q}} = K\boldsymbol{q} + \boldsymbol{u} \tag{4.11}$$

As shown later, the form of (4.11) is completely general, for compartment models with any number of compartments n (K is n by n).

大意是使用两种方法求时域解析解

（1）将两个 ODE 转换为一个二阶 ODE，并求出自由响应和强制响应

q(t)总体解=自由响应+强迫响应

（2）用零输入和零状态(卷积积分)响应写出向量 q(t)的通解

q(t)总体解=零输入响应+零状态响应

该系统方程为 $\begin{cases} q_1' = -(k_{01}+k_{21})q_1 + k_{12}q_2 + u_1 \\ q_2' = k_{21}q_1 - (k_{02}+k_{12})q_2 + u_2 \end{cases}$

令 $u_2=0$，则 $\begin{cases} q_1' = -(k_{01}+k_{21})q_1 + k_{12}q_2 + u_1 & ① \\ q_2' = k_{21}q_1 - (k_{02}+k_{12})q_2 + u_2 & ② \end{cases}$

1) 对方程①两边求导，则 $q_1'' = -(k_{01}+k_{21})q_1' + k_{12}q_2' + u_1'$

代入 q_2' 于②，则 $q_1'' = -(k_{01}+k_{21})\underbrace{q_1'}_{\text{再代回①}} + k_{12}[k_{21}q_1 - (k_{02}+k_{12})q_2 + u_2] + u_1'$，

即 $q_1'' + (k_{01}+k_{21}+k_{02}+k_{12})q_1' + (k_{01}k_{02}+k_{01}k_{12}+k_{21}k_{02})q_1$

$= \cancel{\cdots} - u_1' + (k_{02}+k_{12})u_1$

自由响应：右侧=0 —齐次解
强迫～：右侧保留，解
> 继续得到 $q_1(t)$ 通解，类似 $q_2(t)$ 同理

2)

0-input 响应：$u_1=u_2=0$，则 $\begin{cases} q_1' = -(k_{01}+k_{21})q_1 + k_{12}q_2 \\ q_2' = k_{21}q_1 - (k_{02}+k_{12})q_2 \end{cases}$

求解方法同上：依次求 $q_1(t)$、$q_2(t)$

0-state 响应：

或直接用 LT 求 0-input & 0-state

0-state：假定 $q_1(0)=q_2(0)=0$，则作 LT 变换求出 $Q_1(s)$、$Q_2(s)$

0-input：令 $u_1=u_2=0$，再作 LT 变换求 $Q_1(s)$、$Q_2(s)$

> ILT 再反

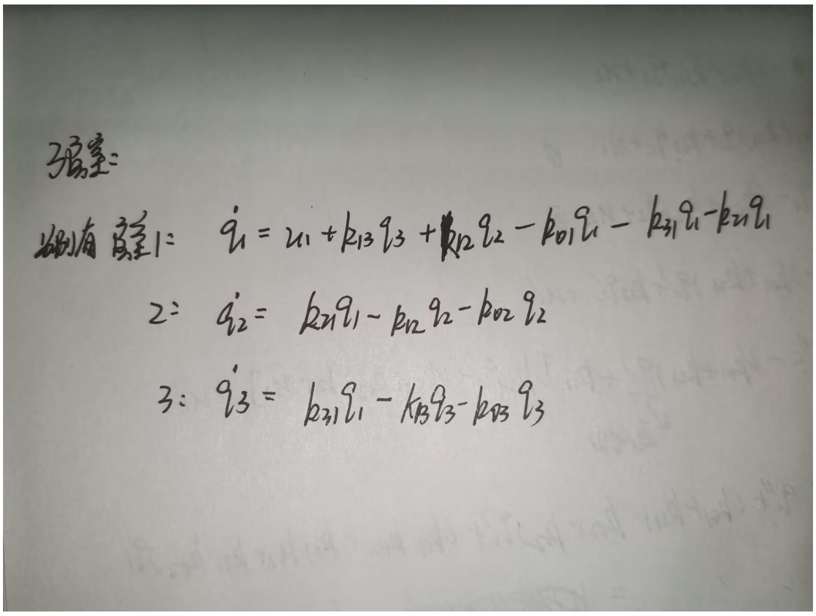
3，哺乳动物 3 房室：

E4.15 ***VisSim or Simulink Experiences***. Implement the three-compartment mammillary model of **Fig. 4.25** in Example 4.11 using a block diagram simulation language, e.g. *VisSim* or *Simulink*. For parameter values, use:

$$V_1 = 20, k_{21} = 1.0, k_{12} = 0.2, k_{31} = 0.1,$$
$$k_{13} = 0.009, k_{01} = 0, k_{02} = 0.02, k_{03} = 0.001$$

a) Run the simulation using the following initial conditions, inputs and simulation algorithm parameters: $q(0) = 0, u_1 = \delta(t)$, over the time interval [0, 600], using all integration algorithms in your package. Use several step-sizes — small to large — for fixed-step methods. Plot all state variables on the same axes, for comparison purposes. Which methods provide consistent results?

b) Repeat (a) for $u_1 = I(t)$, $t \geq 0$ (a unit step), and then again for $u_1 = I(t)$, for $0 < t \leq 20$ and $u_1 = 0$ for $t > 20$ (a unit pulse).



3房室:

则有 房室1: $\dot{q}_1 = u_1 + k_{13}q_3 + k_{12}q_2 - k_{01}q_1 - k_{31}q_1 - k_{21}q_1$

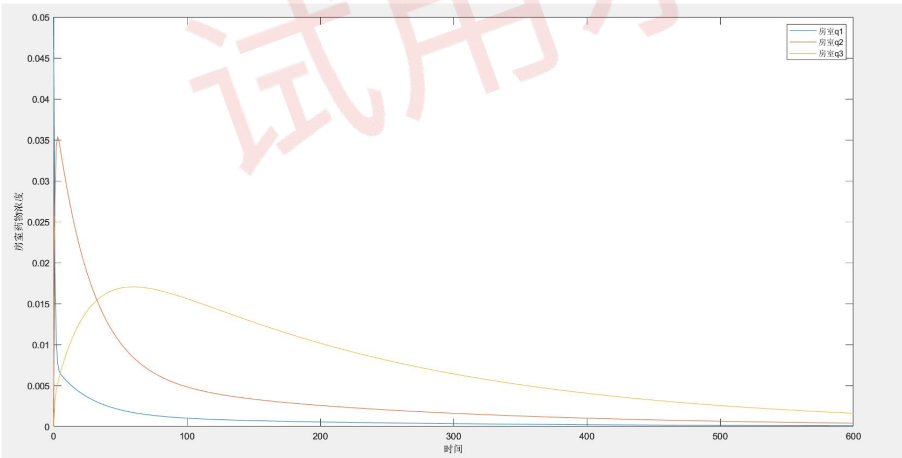
2: $\dot{q}_2 = k_{21}q_1 - k_{12}q_2 - k_{02}q_2$

3: $\dot{q}_3 = k_{31}q_1 - k_{13}q_3 - k_{03}q_3$

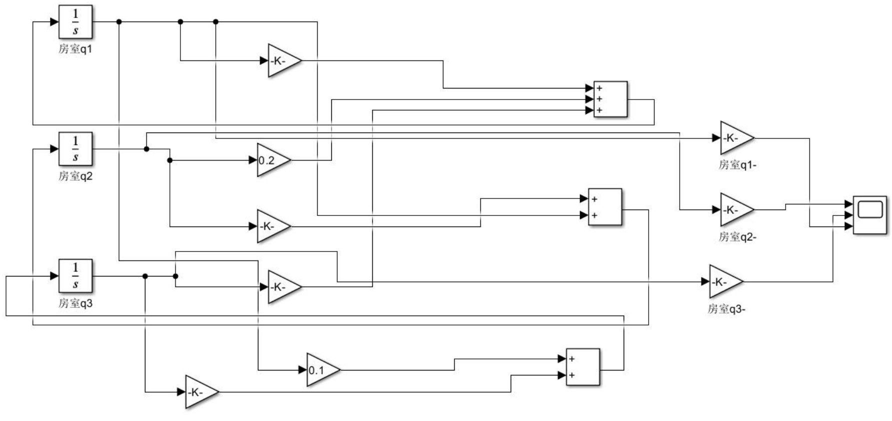
（1）取 u1（t）=dirac（t）情况，默认 3 房室体积一致

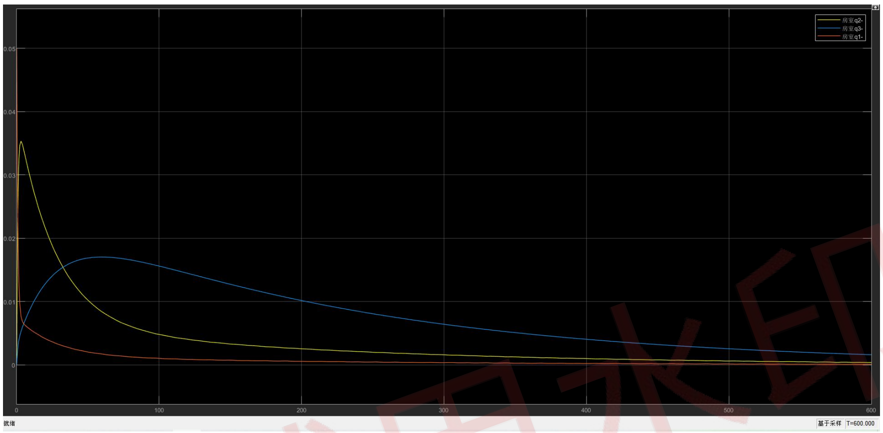
先用 ode45 求解，观察大致分布：见 ex2.m

```
v1=20;k21=1;k12=0.2;k31=0.1;k13=0.009;k01=0;k02=0.02;k03=0.001;
f=@(t,q)[k13*q(3)+k12*q(2)-k01*q(1)-k31*q(1)-k21*q(1);k21*q(1)-k12*q(2)-k02*q(2);k31*q(1)-k13*q(3)-k03*q(3)];
tspan=0:600;
q0=[1,0,0];  %u1=dirac(t)这一块还是体现在初值上算了


[t,q]=ode45(f,tspan,q0);
plot(t,q(:,1)/v1,t,q(:,2)/v1,t,q(:,3)/v1)  %题干中没有提供其他房室的体积。此处默认一致
legend('房室q1', '房室q2','房室q3');
xlabel('时间');
ylabel('房室药物浓度');
```

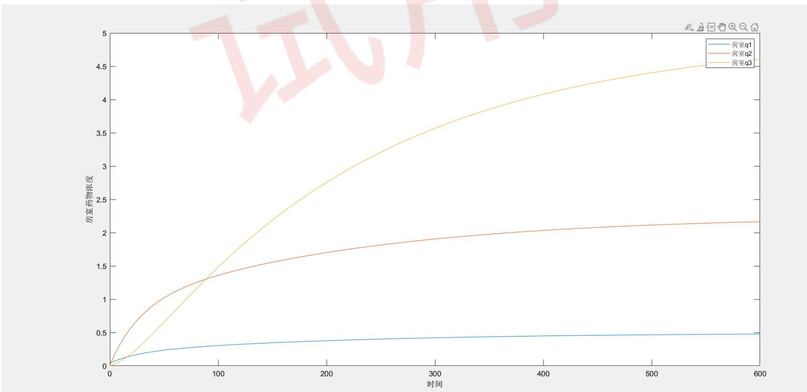



Simulink 仿真:见 ex02.slx



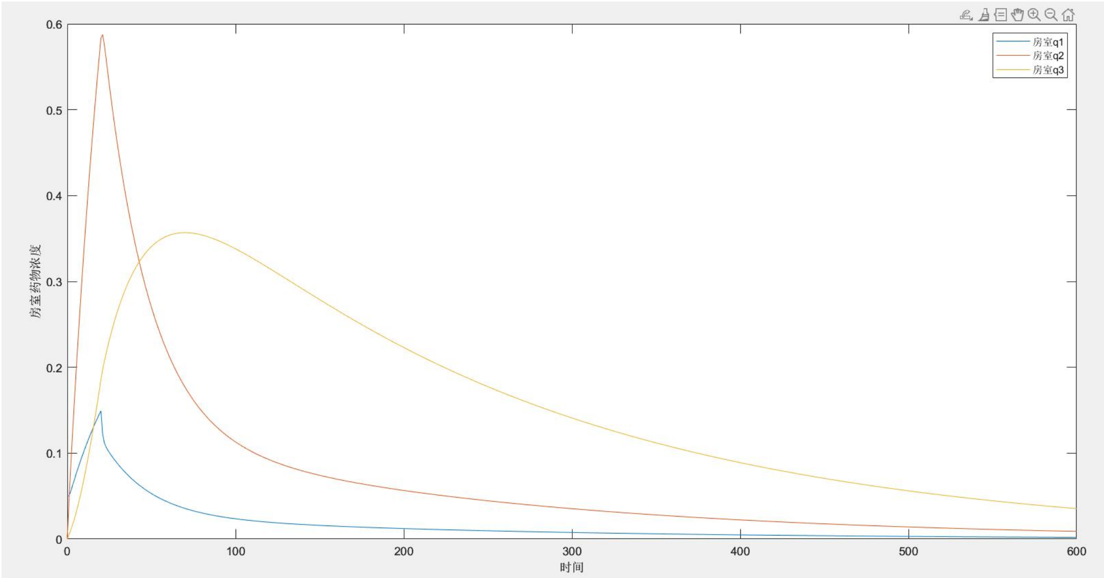


（2）取 u1（t）=相应阶跃函数，一直持续



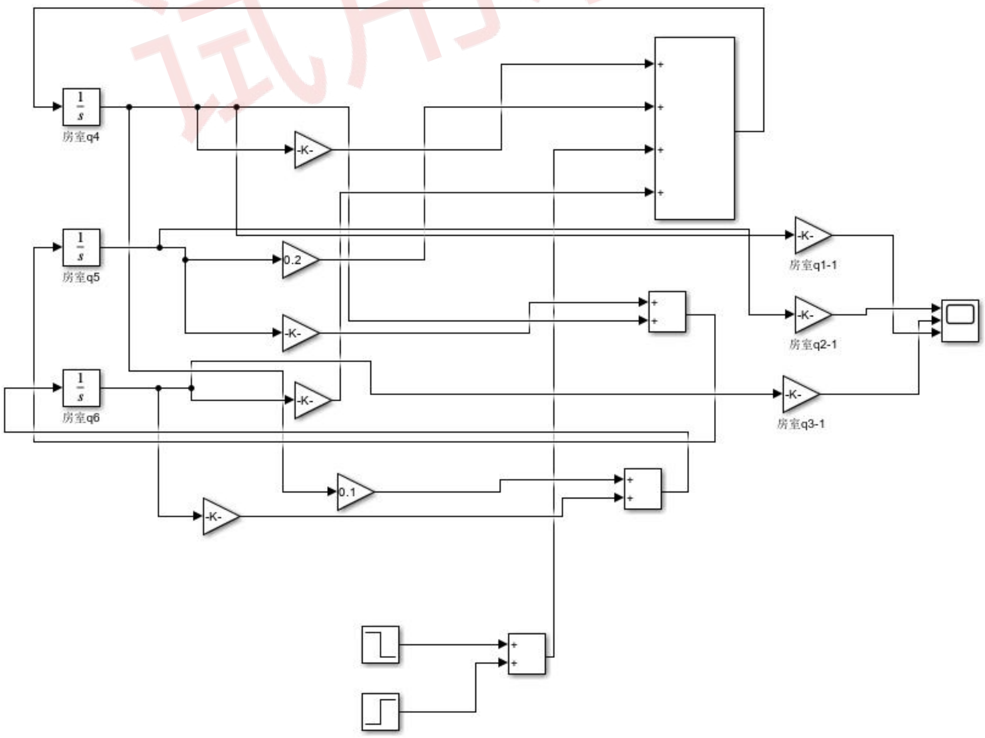

```
%% 阶跃函数注射，且一直保持注射
v1=20;k21=1;k12=0.2;k31=0.1;k13=0.009;k01=0;k02=0.02;k03=0.001;u1=1;
f=@(t,q)[u1+k13*q(3)+k12*q(2)-k01*q(1)-k31*q(1)-k21*q(1);k21*q(1)-k12*q(2)-k02*q(2);k31*q(1)-k13*q(3)-k03*q(3)];
tspan=0:600;
q0=[1,0,0];  %阶跃初值体现

[t,q]=ode45(f,tspan,q0);
plot(t,q(:,1)/v1,t,q(:,2)/v1,t,q(:,3)/v1)  %题干中没有提供其他房室的体积。此处默认一致
legend('房室q1', '房室q2','房室q3');
xlabel('时间');
ylabel('房室药物浓度');
```

（3）阶跃函数注射，但是只在 0-20s 持续

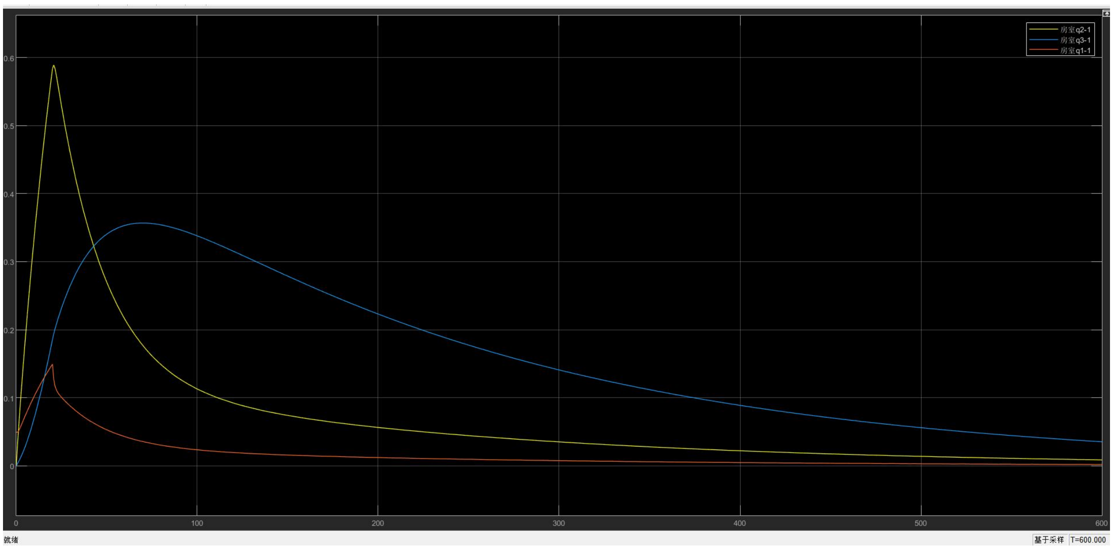



```
%%  阶跃函数注射，仅仅在0-20s内注射
v1=20;k21=1;k12=0.2;k31=0.1;k13=0.009;k01=0;k02=0.02;k03=0.001;
u1 = 0; % 初始时刻开始注射
t_injection = 20; % 阶跃函数注射结束的时间
u1 = 1; % 注射量
f=@(t,q)[(t <= t_injection) * u1+k13*q(3)+k12*q(2)-k01*q(1)-k31*q(1)-k21*q(1);k21*q(1)-k12*q(2)-k02*q(2);k31*q(1)-k13*q(3)-k03*q(3)];
tspan=0:600;
q0=[1,0,0];  %同上阶跃体现初值


[t,q]=ode45(f,tspan,q0);
plot(t,q(:,1)/v1,t,q(:,2)/v1,t,q(:,3)/v1)  %题干中没有提供其他房室的体积。此处默认一致
legend('房室q1', '房室q2','房室q3');
xlabel('时间');
ylabel('房室药物浓度');
```

Simulink 仿真：见 ex02.slx



此处用两个阶跃函数组合来制造短暂阶跃信号


房室q2-1
房室q3-1
房室q1-1
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
就绪
基于采样
T=600.000