Mean of Exponential Distribution :-

$$E(X) = \int_0^\infty x\, f_X(x)\, dx$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = \int_0^\infty \lambda x e^{-\lambda x}\, dx$$

Let $\lambda x = t$

$dx = dt/\lambda$

$$E(X) = \frac{1}{\lambda} \int_0^\infty t e^{-t}\, dt$$

$\Rightarrow$ using gamma function

$$E(X) = \frac{1}{\lambda} \sqrt{2} = \frac{1!}{\lambda}$$

$$\boxed{E(X) = 1/\lambda}$$

Variance of Exponential Distribution

$$Var(X) = E(X^2) - (E(X))^2$$

$$\downarrow$$

$$1/\lambda$$

Now $E(X^2) = \int_0^\infty x^2 \lambda e^{-\lambda x} dx$

$\hookrightarrow \int_0^\infty x^2 f(x) dx$

$\Rightarrow$ Let $\lambda x = t$

$$dx = dt/\lambda$$

$$E(X^2) = \frac{1}{\lambda^2} \int_0^\infty t^2 \lambda e^{-t} \frac{dt}{\lambda}$$

$$= \frac{1}{\lambda^2} \int_0^\infty t^2 e^{-t} dt$$

$$= \frac{1}{\lambda^2} \Gamma 3$$ (using gamma func)

$$= 2/\lambda^2$$

Now $Var(X) = E(X^2) - (E(X))^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} \Rightarrow \boxed{Var(X) = \frac{1}{\lambda^2}}$