6-9-2022

Lect 10

(1)

Binomial Distribution :-

(i) All the trails are indepdent

(ii) Number n of trial is finte

(iii) The Probability p of success is same of each trails.

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

Ex :-

3-Toss

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$2^3 = 8$

$P(2_H) = 3/8$

$p = 1/2 \quad q = 1/2$

$$p(x) = {}^{3}C_{x} (1/2)^{x} (1/2)^{3-x}$$

$$P(2) = {}^{3}C_{2} (1/2)^{2} (1/2)^{1}$$

$$= 3 \times \frac{1}{4} \times \frac{1}{2} = 3/8$$

10- Toss :- & 5 H

$$P(5) = {}^{10}C_{5}\, (1/2)^{5} \cdot (1/2)^{10-5} \Rightarrow$$

Show that $P(x)$ is p.m.f

(i) $P(x) \geq 0$

(ii) $\sum_{x=0}^{n} P(x) = 1$

$$\sum_{x=0}^{n} P(x) = \sum_{x=0}^{n} {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$$= q^{n} + {}^{n}C_{1} \cdot p^{1} \cdot q^{n-1} + \cdots + p^{n}$$

$$= (p+q)^{n}$$

$$= 1^{n} = 1$$

$\boxed{\text{Here } P(x) \text{ is p.m.f}}$

Q: The probability that man aged 60 will live upto 70 is 0.65 out of 10 men, now aged 60 find the probability

(i) at least 7 will live upto 70

(ii) exactly 9 will live upto 70

(iii) At most 9 will live upto 70

Sol^n

$n = 10, \quad p = 0.65, \quad q = 0.35$

(#) $P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= {}^{10}C_{x}\, (0.65)^{x} \cdot (0.35)^{10-x}$

(i) $P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$

$= {}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 \cdot (0.65)^8 (0.35)^2$
$+ {}^{10}C_9 (0.65)^9 (0.35)^1 + {}^{10}C_{10} (0.65)^{10} (0.35)^0$

$= 0.5239$

(ii) $P(9) = {}^{10}C_9 (0.65)^9 (0.35)^1 = 0.0725$

(iii) $P(x \leq 9) = 1 - P(x > 9)$

$= 1 - {}^{10}C_{10} (0.65)^{10} (0.35)^0$

$= 0.9865$

Q:. Out of 800 families with 5 children each, how many families would be expected to have

(i) 3 boys (iii) Either 2 or 3 boys

(ii) 5 Girls (iv) at least 2 Girls.

Sol^n

$N = 800, \quad n = 5 \quad p = 1/2, \quad q = 1/2$

$$P(x) = {}^{n}C_x\, p^x q^{n-x}$$

$$= {}^{5}C_x\, (1/2)^x\, (1/2)^{5-x}$$

$$= {}^{5}C_x \cdot (1/32)$$

(i)

$$P(3) = {}^{5}C_3 \cdot 1/32 = 5/16$$

No. of families $= 800 \times 5/16 = 250$

(ii)

$$P(0) = {}^{5}C_0\, 1/32 = 1/32$$

No. of families $= 800 \times 1/32 = 25$

(iii)

$$P(2) + P(3) = {}^{5}C_2 \cdot 1/32 + {}^{5}C_3 \cdot 1/32$$

$$= \frac{20}{32}$$

No of families $= 800 \times \frac{20}{32} = 500$

(iv)

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$

$$= {}^{5}C_0 \times 1/32 + {}^{5}C_1 \cdot 1/32 + {}^{5}C_2 \cdot 1/32 + {}^{5}C_3\, 1/32$$

$$= 0.5$$

No of family $= 800 \times .5 = 400$

Q # The Probability that a pen manufactured by a Company will be defective is $1/10$. If 12 such pen are manufactured. Find the probability that:

(a) Exactly 2 will be defective
(b) None will be defective
(c) Atleast two will be defective.

Sol^n:

$n = 12$; $p = 1/10$; $q = 1 - 1/10 = 9/10$

$$\boxed{P(x) = {}^{12}C_x \cdot (1/10)^x \cdot (9/10)^{12-x}}$$

(a) $P(2) = {}^{12}C_2 \cdot (1/10)^2 \cdot (9/10)^{10}$

$= 0.2301$

(b) $P(0) = {}^{12}C_0 \cdot (1/10)^0 \cdot (9/10)^{12}$

$= 1\cancel{X} \cdot 1 \times (0.9)^{12} = 0.2824$

(c) $P(x \leq 2) = 1 - [P(0) + P(1)]$

$= 0.3$[illegible]