Lec-11

'Pascal'

⇒ Negative-binomial distribution is applicable when we need to performed an experiment until a total of r success are obtained

⇓
restriction

Note:- if $r=1$, means we perform an experiment till we obtained first success (which is the case of Geometric distribution)

$P(X=x) = P(r=1$ success in first $x-1$ trail AND a success in the x^{th} trail$)$

At x^{th} exp.

r success comp.

1

$r-1$ r success   Bin

$x \longleftrightarrow r$

Ex:-

1
2
3
:
:

4

8

$9^{th} \longleftrightarrow 5$ success

$$P(x) = \left({}^{x-1}C_{r-1} \cdot p^{r-1} \, q^{(x-1)-(r-1)} \right) p$$

$x \Rightarrow$ no. of exp.

$\Rightarrow$

$$\sum_{x=r}^{\infty} P(X=x) = \sum_{x=r}^{\infty} \binom{x-1}{r-1} p^r \cdot q^{x-r}$$

$$= p^r \sum_{x=r}^{\infty} \binom{x-1}{r-1} q^{x-r}$$

Ex 1 If the probability is 0.40 that a child exposed to a certain disease will contain it. What is the probability that the 10th child exposed to the disease will be the 3rd to catch it?

Sol^n

$P(X = 10)$

$$= \left({}^9C_2 \, p^2 q^7 \right) \times p$$

$$= \frac{9 \times 8}{2 \times 1} \cdot (0.4)^3 \, (0.6)^7 = 0.0645$$

1
2
3
.
.
9

9 — 2 child

10 $\Rightarrow$ 3rd child

Ex 2. Let X be the ~~random~~ number of births in a family until the 2^{nd} daughter is born. if the prob. of being a male child is $1/2$. find the probability that the 6^{th} child in the family is the second daughter.

Sol^n

$P(X=6)$ $\quad p = 1/2$

$q = 1/2$

$= ({}^5C_1\, p^1 q^4)\, p$

$= \frac{5 \times \cancel{4}}{\cancel{2 \times 1}} \times \frac{1}{2} \cdot \frac{1}{2}^4 \cdot 1/2$

$= \frac{5}{2^6} \Rightarrow$

$5 \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{cases}$ only 1

$\boxed{6}$ 2^{nd}

Ex 3. In a Company, 5% defective Components are produced. What is the probability that at least 5 Components are to be examined in order to get 3 defective?

Sol^n:

$$P(X \geq 5) = 1 - P(X < 5)$$

$$= 1 - P(X=3) - P(X=4)$$

$$= 1 - {}^2C_2\, p^2 q^0 \times p$$

$$- {}^3C_2\, p^2 \cdot q^1 \times p$$

1

2 $\cancel{X}$

$\boxed{\begin{matrix}3\\4\end{matrix}}$

5

$P(X=3)$ $\left\{\begin{matrix}1\\2\\3\end{matrix}\right.$ $\left.\begin{matrix}1\\2\end{matrix}\right\}$ Bino.

⇒ Find ~~moment Gen. functn of Bino. distr.~~ (5)

Mean & Variance of Bino distr.

$$p(x) = {}^nC_x \, p^x q^{n-x}$$

$$\text{mean} = E(X) = \sum_{x=0}^{n} x \, p(x).$$

$$= \sum_{x=0}^{n} x \cdot {}^nC_x \cdot p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x(x-1)!\,(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n(n-1)! \; p \cdot p^{x-1} \; q^{(n-1)-(x-1)}}{(x-1)! \; ((n-1)-(x-1))!}$$

$$= np \sum_{x=0}^{n} \frac{(n-1)!}{(x-1)! \; ((n-1)-(x-1))!} p^{x-1} q^{(n-1)-(x-1)}$$

$$E(x) = n \cdot p$$

$$\text{Var}(x) = \sigma_x^2 = E(x^2) - (E(x))^2$$

$$= npq$$

Bernaulli's Distribution

1. $X \rightarrow$ Success $= 1$ $\quad P(X=1) = p$

$\quad \hookrightarrow$ failure $= 0$ $\quad P(X=0) = q = 1-p$

② $0 \leq p \leq 1$ $\quad p+q = 1$

" A discrete r.v X is said to have a Bernoulli distribution with parameter p, if its probability mass function is given by "

$$f(x) = p^{x}(1-p)^{1-x} \quad ; \quad x = 0, 1$$

Bernoulli Trail is an experiment with only 2 possible outcomes; $S =$ Success, $F =$ failure $\rightarrow$ b/g; D/L ; c/nc

Bernoulli distribution arises when the follow 3 conditions are satisfied.

1. Each trail of an exp results in an outcome that may be classified as a succes or a failure.

2. The probability of a succes $P(S)=p$ is the same for each trail

3. The trails are independ; that is the outcome of one trail has no effect on the outcome of any other trail.

$E(X)=p$, $Var(X)=pq$; $q=1-p$

Ex1. A 6 sided fair die is tossed, with each element in the sample space having prob. of 1/6. Find the distr. the mean & variance if the value occurs 5

Sol^n $p = 1/6$

$S = \{1, 2, 3, 4, 5, 6\}$

$p = 1/6$, $q = 5/6$

$$P(x) = p^x (1-p)^{1-x}$$

$$= (1/6)^x (1 - 1/6)^{1-x}$$

$$\boxed{P(x) = (1/6)^x (5/6)^{1-x}}$$

mean = $E(x) = p = 1/6$

$\text{Var}(x) = pq = 1/6 \cdot 5/6 = 5/36$

Ex1. 5 white, 3 black ; $\boxed{6 \text{ trails}}$

white ball - succ

(i) ~~Repea~~ replacement $\Rightarrow$ $p = 5/8$, $q = 1 - p = 3/8$

(ii) without replmt

$\Downarrow$

1st $p = 5/8$; $q = 3/8$

2nd $p = \frac{4}{7}$; $q = 3/7$