Discrete Random Variable

Binomial Distribution:

- All the trails are independent
- Number (n) of trails is finite
- The Probability (p) of the success is same of each trials

$$P(x) = C_x^n \, p^x q^{(n-x)}$$

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Example:

a. A coin toss 3-times, find the probability of 2-Heads.

b. A coin toss 10-times, find the probability of 5-Heads.

Binomial Distribution

Q1. The probability that man aged 60 will live up to 70 is 0.65 out of 10 men. Now aged 60, find the probability:

1. At least 7 will live up to 70

2. Exactly 9 will live up to 70

3. At most 9 will live up to 70

Binomial Distribution

Q2. Out of 800 families with 5 children each, how many families would be expected to have

- 3 boys
- 5 girls
- Either 2 or 3 boys
- At least 2 girls

Binomial Distribution

Q3. The Probability that a pen manufactured by a company will be defective is 1/10. If 12 such pen are manufactured. Find the probability that:

1. Exactly 2 will be defective
2. None will be defective
3. At least 2 will be defective

Binomial Distribution

Q4. Medical professionals use the binomial distribution to model the probability that a certain number of patients will experience side effects as a result of taking new medications.

E.g., suppose it is known that 5% of adults who take a certain medication experience negative side effects. We can use a Binomial Distribution Calculator to find the probability that more than a certain number of patients in a random sample of 100 will experience negative side effects.

- $P(X > 5$ patients experience side effects$)$ = **??**
- $P(X > 10$ patients experience side effects$)$ = **??**
- $P(X > 15$ patients experience side effects$)$ = **??**

Binomial Distribution

Q4. Medical professionals

suppose it is known that 5% of adults who take a certain medication experience negative side effects. Find the probability that more than a certain number of patients in a random sample of 100 will experience negative side effects.

- P(X > 5 patients experience side effects) = **0.38400**
- P(X > 10 patients experience side effects) = **0.01147**
- P(X > 15 patients experience side effects) = **0.0004**

$p = 0.05$ $\qquad$ $p(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$q = 0.95$

$n = 100$

$P(X > 5) = 1 - P(X \leq$

$= 1 - [P(0) + P(1) + P(2) + P(3) +$

Binomial Distribution

Q5. Banks use the binomial distribution to model the probability that a certain number of credit card transactions are fraudulent.

E. g., suppose it is known that 2% of all credit card transactions in a certain region are fraudulent. If there are 50 transactions per day in a certain region, we can use a Binomial Distribution Calculator to find the probability that more than a certain number of fraudulent transactions occur in a given day:

- P(X > 1 fraudulent transaction) = ??
- P(X > 2 fraudulent transactions) = **??**
- P(X > 3 fraudulent transactions) = **??**

Binomial Distribution

Q5. Banks use the binomial distribution to model the probability that a certain number of credit card transactions are fraudulent.

E. g., suppose it is known that 2% of all credit card transactions in a certain region are fraudulent. If there are 50 transactions per day in a certain region, we can use a Binomial Distribution Calculator to find the probability that more than a certain number of fraudulent transactions occur in a given day:

- P(X > 1 fraudulent transaction) = **0.26423**
- P(X > 2 fraudulent transactions) = **0.07843**
- P(X > 3 fraudulent transactions) = **0.01776**

$p = 0.02$ $n = 50$

$q = 0.98$

$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$P(X > 1) = 1 - [P(0) + P(1)]$

Negative Binomial Distribution

NBD is applicable when we need to performed an experiment untill a total of r success are obtained

Note: If r = 1, means we perform an experiment till we obtained first success.

Negative Binomial Distribution

Take a standard deck of cards, shuffle them, and choose a card. Replace the card and repeat until you have drawn two Kings.

Y is the number of draws needed to draw two Kings.

As the number of trials isn't fixed (i.e. you stop when you draw the second King), this makes it a negative binomial distribution.

Negative Binomial Distribution

$$P(x) = \left(C_{r-1}^{x-1}\, p^{r-1} q^{(x-1)-(r-1)}\right) p$$

$$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

Negative Binomial Distribution

Q1. If the probability is 0.40 that a child exposed the certain disease will contain it. What is the probability that the 10th child exposed to the disease will be the 3rd to catch ?

$$P(X=10) = \left({}^{9}C_{2} \cdot p^{2} \cdot q^{7}\right) p$$

$$= \frac{\lfloor 9}{\lfloor 7 \lfloor 2} (0.4)^2 (.6)^7 \times 0$$

$$= 0.064$$

Negative Binomial Distribution

Q3. Let x be the number of births in a family until the 2^{nd} daughter is born. If the probability of the having a male child is ½ . Find the probability that the 6^{th} child in the family is the second daughter.

Bernoulli Distribution

A discrete random variable X is said to have a Bernoulli distribution with parameter p. If its probability mass function is given by:

$$P(x) = p^x(1-p)^{1-x} \quad , x = 0, 1$$

Bernoulli distribution

Bernoulli distribution arises when the following 3-conditions are satisfied.

1. Each trail of an experiment results in an outcome that may be classified as a success or failure
2. The probability of a success $P(S) = p$ is the same for each trail.
3. The trails are independent; that is the outcome of one trail have no effect on the outcome of any other trail.

Exercise Problems

Q1. You are surveying people exiting from a polling booth and asking them if they voted independent. The probability (p) that a person voted independent is 20%. What is the probability that 15 people must be asked before you can find 5 people who voted independent?

Exercise Problems

Q3. An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil.

What is the probability that the first strike comes on the third well drilled?

Exercise Problems

Q4. An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil.

What is the probability that the third strike comes on the seventh well drilled?

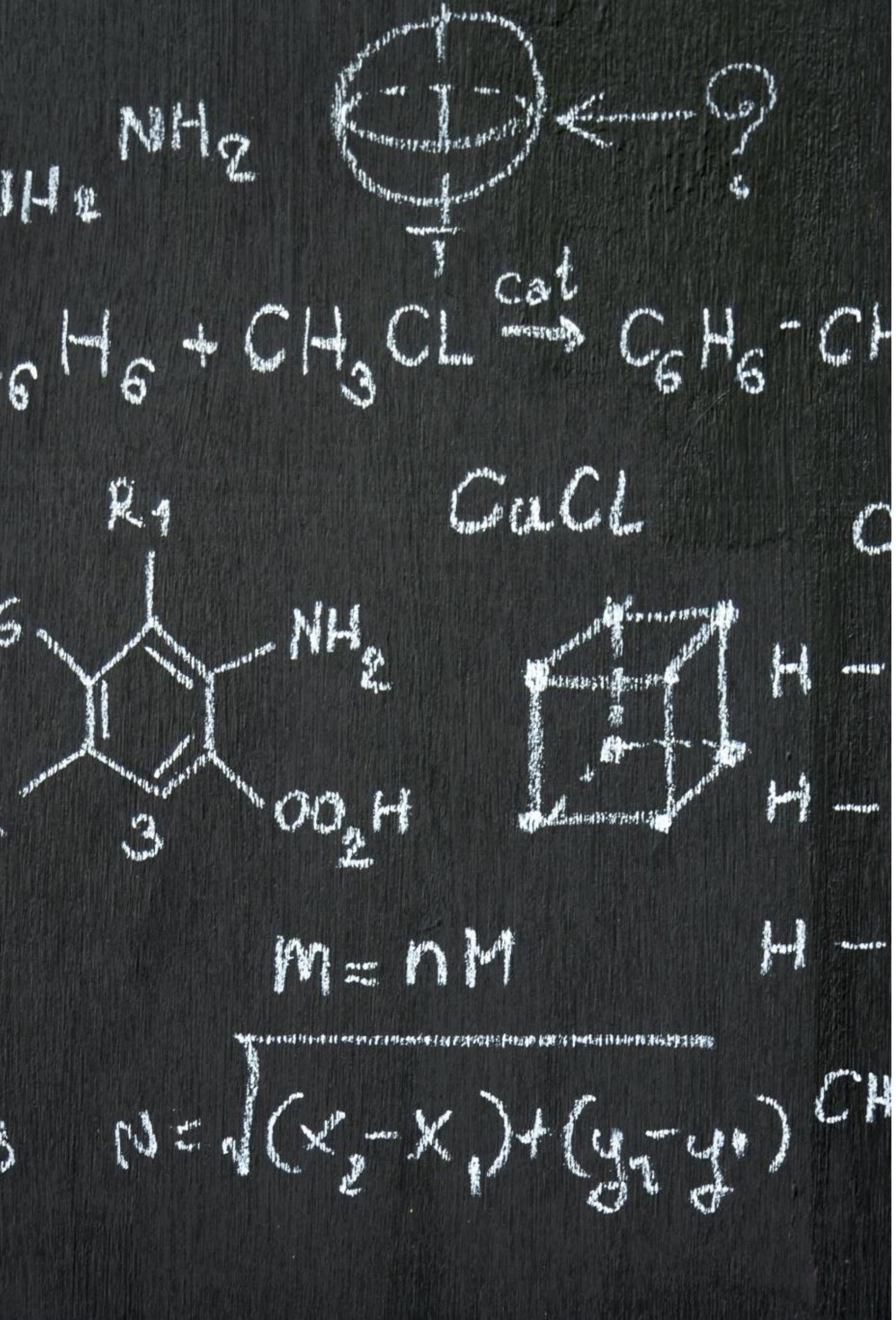
Exercise Problems

Q5. An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil.

What is the mean and variance of the number of wells that must be drilled if the oil company wants to set up three producing wells?

Exercise Problems

Q6. Bob is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70. During the season, what is the probability that Bob makes his third free throw on his fifth shot?



Q6. Solution

This is an example of a negative binomial experiment. The probability of success (P) is 0.70, the number of trials (x) is 5, and the number of successes (r) is 3.

We enter these values into the negative binomial formula.

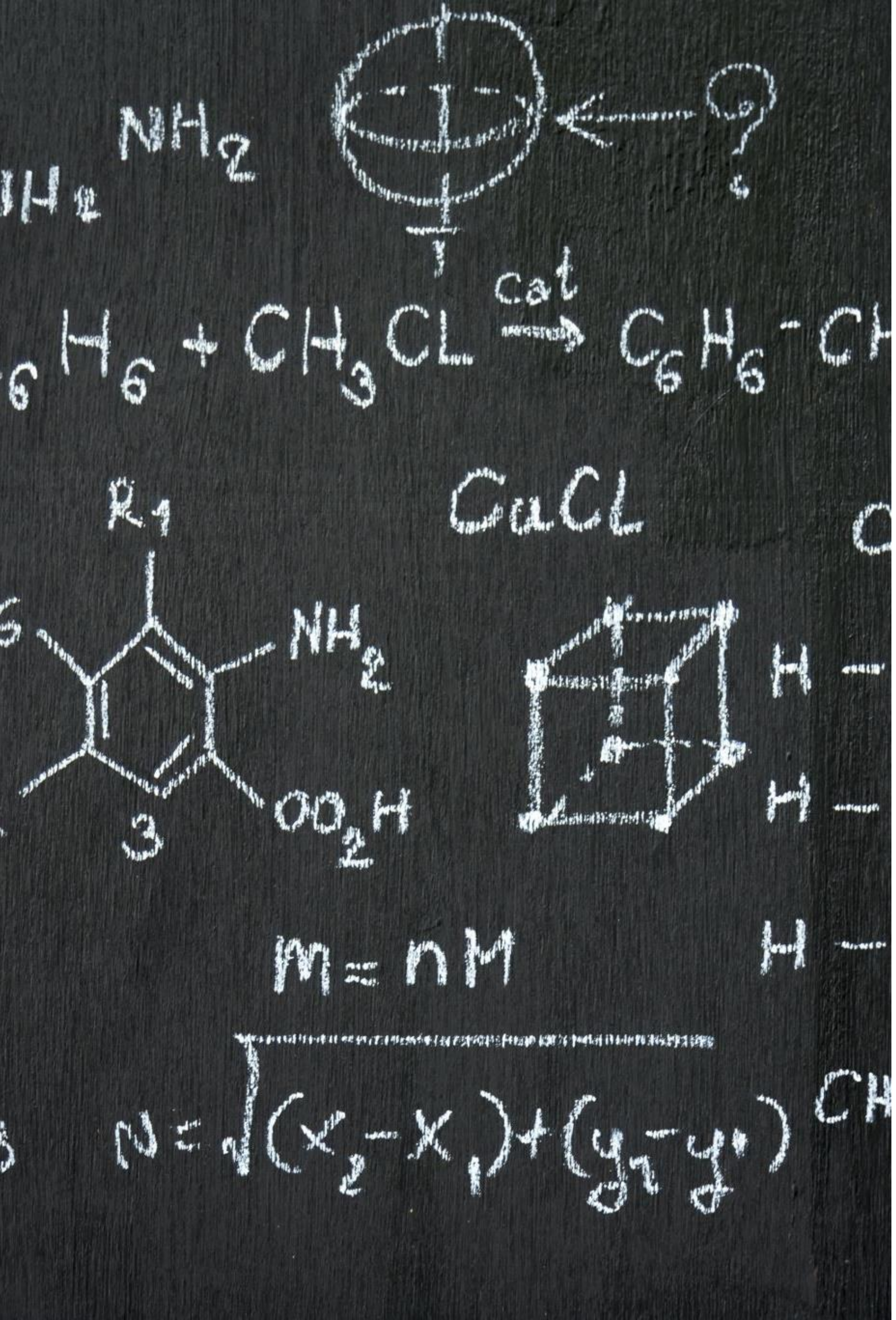
$b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * Q^{x - r}$
$b^*(5; 3, 0.7) = {}_4C_2 * 0.7^3 * 0.3^2$
$b^*(5; 3, 0.7) = 6 * 0.343 * 0.09 = 0.18522$

Thus, the probability that Bob will make his third successful free throw on his fifth shot is 0.18522.

Exercise Problems

Q7. Bob is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70. During the season,

What is the probability that Bob makes his first free throw on his fifth shot?

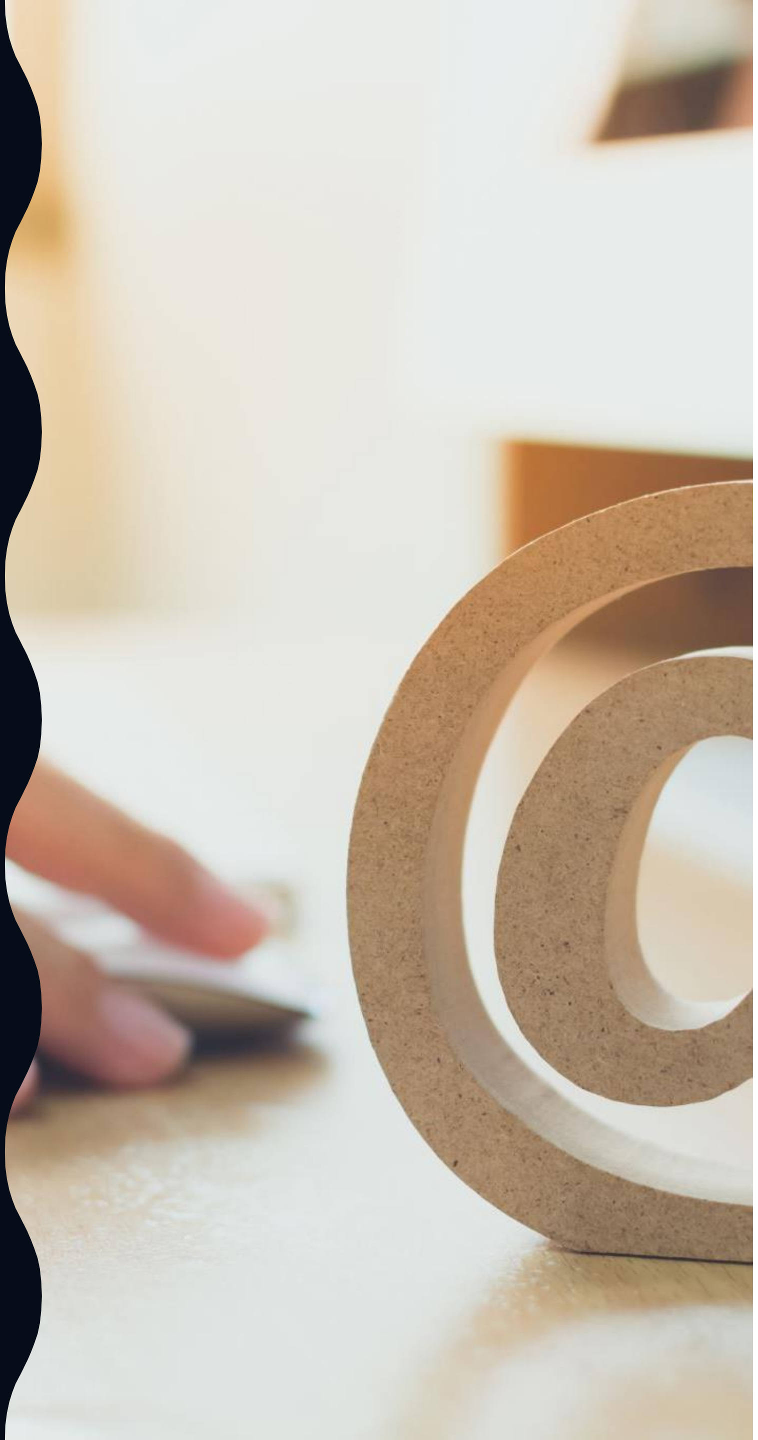


Q7. Solution

- The probability of success (*P*) is 0.70, the number of trials (*x*) is 5, and the number of successes (*r*) is 1. We enter these values into the negative binomial formula.
  - $b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * Q^{x - r}$

    $b^*(5; 1, 0.7) = {}_{4}C_{0} * 0.7^1 * 0.3^4$

    $b^*(5; 3, 0.7) = 0.00567$

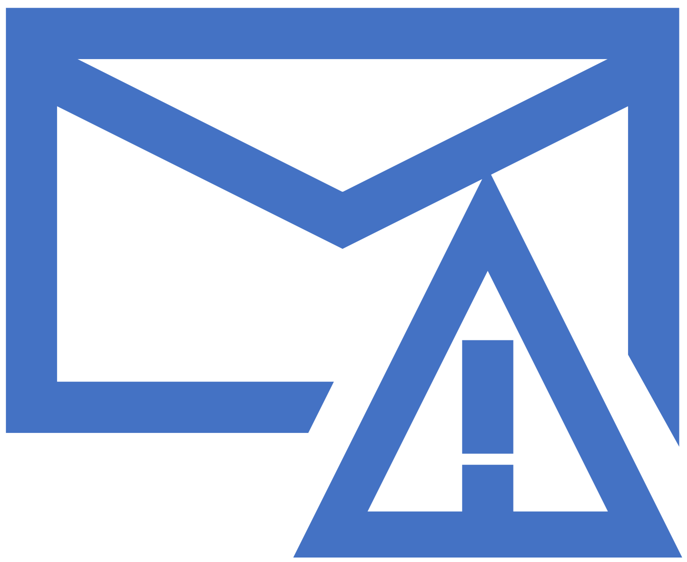


Exercise Problems

Q8. **Number of Spam Emails per Day**

Suppose it is known that 4% of all emails are spam. If an account receives 20 emails in a given day, find the probability that a certain number of those emails are spam:

- P(X = 0 spam emails)
- P(X = 1 spam email)
- P(X = 2 spam emails)



Exercise Problems

Q8. Suppose it is known that 4% of all emails are spam. If an account receives 20 emails in a given day, find the probability that a certain number of those emails are spam:

- P(X = 0 spam emails) = **0.44200**
- P(X = 1 spam email) = **0.36834**
- P(X = 2 spam emails) = **0.14580**

Exercise Problems

Q9. river overflows

suppose it is known that a given river overflows during 5% of all storms. If there are 20 storms in a given year, find the probability that the river overflows a certain number of times:

P(X = 0 overflows)

P(X = 1 overflow)

P(X = 2 overflows)

Exercise Problems

Q9. river overflows

suppose it is known that a given river overflows during 5% of all storms. If there are 20 storms in a given year, find the probability that the river overflows a certain number of times:

- P(X = 0 overflows) = **0.35849**
- P(X = 1 overflow) = **0.37735**
- P(X = 2 overflows) = **0.18868**