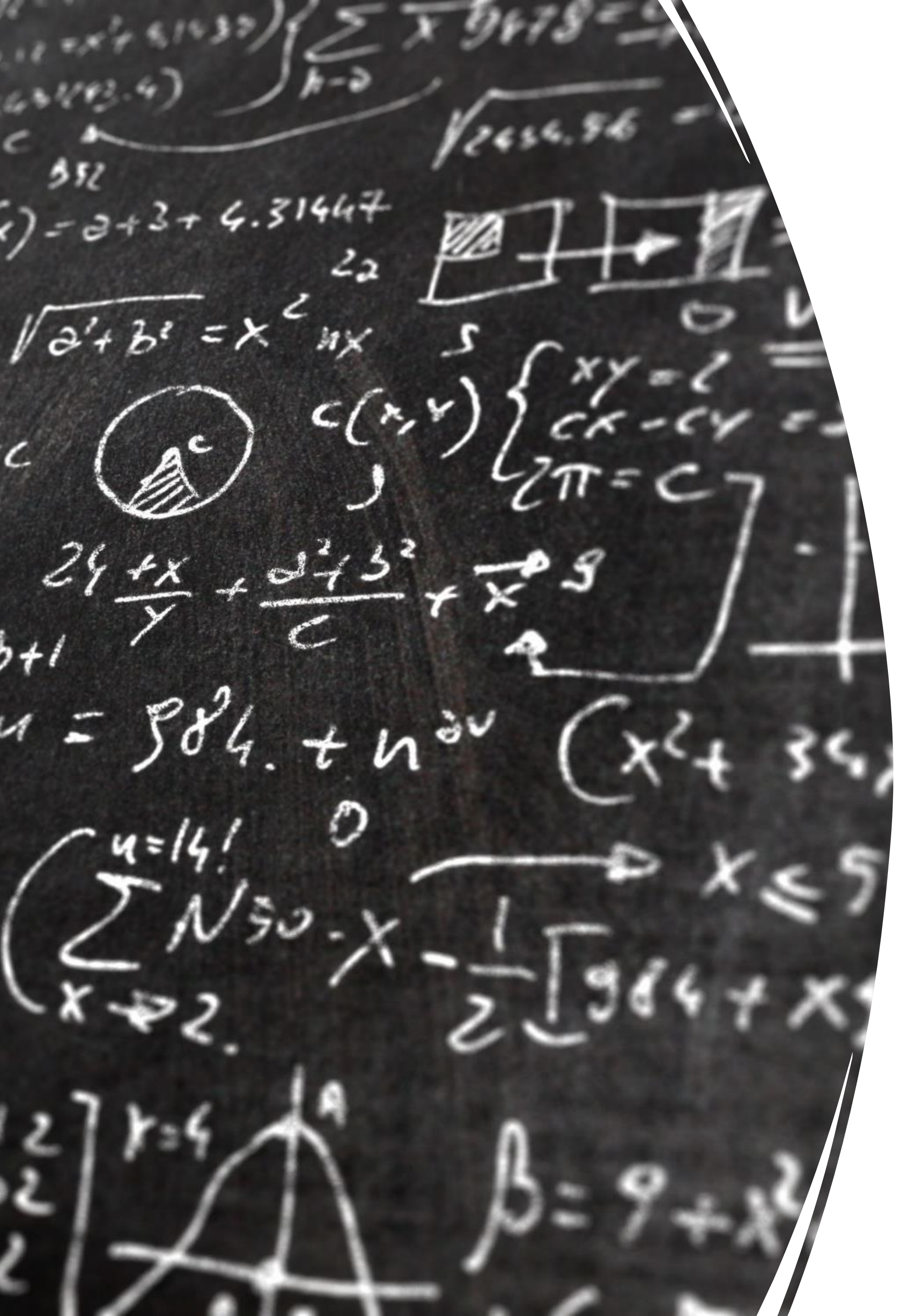


Mathematical Expectation

- Let X be any RV of $\varphi(x)$ be any function of x. Then expectation of $\varphi(x)$ is denoted by $E(x)$ and Defined by:

$$E\big(\varphi(x)\big)$$

- ❑ DRV- $\sum_x \varphi(x)P(x)$
- ❑CRV- $\int_{-\infty}^{\infty} \varphi(x)f(x)dx$

$X(HH) = 2-0=2$

$X(HT) = 1-1=0$

$X(TH) = 1-1=0$

$Var(X) = E(x^2) - (E$

$= 2 -$

Tossing 2 coins

$S = \{HH, HT, TH, TT\}$

X = Number of head – Number of Tail

$X = \{2, 0, 0, -2\} = \{2, 0, -2\}$

$x(TT) = -2$

X	P(X)	X^2
-2	1/4	4
0	2/4	0
2	1/4	4

$4 \times \frac{1}{4}$

$+ 0 \times$

$+ 4$

$E(\phi(X)) = \sum_x \phi(x) \cdot P(x)$

Solution

- $\mu = \sum X\, P(X) = -2 \ * \frac{1}{4} + 0 \ * \frac{2}{4} + 2 \ * \frac{1}{4} = 0$

- $E(X) = 0$

- $E(X^2) = 4 \ * \frac{1}{4} + 0 * \frac{2}{4} + 4 * \frac{1}{4} = 2$

- $Var(x) = E(X^2) - \left(E(X)\right)^2 = 2 - 0 = 2$

Mathematical Expectation

- ❑ DRV- $\sum_{x} \varphi(x)P(x)$
- ❑ CVR- $\int_{-\infty}^{\infty} \varphi(x)f(x)dx$
- ❖ If $\varphi(x) = x$
- ❑ DRV- $\sum_{x} xP(x)$
- ❑ CVR- $\int_{-\infty}^{\infty} xf(x)dx$

Mathematical Expectation

$\bar{x} = E(x)$

Variance-

- $E(x - \bar{x})^2 = E(x^2 - 2x\,\bar{x} + \bar{x}^2)$

$$= E(x^2) - 2\bar{x}\,E(x) + \bar{x}^2$$

$$= E(x^2) - 2\bar{x}\bar{x} + \bar{x}^2$$

$$= E(x^2) - \bar{x}^2$$

$$= E(x^2) - \left(E(x)\right)^2$$

Example-1

Q. Find mean and variance of the probability distribution, given by the following table

x	1	2	3	4	5
P(x)	0.2	0.35	0.25	0.15	0.05

$E(x) = \sum_x x\,p(x) = \ldots$

$= 1 \times 0.2 + 2 \times .35 + 3 \times \ldots$

$+ \ldots$

$Var(x) = E(x^2) - (E(x))^2 \Rightarrow 7.5 - (2\ldots$

$E(x^2) = 1 \times 0.2 + 4 \times .35 + 9 \times .25 + 1\ldots$

$+ 2\ldots$

Solution

- Discuss in class

Solution-1

x	1	2	3	4	5
P(x)	0.2	0.35	0.25	0.15	0.05

- $E(x) = \sum xp(x) = 2.5$
- $E(x^2) = \sum x^2 p(x) = 7.5$
- $Var(x) = E(x^2) - \left(E(x)\right)^2 = 7.5 - 2.5 * 2.5 = 1.25$

Example-2

13 cards are drawn simultaneously from a pack of 52 cards. If Ace count 1 and all face count 10, and other according to their denomination.

Find the expectation of total score in 13 cards

$$E(X) = \sum_x x\,p(x)$$

$$= \frac{1}{13}(1+2+3$$

	A	2	3	4	5	6	7	8	9	10	J	Q	K
x	1	2	3	4	5	6	7	8	9	10	10	10	10
$P(x)$	1/13	1/13	1/13								1/13	1/13	1/13

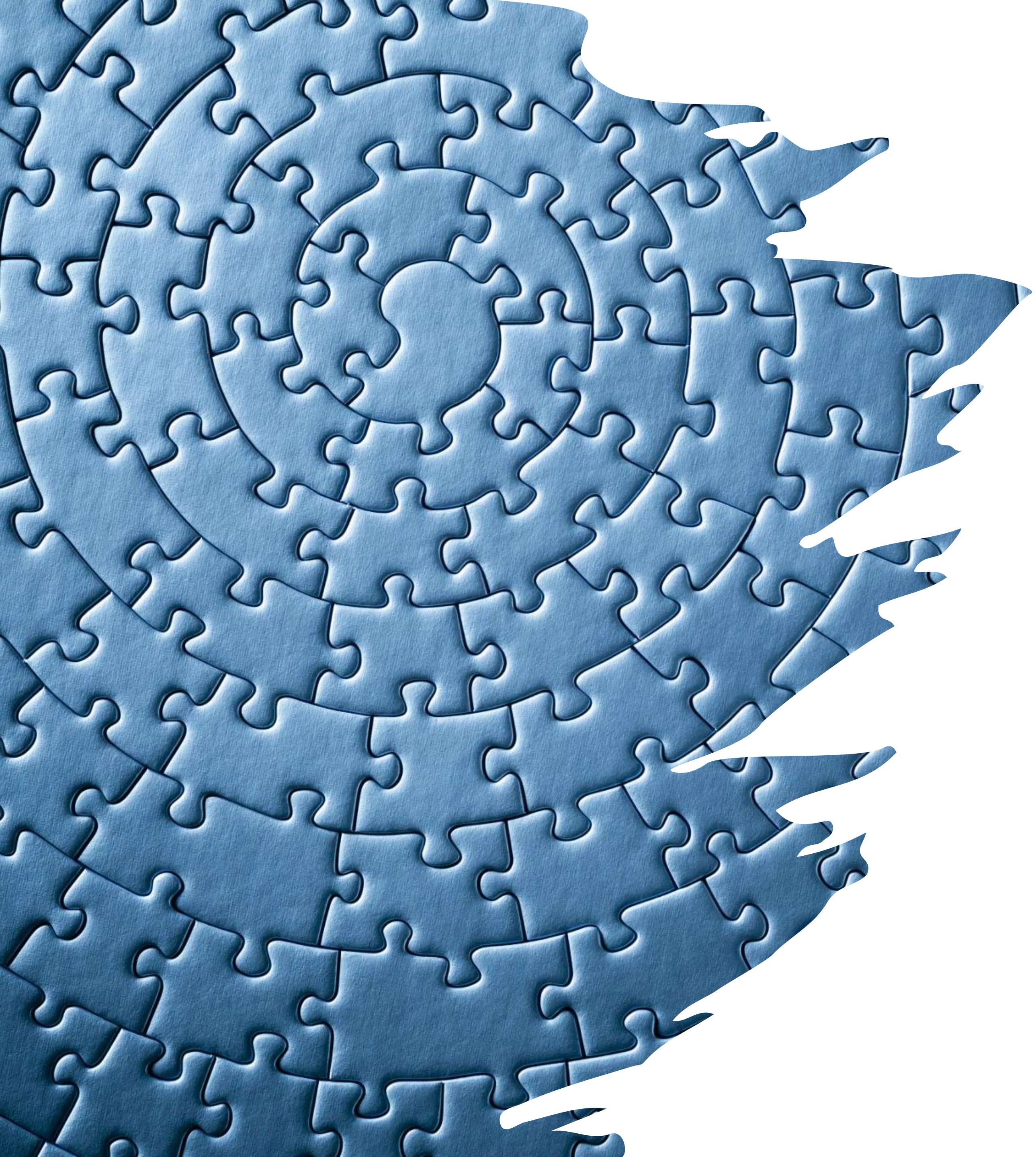
- Discuss in class

Solution-2

- Ace-> 1
- 2 to 10 -> 2 to 10
- Face -> J, Q, K -> 10, 10, 10

X	1	2	3	4	5	6	7	8	9	10	10	10	10
P(x)	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13

- $E(x) = \frac{1}{13}(1 + 2 + 3 \ldots \ldots . +10 + 10 + 10 + 10) = 85/13$



Example-3

Q. A CRV x has density function given by:

$$f(x) = \begin{cases} 2\,e^{-2x} & x > 0 \\ 0 & other\ wise \end{cases}$$

Find expected value and variance of x.

- Discuss in class

Solution-3

•

Qus:- A cummulative RV has density fun^c given by

$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

find expectation value and variance of x.

Sol. $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{0}^{\infty} x f(x)\, dx$

$$= 2\int_0^{\infty} x e^{-2x}\, dx$$

Gamma function

$$\int_0^{\infty} x^{n-1} e^{-ax}\, dx = \frac{\Gamma n}{a^n}$$

and $\Gamma n = (n-1)!$

$$E(x) = \frac{2\,\Gamma 2}{2^2} = \frac{2\times(1)!}{4}$$

$$\boxed{E(x) = 1/2}$$

now $E(x^2) = \int_0^{\infty} x^2 f(x)\, dx$

Solution-3

•

$$E(x^2) = \int_0^\infty x^2 f(x)\,dx$$

$$= 2\int_0^\infty x^2 e^{-2x}\,dx$$

Again, gamma function

$$E(x^2) = 2\,\frac{\Gamma 3}{2^3} = \frac{2\,(2)!}{2^3}$$

$$= \frac{2\times 2\times 1}{8}$$

$$\boxed{E(x^2) = 1/2}$$

$$var(x) = \sigma^2 x = E(x^2) - (E(x))^2$$

$$= 1/2 - (1/2)^2$$

$$= 1/2 - 1/4$$

$$\boxed{var(x) = 1/4}$$

Example: 4

3-Cards are drawn successfully with replacement from a well shuffled pack of 52-cards. Find the probability of number of spades. Here find the mean of the distribution.

- X = number of spades

Solution-4

Discuss in class

Solution-4

3 Cards are drawn successfully with replacement from a well shuffled pack of 52 cards. Find the probability of number of spades. Here find the mean of the distribution.

- X = number of spades

X	P(X)
0	$(\frac{39}{52})^3 = 27/64$
1	$(\frac{13}{52})(\frac{39}{52})^2 = 9/64$
2	$(\frac{13}{52})^2(\frac{39}{52})^1 = 3/64$
3	$(\frac{13}{52})^3 = 1/64$

Solution-4

- $P(Getting\ 1\ spades) = \frac{13}{52} = \frac{1}{4}$
- $P(Not\ Getting\ 1\ spades) = 1\ -\ \frac{13}{52} = \frac{3}{4}\ (or\ we\ can\ write\ \frac{39}{52})$

- $\mu = 0\ * \frac{27}{64} + 1\ * \frac{9}{64} + 2 * \frac{3}{64} + 3 * \frac{1}{64}$
- $\mu = \frac{18}{64}$

Example-5

Two Numbers are selected at random without replacement from positive integers {2, 3, 4, 5, 6, 7}

- X: larger of 2 numbers obtained
- Find means and Variance of X

Solution-5

Discuss in class

Solution-5

-

X	P(X)	X^2
3	$\frac{1}{c_2^6} = \frac{1}{15}$	9
4	$\frac{2}{c_2^6} = \frac{2}{15}$	16
5	$\frac{3}{c_2^6} = \frac{3}{15}$	25
6	$\frac{4}{c_2^6} = \frac{4}{15}$	36
7	$\frac{5}{c_2^6} = \frac{5}{15}$	49

- $E(x) = 17/3$
- $Var(x) = 14/9$

Example-6

- In a game , a man wins 5 rupees for getting a number greater than 4 and loss rupees 1 otherwise, when a fair die is thrown.

- The man decide to throw a die thrice but to quit as and when he wins. Find the expected value of the amount he win/loss.

Solution-6

Discuss in class

Solution-6

X	P(X)
5 {case W}	1/3
4 {case LW}	2/3 * 1/3 = 2/9
3 {case LLW}	2/3 * 2/3 * 1/3 = 4/27
-3 {case LLL}	2/3 * 2/3 * 2/3 = 8/27

X: Amount won/loss

P(w) = 2/6 = 1/3 {getting 5 or 6}

P(L) = 4/6=2/3 {Getting 1, 2, 3, 4}

Through die thrice-> quit if get win: {W, LW, LLW, LLL}

$$E(x) = \sum xp(x) =$$