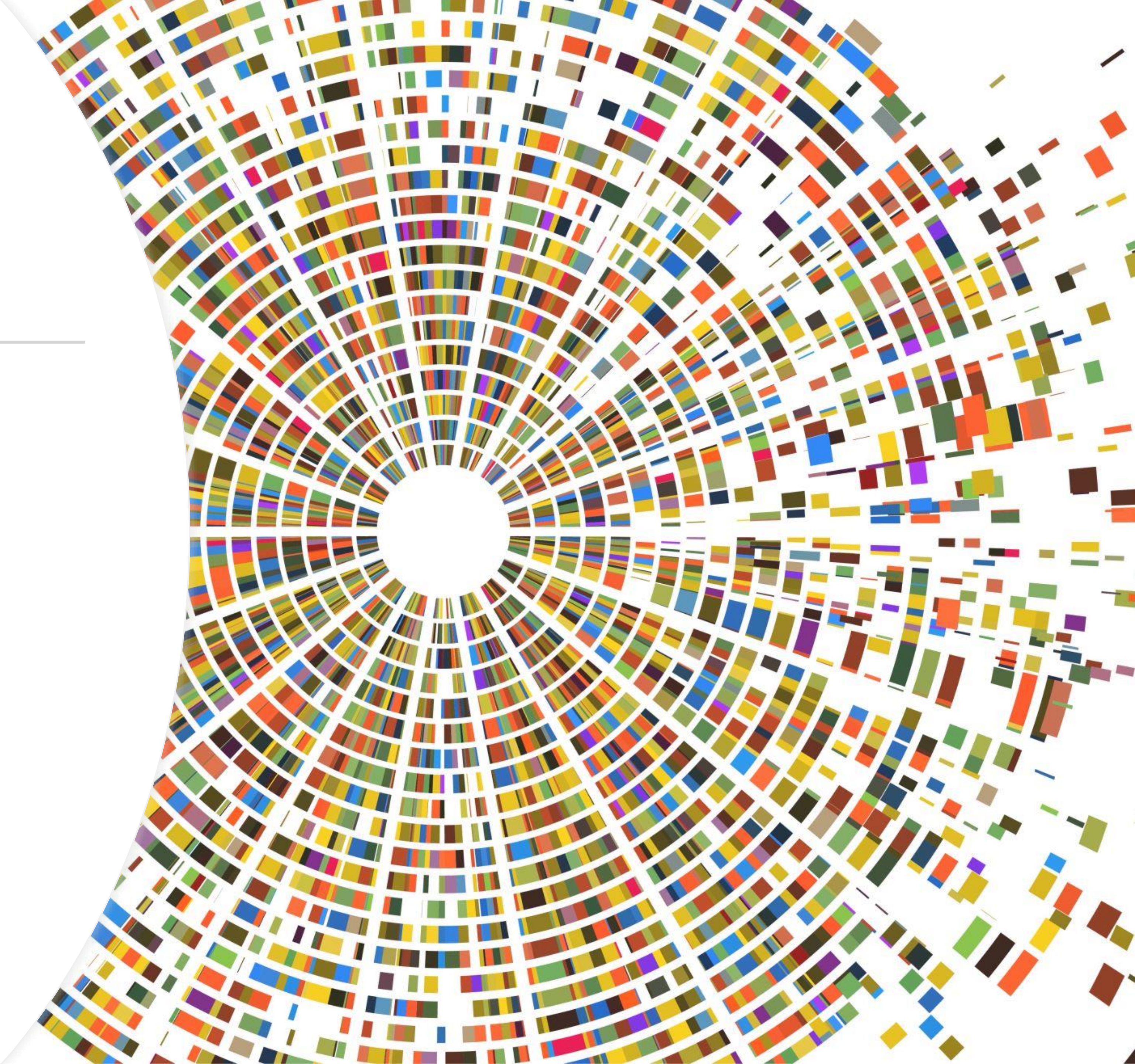
Example-1

In a city with one hundred taxis, 1 is blue and 99 are green. A witness observes a hit-and-run by a taxi at night and recalls that the taxi was blue, so the police arrest the blue taxi driver who was on duty that night. The driver proclaims his innocence and hires you to defend him in court. You hire a scientist to test the witness' ability to distinguish blue and green taxis under conditions similar to the night of accident. The data suggests that the witness sees blue cars as blue 99% of the time and green cars as blue 2% of the time.

Write a speech for the jury to give them reasonable doubt about your client's guilt. Your speech need not be longer than the statement of this question. Keep in mind that most jurors have not taken this course, so an illustrative table may be easier for them to understand than fancy formulas.

Random Variable

- Discrete Random Variable
- Continuous Random Variable



Random Variable

- Discrete Random Variable:

❑ Tossing coin:

A fair coin toss 2-times: S = {HH, HT, TH, TT}

X (no. of head)	0	1	2
P(x)	1/4	2/4	1/4

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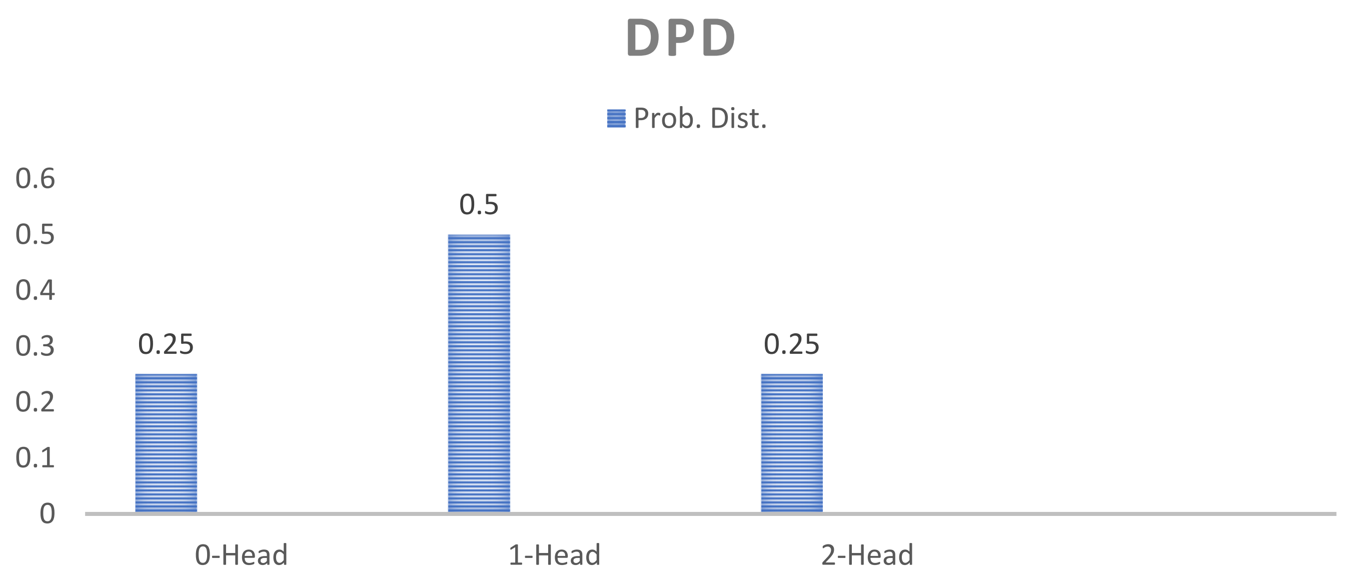
Probability Distribution			

Random Variable



X (no. of head)	**0**	**1**	**2**
P(x)	1/4	2/4	1/4
Probability Distribution			

Random Variable

- Continuous Random Variable
  - ❑ Very Large Sample Space
    - No. of students in university (E.g., 10K). Find the % of students whose weight between 50 kg to 60 kg.
    - Price of plot in city

Random Variable

RV is a real valued function which assign a real number to each sample points in the sample space.

- Tossing a fair coin 3-times then sample space:

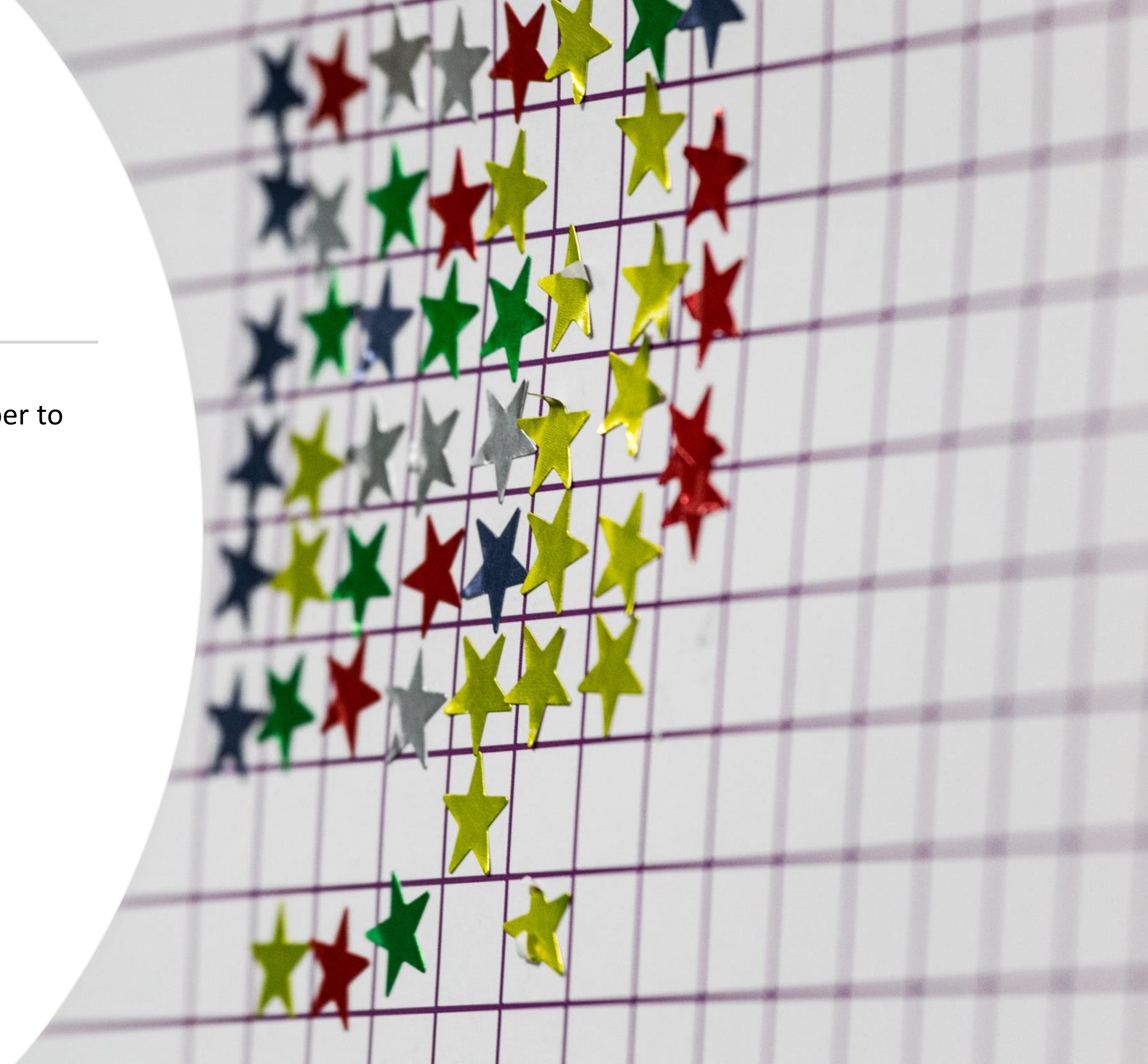
  S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

  X(S1) = 3;

  X(S2) = X(S3) = X(S4) = 2

  X(S5) = X(S6) = X(S7) = 1

  X(S8) = 0



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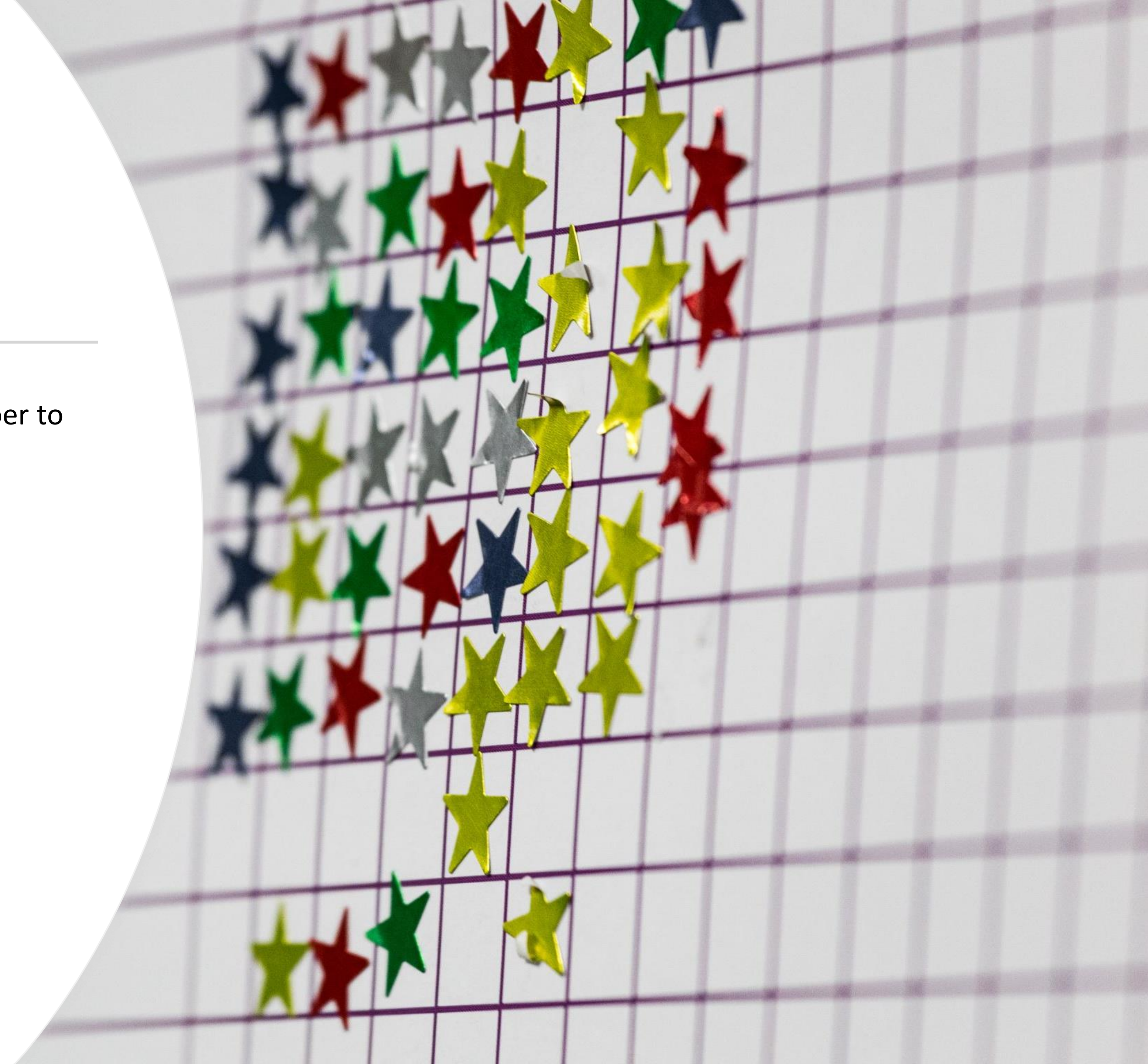
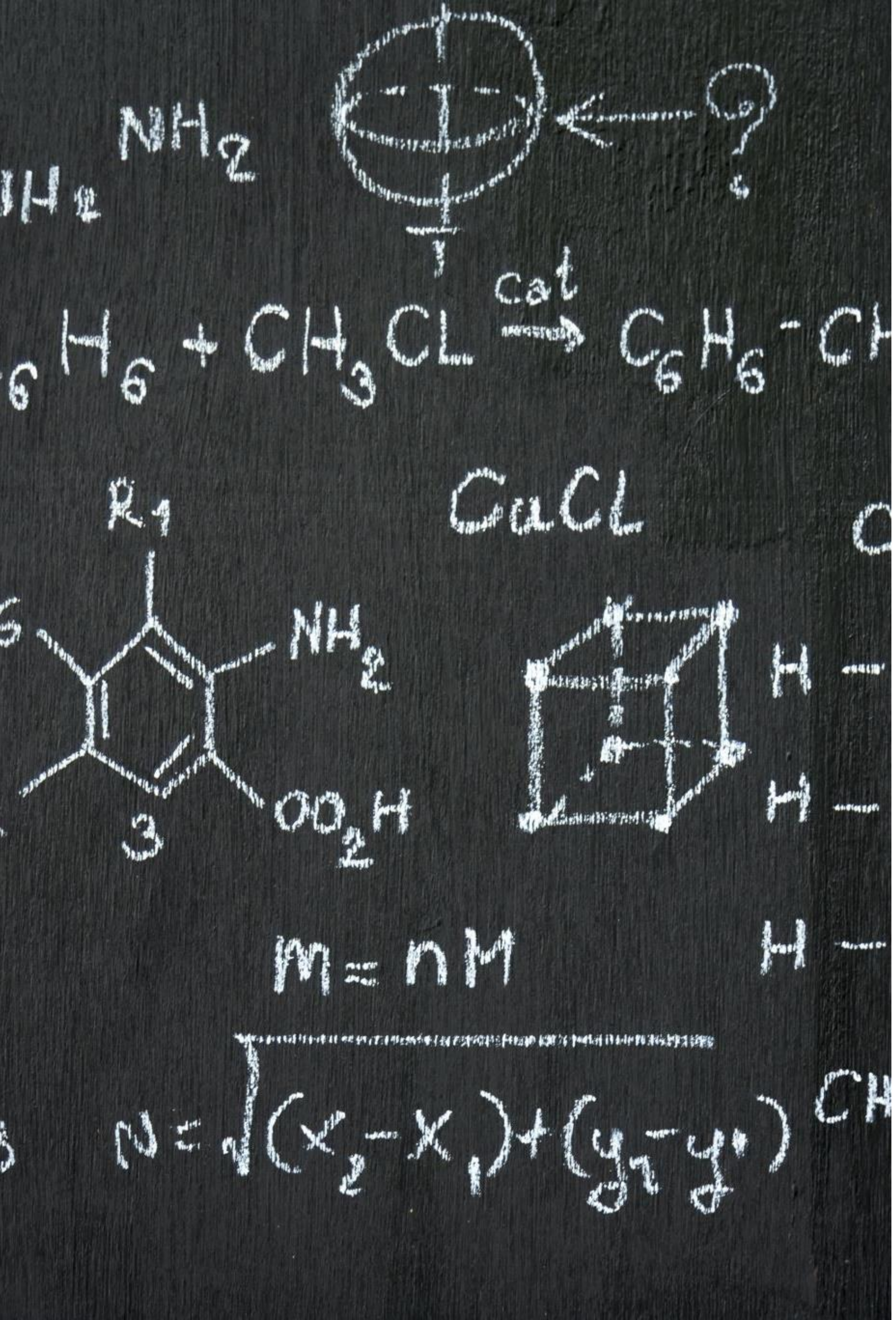
  X(S8) = 0

  Probability Distribution: “Probability with RV”

  {X(S), P}



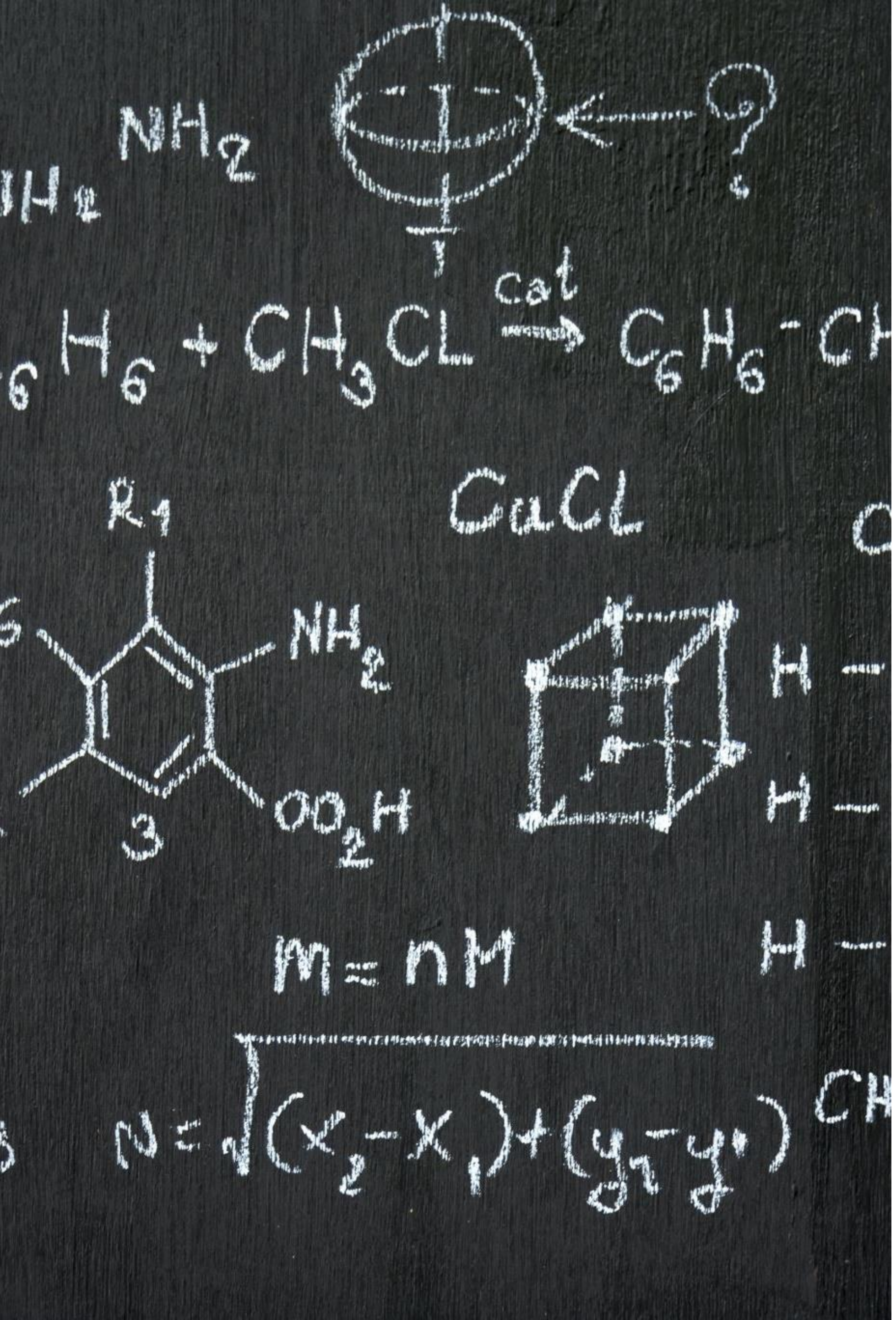


Random Variable

- Discrete Random Variable: A RV which takes finite or at most countable number of values is called DRV.

Example: 1) No. of head obtained when two coin are tossed

2) No. of defective items in a lot



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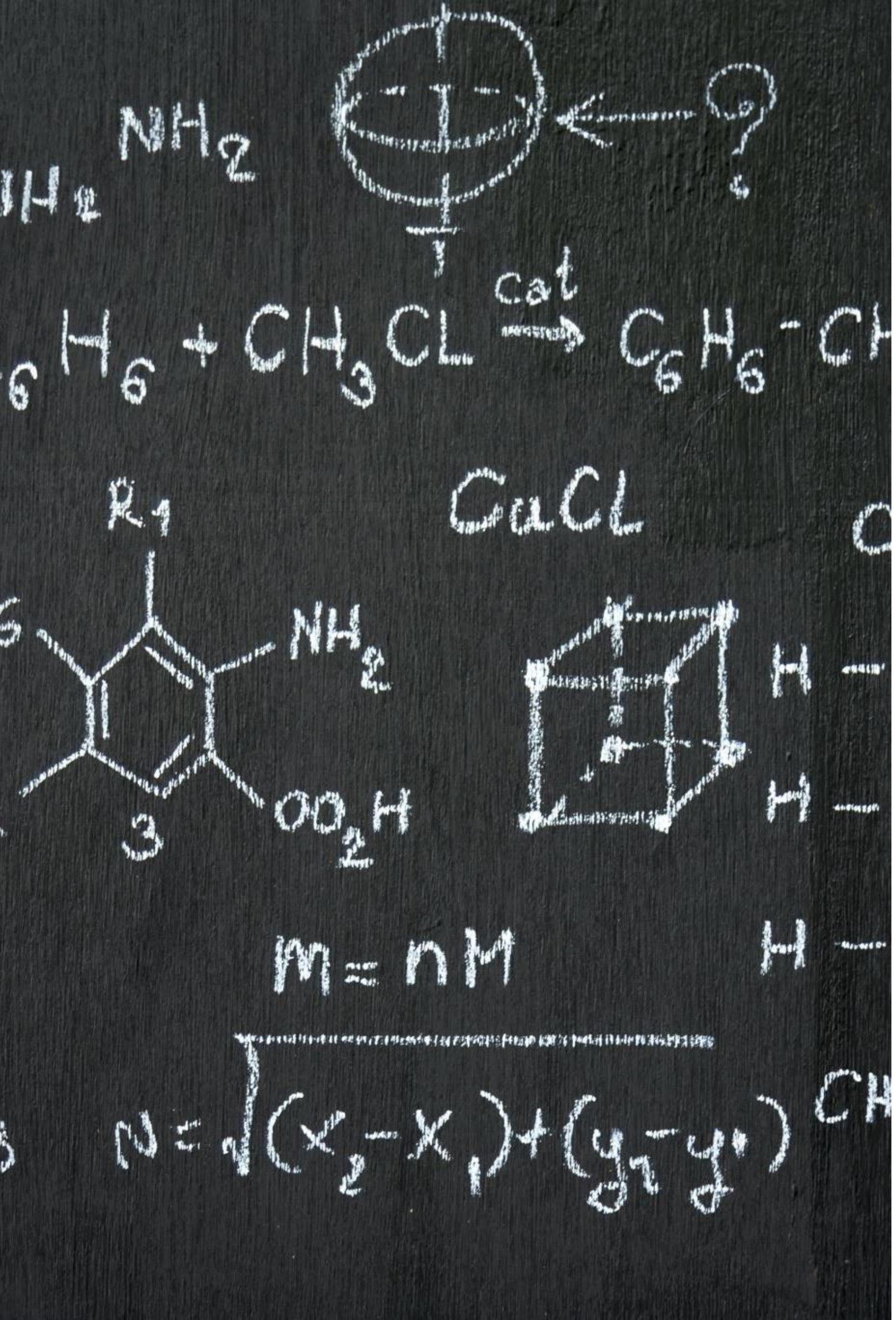

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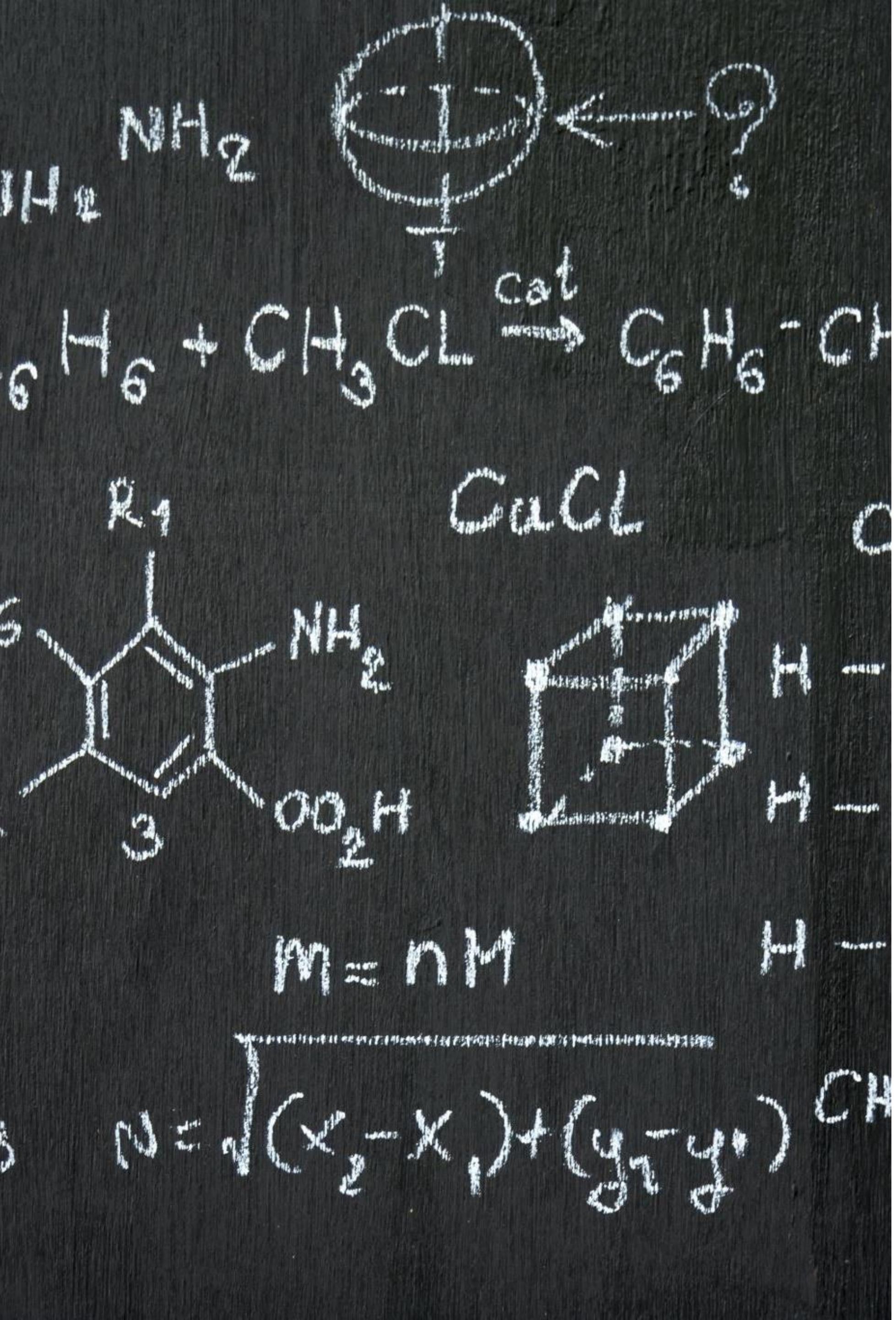
❑ Example: 1) Tossing a fair coin 3-times then sample space:

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

X (No. of Heads)	**0**	**1**	**2**	**3**
P(X)	1/8	3/8	3/8	1/8



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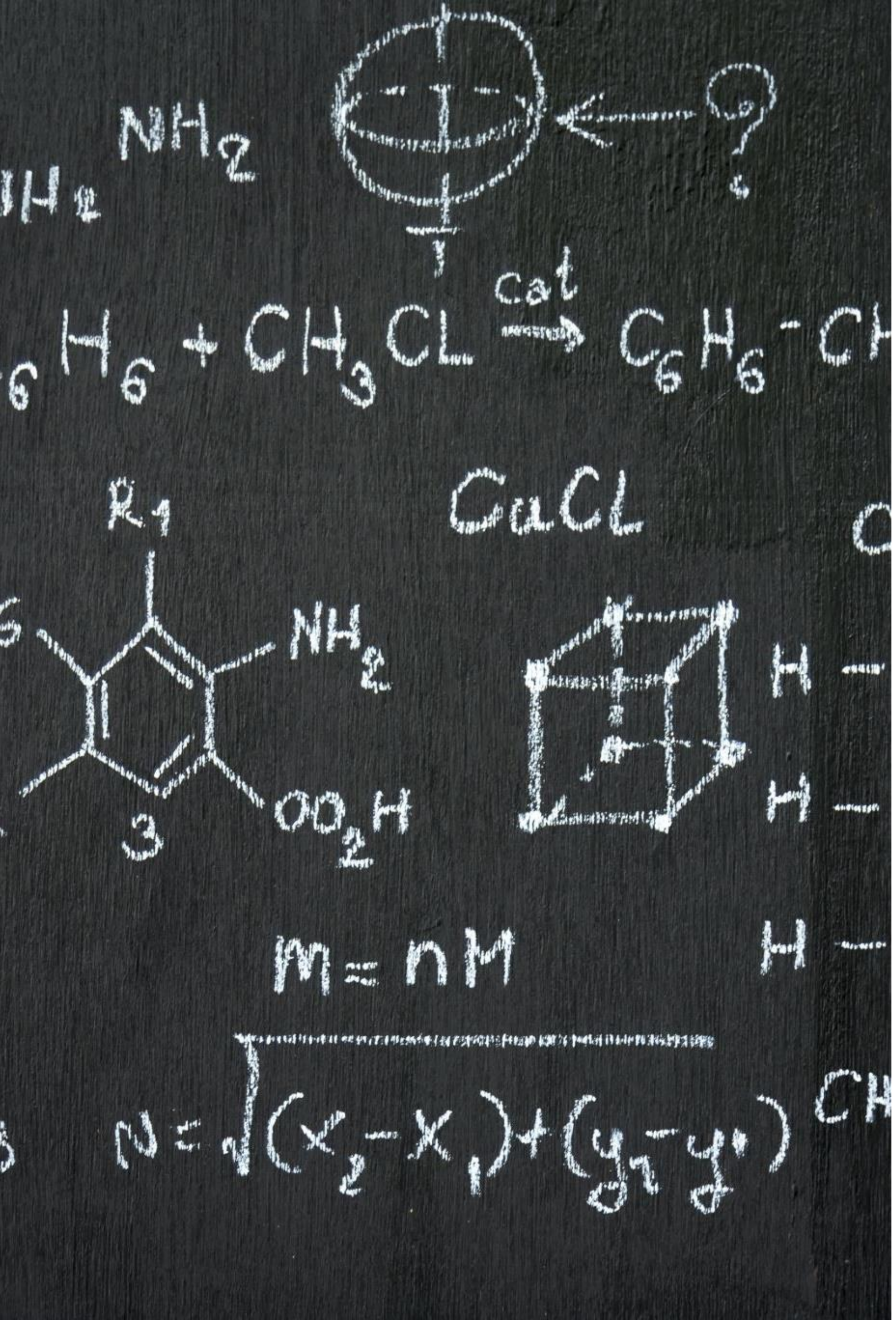
- ❑ Example: 2) Four bad oranges are mixed with 16 good oranges. Find probability distribution of number of bad oranges in drawn of 2 oranges



Random Variable

- Discrete Random Variable: A RV which takes finite or at most countable number of values is called DRV.

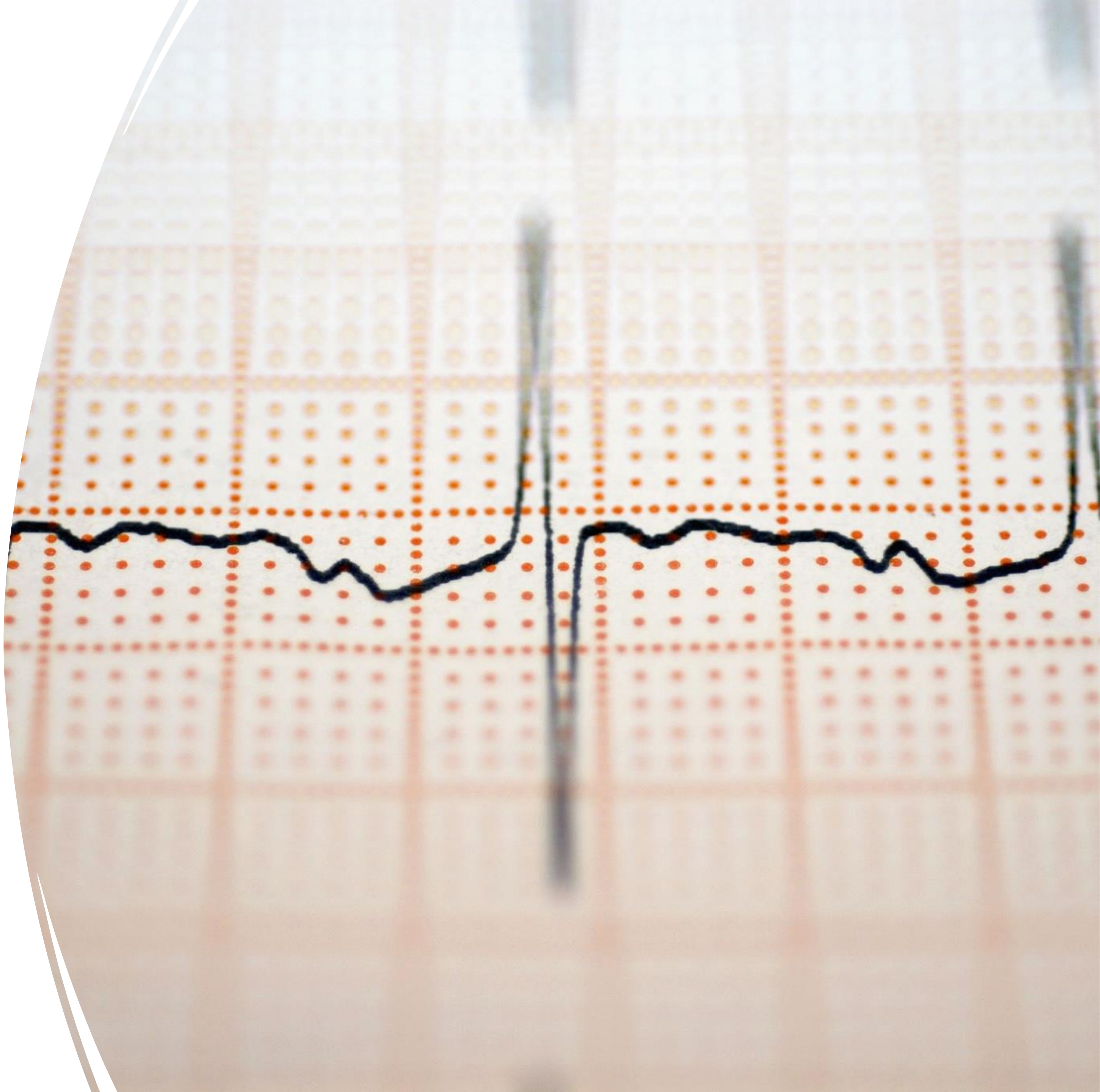
❑ Example: 2) Four bad oranges are mixed with 16 good oranges. Find probability distribution of number of bad oranges in drawn of 2 oranges

X (No. of bad oranges)	0	1	2	3
P(X)	16C2/ 20C2	16C1 4C1/ 20C2	4C2/ 20C2	0

Probability Mass Function

- Let x be DRV such that *P(X=x)* = Pi the Pi is said to be PMF.
- If it satisfy the following condition:

$$(i) Pi \geq 0$$
$$(ii) \sum Pi = 1$$



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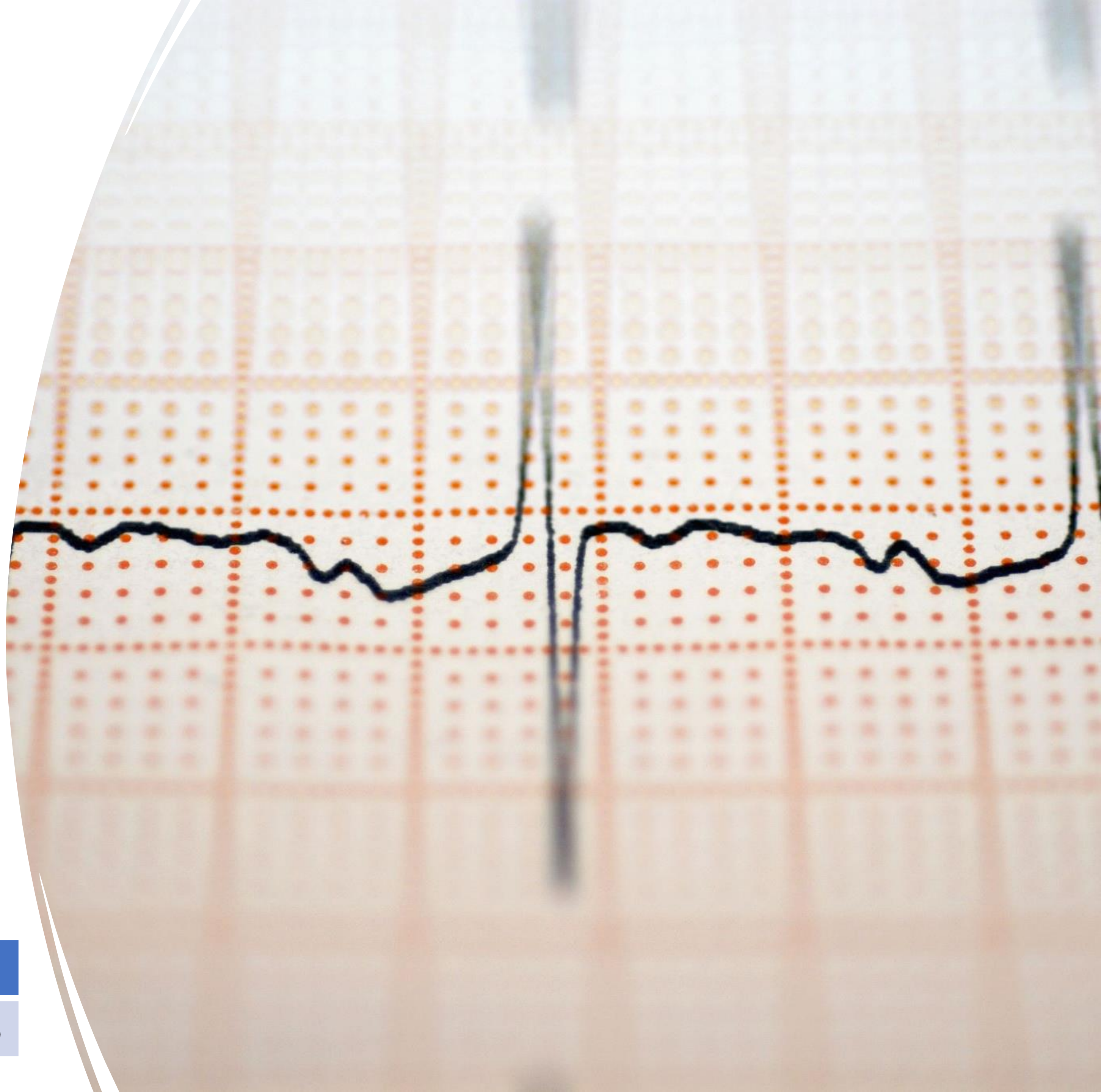
X (No. of Heads)	0	1	2	3
P(X)	1/8	3/8	3/8	1/8



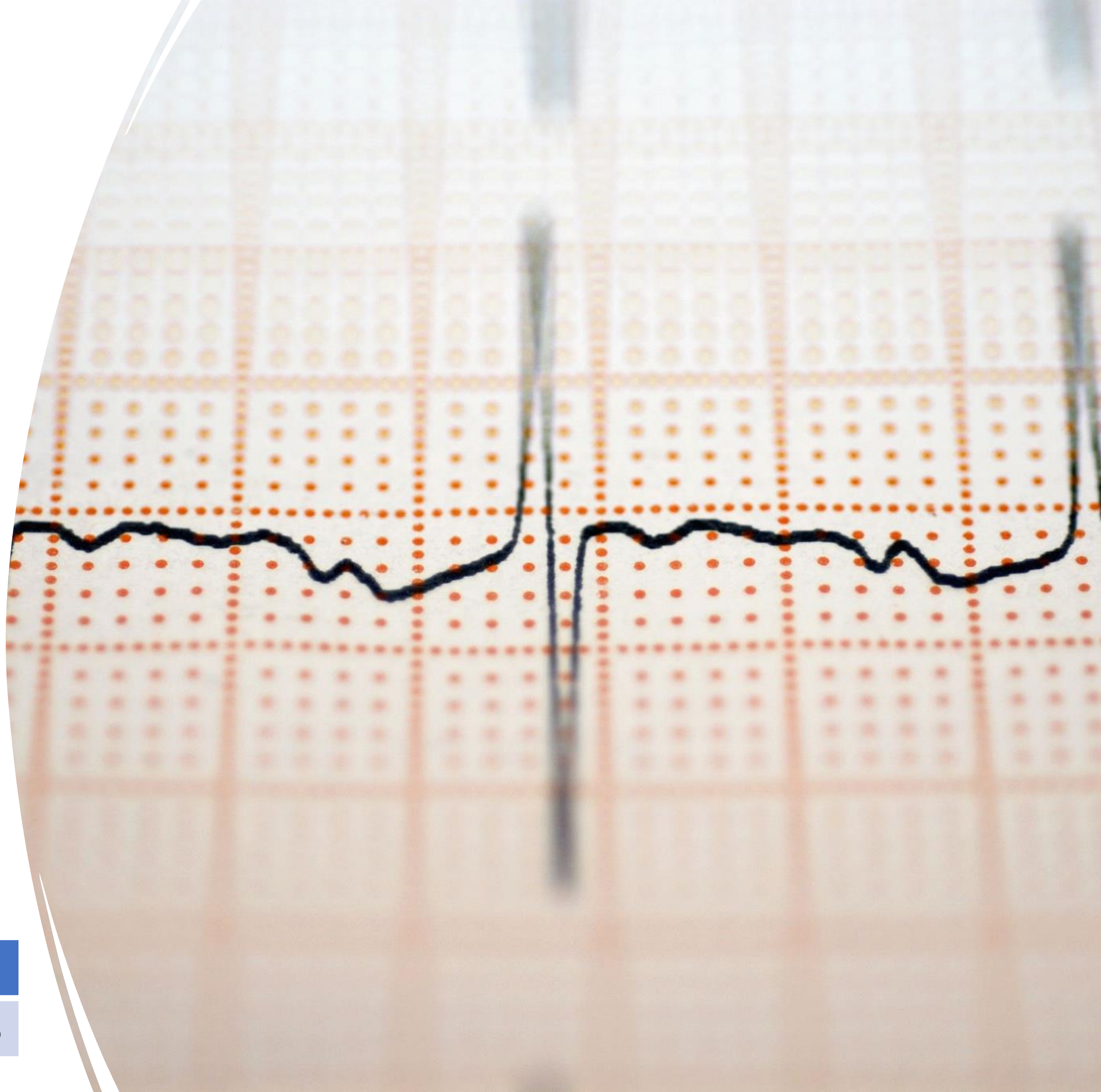
Distribution Function

- Let x be a DRV then its discrete distribution function or cumulative distribution function (CDF) is defined as:

$$f(x) = \sum Pi = P(x \le xi)$$

Example:

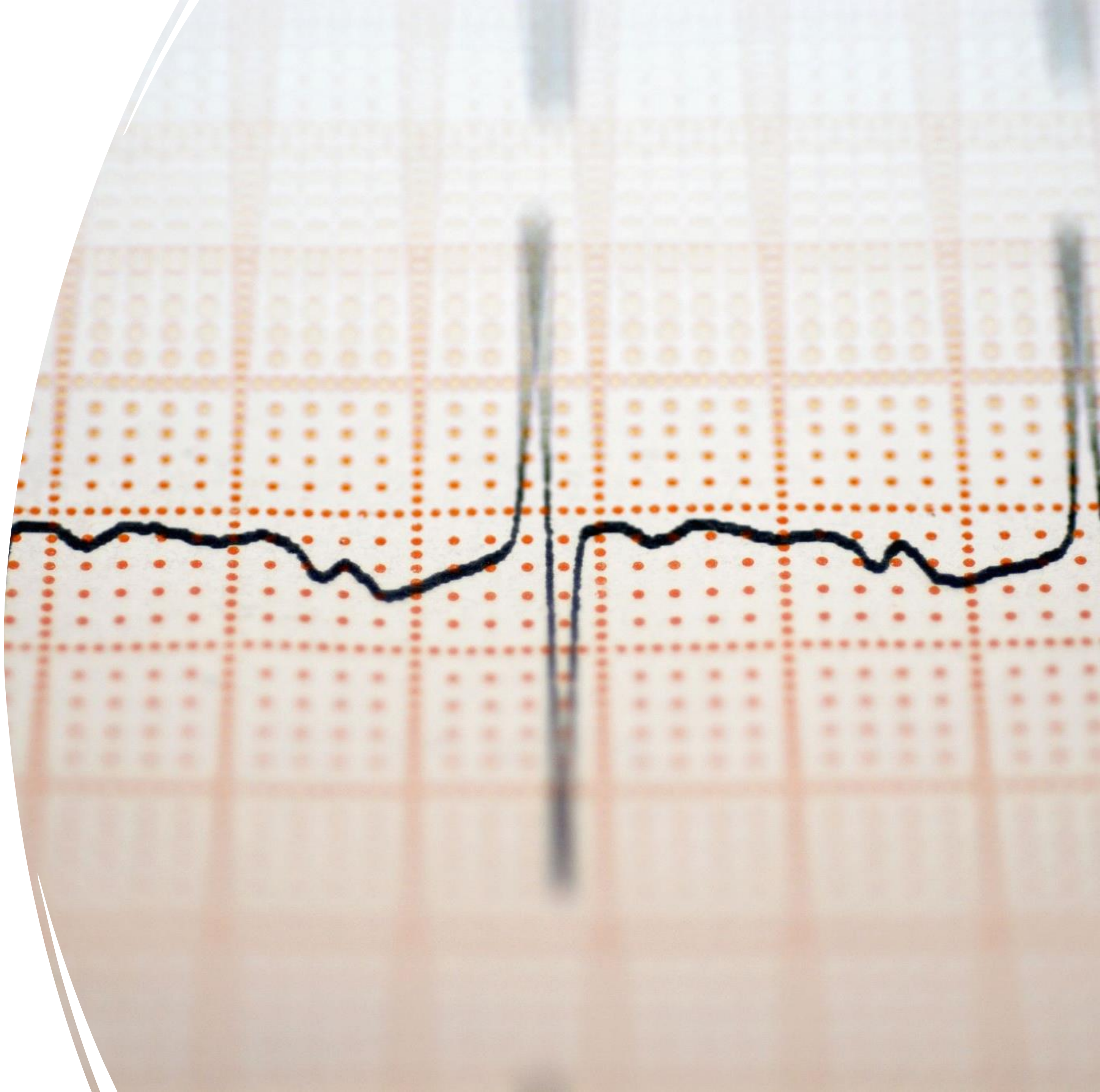
X (No. of Heads)	0	1	2	3
P(X)	1/8	3/8	3/8	1/8



Distribution Function

- Let x be a DRV then its discrete distribution function or cumulative distribution function (CDF) is defined as:

$$f(x) = f(x) = \begin{cases} \frac{1}{8} & x \leq 0 \\ \frac{4}{8} & x \leq 1 \\ \frac{7}{8} & x \leq 2 \\ 1 & x \leq 3 \end{cases}$$



Exercise Set-1

Q1. A Random Variable x has following probability distribution:

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	k^2+k

Find: 1. k

2. $P(x < 6), P(x \geq 6), P(0 < x \ < 5)$

3. Probability distribution

4. if $(P \ \leq c\ ) > \frac{1}{2}$. Find the min value of c

5. Find $P\left(\frac{1.5 < x < 4.5}{x > 2}\right)$