## Example-5

In a game, a man wins 5 rupees for getting a number greater than 4 and loss rupees 1 otherwise, when a fair die is thrown.

The man decide to throw a die thrice but to quit as and when he wins. Find the expected value of the amount he win/loss.

## Expected Value

#### Expected value

**Expected value:** measure of location, central tendency X continuous with range [a,b] and pdf f(x):

$$E(X) = \int_a^b x f(x) \, dx.$$

X discrete with values  $x_1, \ldots, x_n$  and pmf  $p(x_i)$ :

$$E(X) = \sum_{i=1}^{n} x_i p(x_i).$$

View these as essentially the same formulas.

# Variance and Standard Deviation

#### Variance and standard deviation

Standard deviation: measure of spread, scale

For any random variable X with mean  $\mu$ 

$$Var(X) = E((X - \mu)^2), \qquad \sigma = \sqrt{Var(X)}$$

X continuous with range [a, b] and pdf f(x):

$$Var(X) = \int_a^b (x - \mu)^2 f(x) \, dx.$$

X discrete with values  $x_1, \ldots, x_n$  and pmf  $p(x_i)$ :

$$Var(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i).$$

View these as essentially the same formulas.

## Properties

**Properties:** (the same for discrete and continuous)

**1.** 
$$E(X + Y) = E(X) + E(Y)$$
.

**2.** 
$$E(aX + b) = aE(X) + b$$
.

**3.** If 
$$X$$
 and  $Y$  are independent then  $Var(X + Y) = Var(X) + Var(Y)$ .

**4.** 
$$Var(aX + b) = a^2Var(X)$$
.

**5.** 
$$Var(X) = E(X^2) - E(X)^2$$
.

## Covariance

Measures the degree to which two random variables vary together, e.g. height and weight of people.

X, Y random variables with means  $\mu_X$  and  $\mu_Y$ 

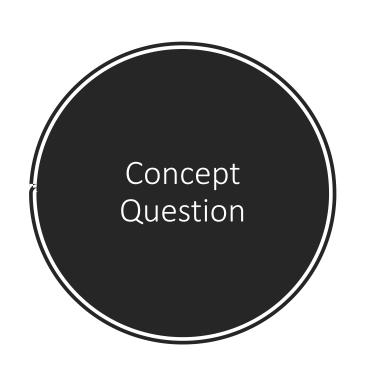
$$Cov(X, Y) = E((X - \mu_X)(Y - \mu_Y)).$$

TX/X/COV(X,Y)=E(X-MX)(X-) 

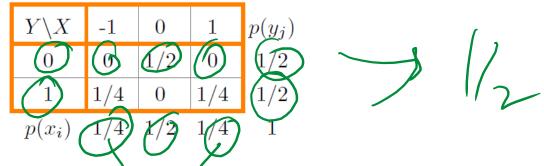
## Properties of Covariance

#### **Properties**

- 1. Cov(aX + b, cY + d) = acCov(X, Y) for constants a, b, c, d.
- 2.  $Cov(X_1 + X_2, Y) = Cov(X_1, Y) + Cov(X_2, Y)$ .
- 3. Cov(X, X) = Var(X)
- 4.  $Cov(X, Y) = E(XY) \mu_X \mu_Y$ .
- 5. If X and Y are independent then Cov(X, Y) = 0.
- 6. Warning: The converse is not true, if covariance is 0 the variables might not be independent.



Suppose we have the following joint probability table.



At your table work out the covariance Cov(X, Y).

Because the covariance is 0 we know that X and Y are independent

1. True 2. False

Key point: covariance measures the linear relationship between X and Y. It can completely miss a quadratic or higher order relationship.

ME (9) = 10 X/2 = 1/2 Cov(x,y)= E(X-(4a) (y-My) = E(x-y-xy)

## Correlation

Like covariance, but removes scale.

The correlation coefficient between X and Y is defined by

$$Cor(X, Y) = \rho = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}.$$

### Properties:

- **1.**  $\rho$  is the covariance of the standardized versions of X and Y.
- **2.**  $\rho$  is dimensionless (it's a ratio).
- **3.**  $-1 \le \rho \le 1$ .  $\rho = 1$  if and only if Y = aX + b with a > 0 and  $\rho = -1$  if and only if Y = aX + b with a < 0.

X! bought of Turner X= hynt of Som  $\times$ 8= Cov(X, y) 502 54 67 68 (ul(x,y)= E(xx) - E(x) [(1 5x = (E(x)) - E(x)) L 70 | 61 | 8y = JE(y") - (EN) 72 | 71 |

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