

7-9-2022

①

lec-11

'Parcal'

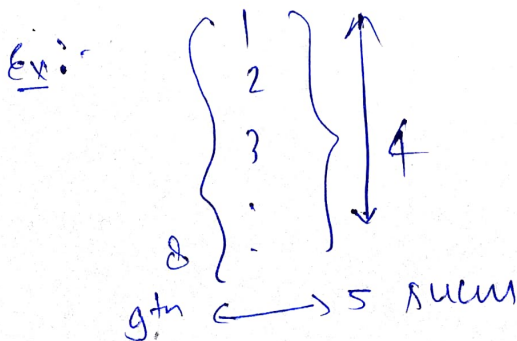
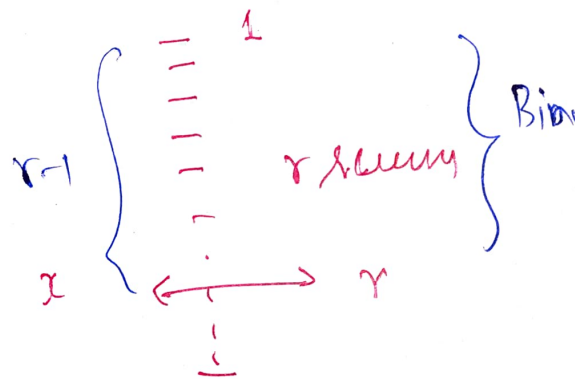
→ Negative binomial distribution is applicable when we need to performed an experiment until a total of r success are obtained

⇓
restriction

Note:- if $r=1$, mean we perform an experiment till we obtained first success (which is the case of Geometric distribution)

$P(X=x) = P(r=1 \text{ success in first } x-1 \text{ trail AND a success in the } x^{\text{th}} \text{ trail})$

At x^{th} exp.
 r success comp.



$$P(x) = \binom{x-1}{r-1} \cdot p^{r-1} \cdot q^{(x-1)-(r-1)} \cdot p$$

②

$x \Rightarrow$ no. of exp.

\Rightarrow

$$\sum_{x=r}^{\infty} P(x=x) = \sum_{x=r}^{\infty} \binom{x-1}{r-1} p^r \cdot q^{x-r}$$

$$= p^r \sum_{x=r}^{\infty} \binom{x-1}{r-1} q^{x-r}$$

Ex 1 if the probability is 0.40 that a child exposed to a certain disease will contain it. What is the probability that the 10th child exposed to the disease will be the 3rd to catch it?

Solⁿ

$$P(x=10) = \binom{9}{2} p^2 q^7 \times p$$

$$= \frac{9 \times 8}{2 \times 1} \cdot (0.4)^3 (0.6)^7 = 0.0648$$

9

1
2
3
⋮
9

 2 child

10 \Rightarrow 3rd child

3

x2. Let X be the ~~random~~ number of births in a family until the 2nd daughter is born. If the prob. of having a male child is $1/2$. Find the probability that the 6th child in the family is the second daughter.

Solⁿ

$$P(X=6) \quad p=1/2 \\ q=1/2 \\ = ({}^5C_1 p^1 q^4) p$$

$$= \frac{5 \times 1}{2^5} \times \frac{1}{2} \cdot \frac{1}{2}^4 \cdot \frac{1}{2} \\ = \frac{5}{2^6} \Rightarrow$$

5 { 1
2 only 1
3
4
5
6 2nd

Ex 2. In a Company, 5% defective Components are produced. What is the probability that at least 5 Components are to be examined in order to get 3 defective?

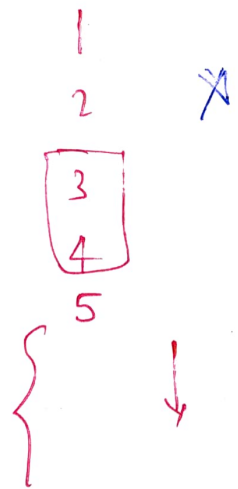
Solⁿ:

$$P(X \geq 5) = 1 - P(X < 5)$$

$$= 1 - P(X=3) - P(X=4)$$

$$= 1 - {}^2C_2 p^2 q^0 \times p$$

$$- {}^3C_2 p^2 \cdot q^1 \times p$$



$$P(X=3) \quad \left\{ \frac{1}{2} \right\}^2 \text{ Bino.}$$

(3)

⇒ Find ~~moment Gen. funth~~ of Bino dist.

(5)

Mean & Variance of Bino dist.

$$P(x) = {}^n C_x p^x q^{n-x}$$

$$\text{mean} = E(x) = \sum_{x=0}^n x P(x)$$

$$= \sum_{x=0}^n x \cdot {}^n C_x \cdot p^x q^{n-x}$$

$$= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^n x \frac{n!}{x(x-1)!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^n \frac{n(n-1)!}{(x-1)!(n-1-(x-1))!} p \cdot p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=0}^n \frac{(n-1)!}{(x-1)!(n-1-(x-1))!} p^{x-1} q^{(n-1)-(x-1)}$$

$$E(x) = n \cdot p$$

$$\text{Var}(x) = \sigma_x^2 = E(x^2) - (E(x))^2$$

$$\hookrightarrow = npq$$

Bernoulli's Distribution

1. $X \rightarrow \begin{cases} \text{Success} = 1 & P(X=1) = p \\ \text{Failure} = 0 & P(X=0) = q = 1-p \end{cases}$

② $0 \leq p \leq 1 \quad p + q = 1$

" A discrete r.v X is said to have a Bernoulli distribution with parameter p if its probability mass function is given by "

$$f(x) = p^x (1-p)^{1-x} \quad ; \quad x=0,1$$

Bernoulli trial is an experiment with only 2 possible outcomes; $S = \text{Success}$
 $F = \text{Failure} \rightarrow \begin{cases} b/g; \\ D/L; C/nc \end{cases}$

Bernoulli distribution arises when the following 3 conditions are satisfied.

- ① Each trial of an exp results in an outcome that may be classified as a success or a failure.
- ② The probability of a success $P(S) = p$ is the same for each trial
- ③ The trials are independent; that is the outcome of one trial has no effect on the outcome of any other trial.

$$E(X) = np, \quad \text{Var}(X) = npq; \quad q = 1 - p$$

Ex 1. A 6 sided fair die is tossed, with each element in the sample space having prob. of $1/6$. find the distn. - the mean & variance of the value occurs 5.

Solⁿ

$$p = 1/6$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$p = 1/6, \quad q = 5/6$$

$$P(x) = p^x (1-p)^{1-x}$$

$$= (1/6)^x (1-1/6)^{1-x}$$

$$P(x) = (1/6)^x (5/6)^{1-x}$$

$$\text{mean} = E(x) = p = 1/6$$

$$\text{var}(x) = pq = 1/6 \cdot 5/6 = 5/36$$

Ex¹. 5 white, 3 black ; 6 trials

white ball - success

① ~~Repea~~ replacement $\Rightarrow p = 5/8, \quad q = 1-p = 3/8$

② without replant

\swarrow
1st $p = 5/8 ; q = 3/8$

\searrow
2nd $p = 4/7 ; q = 3/7$