Probability

Measure of uncertainty of events

Axiomatic theory = Classical theory

Equally likely outcomes

Conditional Probability

?

Conditional Probability

$$P(A|B)$$

Conditional Probability

Conditional probability of A given B

$P(A|B)$

Given

?

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$$P(A|B)$$

Given

Outcome satisfies **B** $\longrightarrow$ How likely it will satisfy **A**?

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Conditional probability of A given B

$$P(A|B)$$

Given

Outcome satisfies **B** $\longrightarrow$ How likely it will satisfy **A**?

Chance of occurrence of **A** $\longrightarrow$ **B** has already occurred

$$A \overset{\textit{Dependent}}{\rightleftarrows} \boxed{B}$$

Cheteshwar Pujara → Scored 50 runs → Chance of scoring 100 runs?

Cheteshwar Pujara $\longrightarrow$ Scored 50 runs $\longrightarrow$ Chance of scoring 100 runs?

$$P(\text{scores 100 runs} \mid \text{scored 50 runs}) = ?$$

Cheteshwar Pujara → Scored 50 runs → Chance of scoring 100 runs?

No. of first innings → 76
50 runs → 31
100 runs → 16

P(scores 100 runs | scored 50 runs) = ?

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B → Pujara scores 100 runs

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$$P(B|A) = \frac{16}{31} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \frac{n(B \cap A)}{n(A)}$$

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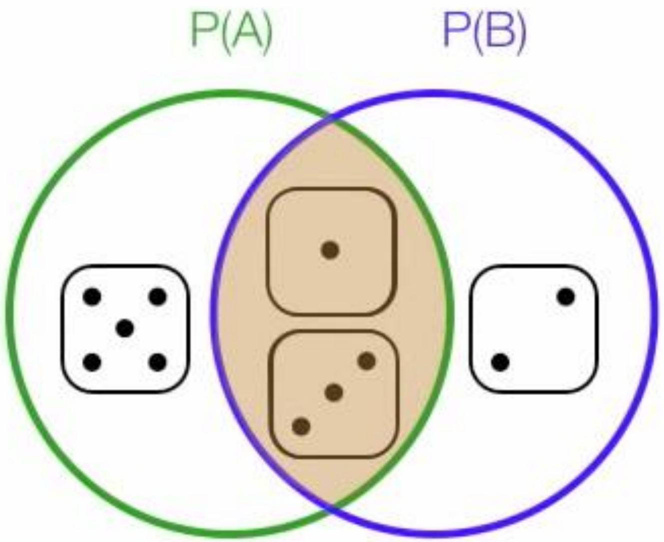
Conditional Probability

What is the Probability of

rolling a dice and it's value is less than 4

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

knowing that the value is an odd number



Example-1: The % of adults who are men and alcoholics is 2.25 %. What is the probability of being an alcoholics, given being man ?

Example-1: Solution

A =: Alcoholic

B =: man

$$P(A \cap B) = 2.25\ \% = 0.0225$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.0225}{0.5} = 4.5\ \%$$

Bayes' theorem

LIKELIHOOD
The probability of "B" being True, given "A" is True

PRIOR
The probability "A" being True. This is the knowledge.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

POSTERIOR
The probability of "A" being True, given "B" is True

MARGINALIZATION
The probability "B" being True.

---------A special case of the Bayes' theorem is when event A is a binary variable.

$$P(A^+/B) = \frac{P(B/A^+)\,P(A^+)}{P(B/A^-)P(A^-) + P(B/A^+)P(A^+)}$$

Where:

- P(B|A−) – the probability of event B occurring given that event A− has occurred

- P(B|A+) – the probability of event B occurring given that event A+ has occurred

- In the special case above, events A− and A+ are mutually exclusive (Mutually exclusive is a statistical term describing two or more events that cannot happen simultaneously) outcomes of event A.

"A" With Three (or more) Cases

$$P(A1/B) = \frac{P(B/A\,1)\;P(A1)}{P(B/A1)P(A1) + \; P(B/A2)P(A2) + P(B/A3)P(A3) + \cdots .etc}$$

When "A" has 3 or more cases we include them all in the bottom line:

Mutually exclusive and Independent event:

Mutually exclusive event:

Two events are mutually exclusive when they cannot occur at the same time.

For example, if we flip a coin it can only show a head OR a tail, not both.

Independent event:

The occurrence of one event does not affect the occurrence of the others.

For example, if we flip a coin two times, the first time may show a head, but the next time when we flip the coin the outcome will be heads also.

From this example, we can see the first event does not affect the occurrence of the next event.

Example-2: What is the probability of two girls given at least one girl?

Example-2: Solution

$$Sample\ Space =: \{GG, GB, BG, BB\}$$

$$P\,(2G \,/\, at\ least\ 1G) = \frac{P(1G|2G)\,.\,P(2G)}{P(1G)}$$

$$= \frac{1\,.1/4}{3/4} = 1/3$$

Example-3: If we randomly draw a blue ball. What is the probability of being in 1st bucket ?

B1: 3Yellow, 3 Blue

B2: 4 Yellow, 2 Blue

Example-3: Solution

B1: 3Yellow, 3 Blue

B2: 4 Yellow, 2 Blue

A =: select a blue ball

P(A | B1) = 1/2

P(A | B2) = 1/3

P(B1) = P(B2) = ½

P(A) = P(A ∩ B1) + P(A ∩ B2)

= P(A|B1) . P(B1) + P(A|B2).P(B2)

= 3/6 . 1/2 + 2/6. 1/2

= 1/4 + 1/6 = 5/12

$$P(B1|A) = \frac{P(A|B1).P(B1)}{P(A)} = \frac{\frac{1}{2} * \frac{1}{2}}{\frac{5}{12}}$$

Example-4: Given that,

B1: 3 Golds

B2: 2 Golds, 1 Silver

B3: 1 Golds, 2 Silver

A person chosen a box at random and takes out a coin. If the coin is Gold. What is the probability that it was drawn from box 3?

Example-4: Solution

Given that,

B1: 3 Golds

B2: 2 Golds, 1 Silver

B3: 1 Golds, 2 Silver

A =: event of getting Gold coin

E1 =: event of getting B1

E2 =: event of getting B2

E3 =: event of getting B3

Example-4: Solution

A =: event of getting Gold coin

E1 =: event of getting B1

E2 =: event of getting B2

E3 =: event of getting B3

$$P(E3\,|A) = \frac{P(E3).\,P(A|E3)}{P(E1).\,P(A\,|E1) + P(E2).\,P(A\,|E2) + P(E3).\,P(A\,|E3)}$$

$$= \frac{\frac{1}{3}.\frac{1}{3}}{\frac{1}{3}.1+\frac{1}{3}.\frac{2}{3}+\frac{1}{3}.\frac{1}{3}} = \frac{1}{6}$$

Example-5 The number of balls in three Jars is as follows:

#Jars	Red	White	Black
1	3	2	1
2	2	1	2
3	4	2	3

One Jar is chosen at random and two balls are drawn. The balls drawn are red and white. What is the probability that they come from Jar 1

Example-5: Solution

#Jars	Red	White	Black
1	3	2	1
2	2	1	2
3	4	2	3

Let – E is the event of drawing 'red and white' ball

$$P(E) = P(E|J1)P(J1) + P(E|J2)P(J2) + P(E|J3)P(J3)$$

P(J1) = P(J2) = P (J3) = 1/3

P (E| J1) = $\frac{C_1^3 C_1^2}{C_2^6} = \frac{6}{15} = \frac{2}{5}$

P (E| J2) = $\frac{C_1^2 C_1^1}{C_2^5} = \frac{2}{10} = \frac{1}{5}$

P (E| J3) = $\frac{C_1^4 C_1^2}{C_2^9} = \frac{2}{9}$

P(J1 | E) = $\frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{5}+\frac{2}{9}} = \frac{18}{37}$

Poker Hands

Deck of 52 cards

- 13 *ranks*: 2, 3, . . . , 9, 10, J, Q, K, A
- 4 *suits*: $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$,

Poker hands

- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other rank
- Example: $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, K\diamondsuit\}$

The probability of a one-pair hand is:

(1) less than 5%
(2) between 5% and 10%
(3) between 10% and 20%
(4) between 20% and 40%
(5) greater than 40%

Solution

$P\ (1\ pair\ in\ a\ Poker\ hand)\ =?$

Sample space = C_5^{52}

Pair:

Pick value =: C_1^{13}

Pick suit =: C_2^4 $C_1^{13} . C_2^4$

Other 3 Cards:

Pick value =: C_3^{12}

Pick suits =: $C_1^4 . C_1^4 . C_1^4$ $C_3^{12} . C_1^4 . C_1^4 . C_1^4$

$$P\ (1\ pair\ in\ a\ Poker\ hand) = \frac{C_1^{13} . C_2^4 . C_3^{12} . C_1^4 . C_1^4 . C_1^4}{C_5^{52}} = 42\ \%$$

Example

I won't wear green and red together; I think black or denim goes with anything;

Here is my wardrobe.

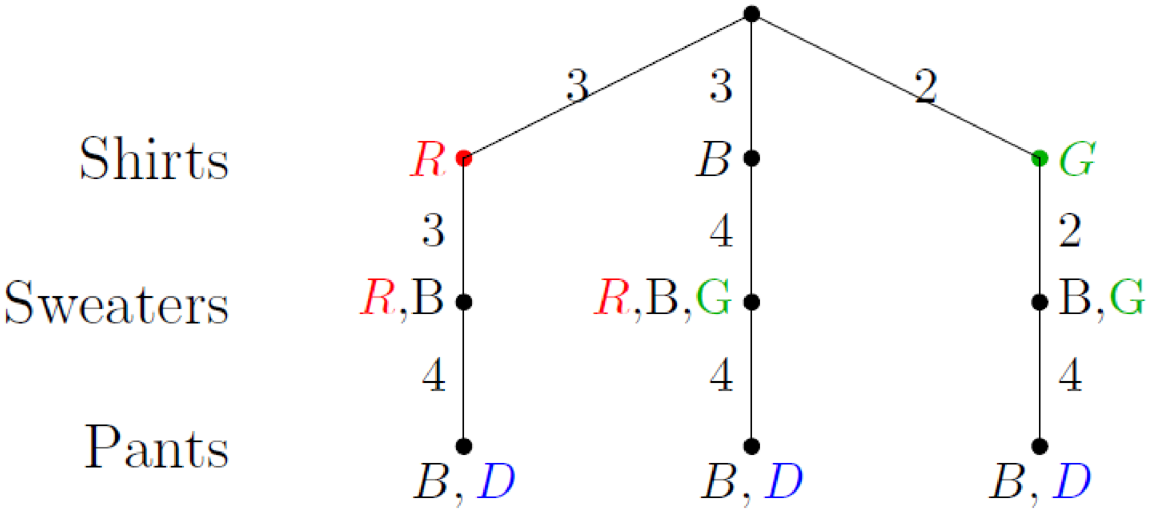
Shirts: 3B, 3R, 2G;

sweaters 1B, 2R, 1G;

pants 2D,2B.

How many different outfits can I wear?

Solution



Multiplying down the paths of the tree:

Number of outfits = $(3 \times 3 \times 4) + (3 \times 4 \times 4) + (2 \times 2 \times 4) = 100$