

Gamma Function

$$\Rightarrow \int_0^{\infty} e^{-x} x^{n-1} dx = \Gamma n$$

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$$\Rightarrow \int_0^{\infty} e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$$

$$ax = t$$

$$a dx = dt$$

$$\int_0^{\infty} e^{-t} \left(\frac{t}{a}\right)^{n-1} \frac{dt}{a}$$

$$\Rightarrow \frac{1}{a^n} \int_0^{\infty} e^{-t} t^{n-1} dt = \frac{\Gamma n}{a^n}$$

$$\Rightarrow \Gamma n = \cancel{n!} (n-1)! = 1 \times 2 \times \dots \times (n-1)$$

$$\Rightarrow \Gamma 1/2 = \sqrt{\pi}$$

$$\Rightarrow \Gamma n/2 = \sqrt{\pi} \frac{(n-2)!!}{2^{(n-1)/2}} \rightarrow \text{Double factorial}$$