Discrete Random Variable

Binomial Distribution:

- All the trails are independent
- Number (n) of trails is finite
- The Probability (p) of the success is same of each trials

$$P(x) = C_x^n \, p^x q^{(n-x)}$$

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Example:

a. A coin toss 3-times, find the probability of 2-Heads.

b. A coin toss 10-times, find the probability of 5-Heads.

Binomial Distribution

Q1. The probability that man aged 60 will live up to 70 is 0.65 out of 10 men. Now aged 60, find the probability:

1. At least 7 will live up to 70

2. Exactly 9 will live up to 70

3. At most 9 will live up to 70

Binomial Distribution

Q2. Out of 800 families with 5 children each, how many families would be expected to have

- 3 boys
- 5 girls
- Either 2 or 3 boys
- At least 2 girls

Binomial Distribution

Q3. The Probability that a pen manufactured by a company will be defective is 1/10. If 12 such pen are manufactured. Find the probability that:

1. Exactly 2 will be defective
2. None will be defective
3. At least 2 will be defective

Binomial Distribution

Q4. Medical professionals use the binomial distribution to model the probability that a certain number of patients will experience side effects as a result of taking new medications.

E.g., suppose it is known that 5% of adults who take a certain medication experience negative side effects. We can use a Binomial Distribution Calculator to find the probability that more than a certain number of patients in a random sample of 100 will experience negative side effects.

- P(X > 5 patients experience side effects) = **??**
- P(X > 10 patients experience side effects) = **??**
- P(X > 15 patients experience side effects) = **??**

Binomial Distribution

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- P(X > 5 patients experience side effects) = **0.38400**
- P(X > 10 patients experience side effects) = **0.01147**
- P(X > 15 patients experience side effects) = **0.0004**

$p = 0.05$

$q = 0.95$

$n = 100$

$$p(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$P(X > 5) = 1 - P(X \le$$

$$= 1 - [P(0) + P(1) + P(2) + P(3) +$$

Binomial Distribution

Q5. Banks use the binomial distribution to model the probability that a certain number of credit card transactions are fraudulent.

E. g., suppose it is known that 2% of all credit card transactions in a certain region are fraudulent. If there are 50 transactions per day in a certain region, we can use a Binomial Distribution Calculator to find the probability that more than a certain number of fraudulent transactions occur in a given day:

- P(X > 1 fraudulent transaction) = ??
- P(X > 2 fraudulent transactions) = **??**
- P(X > 3 fraudulent transactions) = **??**

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- P(X > 1 fraudulent transaction) = **0.26423**
- P(X > 2 fraudulent transactions) = **0.07843**
- P(X > 3 fraudulent transactions) = **0.01776**

$p = 0.02 \qquad n = 50$

$q = 0.98 \qquad p(x) = {}^{n}C_{x}\, p^{x} q^{n-}$

$$P(X > 1) = 1 - [P(0) + P(1)]$$

Negative Binomial Distribution

NBD is applicable when we need to performed an experiment untill a total of r success are obtained

Note: If r = 1, means we perform an experiment till we obtained first success.

Negative Binomial Distribution

$$P(x) = \left(C_{r-1}^{x-1}\, p^{r-1} q^{(x-1)-(r-1)}\right) p$$

$$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

Negative Binomial Distribution

Q1. If the probability is 0.40 that a child exposed the certain disease will contain it. What is the probability that the 10th child exposed to the disease will be the 3rd to catch ?

$P(X=10) = \left({}^{9}C_{2} \cdot p^{2} \cdot q^{7}\right) p$

$= \frac{\lfloor 9}{\lfloor 7 \lfloor 2} (0.4)^{2} (.6)^{7} \times 0$

$= 0.064$

Negative Binomial Distribution

Q3. Let x be the number of births in a family until the 2nd daughter is born. If the probability of the having a male child is ½ . Find the probability that the 6th child in the family is the second daughter.

Bernoulli Distribution

A discrete random variable X is said to have a Bernoulli distribution with parameter p. If its probability mass function is given by:

$$P(x) = p^x(1-p)^{1-x} \ , x = 0, 1$$

Bernoulli distribution

Bernoulli distribution arises when the following 3-conditions are satisfied.

1. Each trail of an experiment results in an outcome that may be classified as a success or failure
2. The probability of a success $P(S) = p$ is the same for each trail.
3. The trails are independent; that is the outcome of one trail have no effect on the outcome of any other trail.