> Expectation :-

Binomial distribution

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$\text{mean} = E(x) = \sum_{x=0}^{n} x\, p(x) = \sum_{x=0}^{n} x \cdot {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!\,(n-x)!}\, p^{x} \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x\,(x-1)!\,(n-x)!}\, p^{x} \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n\,(n-1)!\; p \cdot p^{x-1}\, q^{(n-1)-(x-1)}}{(x-1)!\,(n-1)-(x-1)!}$$

$$= np \sum_{x=0}^{n} \underbrace{\frac{(n-1)!}{[(n-1)-(x-1)]!\,(x-1)!}\, p^{x-1} \cdot q^{(n-1)-(x-1)}}$$

$$= np \cdot (p+q)^{n-1}$$

$$\underbrace{\sum_{x=0}^{n} {}^{n-1}C_{x-1} \cdot p^{x-1} \cdot q^{(n-1)-(x-1)}}_{(p+q)^{n-1}}$$

$$\boxed{\text{mean} = np}$$

Variance: Binomial:

$$\boxed{\sigma^2 \Rightarrow npq}$$

Proof

$$Var\ \sigma_x^2 = E(x^2) - [E(x)]^2$$

$$E[x^2) = \sum_{x=0}^{n} x^2\ p(x)$$

$$= \sum_{x=0}^{n} [x + (x-1)x]\ p(x)$$

$$= \sum_{x=0}^{n} x\ p(x) + \sum_{x=0}^{n} (x-1)\ x.\ p(x)$$

$$= np + \sum_{x=0}^{n} x(x-1)\ ^nC_x .\ p^x\ q^{n-x}$$

$$= np + \sum_{x=0}^{n} x(x-1) . \frac{n!}{(n-x)!\ x!} . p^x . q^{n-x}$$

$$= np + \sum_{x=0}^{n} x(x-1) \frac{n!}{(n-x)!\ x.(x-1)(x-2)!} p^x . q^{n-x}$$

$$= np + \sum_{x=0}^{n} \frac{n!}{(n-x)!\ (x-2)!} p^x . q^{n-x}$$

$$= np + \sum_{x=0}^{n} \frac{n(n-1)(n-2)!}{(n-2)-(x-2)!\ (x-2)!} p^2\ p^{x-2} . q^{(n-2)-(x-2)}$$

$$= np + n(n-1)p^2 \underbrace{\sum \frac{(n-2)!}{[(n-2)-(x-2)]!\,(x-2)!} p^{x-2} \cdot q^{(n-2)-(x-2)}}$$

$$= np + n(n-1)p^2 \cdot (p+q)^{n-2}$$

$(p+q) = 1$

$$= np + n(n-1)p^2$$

$$E(x^2) = np + n^2p^2 - np^2$$

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$= np + n^2p^2 - np^2 - n^2p^2$$

$$= np - np^2$$

$$= np(1-p)$$

$$\boxed{\text{var}(x) = npq}$$

Poisson Distribution

$$\boxed{P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}}$$

mean

$$E(X) = \sum_{x=0}^{\infty} x \cdot p(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \frac{0 \cdot e^{-\lambda} \cdot \lambda^{0}}{0!} + \frac{1 e^{-\lambda} \cdot \lambda^{1}}{1!} + \frac{2 \cdot e^{-\lambda} \lambda^{2}}{2!} + \cdots$$

$$= 0 + \lambda e^{-\lambda} + e^{-\lambda} \cdot \lambda^{2}$$

$$= \lambda e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^{2}}{2!} + \cdots \right]$$

$$= \lambda e^{-\lambda} e^{+\lambda}$$

$$\boxed{E(X) = \lambda}$$

Variance :-

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{x=0}^{\infty} x^2 \cdot P(x)$$

$$= \sum [x + x(x-1)] \, p(x)$$

$$= \sum [x + x(x-1)] \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \sum x \cdot \frac{e^{-\lambda} \lambda^x}{x!} + \sum \frac{x(x-1) \cdot e^{-\lambda} \lambda^x}{x!}$$

$$= \lambda + \left[0 + 0 + \frac{2 \cdot 1 \cdot e^{-\lambda} \lambda^2}{2!} + \frac{3 \cdot 2 \cdot e^{-\lambda} \lambda^3}{3!} + \cdots \right]$$

$$= \lambda + e^{-\lambda} \cdot \lambda^2 \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right]$$

$$= \lambda + e^{-\lambda} \lambda^2 \cdot e^{\lambda}$$

$$E(x^2) = \lambda + \lambda^2$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \lambda + \lambda^2 - \lambda^2 = \lambda$$

$$\boxed{Var(x) = \lambda}$$

Negative Binomial distribution

mean $\Rightarrow$ rq/p

variance $\Rightarrow$ $\dfrac{rq}{p^2}$

Geometric distribution

mean $= \dfrac{1-p}{p}$

Var $= \dfrac{1-p}{p^2}$