Normal Distribution

A CRV which has the following P.D.F

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \left.\begin{array}{l} -\infty < x < \infty \\ -\infty < \mu < 0 \\ \sigma > 0 \end{array}\right\}$$

is called Normal variates &

it's distribution is called ND

& Denoted by $\boxed{X \sim N(\mu, \sigma)}$

Mean & Variance of ND

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$

$$E(x) = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \, e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z \Rightarrow x = \mu + \sigma z$

$\frac{dx}{\sigma} = dz$

$dx = \sigma \, dz$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) \, e^{-z^2/2} \, \sigma \, dz$$

$$= \cancel{\mu + \sigma}$$

$$\Rightarrow \frac{1}{\cancel{\sigma}\sqrt{\pi}} \left[\int_{-\infty}^{\infty} \mu \, e^{-z^2/2} dz + \underbrace{\int_{-\infty}^{\infty} \sigma \, z \, e^{-z^2/2} dz}_{0} \right]$$

odd limit

$$\Rightarrow \frac{2}{\sqrt{2\pi}} \mu \int_{0}^{\infty} e^{-z^2/2} dz$$

Let $z^2/2 = p$

$2z \frac{dz}{2} = dp$

$dz = \frac{dp}{\sqrt{2p}}$

$$\Rightarrow \frac{2}{\sqrt{2\pi}} \mu \int_{0}^{\infty} e^{-p} \frac{dp}{\sqrt{2p}}$$

$$= \frac{1}{\sqrt{\pi}} \mu \int_{0}^{\infty} p^{-1/2} \cdot e^{-p} \, dp$$

gama function

$$= \frac{\mu}{\sqrt{\pi}} \cdot \Gamma\tfrac{1}{2} = \mu$$

$$E(x) = \mu$$

$$E(x^2) = \mu^2 + \sigma^2$$

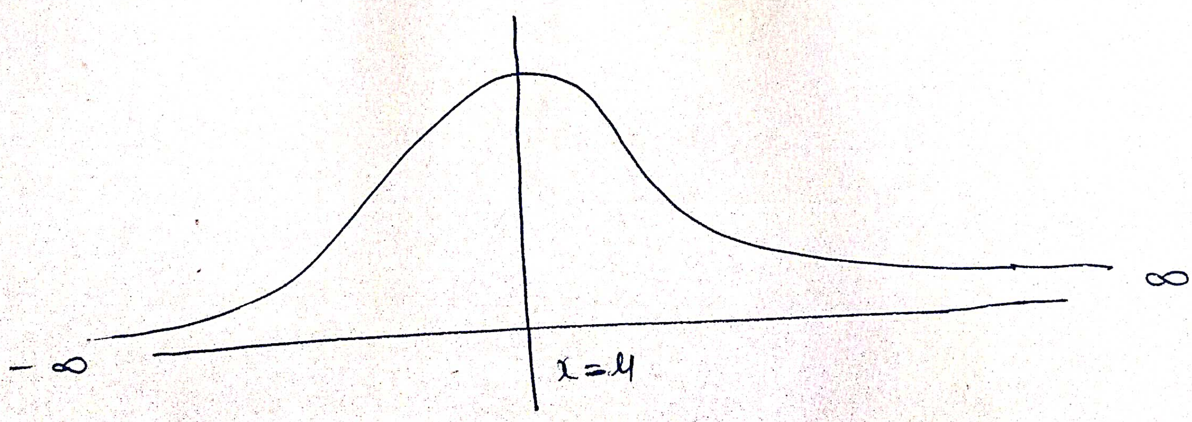
$$Var(x) = E(x^2) - E(x)^2$$

$$= \mu^2 + \sigma^2 - \mu^2$$

$$\boxed{Var(x) = \sigma^2}$$

Area Under the Curve

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



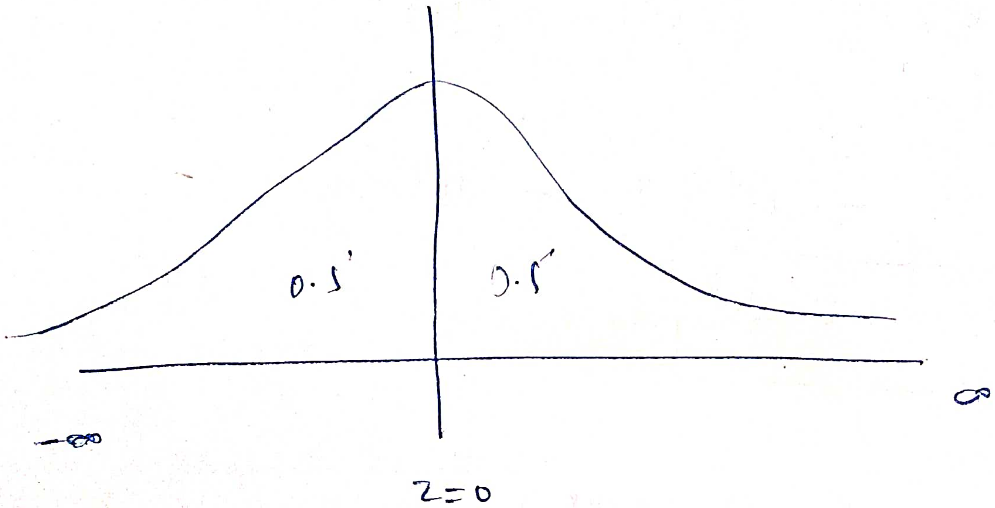
$$-\infty < X < \infty$$

$\mu = 0$

$\sigma = 1$

$$z = \frac{x - \mu}{\sigma}$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \qquad -\infty < z < \infty$$



" $x = \mu$ "

Symmetric

Q1. If the height of 300 students are normally distributed with mean 64.1" inches & S.D. 3.3 inches. How many students have height

1) Less than 5 feet
2) between 5 feet & 5 feet 9 inches

Given

$\mu = 64.5'$

$\sigma = 3.3$

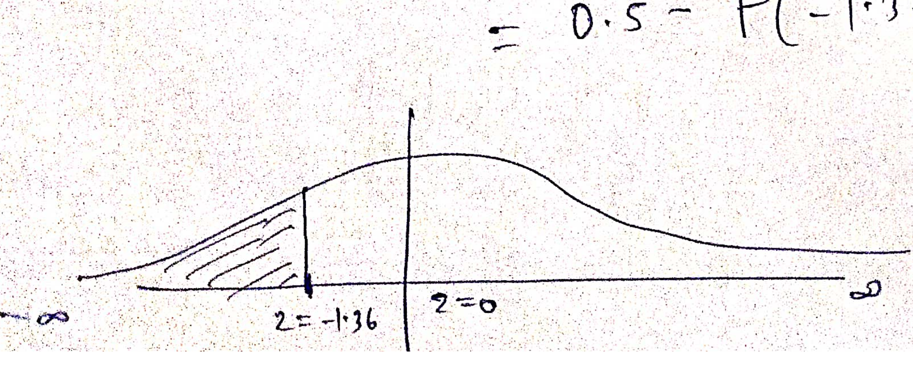
$$z = \frac{x-\mu}{\sigma} = \frac{x - 64.5}{3.3}$$

(1) $P(X < 5) \Rightarrow P(X < 60)$

$$= P\left(z < \frac{60 - 64.5}{3.3}\right)$$

$$= P(z < -1.36)$$

$$= 0.5 - P(-1.36 < z < 0)$$



$P(X < 60) = 0.5 - P(0 < x < 1.36)$

$= 0.5 - 0.4131$

$= 0.0869$

# of students having height less than 5 feet $= 300 \times P(X < 60)$

$= 300 \times 0.0869$

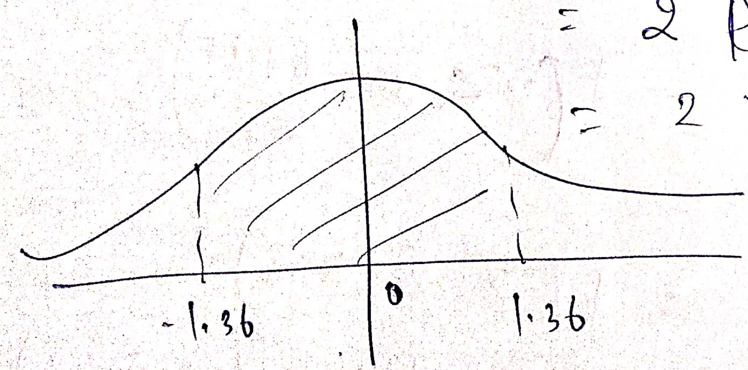
$= 26.07 \simeq 26$ students

(2) $P(60 < X < 69) = P(-1.36 < z < 1.36)$

$= P(-1.36 < z < 0) + P(0 < z < 1.31)$

$= 2\,P(0 < z < 1.36)$

$= 2 \times 0.4131 = 0.8262$



# of students having height between 5 feet & 5 feet'9 $= 300 \times 0.8262$

$= 247.86$

$= 248$ students

Q:. The dist. of 500 workers in a factory is approximately normal with mean & S.D Rs 75 & Rs. 15 resp.

Find the No. of worker who receives weekly wages (i) More than 90

(ii) Less than 45

Given

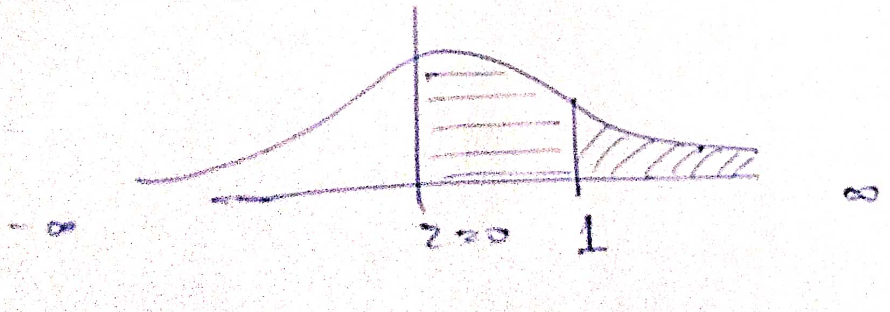
$\mu = 75$

$\sigma = 15$

$$\boxed{z = \frac{x - \mu}{\sigma} = \frac{x - 75}{15}}$$

(i) $P(X > 90) = P(Z > 1)$

$= 0.5 - P(0 < z < 1)$



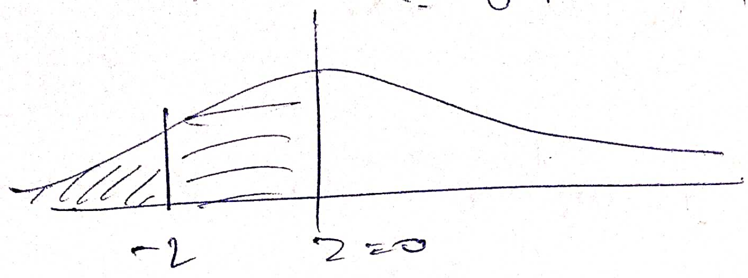
$= 0.5 - 0.3413 = 0.1587$

# of worker who receive weekly wages more than 90/- = $500 \times 0.1587 = 79$

(II) $P(X < 45) = P(Z < -2)$

$= 0.5 - P(-2 < Z < 0)$

$= 0.5 - P(0 < Z < 2)$



$= 0.5 - 0.4772 = 0.0228$

# of workers $= 500 \times 0.0228$

$= 11$ workers