

The auxiliary random variable X is given by

$$\begin{aligned} X &= \|(\mathbf{H}_e + \mathbf{G}_e \mathbf{\Theta} \mathbf{H}) \mathbf{b}\|^2 \\ &= \|\beta_d \mathbf{Z} \mathbf{b} + \beta_r \mathbf{C} \mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2 \\ &= \|\beta_d \mathbf{a} + \beta_r \mathbf{C} \mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2. \end{aligned}$$

where $\mathbf{a} \sim \mathcal{CN}_{N_e,1}(\mathbf{0}, \mathbf{I}_{N_e})$, the mean of X is expressed as

$$\begin{aligned} \mathbb{E}[X] &= \mathbb{E}[\|\beta_d \mathbf{a}\|^2] + \mathbb{E}[\|\beta_r \mathbf{C} \mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2] \\ &= \mathbb{E}[\beta_d^2] \cdot \mathbb{E}[\|\mathbf{a}\|^2] + \mathbb{E}[\beta_r^2] \cdot \mathbb{E}[\|\mathbf{C} \mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2] \\ &= (\beta_d^2 + \beta_r^2 \|\mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2) N_e. \end{aligned}$$

The variance of X is given by

$$\begin{aligned} \text{Var}[X] &= \mathbb{E}[|X|^2] - |\mathbb{E}[X]|^2 \\ &= (\beta_d^4 + \beta_r^4 \|\mathbf{\Theta} \mathbf{H} \mathbf{b}\|^4 + 2\beta_d^2 \beta_r^2 \|\mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2) N_e \end{aligned}$$

Hence, the shape and scale of the Gamma distribution can be given by

$$\begin{aligned} k &= \frac{|\mathbb{E}[X]|^2}{\text{Var}[X]} = N_e, \\ w &= \frac{\text{Var}[X]}{\mathbb{E}[X]} = \beta_d^2 + \beta_r^2 \|\mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2. \end{aligned}$$

With the help of a Gamma distribution, the CDF of X can be expressed as

$$\begin{aligned} F_X(x) &= 1 - \frac{1}{\Gamma(k)} \Gamma\left(k, \frac{x}{w}\right) \\ &= 1 - \frac{1}{\Gamma(N_e)} \Gamma\left(N_e, \frac{x}{\beta_d^2 + \beta_r^2 \|\mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2}\right). \end{aligned}$$

where $\Gamma(m) = \int_0^\infty t^{m-1} e^{-t} dt$ is the Gamma function, and $\Gamma(m, n) = \int_n^\infty t^{m-1} e^{-t} dt$ is the upper incomplete Gamma function. Then, we can deduce an approximate expression of secrecy outage probability as

$$\begin{aligned} P_{\text{out}}(R_s) &= 1 - F_X(\phi) \\ &= \frac{1}{\Gamma(N_e)} \Gamma\left(N_e, \frac{\phi}{\beta_d^2 + \beta_r^2 \|\mathbf{\Theta} \mathbf{H} \mathbf{b}\|^2}\right) \end{aligned}$$

where $\phi = \sigma_e^2 (2^{C_m - R_s} - 1) / P_t$.