

UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

END SEMESTER EXAMINATION –DECEMBER 2022

MC3020 : PROBABILITY AND STATISTICS

Time Allowed: Two (02) Hours

Instructions

1. This paper contains **TWO (2)** parts:
 - (a) **PART 01** contains **15** questions: Each question in this part is a multiple choice with four answer choices. Read each question and answer choice carefully and choose the **ONE** best answer.
 - (b) **PART 02** contains **03** questions.
 2. Answer **all** questions in the answer book provided.
 3. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
 4. This examination accounts for **50%** of module assessment. Total maximum mark attainable is **100**.
 5. Write your **registration number, the module code** and the **title of the paper** in the answer book. Also write your registration number on each additional sheet attached.
 6. Calculators are allowed.
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PART 01 [30 marks]

Underline the correct answer. This part of the question paper should be attached with the answer script.

1. In an area, 80 % of the population is identified to be Covid-19 patients. It is also known that among the identified Covid-19 patients 45 % are infected with the new variant, Omicron. What is the probability of a randomly selected person from that area being a Covid-19 patient infected with the new variant, Omicron?
(a) 0.45 (b) 0.45/0.8 (c) 0.45×0.8 (d) None of the above
2. 10 % of the gas cylinders of the "Blue-gas" company are found to be defective. Retail sellers of the gas cylinder have decided to sell the cylinders until they find a defective cylinder. Calculate the average number of cylinders sold by the retail sellers before they find a defective cylinder.
(a) 100 (b) 10 (c) 1 (d) None of the above

3. Consider ' n ' different objects taken ' r ' at a time and $1 < r < n$; which of the following statement is true

- (a) $nPr > nCr$ (b) $nPr < nCr$ (c) $nPr = nCr$ (d) None of the above

4. If A and B are any two mutually exclusive events, such that $P(A) > 0$, $P(B) > 0$, then;

- (a) A and B are independent events.
 (b) A and B are not independent events.
 (c) A and B are not disjoint events.
 (d) None of the above.

5. If A and B are independent events, then which of the following are not independent events?

- (a) $(B \cap A^c)$ and $(A \cap B^c)$ (c) A and B^c
 (b) B and A^c (d) A^c and B^c

6. Which of the following probability distribution is not related with Bernoulli trials?

- (a) Binomial (b) Geometric (c) Normal (d) All of the above

7. If A and B are any two independent events, such that $0 < P(A) < 1$ and $0 < P(B) < 1$, then; which of the following is true?

- (a) $A \subset B$ (b) $A \cap B = \emptyset$ (c) $A \cap B \neq \emptyset$ (d) None of the above

8. If the occurrence of a rare event follows Poisson distribution, then the probability distribution of the waiting time for the first occurrence is,

- (a) Geometric (b) Normal (c) Uniform (d) Exponential

9. If A , B and C are mutually independent events, then which of the following is not independent with the event A ?

- (a) $(B \cap C)$ (b) $(B \cup C)$ (c) $(A \cup B \cup C)$ (d) None of the above

10. If two events, A and B , are such that $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B^c) = 0.4$, then $P(B \cap A^c | A \cup B)$ is

- (a) 0.1/0.7 (b) 0.2/0.7 (c) 0.3/0.7 (d) None of the above

11. If two events, A and B , are such that $P(A) = x$, $P(B) = y$ and $P(A \cap B) = z$, then $P((B \cap A) | (A \cup B)^c)$ is

- (a) $z/(x + y)$ (b) 0 (c) $y/(x + z)$ (d) 1

12. Assume A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.3$. Then, $P(A|(A \cup B))$ is
- (a) $0.2/0.5$ (b) $0.2/0.44$ (c) $0.2/0.3$ (d) None of the above
13. In a city it is known that the probability of a person infected by COVID-19 is 0.6, and among the infected persons only 5% need ventilator machines. If the total number of persons in that city is 100000, the average number of ventilator machines needed for the city is,
- (a) $100000 * 0.05/0.6$ (c) $100000 * 0.6$
 (b) $100000 * 0.05$ (d) $100000 * 0.6 * 0.05$
14. Which of the following is more evidence against the null hypothesis?
- (a) $P\text{-value} = 0.05$ (c) $P\text{-value} = 0.1$
 (b) $P\text{-value} = 0.001$ (d) None of the above
15. If a Null hypothesis has been rejected with a level of significance α , then which of the following is true?
- (a) Alternative hypothesis is always true
 (b) Null hypothesis is always wrong
 (c) Null hypothesis may still be true
 (d) None of the above

PART 02

Question 1[20 marks]

- (a) The "RAT" test is used to test whether or not a person has COVID-19. The test is not perfect; there is a 30% chance that a person who has COVID -19 is falsely tested negative and a 15% chance that a person who does not have COVID-19 is falsely tested positive. If the COVID-19 is infected among 60% of the tested population.
- A person is selected at random from the population and given the test. If the result is negative, what is the probability that this person has COVID-19?
 - Suppose a person, initially selected at random from the population, is given the test once and result is negative. This person is then given the test, independently, second time and the result is again negative. What is the probability that this person has COVID-19?
- (b) The length of time between breakdowns of an essential piece of equipment is important in the decision of the use of auxiliary equipment. An engineer thinks that the best model for time between breakdowns of a generator is the exponential distribution with a mean of 15 days.
- If the generator has just broken down, what is the probability that it will breakdown in the next 21 days?

- ii. What is the probability that the generator will operate for 30 days without a breakdown?

Question 2 [20 marks]

- (a) In a certain laboratory, the time duration for the "PCR" test follows normal distribution with mean 24 hours and standard deviation 4 hours. A random sample of five "PCR" tests from the laboratory is chosen. Find the probability that exactly 3 of the test's duration is more than 28 hours.
- (b) The time taken for a "RAT" test follows normal distribution with mean " m " minutes and standard deviation 1 minute. It is found that 95% of times taken are longer than 1 minute. Find the value of " m ".
- (c) In a base hospital the number of ventilator machines needed (during the pandemic situation) follows normal distribution with mean 20 and the standard deviation 4. The hospital administration doesn't want to have a shortage for the ventilator machines more often than 2.28% of the time. Then, calculate the minimum number of ventilator machines needed to the hospital?

Question 3 [30 marks]

- (a) A soft drink machine at a steak house is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of 200 milliliters and a standard deviation of 15 milliliters. The machine is checked periodically by taking a sample of 9 drinks and computing the average content. If \bar{x} falls in the interval, $191 < \bar{x} < 209$ then the machine is thought to be operating satisfactorily; Otherwise, we conclude that $\mu \neq 200$ milliliters. Find the probability of committing a type I error when $\mu = 200$ milliliters. Moreover find the probability of committing a type II error and power of the test when $\mu = 215$ milliliters.
- (b) It is claimed that an automobile is driven on the average more than 20000 kilometers per year. To test this claim, a random sample of 100 automobile owners is asked to keep a record of the kilometers they travel. Would you agree with this claim if the random sample showed an average of 23500 kilometers and a standard deviation of 3900 kilometers? Use a p-value in your conclusion. Interpret your results by explaining the meaning of the p-value (Use $\alpha = 5\%$).
- (c) What sample size should be taken in order to estimate the proportion of supports for a political party among the whole population to be within 0.05? It is thought that the proportion is about 0.4, and the confidence level attached to the estimate is to be 95%.