MC8020

Assignment -2 06/06/2023

Q.1)

$N = 100$

$\mu = 12$

$n = 10$

a) $\bar{x} \pm t_{n-1}\, \alpha/2\; S.E(\bar{x})$

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{n} = \frac{126}{10} = 12.6$$

$$S = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}} = \sqrt{\frac{1610 - 10(12.6)^2}{(10-1)}} = \sqrt{2.4889} = 1.5776$$

$\Rightarrow t_{n-1, \alpha/2}$ :

$= t_{10-1, 0.02/2}$

$= t_{9, 0.01}$

$= 2.821$

$\Rightarrow$ 98% C.I estimate for the mean nitrate content

$\Rightarrow \bar{x} \pm t_{n-1, \alpha/2}\; S.E(\bar{x})$

$= \bar{x} \pm t_{9, 0.01} \cdot S/\sqrt{n}$

$= 12.6 \pm 2.821 \times \frac{1.5776}{\sqrt{10}}$

$= (11.1927, 14.0073)$.

b) 95% C.I for the variance content for each 100 kg

$$\left( \frac{(n-1)s^2}{\chi^2_u} , \frac{(n-1)s^2}{\chi^2_L} \right)$$

$n = 10$

$s = 1.5776$

$$\chi^2_u = \chi^2_{9, 0.025}$$

$$= 19.02$$

$$\chi^2_L = \chi^2_{9, 0.975}$$

$$= 2.70$$

$$\left( \frac{(10-1)(1.5776)^2}{19.02} , \frac{(10-1)(1.5776)^2}{2.70} \right)$$

$$= (1.1777, 8.2961)$$

c) 95% C.I for S.D nitrate content for each 100 kg

$$\left( \sqrt{\frac{(n-1)s^2}{\chi^2_u}} , \sqrt{\frac{(n-1)s^2}{\chi^2_L}} \right)$$

$$= \left( \sqrt{1.1777} , \sqrt{8.2961} \right)$$

$$= (1.0852, 2.8803)$$

Q2)

$\mu = 1.20$

$\sigma = 0.32$

$n = 15$

a) $\bar{x} = \dfrac{\sum_{i=1}^{15} x_i}{n}$

$= \dfrac{21.99}{15}$

$= 1.466$

b) 95% C.I on the mean mercury concentration after the accident.

$\bar{x} \pm Z_{\alpha/2} \, S.E(\bar{x})$

$= \bar{x} \pm Z_{0.05/2} \, \sigma/\sqrt{n}$ $\qquad Z_{0.025} = 1.96$

$= 1.466 \pm \left(1.96 \times \dfrac{0.32}{\sqrt{15}}\right)$

$= (1.3041, 1.6280)$

We are 95% confident the true mean mercury concentration lie between (1.3041, 1.6280) after the accident.

a)

Step 1. Hypothesis

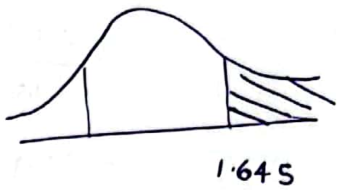
$H_0 : \mu \leq 1.20$

$H_1 : \mu > 1.20$ (Right tail)

Step 2. : Test statistic value calculation

$$z = \frac{\bar{x} - \mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$$= \frac{1.466 - 1.20}{\left(\frac{0.32}{\sqrt{15}}\right)}$$




$$= 3.2194$$

Step 3 :- Critical value.

$$z_\alpha = z_{0.05}$$

$$= 1.645.$$

Step 4 : Test value falls in rejection region,

$1.645 < 3.2194$

So we reject $H_0$

Step 5 :

There is evidence that the mean mercury concentration has increase by the accident, at 5% significance level.

(Mercury concentration has not decrease by the accident at 5% significance level)

Q3)

Let P denote the amount of pollutant in randomly selected automobile in Houston, Texas.

$$P \sim N(\mu = 70, \sigma^2 = 13^2).$$

or

Let X denote the vehicle emission level of nitrogen oxide

$$X \sim N(\mu = 70, \sigma^2 = 13^2)$$

a) $P(X < 60)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{60-70}{13}\right)$

$= P(Z < -0.7692)$

$= P(Z < -0.77)$ $\quad$ $P(Z < -0.7692)$

$= 0.220650$ $\quad$ or $\quad$ $0.22089$.

b) $P(X > 90)$

$= P\left(\frac{X-\mu}{\sigma} > \frac{90-70}{13}\right)$

$= P(Z > 1.5385)$

$= P(Z > 1.54)$

$= P(Z < -1.54)$ $\quad$ $P(Z > 1.5385)$

$= 0.061780$ $\quad$ or $\quad$ $0.061963$

c) $P(60 < x < 90)$

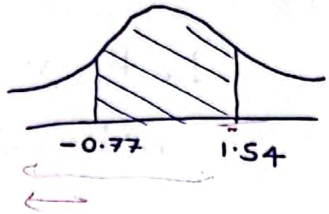
$$= P\left(\frac{60-70}{13} < \frac{x-\mu}{\sigma} < \frac{90-70}{13}\right)$$

$$= P(-0.77 < z < 1.54)$$

$$= P(z < 1.54) - P(z < -0.77)$$

$$= 0.938220 - 0.220650$$

$$= 0.71757.$$



Q4)

Let X be the number of days ahead travelers purchase their airline tickets.

(i). $X \sim \exp(1/15)$

$p.d.f = \lambda e^{-\lambda x}$.

$$P(x < 10) = \int_0^{10} \lambda . e^{-\lambda x} dx$$

$$= \int_0^{10} 1/15 \, e^{-1/15 x} dx$$

$$= 1/15 \int_0^{10} e^{-1/15 x} dx$$

$$= 1/15 \left( \frac{e^{-1/15 x}}{-1/15} \Big|_0^{10} \right)$$

$$= \frac{-15}{15} \left[ e^{-10/15} - e^{0} \right]$$

$$= -1 \, (0.5134 - 1)$$

$$= 0.4866.$$

ii) $P(X < x) = 1/2$.

$$\int_0^x \lambda \cdot e^{-\lambda t} \, dt = 1/2$$

$$\lambda \left[ \frac{e^{-\lambda t}}{-\lambda} \Big|_0^x \right] = 1/2$$

$$\frac{-\lambda}{\lambda} \left[ e^{-\lambda x} - e^{0} \right] = 1/2$$

$$- \left[ e^{-\lambda x} - 1 \right] = 1/2$$

$$e^{-\lambda x} = 1/2$$

$$-\lambda x = \ln(1/2)$$

$$-\frac{1}{15} x = -0.6931$$

$$= 10.3965$$

$$x = 10.3972$$

approximately, half of the travelers will wait 11 days.

Q5)

$\alpha = 92\%$
$E = 2.5$
$\sigma = 9.5$
$Z_{0.08/2} = -1.75$

$$n = \frac{(Z_{\alpha/2})^2 \sigma^2}{E^2}$$

$$= \frac{(-1.75)^2 \times 9.5^2}{2.5^2}$$

$= 44.2225$

$\simeq 45.$

Bonus Question.

Let x denote the amount paid in the estimated tax voucher for first 3 payments.

$X \sim N(\mu = 530, \sigma^2 = 205^2)$

$P(X < x) = 0.75$

$P\left(Z < \frac{x - 530}{205}\right) = 0.75$

$\frac{x - 530}{205} = 0.67$

$x = (0.67 \times 205) + 530$

$= 667.35$