Tutorial - 04

1) 90% C.I for the proportion of workers that prefer union representation.

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= \frac{120}{400} \pm Z_{0.05}\sqrt{\frac{\frac{120}{400}\times\frac{280}{400}}{400}}$$

$$= 0.3 \pm 1.645\sqrt{\frac{0.3\times 0.7}{400}}$$

$$= (0.2623,\ 0.3377)$$

2) $$n = \frac{(Z_{\alpha/2})^2 p(1-p)}{E^2}.$$

E - margin of error / half width

$$n = \frac{(Z_{0.05})^2 (0.6)(0.4)}{(0.02)^2}.$$

$$= \frac{(1.645)^2 (0.6)(0.4)}{(0.02)^2}$$

$$= 1623.615$$

$$\simeq 1624.$$

3)

$$n = \frac{Z_{\alpha/2}^2 \cdot p(1-p)}{E^2}$$

$$= \frac{Z_{0.05}^2 * (0.25)(0.75)}{(0.025)^2}$$

$$= \frac{(1.645)^2 (0.25)(0.75)}{(0.025)^2}$$

$$= 811.8075$$

$$\simeq 812$$

4)

$$n = \frac{(Z_{\alpha/2})^2 \cdot \sigma^2}{E^2}$$

$$= \frac{(Z_{0.025})^2 \times 3^2}{1^2}$$

$$= (1.96)^2 \times 9$$

$$= 34.5744$$

$$\simeq 35$$

5)

$n = 9$

102.01, 103.9, 101.4, 103.7, 102.6, 102.2, 104.2, 101.9, 100.6

a) $s = \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n-1}}$

$\bar{x} = 102.5$

$\sum x^2 = 94568.407$

$\sum x = 922.5$

$= \sqrt{\dfrac{n \sum x^2 - (\sum x)^2}{n(n-1)}}$

$= 1.2159$

95% C.I for the mean rating in ohms of this shipment of resistors.

$\left(\bar{x} - t_{\alpha/2} \dfrac{s}{\sqrt{n}} , \bar{x} + t_{\alpha/2} \dfrac{s}{\sqrt{n}}\right)$

$\left(102.5 - t_{8,0.025} \times \dfrac{1.2155}{\sqrt{9}} , 102.5 + t_{8,0.025} \times \dfrac{1.2155}{\sqrt{9}}\right)$

$\left(102.5 - 2.306 \times \dfrac{1.2155}{\sqrt{9}} , 102.5 + 2.306 \times \dfrac{1.2155}{\sqrt{9}}\right)$

$(101.5657, 103.4343)$

b) 95% C.I for the variance of the ratings in ohms of this shipment of resistors.

$$\left(\frac{(n-1)s^2}{\chi^2_U} , \frac{(n-1)s^2}{\chi^2_L} \right)$$

$$\chi^2_{8,0.025} = 17.53$$

$$\chi^2_{8,0.975} = 2.18$$

$$\left(\frac{8 \times (1.2155)^2}{17.53} , \frac{8 \times (1.2155)^2}{2.18} \right)$$

$$(0.6742 , 5.4218)$$

c) 95% C.I for the standard deviation of the ratings in ohms of this shipment of resistors.

$$\left(\sqrt{\frac{(n-1)s^2}{\chi^2_U}} , \sqrt{\frac{(n-1)s^2}{\chi^2_L}} \right)$$

$$= \left(\sqrt{0.6742} , \sqrt{5.4218} \right)$$

$$(0.8211 , 2.3285)$$

6)

$$n = Z_{\alpha/2}^{2} \frac{\sigma^2}{E^2}$$

$$= \frac{Z_{0.015}^{2} \times (2.5)^2}{(0.5)^2}$$

Range 0-10

$\sigma \simeq \frac{R}{4} = \frac{10}{4} = 2.5$

$$= \frac{2.17^2 \times (2.5)^2}{(0.5)^2}$$

$= 117.7225$

$\simeq 118$

7) $n = 15$

$S = 97$

90% C.I for the pop^n standard deviation of the no. of tissues per box.

$$\left(\sqrt{\frac{(n-1)s^2}{\chi_U^2}} , \sqrt{\frac{(n-1)s^2}{\chi_L^2}} \right)$$

$\chi^2_{0.05, 14} = 23.68$

$\chi^2_{0.95, 14} = 6.57$

$$= \left(\sqrt{\frac{14 \times 97^2}{\chi^2_{0.05, 14}}} , \sqrt{\frac{14 \times 97^2}{\chi^2_{0.95, 14}}} \right)$$

$= (74.5839 \ , \ 141.5967)$

90% C.I for the population variance of the number of tissues per box.

$$\left(\frac{(n-1)s^2}{\chi^2_{0.05,14}} , \frac{(n-1)s^2}{\chi^2_{0.95,14}} \right)$$

$$= (74.5839^2 , 141.5967^2)$$

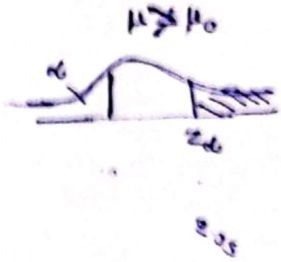
$$= (5562.7581 , 20049.6255)$$

08)

Step 1:

$H_0 : \mu \leq \$42,000$

$H_1 : \mu > \$42,000$ (Right tail)



Step 2:

Test statistic $Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$

$$= \frac{43500 - 42000}{900/\sqrt{36}}$$

$$= 10$$

Step 3

Critical value $Z_{0.02} = 2.05$.

Step 4:

test value falls in the critical region $10 > 2.05$, So reject $H_0 ; \mu \leq \$42,000$

Step 5:

Mean income is significantly higher than \$42,000

09) Step 1

$H_0 : \mu \geq 137.5$

$H_1 : \mu < 137.5$ (left tail)

$\sigma = 24$
$n = 16$
$\bar{x} = 132$

Step 2.

Test statistic, $Z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

$= \dfrac{132 - 137.5}{24/\sqrt{16}}$

$= -0.92$

left → $P(Z < Z_0)$
Right $P(Z > Z_0)$
two tail $2 * P(Z > |Z_0|)$
$P(Z < -|Z_0|)$ or $P(Z > Z_0)$

Step: 3

P-Value $= P(Z < -0.92)$

$= 0.1788$

Step 4:

if p-value ≤ 0.05 reject H_0.

Here p-value $= 0.1788 \geq 0.05$, So donot reject H_0

Step 5:

So we can conclude that the mean weight is not significantly lower than 137.5gm.

∴ Consumers receiving fair measure.

OR

Critical value $Z_{0.05} = -1.645$

10) Step 1:

$H_0: \mu = 950$

$H_1: \mu \neq 950$ (two sided test)

$n = 81$

$\bar{x} = 915$

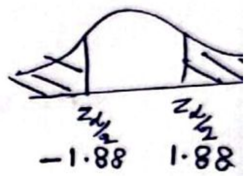
$s = 45$

$\alpha = 0.06$

Step 2:

Test statistic, $Z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$= \dfrac{915 - 950}{45/\sqrt{81}} = -7$



Step 3:

Critical value: $Z_{\frac{0.06}{2}} = 1.88$.

Step 4:

if, $|Z| > 1.88$ Reject H_0

Here $|-7| > 1.88$, So we reject H_0.

Test value falls in the rejection region.

Step 5:

$\therefore$ the mean salary is significantly different than \$950 and the test is carried out 0.06 level of significance.

11)

Step 1:

$H_0 : \mu = 120$

$H_1 : \mu \neq 120$

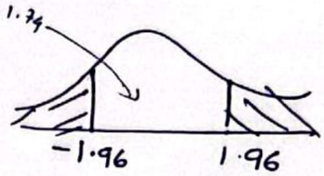
Step 2: (Since sample size is small & normal popⁿ but σ is given so t-test is not used.

Test statistic $Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$,

from data $\bar{x} = \dfrac{1189}{10} = 118.9$.

$= \dfrac{118.9 - 120}{2/\sqrt{10}}$

$= -1.74$



Step 3:

Critical value $Z_{0.025} = 1.96$.

Step 4:

$|Z| = |-1.74| < 1.96$

Test value falls in acceptance region. So we donot reject H_0

Step 5:

So, we can conclude that the mean soap production is not significantly different than 120, and the test is carried out with 95% confidence.

12)

a) $H_0 : P \leq 0.06$

$H_1 : P > 0.06$

$\hat{p} \geq \frac{8}{100} = 0.08$

Reject H_0 if $\hat{p} \geq 0.08$

b) Type I error

$= P(\text{Reject } H_0 \text{ when } H_0 \text{ is true})$

$= P(\hat{p} \geq 0.08 \mid P_0 = 0.06)$

$= P\left(\frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \geq \frac{0.08-0.06}{\sqrt{\frac{0.06(1-0.06)}{100}}}\right)$

$= P(Z > 0.8421)$

$= P(Z > 0.8421)$

$= 1 - P(Z < 0.84)$ or $P(Z < -0.84)$

$= 0.2005$

c) The Type I error indicates that the supplier incurs loss after the canning company wrongly rejects the lot because the sample has a higher number of defectives when in fact the lot has fewer than 6% defectives

The type II error indicates wrongly accepting the lot due to the sample having a lower number of defectives when in fact the lot has higher than 6% defective. Therefore, the canning company incurs loss.

13)

Step 1.

$H_0: \sigma \leq 0.58$

$H_1: \sigma > 0.58$. (Right sided test)

$n=36$ $\bar{x}=8.5$
$s=0.6$

Step 2:

Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}.$$

$$= \frac{(36-1)(0.6)^2}{(0.58)^2}.$$

$$= 37.4554.$$

Step 3:

$$\text{Critical value} = \chi^2_{n-1,\alpha}$$

$$= \chi^2_{35,\,0.05}$$

$$= \frac{43.77+55.76}{2}$$

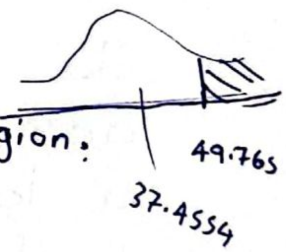
$$= 49.765.$$

$\nu \backslash \alpha$	0.05
30	43.77
40	55.76

Step 4: Test value=37.4554 < 49.765

Test value falls in Acceptance region.

∴ Donot reject H_0.



Step 5:

Population standard deviation is not significantly higher than 0.58.