MC3020 : Probability & Statistics.
Tutorial-1 Solution.

1) C be the event that the person has cancer.
D be the event the doctor diagnoses cancer.

$P(C) = 0.05$ $\quad\quad P(C') = 0.95$.

$P(D|C) = 0.78$

$P(D|C') = 0.06$.

a) $P(D) = P(D|C) \cdot P(C) + P(D|C') \cdot P(C')$

$= 0.78 \times 0.05 + 0.06 \times 0.95$

$= 0.096$.

b) $P(C'|D) = \dfrac{P(D|C') \cdot P(C')}{P(D)}$

$= \dfrac{0.06 \times 0.95}{0.096}$

$\doteq 0.5938$.

2) O : Overrun
A : Consulting firm A
B : Consulting firm B.
C : Consulting firm C.

$P(A) = 0.4$ $\quad P(B) = 0.35$ $\quad P(C) = 0.25$.

$P(O|A) = 0.05$ $\quad P(O|B) = 0.03$ $\quad P(O|C) = 0.15$.

a) $P(C|O) = \dfrac{P(O|C) \cdot P(C)}{P(O|A) \cdot P(A) + P(O|B) \cdot P(B) + P(O|C) \cdot P(C)}$

$= \dfrac{0.15 \times 0.25}{0.05 \times 0.4 + 0.03 \times 0.35 + 0.15 \times 0.25}$

$= \dfrac{0.0375}{0.068}$

$= 0.5515$

b) $$P(A|O) = \frac{P(O|A).P(A)}{P(O|A).P(A) + P(O|B).P(B) + P(O|C).P(C)}$$

$$= \frac{0.05 \times 0.4}{0.0680}$$

$$= 0.2941$$

3)

A	B	C
10	6	8
4D	1D	3D

$$P(D) = P(D|A).P(A) + P(D|B).P(B) + P(D|C).P(C).$$

$$= \frac{4}{10} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{3} + \frac{3}{8} \times \frac{1}{3}$$

$$= \frac{2}{15} + \frac{1}{18} + \frac{1}{8}$$

$$= 0.3139.$$

4) A: Production of certain article from machine A
B: " machine B
D: Defective item.

$P(A) = 0.1$ $P(B) = 0.9$
$P(D|A) = 0.01$ $P(D/B) = 0.05.$

$$P(D) = P(D/A).P(A) + P(D/B).P(B)$$

$$= 0.01 \times 0.1 + 0.05 \times 0.9$$

$$= 0.001 + 0.045$$

$$= 0.046.$$

b) $P(A/D) = \dfrac{P(D/A) \cdot P(A)}{P(D)}$

$= \dfrac{0.01 \times 0.1}{0.046}$

$= 0.0217$

5) $P(A \cap B \cap (C \cup D))$

$= P(A) \cdot P(B) \cdot P(C \cup D)$

$= P(A) \cdot P(B) [1 - P(C' \cap D')]$

$= P(A) \cdot P(B) [1 - P(C') \cdot P(D')]$

$= 0.9 \times 0.9 [1 - (1-0.9)(1-0.9)]$

$= 0.81 \times 0.99$

$= 0.8019$

OR

$P(A \cap B \cap (C \cup D))$

$= P(A) \cdot P(B) \cdot P(C \cup D)$

$= P(A) \cdot P(B) [P(C) + P(D) - P(C \cap D)]$

$= P(A) \cdot P(B) [P(C) + P(D) - P(C) \cdot P(D)]$

$= 0.9 \times 0.9 \; (0.9 + 0.9 - (0.9 \times 0.9)]$

$= 0.81 \quad (1.8 - 0.81)$

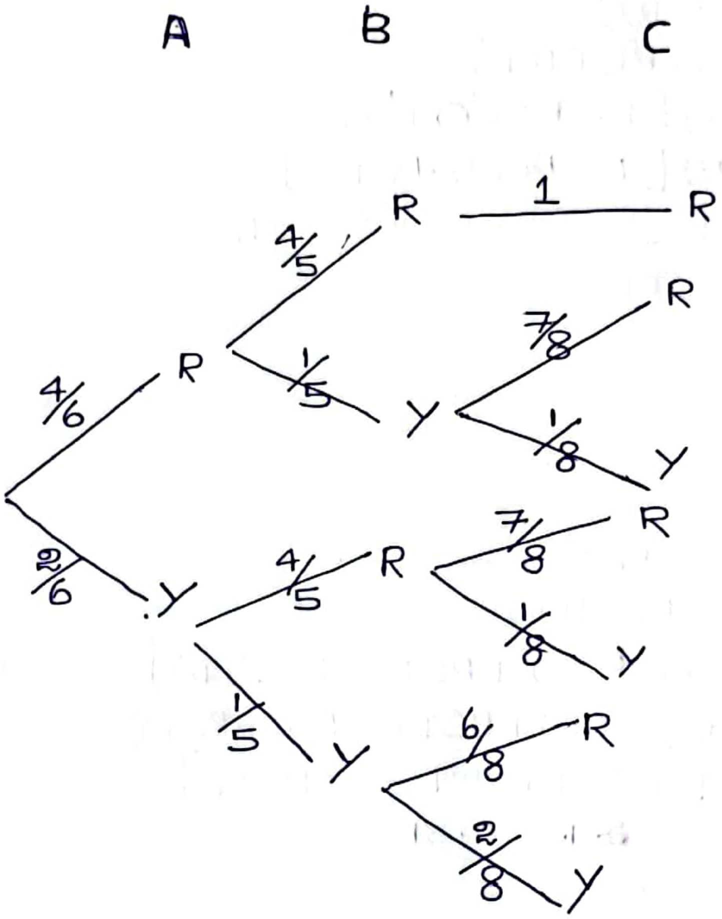
$= 0.81 \times 0.99$

$= 0.8019$

6)

A	B	C
4R	4R	6R
2Y	1Y	

a)



b) P(Record yellow exactly twice)

$$= P(RYY) + P(YRY) + P(YYR)$$

$$= \frac{4}{6} \times \frac{1}{5} \times \frac{1}{8} + \frac{2}{6} \times \frac{4}{5} \times \frac{1}{8} + \frac{2}{6} \times \frac{1}{5} \times \frac{6}{8}$$

$$= \frac{1}{60} + \frac{1}{30} + \frac{1}{20}$$

$$= \frac{1}{10}.$$

c) Let x be the event A is red
Let z be the event exactly 2 yellow balls.

$x = \{\{RRR\}, \{RYR\}, \{RYY\}\}$.

$Z = \{\{RYY\}, \{YRY\}, \{YYR\}\}$.

$x \cap Z = \{\{RYY\}\}$.

$$P(x|z) = \frac{P(x \cap Z)}{P(Z)}$$

$$= \frac{P(RYY)}{P(Z)}$$

$$= \frac{\frac{4}{6} \times \frac{1}{5} \times \frac{1}{8}}{\frac{1}{10}}$$

$$= \frac{1}{6}$$

$$= 0.1667.$$

7) a) R be Republican.
M be Male.

	R	D	I	
M	46	39	1	86
F	5	9	0	14
	51	48	1	

$$P(R/M) = \frac{P(R \cap M)}{P(M)}$$

$$= \frac{\frac{46}{100}}{\frac{86}{100}}$$

$$= \frac{23}{43}$$

$$= 0.5349.$$

OR, $P(R/M) = \frac{46}{86}$.

$= 0.5349$

b) $$P(M/R) = \frac{P(M \cap R)}{P(R)}$$

$$= \frac{46/100}{51/100}$$

$$= \frac{46}{51}$$

$$= 0.90196$$

$$= 0.902$$

No.

c) $P(F/I) = 0$

d) A: Selected senator is a female.
B: Selected Senator is a Republician.

$P(A \cap B) = 5/100 \neq 0$

A & B are not mutually exclusive.

8) 4 Women & 3 men $\Rightarrow$ 3 are selected.

a) P(All 3 selected will be woman)

$$= \frac{4C_3 \times 3C_0}{7C_3}$$

$$= \frac{\frac{4!}{3!\,1!} \times \frac{3!}{3!\,0!}}{\frac{7!}{4!\,3!}}$$

$$= 4/35$$

$$= 0.1143$$

b) P(All 3 selected will be men)

$$= \frac{3C_3 \times 4C_0}{7C_3}$$

$$= 1/35$$

$$= 0.0286$$

c) P(2M and 1W)

$$= \frac{3C_2 \times 4C_1}{7C_3}$$

$$= \frac{\frac{3!}{2!\,1!} \times \frac{4!}{3!\,1!}}{\frac{7!}{4!\,3!}}$$

$$= 12/35$$

$$= 0.3429$$

d) P(1M and 2W)

$$= \frac{3C_1 \times 4C_2}{7C_3}$$

$$= \frac{3 \times 6}{35}$$

$$= 18/35$$

$$= 0.5143$$

9)

a) A : Employee smoke.

B : Graduated from college.

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{19/89}{44/89}$

$= 19/44$

$= 0.4318$

b) A : Employee smoke.

H : Employee didnot graduate from high school.

$P(A/H)$

$= \frac{P(A \cap H)}{P(H)}$

$= \frac{25/89}{68/89}$

$= \frac{25}{68}$

$= 0.3676.$