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MC 3020 - Assignment 02

50 minutes **06 - 06 - 2023**

Answer all the questions(1-5), and a bonus question is given on the next page, giving you a chance to gain extra five marks.

1. A fertilizer mixing machine is set to give 12 kg of nitrate for every quintal bag of fertilizer. Ten 100 kg bags are randomly selected and examined. The percentages of nitrate in the sample are as follows:

$$11, 14, 13, 15, 13, 11, 13, 14, 10, 12$$

Assume that the sample is from a normally distributed population.

   (a) Construct a 98% confidence interval estimate for the mean nitrate content for each 100 kg bag in the company.

   (b) Construct a 95% confidence interval for the variance nitrate content for each 100 kg bag in the company.

   (c) By using the results from part(b), construct a 95% confidence interval for the standard deviation nitrate content for each 100 kg bag in the company.

2. The concentration of mercury in a lake has been monitored for a number of years. Measurements taken on a weekly basis yielded an average of $1.20mg/m^3$ (milligrams per cubic meter) with a standard deviation of $0.32mg/m^3$. Following an accident at a smelter on the shore of the lake, 15 measurements produced the following mercury concentrations.

$$1.60, 1.77, 1.61, 1.08, 1.07, 1.79, 1.34, 1.07, 1.45, 1.59, 1.43, 2.07, 1.16, 0.85, 2.11$$

   (a) Give a point estimate (samples mean) of the mean mercury concentration after the accident.

   (b) Construct a 95% confidence interval on the mean mercury concentration after the accident. Interpret this interval.

   (c) Is there sufficient evidence that the mean mercury concentration has increased since the accident? Use a $\alpha = 0.05$.

3. Ahigh accumulation of ozone gas in the lower atmosphere at ground level is air pollution and can be harmful to people, animals, crops, and various materials. Elevated levels above the national standard may cause lung and respiratory disorders. Nitrogen oxides and hydrocarbons are known as the chief "precursors" of ozone. These compounds react in the presence of sunlight to produce ozone. The sources of these precursor pollutants include cars, trucks, power plants, and factories. Large industrial areas and cities with heavy summer traffic are the main contributors to ozone formation. The United States Environmental Protection Agency (EPA) has developed procedures for measuring vehicle emission levels of nitrogen oxide. Let P denote the amount of this pollutant in a randomly selected automobile in Houston, Texas. Suppose the distribution of P can be adequately modelled by a normal distribution with a mean level of $\mu = 70$ ppb (parts per billion) and standard deviation of $\sigma = 13$ ppb.

(a) What is the probability that a randomly selected vehicle will have emission levels less than 60 ppb?

(b) What is the probability that a randomly selected vehicle will have emission levels greater than 90 ppb?

(c) What is the probability that a randomly selected vehicle will have emission levels between 60 and 90 ppb?

4. The number of days ahead travelers purchase their airline tickets can be modeled by an exponential distribution with the average amount of time equal to 15 days. Find the probability that a traveler will purchase a ticket fewer than ten days in advance. How many days do half of all travelers wait?

5. A survey is to be conducted to determine the average driving in miles by Faculty of Engineering students at Ariviyal Nagar. The investigator wants to know how large the sample should be so that he/she can be 92% confident on the estimate and the estimate is within 2.5 miles of the true average. A similar study was conducted in past and it was found that the standard deviation of the students' driving distances was 9.5 miles.

***Bonus Question (5 Marks)**

Analysis of income tax returns from the previous year indicates that for a given income classification, the amount of money owed to the government over and above the amount paid in the estimated tax vouchers for the first three payments is approximately normally distributed with a mean of \$530 and a standard deviation of \$205. Find the 75th percentile for this distribution of measurements. The government wants to target that group of returns having the largest 25% of amounts owed.

****The maximum mark possible to obtain for this assignment is 100.***

Some useful formulas:

1. If X follows normal distribution with parameter μ and σ then $\left(\frac{X-\mu}{\sigma}\right) = Z$ follows standard normal distribution with $\mu = 0$ and $\sigma = 1$.

2.

Parameter	$(1-\alpha) * 100\%$ confidence interval	Test statistic value
Mean (μ)	Case 1: $\left(\bar{X} \mp Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}\right)$	Case 1: $Z = \frac{\bar{X}-\mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}$
	Case 2: $\left(\bar{X} \mp t_{\frac{\alpha}{2},df} * \frac{S}{\sqrt{n}}\right)$	Case 2: $T = \frac{\bar{X}-\mu_0}{\left(\frac{S}{\sqrt{n}}\right)}$
	Case 3: $\left(\bar{X} \mp Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}\right)$	Case 3: $Z = \frac{\bar{X}-\mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}$
	Case 4: $\left(\bar{X} \mp Z_{\frac{\alpha}{2}} * \frac{S}{\sqrt{n}}\right)$	Case 4: $Z = \frac{\bar{X}-\mu_0}{\left(\frac{S}{\sqrt{n}}\right)}$
Variance (σ)	$\left(\frac{(n-1)*S^2}{\chi^2_U}, \frac{(n-1)*S^2}{\chi^2_L}\right)$	

3. Sample size calculation based on mean confidence interval : $n = \left(Z_{\frac{\alpha}{2}} * \frac{\sigma}{E}\right)^2$ (where $E-$ margin error)

4. Sample variance (S) can be estimated

$$S = \sqrt{\frac{\sum\left(X-\bar{X}\right)^2}{n-1}} = \sqrt{\frac{n\sum X^2 - \left(\sum X\right)^2}{n(n-1)}}$$

5. Case 1: when population is normal and σ is known, case 2: when population is normal and σ is unknown, case 3: when population is not normal and σ is known, sample size n is large and case 3: when population is not normal and σ is known, sample size n is large