UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

Mid Semester Examination - May 2023

MC3020: Probability and Statistics

(Duration: 1 hour)

Answer all the questions(1-5), and a bonus question is given on the next page, giving you a chance to gain extra five marks.

---

1. Use the following data from the 100 senators from the 108th Congress of the United States:

	Republican	Democrat	Independent
Male	47	38	1
Female	6	8	0

   (a) If we randomly select one senator, what is the probability of getting a Republican, given that a male was selected?

   (b) If we randomly select one senator, what is the probability of getting a male, given that a Republican was selected? Is this the same result found in (a)?

   (c) If we randomly select one senator, what is the probability of getting a female, given that an Independent was selected?

   (d) If we randomly select one Senator, let A be the event that the selected Senator is a female and B be the event that the selected Senator is a Republican. Are events A and B mutually exclusive?

2. Suppose that 25% of all subscribers to an internationally circulated engineering magazine (IEEE Spectrum) earn an annual income in excess of $\$100,000$. The magazine polls 24 subscribers at random to determine the income category into which each fall.

   (a) What is the probability that none of the 24 subscribers earns more than $\$100,000$?

   (b) Find the probability that exactly half of those chosen earns in excess of $\$100,000$.

   (c) Find the probability that three or fewer earn in excess of $\$100,000$.

   (d) Find the expected number of subscribers with income exceeding $\$100,000$ one would expect to find in the random sample of size 24.

3. A certain material is fed to a two-step process. For this process, the probabilities of a malfunction are $P(B_1) = 0.03$ and $P(B_2) = 0.05$, where the factors B_1 and B_2 represent a malfunction in Steps 1 and 2, respectively. A sample of the final

product is taken and found to be unacceptable. Our experience over the previous two months indicates that a defective product will be obtained 20% of the time if Section 1 of the process malfunctions and 36% of the time if Section 2 malfunctions. That means $P(E|B_1) = 0.20$ and $P(E|B_2) = 0.36$. In which part of the process does the fault probably lie?

4. The traffic Police officer's radar speed gun causes grief for many drivers who use the Jaffna-Kandy (A9) road. A particular location between Ariviyal Nagar and Iranaimadu junction in Kilinochchi catches six speeding motorists per hour. This figure is an average of all hours between 9am and 5pm.

    (a) How many motorists would you expect to be caught speeding by the Police officer in any given 30 minutes period? And which distribution would best be used to model the number of speeding motorists in a 30 minutes period?

    (b) What proportion of 30 minutes period would you expect the Police officer's radar speed gun to remain unused to deliver a fine? That is, what is the probability that the Police officer catches no motorists in 30 minutes period?

5. The IQ scores of 1800 applicants for admission to a tuition free graduate school are normally distributed with a mean of 125 and a standard deviation of 10.

    (a) If an applicant is chosen at random, what is the probability that the IQ score is less than 142?

    (b) What is the 92% of these score?

    (c) If the admission policy is to refuse entry to any applicant with an IQ below 110, how many applicants will not be admitted?

    (d) Between which two IQ scores symmetrically located about the mean are 94% of the score?

***Bonus Question (5 Marks)**

The time intervals between successive barges passing a certain point on a busy waterway have an exponential distribution with mean 8 minutes. Find the probability that the time interval between two successive barges is less than 5 minutes.

****The maximum marks possible to obtain for this exam is 100.***

Some useful formulas:

1. If X follows binomial distribution with parameters n, p. Then, the probability mass function is given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x};\ x = 0, 1, 2, \cdots n$$

2. If X follows Poisson distribution with parameters λ. Then, the probability mass function is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!};\ x = 0, 1, 2, \cdots\cdots$$

3. If X follows exponential distribution with parameter λ. Then, the probability density function is given by
$$f(x) = \lambda e^{-\lambda x}; \quad x > 0$$

4. If X follows normal distribution with parameter μ and σ then $\left(\frac{X-\mu}{\sigma}\right) = Z$ follows standard normal distribution with $\mu = 0$ and $\sigma = 1$

5. If A and B are two events in a sample space. The conditional probability of event B occurring, given that event A has already occurred, is given by
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

6. Let us consider that a sample space S is divided into two mutually exclusive partitions S_1 and S_2. An event H has occurred, and $P(S_1|H)$ can be written as
$$P(S_1|H) = \frac{P(S_1)P(H|S_1)}{P(S_1)P(H|S_1) + P(S_2)P(H|S_2)}$$

3. If X follows exponential distribution with parameter λ. Then, the probability density function is given by
$$f(x) = \lambda e^{-\lambda x}; \quad x > 0$$

4. If X follows normal distribution with parameter μ and σ then $\left(\frac{X-\mu}{\sigma}\right) = Z$ follows standard normal distribution with $\mu = 0$ and $\sigma = 1$

5. If A and B are two events in a sample space. The conditional probability of event B occurring, given that event A has already occurred, is given by
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

6. Let us consider that a sample space S is divided into two mutually exclusive partitions S_1 and S_2. An event H has occurred, and $P(S_1|H)$ can be written as
$$P(S_1|H) = \frac{P(S_1)P(H|S_1)}{P(S_1)P(H|S_1) + P(S_2)P(H|S_2)}$$