Tutorial-05

1)

a) Jone $P_1 = \frac{110}{600}$ $\quad\quad \alpha = 0.1$

Smith $P_2 = \frac{80}{600}$ $\quad\quad Z_{0.05} = 1.645$

90% C.I for the true difference in proportions of E's given by prof.smith & prof.Jone.

90% C.I for $P_1 - P_2$ is

$$= \left((\hat{P}_1 - \hat{P}_2) - Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}} , \right.$$

$$\left. \hat{P}_1 - \hat{P}_2 + Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}} \right)$$

$$= \left[\left(\frac{110}{600} - \frac{80}{600}\right) - 1.645\sqrt{\frac{\frac{110}{600}\times\frac{490}{600}}{600} + \frac{\frac{80}{600}\times\frac{520}{600}}{600}} , \right.$$

$$\left. \left(\frac{110}{600} - \frac{80}{600}\right) + 1.645\sqrt{\frac{\frac{110}{600}\times\frac{490}{600}}{600} + \frac{\frac{80}{600}\times\frac{520}{600}}{600}} \right]$$

$$= (0.05 - 1.645\times 0.02102 , 0.05 + 1.645\times 0.02102)$$

$$= (0.0154 , 0.0846)$$

b)

Step 1.

$H_0 : P_1 \leq P_2$

$H_1 : P_1 > P_2$ (Right sided test)

Step 2.

Test statistic

$$Z = \frac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{\bar{P}(1-\bar{P})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$\bar{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{110+80}{600+600} = \frac{190}{1200} = 0.1583.$$

$$= \frac{\frac{110}{600} - \frac{80}{600} - 0}{\sqrt{\frac{190}{1200} \times \frac{1010}{1200}\left(\frac{1}{600} + \frac{1}{600}\right)}}$$

$$= 2.3725.$$

Step 3:

Critical value $Z_{0.03} = 1.88$.

Step 4:

Test value > Critical value

Test value falls on the rejection region. So reject H_0

Step 5:

We conclude that the rate of E's professor Jones is significantly higher than that of professor smith & the test is carried out with 3% level of significance.

2)

$$P_1 = \frac{176}{400} = 0.44$$

$$P_2 = \frac{216}{600} = 0.36$$

Step 1

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ (two sided test)

Step 2.

Test statistic $$Z = \frac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{\bar{P}(1-\bar{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\bar{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{176+216}{400+600} = \frac{392}{1000} = 0.392$$

$$Z = \frac{0.44 - 0.36 - 0}{\sqrt{0.392(1-0.392)\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= 2.5386$$

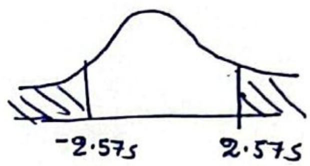
Step 3:

Critical value $= Z_{0.005} = 2.575$

Step 4:

Test value $= |2.5386| < $ Critical value. 2.575

Test value falls on the acceptance region. So donot reject H_0.



Step 5:

We can conclude that percentages of students in two types of schools are not differed in their opinion at 1% level of significance.

b)

$Z_{\alpha/2} = Z_{0.035} = 1.81$

93% C.I for the difference between the percentages $P_1 - P_2$ is

$$= \left(\hat{P}_1 - \hat{P}_2 - Z_{0.035}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}},\ \hat{P}_1 - \hat{P}_2 + Z_{0.035}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\right)$$

$$= \left(0.44 - 0.36 - 1.81\sqrt{\frac{0.44(1-0.44)}{400} + \frac{0.36(1-0.36)}{600}},\right.$$
$$\left. 0.44 - 0.36 + 1.81\sqrt{\frac{0.44(1-0.44)}{400} + \frac{0.36(1-0.36)}{600}}\right)$$

$$= (0.08 - 0.0572,\ 0.08 + 0.0572)$$

$$= (0.0228,\ 0.1372)$$

3) $D = A - B = 25 \quad 32 \quad 16 \quad 17 \quad -3 \quad 26 \quad 22 \quad -1 \quad 13 \quad 15 \quad 21 \quad 4$

a) Step 1: $n = 12$

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$ (two sided test)

$$\bar{D} = \frac{\Sigma D}{n} = \frac{187}{12} = 15.5833$$

Step 2:

Test statistic

$$T = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}}$$

$$S_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}} = \sqrt{\frac{12 \times 4215 - (187)^2}{12 \times 11}} = 10.87497 = 10.875$$

$$= \frac{15.5833 - 0}{\frac{10.875}{\sqrt{12}}} = 4.9639$$

Step 3:

Critical value $t_{n-1,\alpha/2} = t_{11,0.025} = 2.201$.

Step 4:

Test value $= |4.9639| > 2.201 =$ Critical value

Test value falls in rejection region.

So reject H_0 at 5% level of significance.

Step 5:

So we conclude that mean sales for two salespersons are significantly different, and the test is carried out at 5% level of significance.

b)

95% CI for $\mu_A - \mu_B$ is.

$$\left(\bar{D} - t_{n-1,\alpha/2}\cdot\frac{S_D}{\sqrt{n}},\ \bar{D} + t_{n-1,\alpha/2}\cdot\frac{S_D}{\sqrt{n}}\right)$$

$$\doteq\left(15.5833 - 2.2010\times\frac{10.875}{\sqrt{12}},\ 15.5833 + 2.201\times\frac{10.875}{\sqrt{12}}\right)$$

$$=(8.6736,\ 22.49299)$$

$$=(8.6736,\ 22.4930)$$

Interpretation

We are 95% confident that the true mean difference is between 8.6736 and 22.4930.

4) Two are independent pop^n dist^n not normal, σ_1^2, σ_2^2 are unknown. n_1, n_2 are large.

a)

Step 1.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ (two sided test)

$n_1 = 80$ $\quad$ $n_2 = 100$

$\bar{x}_1 = 10.0$ $\quad$ $\bar{x}_2 = 10.25$

$s_1 = 0.6$ $\quad$ $s_2 = 0.72$

Step 2:

Test statistic $Z = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$$= \frac{10.0 - 10.25 - (0)}{\sqrt{\dfrac{0.6^2}{80} + \dfrac{0.72^2}{100}}}$$

$$= -2.5405.$$

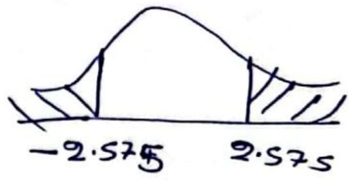
Step 3:

Critical value $= Z_{\frac{0.01}{2}} = 2.575$

Step 4:

Test value falls in acceptance region. $|-2.5405| < 2.575$

So, we donot reject H_0.



Step 5:

There is no significant change in the interest rates for large retailers at 0.01 level of significance.

b)

$Z_{\frac{0.04}{2}} = 2.05$

96% C.I for $\mu_1 - \mu_2$ is

$$\left(\bar{x}_1 - \bar{x}_2 - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \, , \, \bar{x}_1 - \bar{x}_2 + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

$$= \left(10 - 10.25 - 2.05\sqrt{\frac{0.6^2}{80} + \frac{0.72^2}{100}} \, , \, 10 - 10.25 + 2.05\sqrt{\frac{0.60^2}{80} + \frac{0.72^2}{100}} \right)$$

$$= (-0.4517 \, , -0.0483)$$

96% C.I for $\mu_2 - \mu_1$ is $(0.0483, 0.4517)$.

5)

Corn	5.6	7.1	4.5	6.0	7.9	4.8	5.7
Cauliflower	15.9	13.4	17.6	16.8	15.8	16.3	17.1

Samples from independent normal pop^n given that assume that the pop^n variances are unknown and equal

a)

$$\bar{x}_1 = \frac{\sum x_1}{n} = \frac{41.6}{7} = 5.9429.$$

$$s_1^2 = \frac{n\sum x_1^2 - (\sum x_1)^2}{n(n-1)} = \frac{7 \times (255.96) - (41.6)^2}{7 \times (7-1)} = 1.4562$$

$$\bar{x}_2 = \frac{\sum x_2}{n} = \frac{112.9}{7} = 16.1286$$

$$s_2^2 = \frac{n\sum x_2^2 - (\sum x_2)^2}{n(n-1)} = \frac{7 \times 1832.11 - (112.9)^2}{7 \times 6} = 1.8657$$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1) \times 1.4561 + (7-1)1.8657}{7+7-2}}$$

$$= 1.2888 \qquad t_{12,0.05} = 1.782$$

90% C.I for $\mu_1 - \mu_2$ is

$$\left(\bar{x}_1 - \bar{x}_2 - t_{n_1+n_2-2,\alpha/2} S_p\sqrt{1/n_1 + 1/n_2},\ \bar{x}_1 - \bar{x}_2 + t_{n_1+n_2-2,\alpha/2} S_p\sqrt{1/n_1 + 1/n_2}\right)$$

$$= \left(5.9429 - 16.1286\} - 1.782 \times 1.2888\sqrt{1/7 + 1/7},\right.$$

$$\left. 5.9429 - 16.1286 + 1.782 \times 1.2888\sqrt{1/7 + 1/7}\right)$$

$$= (-11.4133, -8.9579)$$

90% C.I for $\mu_2 - \mu_1$ is $(8.9579, 11.4133)$

b) Given that population's have equal variances.

Step 1:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two sided test

Step 2.

Test statistic $T = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$

$= \dfrac{5.9429 - 16.1286 - 0}{1.2888 \sqrt{1/7 + 1/7}}$

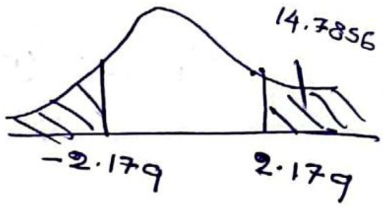
$= -14.7856$

Step 3:

Critical value $t_{n_1+n_2-2, \alpha/2} = t_{12, 0.025} = 2.179$

Step 4:

$|-14.7856| > 2.179$



Test value falls in rejection region. So, we can reject H_0.

Step 5:

We can conclude that there is a significant different between the means at 5% level of significance.

c) (population variances are unknown and notequal)

90% C.I for $\mu_1-\mu_2$ is

$$\left(\bar{x}_1-\bar{x}_2-t_{v,\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\ ,\ \bar{x}_1-\bar{x}_2+t_{v,\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\right)$$

$$A=\frac{s_1^2}{n_1}=\frac{1.4562}{7}=0.2080 \qquad B=\frac{s_2^2}{n_2}=\frac{1.8657}{7}=0.2665$$

$$v=\frac{(A+B)^2}{\frac{A^2}{n_1-1}+\frac{B^2}{n_2-1}}=\frac{(0.2080+0.2665)^2}{\frac{0.2080^2}{6}+\frac{0.2665^2}{6}}=11.8203 \simeq 12$$

$$t_{12,0.10/2}=1.782$$

$$=\left(5.9429-16.1286-1.782\times\sqrt{\frac{1.4562}{7}+\frac{1.8657}{7}}\ ,\ 5.9429-16.1286+1.782\sqrt{\frac{1.4562}{7}+\frac{1.8657}{7}}\right)$$

$$=(-11.4133,\ -8.9581)$$

90% C.I for $\mu_2-\mu_1$ is

$(8.9581,\ 11.4133)$

d)

Population variances are unknown & unequal.

Step 1.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$. (two sided test)

Step 2:

Test statistic

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{5.9429 - 16.1286 - 0}{\sqrt{\frac{1.4562}{7} + \frac{1.8657}{7}}}$$

$$= -14.7859.$$

Step 3:

Critical value $t_{v, \alpha/2} = t_{12, 0.025} = 2.179$

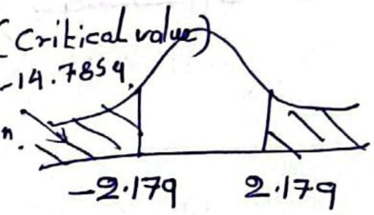
d.f $= 12.8203 \approx 12$ (from part c)

Step 4.

Test value $|t| = |-14.7859| > 2.179$ (Critical value)

Test value falls rejection region.

So reject H_0.



Step 5:

We can conclude that there is a significant different between the means at 5% level of significance.

2)

F_{α, v_1, v_2} or $F^{v_1}_{v_2, \alpha}$

$\alpha = 0.05$

$\alpha = 10\%$.

$v_2 \backslash v_1$	Numerator d.f
denominator d.f	

Step 1.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2:

Test statistic $F = \dfrac{s_1^2}{s_2^2} = \dfrac{1.4562}{1.8657}$

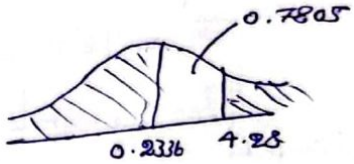
$= 0.7805$

$F_{\alpha, v_1, v_2} = \dfrac{1}{F_{1-\alpha, v_2, v_1}}$

Step 3:

Critical value $= F'_{0.05, 6, 6} = 4.28$

$F_{0.95, 6, 6} = \dfrac{1}{F_{0.05, 6, 6}} = \dfrac{1}{4.28} = 0.2336$



Step 4: $0.2336 < F < 4.28$

Test value falls in acceptance region

So, Donot reject H_0.

Step 5:

We conclude that the variances are not significantly different & the test is carried out 10% level of significance.

6)

Samples are from normal pop^n

. σ_1^2, σ_2^2 are unknown. . Sample size is small

$F =$

At $26^\circ c$	At $5^\circ c$
$n_1 = 6$	$n_2 = 6$
$S_1 = 14.7705$	$S_2 = 23.9562$
$\Sigma x_1^2 = 166095$	$\Sigma x_2^2 = 862443$
$\Sigma x_1 = 995$	$\Sigma x_2 = 2271$
$\bar{x}_1 = 165.833$	$\bar{x}_2 = 378.5$

We want to check variances are equal or unequal.

Step 1:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

Step 2:

Test statistic $F = \frac{S_1^2}{S_2^2} = \frac{14.7705^2}{23.9562^2} = 0.3801$

Step 3:

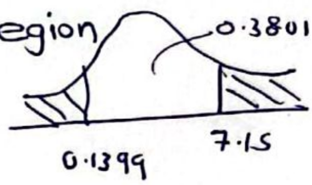
Critical value $F_{0.025, 5, 5} = 7.15$

$F_{0.975, 5, 5} = \frac{1}{7.15} = 0.1399$

Step 4: $0.1399 < F = 0.3801 < 7.15$

Test value falls in acceptance region

Donot reject H_0.



Step 5:

We can conclude that the variances are not significantly different

Step 1:

$H_0 : \mu_{26} - \mu_5 \geq 0$ OR $\mu_{26} \geq \mu_5$

$H_1 : \mu_{26} - \mu_5 < 0$ $\quad \mu_{26} < \mu_5$ (left sided test)

Step 2:

Test statistic

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_{26} - \mu_5)}{s_p \sqrt{1/n_1 + 1/n_2}}$$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{5 \times 14.7705^2 + 5 \times 23.9562^2}{10}}$$

$$= 19.9006$$

$$= \frac{165.833 - 378.5 - 0}{19.9006 \sqrt{1/6 + 1/6}}$$

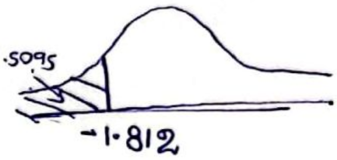
$$= -18.5095$$

Step 3:

Critical value $= t_{0.05,10} = -1.812$.

Step 4:

Test value $= -18.5096 <$ Critical value $= -1.812$



Test value falls in rejection region.

So reject H_0.

Step 5:

We can conclude that μ_{26} is less than μ_5. at 5% level of significant.

b)

$t_{n_1+n_2-2,\alpha/2} = t_{10, 0.025} = 2.228$

95% C.I on the difference in two pop^n mean $\mu_{26} - \mu_5$ is

$$\left(\bar{x}_1 - \bar{x}_2 - t_{n_1+n_2-2,\alpha/2}\, S_p \sqrt{\tfrac{1}{n_1} + \tfrac{1}{n_2}},\ \bar{x}_1 - \bar{x}_2 + t_{n_1+n_2-2,\alpha/2}\, S_p \sqrt{\tfrac{1}{n_1} + \tfrac{1}{n_2}} \right)$$

$$= \left(165.833 - 378.5 - 2.228 \times 19.9006 \sqrt{\tfrac{1}{6} + \tfrac{1}{6}},\ 165.833 - 378.5 + 2.228 \times 19.9006 \sqrt{\tfrac{1}{6} + \tfrac{1}{6}} \right)$$

$$= (-238.2659,\ -187.0681)$$

It indicates that the average warm temperature rat blood pressure is between 187 and 238 units lower than the average 5°C rat blood pressure.

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