Tutorial - 02 - Solutions

01)

(a) $\sum P(X=x) = 0$

$0.20 + 0.30 + c + 0.10 + 0.15 = 1$

$0.75 + c = 1$

$c = 0.25$

(b) $E(X) = \sum x . P(X=x) = \mu$

$\mu = 1 * 0.20 + 2 * 0.30 + 3 * 0.25 + 4 * 0.10 + 5 \times 0.15$

$= 0.20 + 0.60 + 0.75 + 0.40 + 0.75$

$= 2.7$

(c) $Var(X) = \sigma^2 = \sum_{\forall x} (x-\mu)^2 . P(X=x)$

$= (1-2.7)^2 \times 0.20 + (2-2.7)^2 \times 0.30 + (3-2.7)^2 \times 0.25$
$+ (4-2.7)^2 * 0.10 + (5-2.7)^2 \times 0.15$

$= 0.578 + 0.147 + 0.0225 + 0.169 + 0.7935$

$= 1.71$

(d) Standard deviation $= \sigma = \sqrt{1.71}$

$= 1.3077$

(e) Mean Absolute deviation = MAD

$MAD = E|x - E(x)|$

$= \sum |x - E(x)| . P(X=x)$

$= |1-2.7| \times 0.20 + |2-2.7| * 0.30 + |3-2.7| * 0.125$

$+ |4-2.7| \times 0.10 + |5-2.7| \times 0.15$

$= 0.34 + 0.21 + 0.075 + 0.13 + 0.345$

$= 1.1$

(f) P(fewer than two patients are waiting for treatment)

$= P(X < 2)$

$= P(X = 1) = 0.20$

(g) P (at least ~~four~~ three patients are waiting)

$= P(X \geq 3) = P(X=4) + P(X=5) + P(X=3)$

$= 0.10 + 0.15 + 0.25$

$= 0.25 + 0.25 = 0.50$

$= 0.50$

2) (a) Expected number of undershirts produced in a week

$= \sum x \cdot P(X = x)$

$= 10000 \times 0.30 + 20000 \times 0.40 + 40000 \times 0.20 + 70000 \times 0.10$

$= 3000 + 8000 + 8000 + 7000$

$= 26000$

(b) Expected weekly profit = profit on each undershirt * Expected number

$= 26000 \times \$2.25$

$= \$\ 58500$

(c) Expected profit per undershirt after adding sequins

$= \$2.25 - (\$10000 / 26000)$

$= \$\ 1.87$

$\therefore$ Expected weekly profit $= 26000 * \$1.87$

$= \$48620$

(3)

(a) $P(X=12) = 0.7520$

(b) To check this is legitimate probability distribution we need to verify the $\sum_{\forall x} P(X=x) = 1$

$0.10 + 0.007 + 0.007 + 0.013 + 0.032 + 0.068 + 0.070 + 0.041$
$+ 0.752 = 1$

$\therefore$ this is legitimate probability dist^{n}

(c) $P(X \geq 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9)$
$+ P(X=10) + P(X=11) + P(X=12)$

$= 1 - P(X < 6)$

$= 1 - P(X \leq 5)$

$= 1 - [P(X=5) + P(X=4)]$

$= 1 - 0.010 - 0.007$

$= 1 - 0.017$

$= 0.9830$

(d) $P(X > 6) = 1 - P(X \leq 6)$

$= 1 - [P(X=6) + P(X=5) + P(X=4)]$

$= 0.9830 - 0.007$

$= 0.9760$

(e) P(student completed at least one year of high scool)

$= P(X \geq 9)$

$= 0.068 + 0.070 + 0.041 + 0.752$

$= 0.9310$

(4) ~~Ra~~ part (a) and (d) are not an example of binomial experiment.

Resons:

part (a): does not satisfy the condition that each trial can result in one of two outcomes.

part (d) is also not an example of a binomial experiment because it does not satisfy the first requirement of a fixed number of trials.

(5) Let x be the number of subscribers to a nationally circulated bussiness magazine earn an income in excess of \$ 45000.

$$x \sim Bin(n = 20, p = 0.25)$$

The probability mass function is given by

$$P(X=x) = {}_{20}C_x \cdot (0.25)^x (0.75)^{20-x} \; ; \; x = 0,1,2 \cdots 20$$

(a) $P(X=0) = {}_{20}C_0 \, (0.25)^0 (0.75)^{20}$

$= 0.0032$

(b) $P(X=10) = {}_{20}C_{10} \, (0.25)^{10} (0.75)^{10}$

$= 0.0099$

(c) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \sum_{x=0}^{3} {}_{20}C_x \cdot (0.25)^x (0.75)^{20-x}$

$= 0.0032 + 0.0211 + 0.0669 + 0.1339$

$= 0.2251$

d) $E(x) = n \cdot p$

$= 20 (0.25) = 5$

(e) $Var(x) = n \cdot p \cdot q = 20 * 0.25 * 0.75$

$= 3.75$

(6) Let X be the number of selected parts are good (non-defective)

$$X \sim \text{Binomial}(n=4,\ p=0.9)$$

$$\Rightarrow p(X=x) = {}^4C_x \cdot (0.9)^x (1-0.9)^{4-x}$$

A

Accept the lot : if all four selected parts are good

$$p(\text{accept}) = p(X=4)$$

$$= {}^4C_4 \cdot (0.9)^4 (0.1)^{4-4}$$

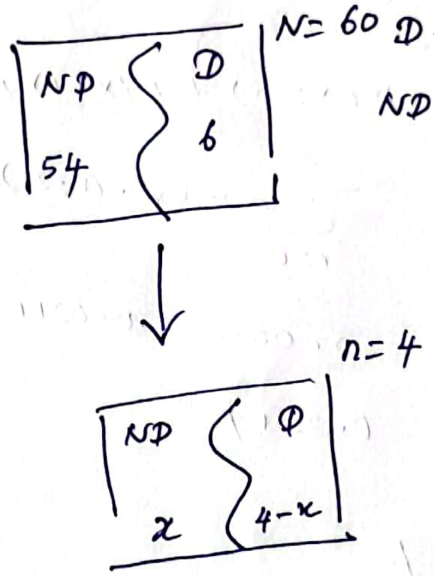
$$= 0.6561$$

(7) ~~Let the random variable X~~ ~~same as 6~~

~~$X \sim Bn$~~

N= 60 D - Defective

ND - Non-defective



Let x be the number of selected parts are non-defective

$$P(X=x) = \frac{{}^{54}C_x * {}^6C_{4-x}}{{}^{60}C_4}$$

$$p(\text{accept}) = p(X=4) = \frac{{}^{54}C_4 * {}^6C_{4-4}}{{}^{60}C_4}$$

$$= \frac{54!/50! \times 4!}{60!/56! \times 4!}$$

$$= \frac{54! \times 56!}{50! \times 60!}$$

$$= \frac{54 \times 53 \times 52 \times 51}{60 \times 59 \times 58 \times 57}$$

$$= 0.6670$$

Therefore, the probability that the quality control technician will accept the lot is approximately 0.6670.

08) Let x be the number of patients arrival at the emergency room of a government hospital in any hour.

$x \sim$ poisson $(\lambda = 2/\text{hour})$

$$p(x=x) = \frac{e^{-2} \cdot 2^{x}}{x!} \; ; \; x = 0, 1, 2, \ldots$$

(a) $p(x=2) = \frac{e^{-2} \cdot 2^{2}}{2!} = 0.2707$

(b) $p(x \geq 1) = 1 - p(x < 1) = 1 - p(x=0)$

$= 1 - 0.1353 = 0.8647.$

(c) Find 95th percentile, ← 95% data falls →

$P(X \leq x_0) = 0.95$ $\qquad$ $P_{95} = x_0$ (say)

So we need to find x_0 value such that $P(X \leq x_0) = 0.95$

$P(X \leq 0) = 0.1353 \simeq 0.13$

$P(X \leq 1) = 0.4060 \simeq 0.41$

$P(X \leq 2) = 0.6767 \simeq 0.68$

$P(X \leq 3) = 0.8571 \simeq 0.89$

$P(X \leq 4) = 0.9473 \simeq 0.95$

$P(X \leq 5) = 0.9834 \simeq 0.98$

0.9473 is very close to 0.95. Therefore, we can say $x_0 = 4$

$\therefore$ $x = 4$ is the 95th percentile.

(9) Let x be the number of flows in given meter

$X \sim$ poisson ($\lambda = 0.2$/meter)

$$P(X=x) = \frac{e^{-0.2} \cdot (0.2)^x}{x!} \quad ; \quad x = 0, 1, 2, \ldots$$

$$P(X=0) = \frac{e^{-0.2} * (0.2)^0}{0!} = 0.8187$$

(10) <u>method I</u>

In this case, each trials are not independent, ~~thats why each~~ So we can't use Binomial distr

Let c be the correct amperage

(a) P(accept the lot)

= P(all three blows at the correct amperage)

= P(CCC)

$= \frac{16}{20} \times \frac{15}{19} \times \frac{14}{18} = \frac{28}{57} = 0.4912$

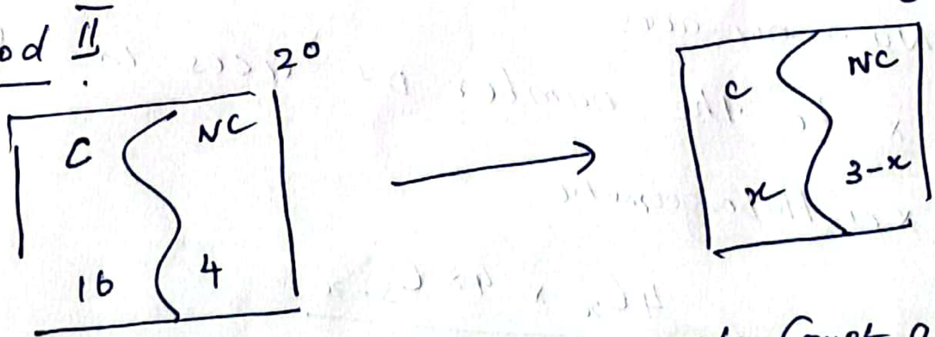
(b) P(2 out of 3 are correct)

$= {}^3C_2 * \frac{16}{20} \times \frac{15}{19} \times \frac{4}{18} = \frac{8}{19} = 0.4210$

(c) P(1 out of 3 are correct)

$= {}^3C_1 * \frac{16}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{8}{95} = 0.0842$

<u>Method II</u>



C - Correct amperage   NC - not Correct amperage.

Let X be the number of correct amperage in sample.

$X \sim$ Hypergeom dist^n $\quad P(X=x) = \dfrac{16C_x \cdot 4C_{3-x}}{20C_3}$; $x = 0,1,2,3$

(a) $P(X=3) = \dfrac{16C_3 \cdot 4C_0}{20C_3}$

$= 0.4912$

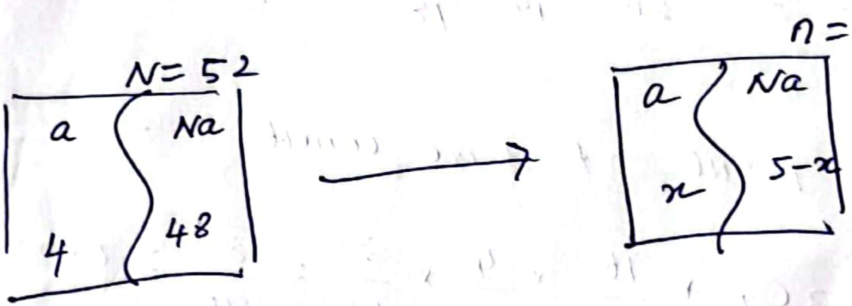
(b) $P(X=2) = \dfrac{16C_2 \cdot 4C_1}{20C_3} = \dfrac{120 \times 4}{1140}$

$= 0.4210$

(c) $P(X=1) = \dfrac{16C_1 \cdot 4C_2}{20C_3} = \dfrac{16 \times 6}{1140}$

$= 0.0842$

(II)



a - aces

Na - non-aces

Let X be the number of aces in sample

$X \sim$ Hypergeometric

$P(X=x) = \dfrac{4C_x \times 48C_{5-x}}{52C_5}$

$P(X=3) = \dfrac{4C_3 \times 48C_2}{52C_5} = 0.0017$

12) Let X be the non-defective chips.

$X \sim Bin(12, 0.95)$

$P_X(x) = \binom{n}{x} p^x q^{n-x}.$

$P(X=12) = \binom{12}{12}(0.95)^{12}(0.05)^0$

$= (0.95)^{12}$

$= 0.54036$

13) Let X be no. of passengers arrival given minutes

$X \sim Poi(10/min).$

$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2 \ldots$

a) $P(X=0) = \frac{e^{-10} 10^0}{0!}$

$= e^{-10}$

$= 4.54 \times 10^{-5}.$

b) $P(X \leq 3) = \sum_{x=0}^{3} \frac{e^{-10} 10^x}{x!}$

$= \frac{e^{-10}10^0}{0!} + \frac{e^{-10}10^1}{1!} + \frac{e^{-10}10^2}{2!} + \frac{e^{-10}10^3}{3!}$

$= e^{-10}\left(1 + 10 + 50 + \frac{1000}{6}\right)$

$= 0.0103.$

c) 10 passenger per minutes

1 min $\rightarrow$ 10

15 sec $\rightarrow$ $\frac{10}{60} \times 15$

$= 2.5$

$\therefore$ $X \sim Poi(2.5 / sec)$.

$$P(x=0) = \frac{e^{-2.5} \, 2.5^0}{0!}$$

$$= e^{-2.5}$$

$$= 0.0821$$

d) $P(x \geq 1) = 1 - P(x=0)$

$$= 1 - e^{-2.5}$$

$$= 0.9179.$$

14) Let X be the no. of flaw on a surface of glass per square meter.

$X \sim Poi(4 / \text{square meter})$.

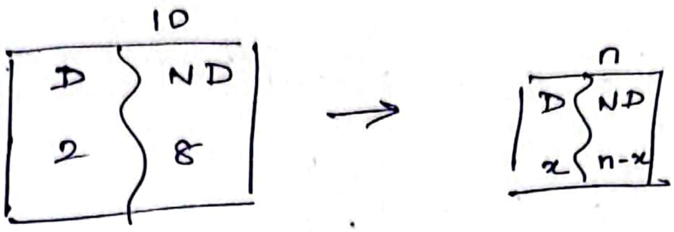
for 2 square meters.

$X \sim Poi(8)$.

$$P(x=5) = \frac{e^{-8} \, 8^5}{5!}$$

$$= 0.0916.$$

15)



a) If $n=3$

Let X be the no. of defective unit in a sample

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\binom{2}{0}\binom{8}{3}}{\binom{10}{3}}$$

$$= 1 - 0.4667$$

$$= 0.5333.$$

b) If $n=4$

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\binom{2}{0}\binom{8}{4}}{\binom{10}{4}}$$

$$= 1 - 0.3333$$

$$= 0.6667.$$

c) If $n=5$

$$P(x \geq 1) = 1 - P(x=0)$$

$$= 1 - \frac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}}$$

$$= 1 - 0.2222$$

$$= 0.7778.$$

d) When there is n, $P(n) = 0.9$

$$0.9 = 1 - \frac{\binom{2}{0}\binom{8}{n}}{\binom{10}{n}}$$

$$0.1 = \frac{\binom{8}{n}}{\binom{10}{n}}$$

$$= \frac{\cancel{\frac{8!}{(8-n)!\,n!}}}{\cancel{\frac{10!}{(10-n)!\,n!}}}$$

$$= \frac{(10-n)(9-n)\cancel{(8-n)!}\ \cancel{8!}}{\cancel{(8-n)!}\ \cancel{8!} \times 9 \times 10}$$

$$9 = (10-n)(9-n)$$

$$n^2 - 19n + 81 = 0$$

$$n = \frac{-(-19) \pm \sqrt{19^2 - 4(1)(81)}}{2(1)}$$

$n = 12.54$
or
$n = 6.46$

16) Let x be the no. of fatalities involved by driver

$X \sim bin(15, 0.7)$

a) $P(X=12) = \binom{15}{12}(0.7)^{12}(0.3)^{15-12}$

$= \binom{15}{12}(0.7)^{12}(0.3)^{3}$

$= 0.1700.$

b) $P(X \geq 13) = 1 - P(X \leq 12)$

$= 1 - \sum_{x=0}^{12}\binom{15}{x}(0.7)^{x}(0.3)^{15-x}$

$= 1 - 0.8732$

$= 0.1268$

c) $E(x) = np$

$= 15 \times 0.7$

$= 10.5.$