Department of Inter-Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 3020: Probability and Statistics

Tutorial: 03 **May 2023**

1) The weights of frying chickens delivered to a fast-food outlet are normally distributed with a mean of 500 gms and a standard deviation of 100 gms. If a chicken is chosen at random, what is the probability that:
   a) The weight is less than 650 gms?
   b) The weight is less than 345 gms?
   c) The weight is between 480 and 710 gms?
   d) The weight is more than 590 gms?

2) The IQ scores of 1,500 applicants for admission to a tuition-free graduate school are normally distributed with a mean of 125 and a standard deviation of 10.
   a) If an applicant is chosen at random, what is the probability that the IQ score is less than 142?
   b) If an applicant is chosen at random, what is the probability that the IQ score is more than 108?
   c) What is the 88th percentile of these scores?
   d) What is the 36th percentile of these scores?
   e) If the admissions policy is to refuse entry to any applicant with an IQ below 100, how many applicants will not be admitted?
   f) Between which two IQ scores symmetrically located about the mean are 95% of the scores?

3) The average annual rainfall in our region is 40 inches, with a standard deviation of 5 inches. Assume that annual rainfall is normally distributed.
   a) If it takes at least 36 inches to make a "good" rice crop, what is the probability that rice farmers will have enough rain for a "good" crop?
   b) The apple crop will suffer if it rains less than 38 inches or more than 50 inches. What is the probability that the crop will suffer because of rain?

4) The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.
   a) A letter to "Dear Abby" inspired one classical use of the normal distribution. A wife claimed to have given birth 308 days after a brief visit from her husband, who was serving in the Navy. Given this information, find the probability of a pregnancy lasting 308 days or longer. What does the result suggest?
   b) If we stipulate that a baby is premature if the length of pregnancy is in the lowest 4%, find the length that separates premature babies from those who are not

premature. Premature babies often require special care, and this result could be helpful to hospital administrators in planning for that care.

5) The ages of the thousands of residents of a retirement community are normally distributed with a mean of 70 and a standard deviation of 4 years.
   a) What proportion of this population is between 60 and 80?
   b) Samples of nine residents are chosen at random. What is the mean of the means of the samples?
   c) Refer to part b. What is the standard deviation of the sample means?
   d) If one sample of nine residents is chosen at random, what is the probability that the sample mean age will be between 68.5 and 71.75?
   e) Between which two symmetric limits are 95% of all the possible values of the sample means?

6) The mean number of photocopies made each working day at a buying house is 250 with a standard deviation of 50 copies. Records show that the number of copies per day is normally distributed. Knowing that each copy costs the firm $0.05, what is the probability that the mean daily cost for 25 days will be less than $11.85?

7) A laundry detergent manufacturing industry produces a machine that fills boxes of laundry flakes. The machine is set for 16 oz net weight. The net weights of boxes filled from the machine have a mean of 16 oz and a standard deviation of 0.3 oz, and they are normally distributed.
   a) What proportion of the boxes will have net weights of more than 15 oz?
   b) What proportion of the boxes will have net weights of more than 16.5 oz?
   c) What proportion of the boxes weights differing from the mean weight for all boxes by more than 0.4 oz?
   d) If a sample of 32 boxes is drawn, what is the probability that its mean net weight will be greater than 16.3 oz?
   e) If a sample of 32 boxes is drawn, what is the probability that its mean net weight will be less than 15.8 oz?

8) In a survey of the preferences of 400 consumers, 90 consumers preferred frozen pizza brand A. If the percentage of consumers that prefer brand A is actually 20%, find the approximate probability of observing 90 or less in a sample of 400 that prefer brand A. Based on this probability, comment on the claimed percentage of 20%.

9) Forty-five percent of all customers of a dairy company prefer large curd cottage cheese to small curd cottage cheese. What is the approximate probability that a simple random sample of 100 of these customers would contain at least 57 persons who preferred small curd cottage cheese?

10) An agricultural industry finds that the length of the pea pods for its olive-green pea crop is normally distributed with a mean of 11.5 cm and a standard deviation of 1.6 cm. A sample of 16 pea pods is chosen at random.
    a) What is the standard deviation of the sampling distribution of the mean of this sampling process?
    b) What is the probability that the sample mean will exceed 12.3 cm?
    c) Between which two symmetrically located values will 68% of the sample means lie?

11) A selective college would like to have an entering class of 1,200 students. Because not all students who are admitted accept, the college admits more than 1,200 students. Past experience shows that about 70% of the students admitted will accept. The college decides to admit 1,500 students. Assume that students make their decisions independently.
    a) What are the mean and the standard deviation of the number X of students who accept?
    b) Use the normal approximation to find the probability that at least 1,000 students accept.
    c) The college does not want more than 1,200 students. What is the probability that more than 1,200 will accept?
    d) If the college decides to increase the number of admissions offers to 1,700, what is the probability that more than 1,200 will accept?

12) An engineer constructing a bridge across a river is concerned of the possible occurrence of a flood exceeding 100 m3/s which can seriously affect his work. If a flow of such magnitude is exceeded once in 5 years on average, on the basis of recorded data, what is the chance that the work which is scheduled to last 14 months can proceed without interruption or detrimental effects?

13) Commonly, car cooling systems are controlled by electrically driven fans. Assuming that the lifetime T in hours of a particular make of fan can be modelled by an exponential distribution with $\lambda = 0.0003$ find the proportion of fans which will give at least 10000 hours service. If the fan is redesigned so that its lifetime may be modelled by an exponential distribution with $\lambda = 0.00035$, would you expect more fans or fewer to give at least 10000 hours service?

14) The time intervals between successive barges passing a certain point on a busy waterway have an exponential distribution with mean 8 minutes.
    a) Find the probability that the time interval between two successive barges is less than 5 minutes.
    b) Find a time interval t such that we can be 95% sure that the time interval between two successive barges will be greater than t.