UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

END SEMESTER EXAMINATION - FEBRUARY 2020

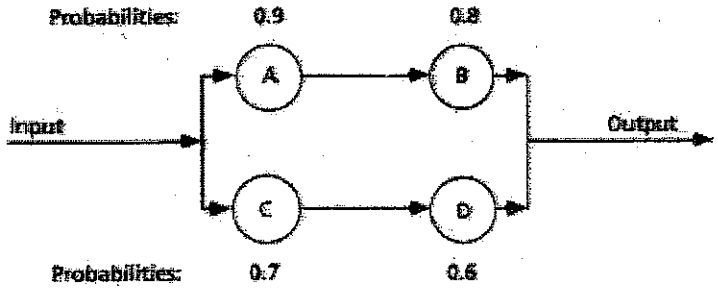
MC 3020 : PROBABILITY AND STATISTICS

(Duration: 2 hours)

This question paper contains 4 questions. Answer ALL questions.

Question 1[20 marks]

1. A system consists of two branches in parallel, each branch having two components. The probabilities of successful operation of components A, B, C,and D are $0.9, 0.8, 0.7$, and 0.6, as shown below. If a component fails, the output from its branch is zero. If only one branch operates, the output is 50%. If both branches operate, the output is 100%. Find the probability of zero output.



2. Machines A and B turn respectively 10% and 90% of total production of a certain type of article. The probability that machine A turns out a defective item is 0.01 and the probability that machine B turns out a defective item is 0.05.

   (a) What is the probability that an article taken at random from the production line is defective?

   (b) What is the probability that an article taken at random from the production line was made machine A, given that it is defective?

Question 2[20 Marks]

1. A large industrial firm purchases several new word processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of word processors X, purchased each year has the following probability distribution:

$X = x$	3	4	5	6	7	8
$P(X = x)$	0.15	0.30	c	0.18	0.10	0.02

   (a) compute the value of c?

   (b) Compute the mean of the random variable X.

   (c) Compute the standard deviation of the random variable X.

   (d) What is the probability that the industrial firm purchases more than 4 word processors in a given year?

2. A report from the Secretary of Health and Human Services stated that 75% of single vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 15 single vehicle traffic fatalities that occur at night on weekend is selected.

   (a) Find the probability that exactly 12 involves a driver who is intoxicated.

   (b) Find the probability that at least 12 involves a driver who is intoxicated.

   (c) What is the expected and variance number of driver who is intoxicated.

Question 3[30 Marks]

1. The number of customers arriving per hour at a certain automobile service facility is assumed to follow a Poisson distribution with mean $\lambda = 420$.

   (a) What is the probability that at least three customers will arrive in a 30 seconds period?

   (b) What is the expected number of customers ($E(X)$) arrived in given day?

2. A gas supplier maintains a team of engineers who are available to deal with leaks reported by customers. Most reported leaks can be dealt with quickly but some require a long time. The time (excluding travelling time) taken to deal with reported leaks is found to have a mean of 65 minutes and a standard deviation of 60 minutes. Assuming that the times may be modeled by a normal distribution.

   (a) Find the probability that it will take more than 185 minutes to deal with a reported leak.

   (b) Find the probability that it will take between 50 minutes and 125 minutes to deal with a reported leak.

3. A study was conducted by the Department of Zoology at the Virginia Tech to estimate the difference in the amounts of the chemical orthophosphorus measured at two different station on the James River. Orthophosphorus was measured in milligrams per liter. Eighteen samples were collected from station 1 and 15 samples were obtained from station 2. The 18 samples from station 1 had an average orthophosphorus content of 3.84 milligrams per liter and a standard deviation of 3.07 milligrams per liter, while the 15 samples from station 2 had an average orthophosphorus content of 1.49 milligrams per liter and a standard deviation of 0.80 milligram per liter.

   (a) Find the margin of error.
   (Hint:Margin of error = Critical value × Standard deviation)

   (b) Find a 95% confidence interval for the difference in the true average orthophosphorus contents at these two station.

Question 4[30 marks]

1. The following data are a result of an investigation as to the effect of reaction temperature x on percent conversion of a chemical process y.

Observation	Temperature ($°C$)	Conversion (%)
1	200.00	43
2	250.00	78
3	200.00	69
4	250.00	73
5	189.65	48
6	260.35	78
7	225.00	65
8	225.00	74
9	225.00	76
10	225.00	79
11	225.00	83
12	225.00	81

   (a) Taking temperature as an independent variable and conversion as a dependent variable, compute SS_{xx}, SS_{yy}, and SS_{xy}.

   (b) Find the least squares regression line.

   (c) Briefly explain the meaning of the values of $\hat{\beta}_1$ and $\hat{\beta}_0$.

   (d) Calculate r and r^2 and explain what they mean.

   (e) Predict the conversion of a chemical process at the temperature $260°C$.

   (f) Construct a 95% confidence interval for β_1.

   (g) Test at the 5% significance level if β_1 is positive.