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MC 3020 - Assignment 01 - Model Answers

50 minutes **02 - 05 - 2023**

Answer all the questions(1-6), and a bonus question is given on the next page, giving you a chance to gain extra five marks.

1. The number of patients waiting for acupuncture treatments at a clinic has the following probability distribution:

x	1	2	3	4	5	6
$P(X = x)$	0.20	k	0.25	0.10	0.10	0.05

   (a) Determine the value of k?

   (b) What is the expected number of patients waiting for treatment?

   (c) What is the standard deviation of the number of patients?

   (d) What is the probability that fewer than three patients are waiting for treatment?

   (e) What is the probability that at least four patients are waiting for treatment?

Solutions:

(a) By the property of the discrete probability mass function,

$$\sum_{\text{for all } x} P(X = x) = 1$$

This implies, $0.20 + k + 0.25 + 0.10 + 0.10 + 0.05 = 1$
$k = 1 - (0.20 + 0.25 + 0.10 + 0.10 + 0.05) = 0.30$
$k = 0.30$

(b) The expected number of patients waiting for treatment is given by

$$\mu = E(X) = \sum_{\text{for all} x} xP(X = x)$$

$$\begin{aligned} E(X) &= \sum_{\text{for all} x} xP(X = x) \\ &= 1 \times 0.20 + 2 \times 0.30 + 3 \times 0.25 + 4 \times 0.10 + 5 \times 0.10 + 6 \times 0.05 \\ &= 2.75 \end{aligned}$$

(c) The variance of the number of patients is given by

$$\sigma^2 = Var(X) = \sum_{\text{for all}} (x - \mu)^2 P(X = x)$$

$$
\begin{aligned}
Var(X) &= \sum_{\text{for all}} (x-\mu)^2 P(X=x) \\
&= (1-2.75)^2 \times P(X=1) + (2-2.75)^2 \times P(X=2) + (3-2.75)^2 \times P(X=3) + \\
&\quad (4-2.75)^2 \times P(X=4) + (5-2.75)^2 \times P(X=5) + (6-2.75)^2 \times P(X=6) \\
&= (1-2.75)^2 \times 0.20 + (2-2.75)^2 \times 0.30 + (3-2.75)^2 \times 0.25 + \\
&\quad (4-2.75)^2 \times 0.10 + (5-2.75)^2 \times 0.10 + (6-2.75)^2 \times 0.05 \\
&= 1.9875
\end{aligned}
$$

(d) $P(X < 3) = P(X = 2) + P(X = 1) = 0.20 + 0.30 = 0.50$

(e) $P(X \geq 4) = 1 - P(X < 4) = 1 - (P(X = 3) + P(X = 2) + P(X = 1))$
$= 1 - (0.20 + 0.30 + 0.25) = 0.25$

2. You have submitted five proposals for upgrading the manufacturing facilities in your process area. From past experience you feel that the chance for any one project to be approved by the Finance Committee is 0.6. Accepting that $p = 0.6$ and that selection is a random event, what are the chances

   (a) that one project will be approved?

   (b) that at least one project will be approved?

   (c) that at most three projects will be approved?

   Solutions: Let X be the number of proposals which is approved by Finance Committee. Then, X follows binomial distribution with parameters $n = 5, p = 0.6$ and probability mass function is given by

$$P(X = x) = \binom{5}{x} 0.6^x 0.4^{5-x}; \quad x = 0, 1, 2, 3, 4, 5$$

   (a) $P(X = 1) = \binom{5}{1} 0.6^1 0.4^{5-1} = 0.0768$

   (b) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \binom{5}{0} 0.6^0 0.4^{5-0} = 1 - 0.0102 = 0.9898$

   (c) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
   $= \binom{5}{0} 0.6^0 0.4^{5-0} + \binom{5}{1} 0.6^1 0.4^{5-1} + \binom{5}{2} 0.6^2 0.4^{5-2} + \binom{5}{3} 0.6^3 0.4^{5-3}$
   $= 0.6630$

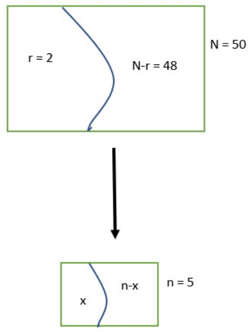
3. In the production of avionics equipment for civilian and military use, one manufacturer randomly inspects 10% of all incoming parts for defects. If any of the parts is defective, all the rest are inspected. If two of the next box of 50 diodes are actually defective, what is the probability that all of the diodes will be checked before use? This question is really whether the quality control sample of five will contain at least one of the defective parts.

   Solutions:

   Let X be the number of defective diode in the sample.

   Then X follows Hypergeometric distribution and probability mass function is given by

$$P(X = x) = \frac{\binom{2}{x}\binom{48}{5-x}}{\binom{50}{5}}; \quad x = 0, 1, 2$$



The probability that all of the diodes will be checked before use = Probability of at least one of the parts defective. Therefore, this probability should be equal to $P(X \geq 1)$

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\begin{aligned}
P(X \geq 1) &= 1 - P(X < 1) = 1 - P(X = 0) \\
&= 1 - \frac{\binom{2}{0}\binom{48}{5-0}}{\binom{50}{5}} \\
&= 1 - 0.8082 \\
&= 0.1918
\end{aligned}
$$

Hence, with the current sampling procedure, there is approximately a 20% chance of finding a defective part.

4. An ultrasonogram machine identifies 46.25% of the babies as boy. A baby is a boy when ultrasonogram identifies boy in 94.67% of the times and a baby is a girl when ultrasonogram identifies girl in 99.88% of the times. A baby born is a girl, what is the probability that the baby was predicted to be a boy?

   Solutions:

   Let S_1 be the ultrasonogram machine identifies babies as boy, S_2 be the ultrasonogram machine identifies babies as girl and H be the event baby born is a girl.

   The question is asking to find $P(S_1|H)$, conditional probability of the baby predicted as a boy given that baby born is a girl.

   $P(S_1) = 0.4625, P(S_2) = 1 - 0.4625 = 0.5375$

   $P(H|S_1) = 1 - 0.9467 = 0.0533$ born girl when predicted to be a boy.

   $P(H|S_2) = 0.9988$ born girl when predicted to be a girl.

   By using Baye's theorem

$$
\begin{aligned}
P(S_1|H) &= \frac{P(H|S_1) \times P(S_1)}{P(H|S_1) \times P(S_1) + P(H|S_2) \times P(S_2)} \\
&= \frac{0.0533 \times 0.4625}{0.0533 \times 0.4625 + 0.9988 \times 0.5375} \\
&= \frac{0.0247}{0.5615} = 0.0439
\end{aligned}
$$

5. A computer system is built so that if component K_1 fails, it is by passed and K_2 is used. If K_2 fails, then K_3 is used. Suppose that the probability that K_1 fails is 0.01, that K_2 fails is 0.03, and that K_3 fails is 0.08. Moreover, we can assume that the failures are mutually independent events. Then what is the probability that the system does not failure?

Solutions:

Let A be the event that the system fails. Then we have:

$$P(A) = P(K_1 \text{ fails})\ P(K_2 \text{ fails})\ P(K_3 \text{ fails})$$

$P(K_1 \text{ fails}) = 0.01$

$P(K_2 \text{ fails}) = 0.03$

$P(K_3 \text{ fails}) = 0.08$

Therefore,

$$P(A) = 0.01 \cdot 0.03 \cdot 0.08 = 0.000024$$

So the probability that the system does not fail is:

$$P(\text{System does not fail}) = 1 - P(A) = 1 - 0.000024 = 0.999976$$

Therefore, the probability that the system does not fail is 0.999976.

[Comments: students may use any other suitable approach to obtain the answer, such as a tree diagram. If anyone obtains the correct answer or an answer close to it using a different approach, please provide them with full marks]

6. On an average in 2010, there were 842 traffic crashes occurred on Jaffna roadways (Road Accident Data by Vehicle type – 2010, http://www.data.gov.lk). (there are 365 days in 2010)

   (a) What is the expected number of traffic crashes on any given day of 2010?

   (b) What is the standard deviation of the number of the traffic crashes occurred on Jaffna roadways on any given month of 2010?

   (c) What is the probability that there were exactly 20 traffic crashes on Jaffna roadways in any given day of 2010?

   (d) What is the probability that there were at least 5 traffic crashes on Jaffna roadways in any given day of 2010?

Solutions:

Let X be the number of the traffic crashes occurred on Jaffna roadways on any given day. Then X follows Poisson distribution with parameter $\lambda = 842/365 = 2.3068$.

The probability mass function is given by

$$P(X = x) = \frac{e^{-2.3068} 2.3068^x}{x!}; \quad x = 0, 1, 2, \cdots$$

(a) Expected number of traffic crashes $= \lambda = 2.3068$

(b) Let Y be the number of the traffic crashes occurred on Jaffna roadways on any given day. Then Y follows Poisson distribution with parameter $\lambda = 30 * 842/365 = 69.2055$. So, standard deviation is $\sqrt{\lambda} = \sqrt{60.2055} = 8.3190$

(c) $P(X = 20) = \frac{e^{-2.3068} 2.3068^0}{0!} \simeq 0$

(d) $P(X \geq 5) = 1 - P(X < 5)$
$= 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4))$
$= 1 - 0.9154 = 0.0846$

***Bonus Question (5 Marks)**
A certain material is fed to a two-step process. For this process, the probabilities of a malfunction are $P(B_1) = 0.03$ and $P(B_2) = 0.05$, where the factors B_1 and B_2 represent a malfunction in Steps 1 and 2, respectively. A sample of the final product is taken and found to be unacceptable. Our experience over the previous two months indicates that a defective product will be obtained 20% of the time if Section 1 of the process malfunctions and 36% of the time if Section 2 malfunctions. That means $P(E|B_1) = 0.2$ and $P(E|B_2) = 0.36$. In which part of the process does the fault probably lie?
Solutions:
By using Bay's law,

$$\begin{aligned} P(B_1|E) &= \frac{P(E|B_1) \times P(B_1)}{P(E|B_1) \times P(B_1) + P(E|B_2) \times P(B_2)} \\ &= \frac{0.03 \times 0.20}{0.03 \times 0.20 + 0.05 \times 0.36} \\ &= 0.25 \end{aligned}$$

And

$$\begin{aligned} P(B_2|E) &= \frac{P(E|B_2) \times P(B_2)}{P(E|B_1) \times P(B_1) + P(E|B_2) \times P(B_2)} \\ &= \frac{0.05 \times 0.36}{0.03 \times 0.20 + 0.05 \times 0.36} \\ &= 0.75 \end{aligned}$$

From these results, Section 2 of the process is the more likely to need corrective maintenance.