Tutorial-03

1. Let x be the weights of frying chicken.

$X \sim N(500, 100^2)$. pdf of Normal distribution

$$f_x(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$X \sim N(\mu, \sigma^2)$$

a) $P(X < 650)$

$= P\left(\frac{x-\mu}{\sigma} < \frac{650-500}{100}\right)$

$= P(Z < 1.5)$

$= 0.9332$.

b) $P(X < 345)$

$= P\left(\frac{x-\mu}{\sigma} < \frac{345-500}{100}\right)$

$= P(Z < \overset{-1.55}{\cancel{0.0069}})$ OR $1 - P(Z < 1.55)$

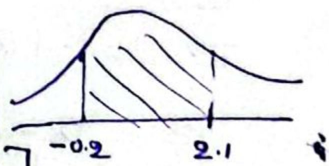
$1 - 0.9394$

$= 0.0606$

c) $P(480 < X < 710)$

$= P\left(\frac{480-500}{100} < Z < \frac{710-500}{100}\right)$

$= P(-0.2 < Z < 2.1)$



$= 1 - [P(Z < -0.2) + P(Z > 2.1)]$

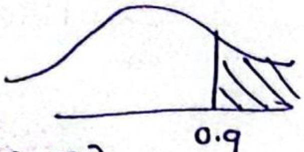
$= 1 - (P(Z < -0.2) + P(Z < -2.1)]$

$= 1 - [0.4207 + 0.0179)$

$= 0.5614$

d) $P(X > 590)$

$= P\left(\frac{x-\mu}{\sigma} > \frac{590-500}{100}\right)$



$= P(Z > 0.9)$ $= 1 - P(Z < 0.9)$

$= P(Z < -0.9) = 0.1841$

2) Let X be the IQ scores.

$X \sim N(125, 10^2)$ $\qquad f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$X \sim N(\mu, \sigma^2)$.

a) $P(X < 142)$

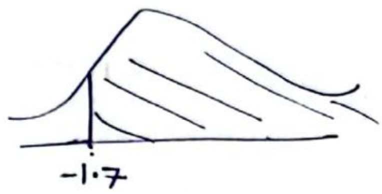
$= P\left(\frac{X-\mu}{\sigma} < \frac{142-125}{10}\right)$

$= P(Z < 1.7)$

$= 0.9554$

b) $P(X > 108)$

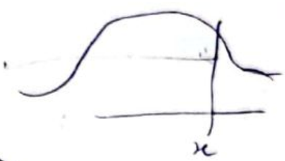
$= P\left(\frac{X-\mu}{\sigma} > \frac{108-125}{10}\right)$

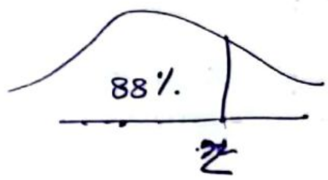


$= P(Z > -1.7)$

$= 1 - P(Z < -1.7)$

$= 1 - 0.0446 = 0.9554$





c) $P(X < x) = 0.88$

$P(Z < z) = 0.88$ $\qquad P\frac{X-\mu}{\sigma}$

$Z = 1.175$

$z = \frac{x-\mu}{\sigma}$

$x = \mu + \sigma z$

$= 125 + 10 \times 1.175$

$= 136.75$.

d) $P(X < x) = 0.36$

$P(Z < z) = 0.36$

$z = -0.36$

$x = \mu + \sigma z$

$= 125 + 10 \times -0.36$

$= 121.4$

e) $P(X < 100)$

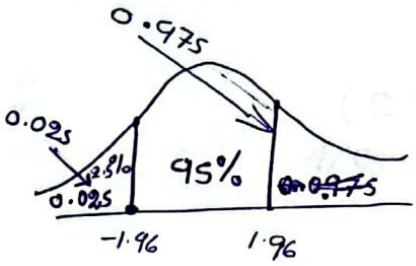
$= P\left(\frac{X-\mu}{\sigma} < \frac{100-125}{10}\right)$

$= P(Z < -2.5)$

$= 0.0062$

Then the no. of applicants will not be admitted is $1500 \times 0.0062 = 9.3 \simeq 9$.

f)



$z_{0.025} = -1.96$

$z_{0.975} = 1.96$

$P(x_1 < X < x_2) = 0.95$

$P(-1.96 < Z < 1.96) = 0.95$

$x_1 = \mu + \sigma z$

$= 125 + 10(-1.96)$

$= 105.4$

$x_2 = \mu + \sigma z$

$= 125 + 10(1.96)$

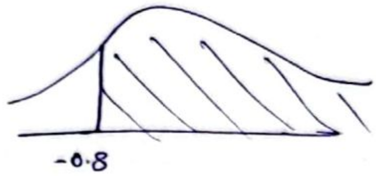
$= 144.6$

3) Let x be the annual rainfall.

$X \sim N(40, 5^2)$

a) $P(X \geq 36)$

$= P\left(\frac{x-\mu}{\sigma} > \frac{36-40}{5}\right)$



$= P(Z > -0.8)$

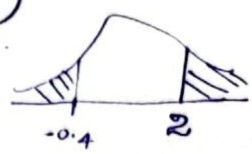
$= 1 - P(Z < -0.8)$

$= 1 - 0.2119$

$= 0.7881$

b) $P(X < 38) + P(X > 50)$

$= P\left(\frac{x-\mu}{\sigma} < \frac{38-40}{5}\right) + P\left(\frac{x-\mu}{\sigma} > \frac{50-40}{5}\right)$



$= P(Z < -0.4) + P(Z > 2)$

$= P(Z < -0.4) + P(Z < -2)$

$= 0.3446 + 0.0228$

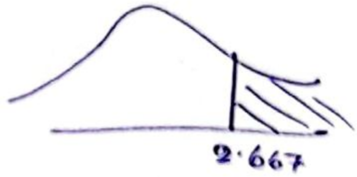
$= 0.3674$

4) a) Let x be the length of pregnancies

$X \sim N(268, 15^2)$

a) $P(X > 308)$

$= P\left(\frac{x-\mu}{\sigma} > \frac{308-268}{15}\right)$



$= P(Z > 2.667)$

$= P(Z < -2.667)$

$= 0.0038$

b) $P(X < x_1) = 0.04$

$P(Z < z) = 0.04$

$z = -1.75$

$x_1 = \mu + \sigma z$

$= 268 + 15(-1.75)$

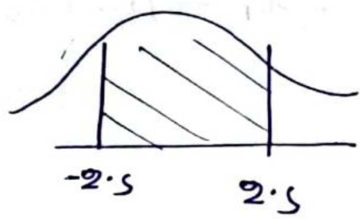
$= 241.75$

5) Let X be ages of the residents.

$X \sim N(70, 4^2)$

a) $P(60 < X < 80)$

$= P\left(\frac{60-70}{4} < Z < \frac{80-70}{4}\right)$



$= P(-2.5 < Z < 2.5)$

$= 1 - 2 \times P(Z < -2.5)$

$= 1 - 2 \times 0.0062$

$= 0.9876$

b) $n = 9$

~~Sample~~ mean of the means of the sample $= 70$

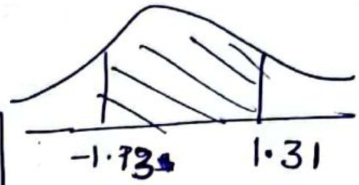
$\mu_{\bar{x}} = 70$

c) S.D $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{9}} = 1.333$

d) $P(68.5 < \bar{X} < 71.75)$

$$= P\left(\frac{68.5-70}{4/\sqrt{9}} < Z < \frac{71.75-70}{4/\sqrt{9}}\right)$$

$$= P(-1.125 < Z < 1.3125)$$



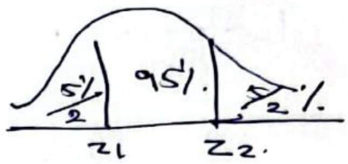
$$= 1-[P(Z<-1.13)+P(Z<-1.31)]$$

$$= 1-[0.1292+0.0951]$$

$$= 0.7757$$

e)

$P(\bar{x}_1 < \bar{X} < \bar{x}_2) = 0.95$



$P(Z<z_1) = 0.025.$

$z_1 = -1.96$

$P(-1.96 < Z < 1.96) = 0.95$

$\bar{x}_1 = \mu_{\bar{x}} + \sigma_{\bar{x}} Z$

$= 70 + 1.33(-1.96)$

$= 67.39$

$\bar{x}_2 = \mu_{\bar{x}} + \sigma_{\bar{x}} Z$

$= 70 + 1.33(1.96)$

$= 72.61$

6)

* $P(\text{cost is less than } \$11.5)$

$= P(\text{No. of copies below } \overset{\text{they total cost value}}{\$11.85} / \$0.05 = 237)$.

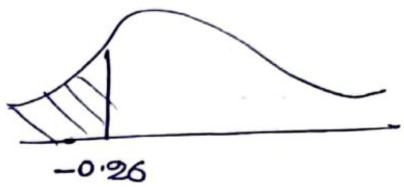
$= P(X < 237)$

$= P\left(\frac{X-250}{50} < \frac{237-250}{50}\right)$

$= P(Z < -0.26)$

$= 0.3974$

$= P(Z < -0.26)$
$1 - P(Z < 0.26)$
$= 1 - 0.6026$
$= 0.3974$



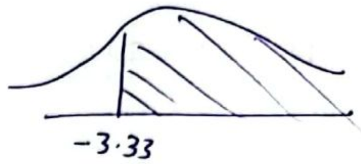
7) Let x be the net weights of boxes filled from the machine.

$$x \sim N(16, 0.3^2)$$

a) $P(x > 15)$

$$= P\left(\frac{x-\mu}{\sigma} > \frac{15-16}{0.3}\right)$$

$$= P(Z > -3.333)$$



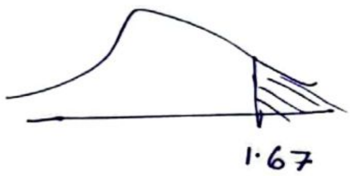
$$= 1 - P(Z < -3.33)$$

$$= 1 - 0.0005$$

$$= 0.9995$$

b) $P(x > 16.5)$

$$= P\left(\frac{x-\mu}{\sigma} > \frac{16.5-16}{0.3}\right)$$



$$= P(Z > 1.6667)$$

$$= \cancel{P(} \ 1 - P(Z < 1.67)$$

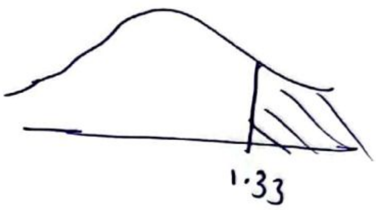
$$= 1 - 0.9525$$

$$= 0.0475$$

c) $P(x > 16.4)$

$$= P\left(\frac{x-\mu}{\sigma} > \frac{16.4-16}{0.3}\right)$$

$$= P(Z > 1.3333)$$



$$= P(Z < -1.33)$$

$$= 0.0918$$

d) $n = 32$

$P(\bar{x} > 16.3)$

$$= P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{16.3-16}{\frac{0.3}{\sqrt{32}}}\right)$$

$$= P(Z > 5.6568)$$

$$= 0.0$$

$=$

e) $n = 32$

$P(\bar{x} < 15.8)$

$$= P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{15.8-16}{\frac{0.3}{\sqrt{32}}}\right)$$

$$= P(Z < -3.7712)$$

$$= 0.0001$$

8) $P(X \leq 90) \doteqdot P(X < 90.5)$, a continuity correction is needed as the responses are binary & the counts have a binomial distribution with $n = 400$, $p = 0.20$.

$\mu = np$

$= 400 \times 0.2$

$= 80$

$\sigma = \sqrt{npq}$

$= \sqrt{400 \times 0.2 \times 0.8}$

$= 8$.

$np \geq 5$, $nq \geq 5$.

we can use normal approximation.

$P(X \leq 90) \doteqdot P(X < 90.5)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{90.5-80}{8}\right)$

$= P(Z < 1.31)$

$= 0.9049$

The sample event is not unlikely assuming the claimed percentage of 20%, hence the claim is not unreasonable.

9)

$np = 45 \geq 5$ & $nq > 5$ So we can use normal approximation.

$P(X \geq 57) \doteq P(X > 56.5)$ ~~Bay~~, a continuity correction is needed as the response are binary & the counts have a Binomial dist$^{\underline{n}}$ with $n = 100$, $p = 0.45$.

$\mu = np$
$= 100 \times 0.45$
$= 45$

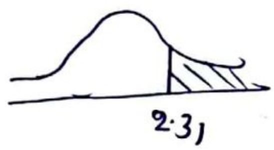
$\sigma = \sqrt{npq}$
$= \sqrt{100 \times 0.45 \times 0.55}$
$= 4.9749$.

$P(X \geq 57) \doteq P(X > 56.5)$

$= P\left(\frac{X - 45}{4.9749} > \frac{56.5 - 45}{4.9749}\right)$

$= P(Z > 2.31)$

$= P(Z < -2.31)$



$= 0.0104$

10) Let X be the length of the pea pods for its olive-green pea crop.

$$X \sim N(11.5, 1.6^2)$$

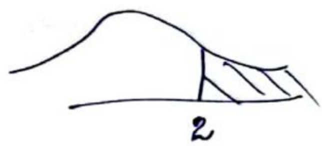
$n = 16$

a) $\sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{1.6}{\sqrt{16}} = 0.4$

b) $P(\bar{X} > 12.3)$

$= P\left(\frac{\bar{X} - 11.5}{0.4} > \frac{12.3 - 11.5}{0.4}\right)$

$= P(Z > 2)$

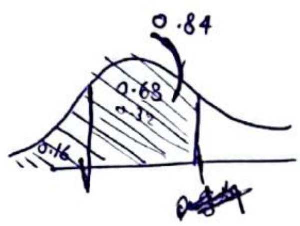


$= 1 - P(Z < 2)$

$= 1 - 0.9772$

$= 0.0228$

c) $P(\bar{x}_1 < \bar{X} < \bar{x}_2) = 0.68$



$P(z_1 < Z < z_2) = 0.68$

$\cancel{z_1 = -2.145}$ $\quad$ $\cancel{z_2 = 0.99}$

$z = \frac{\bar{x} - \mu_x}{\sigma_x}$ $\quad$ $P(Z < z_1) = 0.16$ $\quad$ $P(Z < z_2) = 0.84$

$z_1 = -1.0$ $\quad$ $z_2 = 1$

$\bar{x}_1 = \mu_x + z\sigma_x$

$= 11.5 + (-1)0.4$

$= 11.1$

$\bar{x}_2 = \mu_x + z\sigma_x$

$= 11.5 + 1(0.4)$

$= 11.9$

11)

a) Mean $= np$

$= 1500 \times 0.7$

$= 1050$

standard deviation $= \sqrt{np(1-p)}$

$= \sqrt{1500 \times 0.7 \times 0.3}$

$= 17.7482$

b) $np \geq 5$, $nq \geq 5$

So, we can use normal approximation.

$X \sim N(1050, 17.7482^2)$

$$P(X \geq 1000) = P\left(\frac{X-\mu}{\sigma} \geq \frac{\overset{999.5}{1000} - 1050}{17.7482}\right)$$

$\doteq P(X > 999.5)$

$$= P(Z \geq \overset{-2.8453}{\cancel{-2.8171}})$$

$= 1 - P(Z < -2.85)$

$= 1 - 0.0022$

$= 0.9978$

c) $$P(X > 1200) = P\left(\frac{X-\mu}{\sigma} > \frac{1200.5 - 1050}{17.7482}\right)$$

$\doteq P(X > 1200.5)$

$$= P(Z > \overset{8.4797}{\cancel{8.4516}})$$

$= 0$

d) $N = 1700$

$$\text{mean} = 1700 \times 0.7$$
$$= 1190$$

$$S.D = \sqrt{1700 \times 0.7 \times 0.3}$$
$$= 18.8944$$

$$P(x > 1200)$$
$$\doteq P(x > 1200.5) = P\left(\frac{x-\mu}{\sigma} > \frac{1200.5 - 1190}{18.8944}\right)$$
$$= P(Z > \cancel{0.5293}\ 0.5557)$$
$$= 1 - P(Z < 0.56)$$

$$= 1 - \cancel{0.7019}\ 0.7123$$
$$\cancel{\neq\ 0.2981}$$
$$= 0.2877$$

12)

*

$$f_X(x) = \lambda e^{-\lambda x} \qquad x > 0$$

$$\lambda = \frac{1}{5} = 0.2/\text{year}$$

$$\frac{14}{12} = 1.167 \text{ years}$$

$$P\left(T < \frac{14}{12}\right) = \int_0^{14/12} 0.2 e^{-0.2 x} \, dx$$
$$= -e^{-0.2x} \Big|_0^{14/12}$$
$$= -e^{-0.2 \times \frac{14}{12}} + 1$$
$$= -0.7919 + 1 \qquad = 0.2081$$

13>

Let X be the lifetime of a particular fan.

$X \sim exp(\lambda = 0.0003)$

p.d.f $= \lambda e^{-\lambda x}$ $\quad x > 0$

$$P(X > 10000) = \int_{10,000}^{\infty} \lambda e^{-\lambda x} dx$$

$$= -e^{-\lambda x} \Big|_{10,000}^{\infty}$$

$$= 0 + e^{-0.0003 \times 10,000}$$

$$= e^{-3}$$

$$= 0.04978$$

$$= 0.0498$$

$\lambda = 0.00035$

$$P(X > 10,000) = \int_{10,000}^{\infty} \lambda e^{-\lambda x} dx$$

$$= -e^{-\lambda x} \Big|_{10,000}^{\infty}$$

$$= 0 + e^{-0.00035 \times 10,000}$$

$$= e^{-3.5}$$

$$= 0.030197$$

$$= 0.0302.$$

It will fewer fans compare with earlya one.

14) Let X be the time interval between successive barges.

$$\text{mean} = 8$$
$$\frac{1}{\lambda} = 8$$
$$\lambda = \frac{1}{8} = 0.125$$
$$X \sim \exp\left(\frac{1}{8}\right)$$
$$f_X(x) = \lambda e^{-\lambda x} \quad x > 0$$

a) $$P(X < 5) = \int_0^5 \lambda e^{-\lambda x} dx$$
$$= -e^{-\lambda x}\Big|_0^5$$
$$= -e^{-\frac{1}{8}\cdot 5} + 1$$

check $$= 0.4647$$

b) $$P(T > t) = \int_t^\infty 0.125 e^{-0.125t} \cdot dt$$
$$= -e^{-0.125t}\Big|_t^\infty$$
$$= e^{-0.125t}$$
$$e^{-0.125t} = 0.95$$
$$e^{-t/8} = 0.95$$
$$-\frac{t}{8} = \ln 0.95$$
$$t = -8 \ln 0.95$$
$$= 0.4103$$