Tutorial: 02   **April 2023**

1) The number of patients waiting for acupuncture treatments at a clinic has the following probability distribution:

x	$P(X = x)$
1	0.20
2	0.30
3	c
4	0.10
5	0.15

a) Determine the value of c?
b) What is the expected number of patients waiting for treatment?
c) What is the variance of the number of patients?
d) What is the standard deviation of the number of patients?
e) What is the mean absolute deviation of the number of patients?
f) What is the probability that fewer than two patients are waiting for treatment?
g) What is the probability that at least three patients are waiting for treatment?

2) Each week a garment factory makes one production run of their popular tie-dyed cheesecloth undershirts. The size of the run is either 10,000, 20,000, 40,000 or 70,000 undershirts, based on field sale reports of projected demand. Although demand fluctuates considerably, the long-term market for the undershirts has held up well for many months. The probability distribution based on the factory experience is:

x	$P(X = x)$
10000	0.30
20000	0.40
40000	0.20
70000	0.10

a) What is the expected number of undershirts produced in a week?
b) If the profit on each undershirt is $2.25, what is the expected weekly profit?
c) To hold their market at a constant level, the factory must add sequins to the undershirts. This will cost an additional $10,000 per run, for any size of run. What is the expected profit if they make this change and the market demand remains the same?

3) A study of education followed a large group of fifth-grade children to see how many years of school that a randomly chosen fifth grader completes. Let X be the highest year of school that a randomly chosen fifth grader completes. (Students who go on to college are included in the outcome $X = 12$.) The study found the probability distribution of X:

Years	4	5	6	7	8	9	10	11	12
Prob.	0.010	0.007	0.007	0.013	0.032	0.068	0.070	0.041	0.752

a) What percent of fifth graders eventually finished twelfth grade?
b) Check that this is a legitimate probability distribution.
c) Find $P(X \geq 6)$.
d) Find $P(X > 6)$.
e) What values of X make up the event "the student completed at least one year of high school"? (High school begins with the ninth grade.) What is the probability of this event?

4) Which of the following is NOT an example of a binomial experiment? Why?
a) Whether or not mass-produced parts are defective or not defective.
b) Determining whether or not a certain task is accomplished in 10 minutes.
c) Determining if the average weekly pay rate per employee last year was greater or was not greater than the same period in the previous year.
d) Determining the average number of words typed before a typist makes an error.

5) Suppose that 25% of all subscribers to a nationally circulated business magazine earn an income in excess of $45,000. The magazine polls 20 subscribers at random to determine the income category into which each fall.
a) What is the probability that none of the 20 subscribers earns more than $45,000?
b) Find the probability that exactly half of those chosen earns in excess of $45,000.
c) Find the probability that three or fewer earn in excess of $45,000.
d) Find the expected number of subscribers with income exceeding $45,000 one would expect to find in the random sample of size 20.
e) Find the variance of the number of people in a sample of 20 who earn in excess of $45,000.

6) Ten percent of the items in a large lot of parts are defective. The quality control technician decides to randomly select four items and accept the lot if all four selected parts are good. What is the probability that he will accept the lot?

7) Ten percent of the items in a lot of 60 parts are defective. The quality control technician decides to randomly select four items and accept the lot if all four selected parts are good. What is the probability he will accept the lot?

8) A Poisson distribution with a mean of two arrivals per hour models arrivals at the emergency room of a government hospital well.
   a) What is the probability that exactly two patients will arrive in any hour?
   b) What is the probability that at least one patient will arrive in any hour?
   c) To plan service in the emergency room, find the 95th percentile.

9) A textile industry produces the basic fabric for conversion into jockey caps. The industry's output has been found to have a mean of two flaws every 10 meters, or 0.2 flaws per meter. The quality control manager assumes that the Poisson distribution is applicable and wants to know the probability that a meter of output will have no flaw.

10) A lot, consisting of 20 fuses, is inspected by the following procedure. Three fuses are selected at random and without replacement from the lot and tested. If all three blows at the correct amperage, the lot is accepted. Suppose the lot contains four defective fuses.
    a) Obtain the probability of accepting the lot.
    b) Obtain the probability of two out of three blows at the correct amperage.
    c) Obtain the probability of one out of three blows at the correct amperage.

11) What is the probability of getting exactly three aces in a five-card poker hand dealt from an ordinary 52-card deck? Note that there are four aces and 48 non-aces in a 52-card deck, and the selection is without replacement.

12) An automobile manufacturer buys computer chips from a supplier. The supplier sends a shipment containing 5% defective chips. Each chip chosen from this shipment has a probability of 0.05 of being defective, and each automobile uses 12 chips selected independently. What is the probability that all 12 chips in a car will work properly?

13) Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.
    a) What is the probability of no arrivals in a one-minute period?
    b) What is the probability that three or fewer passengers will arrive in a one-minute period?
    c) What is the probability of no arrival in a 15-second period?
    d) What is the probability of at least one arrival in a 15-second period?

14) Suppose flaws (cracks, chips, specks, etc.) occur on the surface of glass with a density of four per square meter. What is the probability of there being exactly five flaws on a sheet of glass of area 2 square meters?

15) A shipment of 10 items has two defective and eight non-defective units. In the inspection of the shipment, a sample of units will be selected and tested. If a defective unit is found, the shipment of 10 units will be rejected.

a) If a sample of three items is selected, what is the probability that the shipment will be rejected?
b) If a sample of four items is selected, what is the probability that the shipment will be rejected?
c) If a sample of five items is selected, what is the probability that the shipment will be rejected?
d) If management would like a 0.90 probability of rejecting a shipment with two defective and eight non-defective units, how large a sample would you recommend?

16) A report from the Secretary of Health and Human Services stated that 70% of single vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 15 single-vehicle traffic fatalities that occur at night on a weekend is selected,
   a) Find the probability that exactly 12 involve a driver who is intoxicated.
   b) Find the probability that at least 13 involve a driver who is intoxicated.
   c) What is the expected number of drivers who is intoxicated?

17) The American Automobile Association reports that the average time it takes to respond to an emergency call is 25 minutes. Assume the variable is approximately normally distributed and the standard deviation is 4.5 minutes. If 80 calls are randomly selected, approximately how many will be responded to in less than 15 minutes?