UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

MID SEMESTER EXAMINATION– APRIL, 2024

MC3020- PROBABILITY AND STATISTICS

Date: 01 - 04 -2024 Duration: 50 Minutes

Instructions

1. This paper contains **FIVE (5)** questions:
2. Answer **all** the questions.
3. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
4. This examination accounts for **20%** of module assessment. Total maximum mark attainable is **100**.

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1. In the civil engineering laboratory, there are 15 testing machines, pivotal for material analyses essential to construction projects. Unfortunately, it's uncovered that 6 of these machines have faulty calibrations, posing a potential risk to the accuracy of test results. With 5 machines to be randomly selected for testing:
   (a) What is the probability that precisely 2 of them will exhibit faulty calibrations?
   (b) What is the probability that none of the selected machines have faulty calibrations?
2. Amidst the jubilation of the Engineering faculty's 10th-anniversary event held on March 23rd, organizers meticulously conducted a satisfaction survey among attendees. The survey revealed a resounding 65% satisfaction rate, a testament to the meticulous planning and execution that went into the celebrations. However, amidst the celebration, organizers were intrigued by the potential variations in attendee satisfaction. To delve deeper into this aspect, they sought to understand the probabilities associated with certain attendee scenarios. Specifically, they were interested in knowing the likelihood of encountering particular outcomes in randomly selected groups of 12 attendees:
   (a) What is the probability of having exactly five satisfied attendees.
   (b) Find the probability of encountering more than five satisfied attendees.
   (c) Find the probability that three or fewer satisfied attendees.

3. In an advanced software development course at the Faculty of Engineering, University of Jaffna, three coding teams, Alpha, Beta, and Gamma, contribute respectively $15\%, 45\%$, and 40% of the overall codebase for a complex software project. Team Alpha is renowned for its stringent coding standards, with only 0.5% of its code containing bugs. Conversely, teams Beta and Gamma have higher bug rates, with 4% and 6% of their code containing bugs, respectively. All code contributions are integrated into a central repository. During a rigorous code audit, one segment of code is randomly selected and identified to contain a bug.

   (a) What is the conditional probability that it originated from team Alpha?

   (b) What is the probability that it came from team Beta?

4. In a laboratory experiment conducted within the Electrical Engineering department of the University of Jaffna, researchers meticulously gathered data on the resistance values of 2400 electronic components. These components were subjected to rigorous testing, and their resistance values were found to conform to a normal distribution with a mean of 112 ohms and a standard deviation of 18 ohms.

   (a) Considering the random selection of a component from this dataset, what is the probability that its resistance value falls below 152 ohms?

   (b) Delving deeper into the dataset, what specific resistance value corresponds to the 90th percentile of these components?

   (c) Adhering to the quality standards of the department's projects, if components with resistance values below 102 ohms are deemed unsuitable, how many components would be disqualified based on this criterion?

   (d) Seeking to analyze the distribution comprehensively, between which two resistance values, symmetrically positioned around the mean, do we find that approximately 95% of the resistance values lie?

5. In the MC3020 class focused on attendance monitoring and preventing impersonations, the frequency of student check-ins follows a Poisson distribution. On average, there are four check-ins per minute, and the time between check-ins follows an exponential distribution. Ensuring the integrity of attendance records is crucial in such a setting, as any unusual patterns or discrepancies could indicate potential impersonations or fraudulent activities.

   (a) Find the probability that exactly five check-ins occur within a minute.

   (b) Find the probability that less than five check-ins occur within a minute.

   (c) Find the average time between two successive check-ins.

   (d) Find the probability that after a check-in, the next check-in occurs in less than ten seconds.

   (e) By using normal distribution approximation, find the probability that more than 40 check-ins occur in an eight-minute period. (*Hint:* If X follows a Poisson distribution with parameter λ, then the mean and variance of the Poisson random variable are both equal to λ).

Some useful formulas:

1. If X follows binomial distribution with parameters n, p. Then, the probability mass function is given by $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x};\ x = 0, 1, 2, \cdots n$
2. If X follows Poisson distribution with parameter λ. Then, the probability mass function is given by
$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!};\ x = 0, 1, 2, \cdots\cdots$$
3. If X follows exponential distribution with parameter λ. Then, the probability density function is given by $f(x) = \lambda e^{-\lambda x};\quad x > 0$
4. If X follows Hyper-geometric with parameters N, r, n. Then, the probability mass function is given by
$$P(X=x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}};\ x = 0, 1, 2 \cdots r$$
5. If X follows normal distribution with parameters μ and σ then $\left(\frac{X-\mu}{\sigma}\right) = Z$ follows standard normal distribution with $\mu = 0$ and $\sigma = 1$
6. Let us consider that a sample space S is divided into two mutually exclusive partitions S_1 and S_2. An event H has occurred, and $P(S_1|H)$ can be written as
$$P(S_1|H) = \frac{P(S_1)P(H|S_1)}{P(S_1)P(H|S_1) + P(S_2)P(H|S_2)}$$

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