Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 3020 - Assignment 01

50 minutes **12 - 03 - 2024**

Answer all the questions(1-6). If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, <u>without</u> any exceptions or considerations.

1. Consider a civil engineering project where traffic flow at a certain intersection needs to be analyzed. The number of vehicles waiting at a traffic signal follows the following probability distribution:

x	1	2	3	4	5	6
$P(X = x)$	0.20	k	0.25	0.10	0.10	0.05

Here, x represents the number of vehicles waiting, and $P(X = x)$ is the probability of observing x vehicles waiting. Given this information, let's solve the following questions:

(a) Determine the value of k.

(b) Estimate the expected number and standard deviation of vehicles waiting at the traffic signal?

(d) What is the probability that fewer than three vehicles are waiting at the traffic signal?

(e) What is the probability that at least four vehicles are waiting at the traffic signal?

2. The Faculty of Engineering at the University of Jaffna is conducting an evaluation of the attendance of students from the E22 batch in the MC3020 course. The goal is to achieve an attendance rate of 65% among the students. To assess whether this goal is being met, a sample of 20 students from the E22 batch has been selected. These students are representative of the broader E22 cohort. Each of these 20 students is expected to attend the MC3020 course regularly, as part of their academic requirements. The attendance of each student is meticulously recorded over a designated period to determine whether the desired 65% attendance rate is being maintained. We aim to analyze the probabilities associated with the attendance of these 20 sampled students:

(a) What is the probability that exactly half of the sampled students will attend the MC3020 course.

(b) What is the probability that 18 or more of the 20 sampled students will attend the MC3020 course consistently, in accordance with the desired attendance rate.

3. Suppose there has been a recent incident of raggiong (a term used to describe the act of forcefully entering and causing damage or theft) reported at the Faculty of Engineering, University of Jaffna. As a precautionary measure, security personnel are instructed to randomly inspect 10% of all incoming bags for any suspicious items. If any of the inspected bags are found to contain suspicious items, all remaining bags are inspected. Given that

two out of the next box of 50 bags are found to contain suspicious items, what is the probability that all the bags will be inspected before use? This question is essentially asking for the probability that at least one bag among the first 5 inspected bags contains suspicious items.

4. In the lecture class of Probability and Statistics (MC3020) at the Faculty of Engineering, there have been reported instances of students attempting to impersonate others by signing their names on attendance sheets. Occasionally, these impersonation attempts are detected by the Dr T. Mayooran who is the Lecturer of this course. Specifically, when a student attempts to impersonate another by signing their name on the attendance sheet, it is detected in 95% of cases. Overall, there is a 2% chance that a student's signature is impersonated during the course of the lecture. Additionally, there's a 5% chance of detecting an impersonation signature when a student's signature is not impersonated. Given that an impersonation signature is detected during the lecture, what is the probability that a student's signature was impersonated? To address this question, we will analyze the probabilities provided and calculate the probability that a student's signature is impersonated given that an impersonation signature is detected.

5. A computer system is built so that if component K_1 fails, it is by passed and K_2 is used. If K_2 fails, then K_3 is used. Suppose that the probability that K_1 fails is 0.01, that K_2 fails is 0.03, and that K_3 fails is 0.08. Moreover, we can assume that the failures are mutually independent events. Then what is the probability that the system does not failure?

6. As part of the E23 batch orientation program at the Faculty of Engineering, workshops are organized to familiarize incoming students with their curriculum. On average, one workshop experiences a disruption every ten days. The faculty is particularly concerned about ensuring a smooth start on the first day of the orientation program.

   (a) What is the probability that a randomly selected workshop will not experience a disruption on the first day of the orientation program?

   (b) Considering the importance of a successful start, what is the probability of having at least one workshop without disruption on the first day of the orientation program?

<u>Some useful formulas:</u>

1. If X follows binomial with parameters n, p. Then, the probability mass function is given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x};\ x = 0, 1, 2, \cdots n$$

2. If X follows Poisson with parameters λ. Then, the probability mass function is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!};\ x = 0, 1, 2, \cdots\cdots$$

3. If X follows Hyper-geometric with parameters N, r, n. Then, the probability mass function is given by

$$P(X = x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}};\ x = 0, 1, 2 \cdots r$$

4. $\mu = E(X) = \sum_{\text{for all x}} xP(X = x)$ and $\sigma^2 = Var(X) = E(X^2) - (E(X))^2$
   or $Var(X) = E(X-\mu)^2 = \sum_{\text{for all x}} (x-\mu)^2 P(X = x)$

5. Let us consider that a sample space S is divided into two mutually exclusive partitions S_1 and S_2. An event H has occurred, and $P(S_1|H)$ can be written as

$$P(S_1|H) = \frac{P(S_1)P(H|S_1)}{P(S_1)P(H|S_1) + P(S_2)P(H|S_2)}$$