Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 3020 - Assignment 02

40 minutes **24 - 04 - 2024**

Answer all the questions(1-4). If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, without any exceptions or considerations.

1. As part of their Design and Prototyping (ID3020) project, a team of students at the University of Jaffna has developed a cutting-edge smart irrigation system designed to optimize water usage in agricultural fields. The system incorporates sensors and actuators to precisely control the amount of water delivered to crops, enhancing crop yield while conserving water resources. To ensure the system's effectiveness, the team conducts a series of tests on randomly selected plots of land. The team measures the soil moisture levels in ten randomly selected plots and records the following percentages:

   $$30, 35, 32, 38, 33, 31, 36, 34, 37, 29.$$

   Assuming the soil moisture levels follow a normal distribution

   (a) The team aims to estimate the mean soil moisture content per plot and construct a 98% confidence interval for each plot.

   (b) Additionally, the team seeks to assess the variability of soil moisture content across the plots. Construct a 95% confidence interval for the variance of soil moisture content per plot using the recorded data.

   (c) Using the results obtained in part (b), constructs a 95% confidence interval for the standard deviation of soil moisture content per plot, providing valuable insights into the consistency of water distribution achieved by their innovative smart irrigation system.

2. High accumulation of certain pollutants in the atmosphere can adversely affect the environment and human health, which is a concern for the Ariviyal Nagar region, particularly the vicinity of the University of Jaffna. Elevated levels of pollutants such as particulate matter, sulfur dioxide, and nitrogen oxides can lead to respiratory issues and environmental degradation. Nitrogen oxides (NOx) are primary contributors to air pollution, originating mainly from vehicle emissions, industrial processes, and combustion of fossil fuels. Suppose the distribution of nitrogen oxide emissions from vehicles in the Ariviyal Nagar region can be modeled adequately by a normal distribution with a mean emission level (μ) of 80 parts per billion (ppb) and a standard deviation (σ) of 12 ppb.

   (a) What is the probability that a randomly selected vehicle from the Ariviyal Nagar region will have emission levels less than 50 ppb?

   (b) What is the probability that a randomly selected vehicle from the Ariviyal Nagar region will have emission levels greater than 100 ppb?

   (c) What is the probability that a randomly selected vehicle from the Ariviyal Nagar region will have emission levels between 40 ppb and 116 ppb?

3. If the lifetime of a certain type of bearing in a machine follows an exponential distribution with a mean lifetime of 5000 hours,

   (a) What is the probability that a bearing will fail before 2000 hours?

   (b) At what time will half of all bearings have failed?

4. Suppose you are a civil engineer tasked with estimating the average compressive strength of a certain type of concrete used in construction projects in Kilinochchi area. In a previous study, it was found that the standard deviation of the compressive strengths of this type of concrete is 500 psi. To ensure a high level of confidence in your estimate, you aim to conduct a survey with a 92% confidence level. You want the estimate to be within 100 psi of the true average compressive strength. Based on this information, determine the sample size needed for your survey

Some useful formulas:

1. If X follows exponential distribution with parameter λ. Then, the probability density function is given by $f(x) = \lambda e^{-\lambda x}; \quad x > 0$

2. If X follows normal distribution with parameters μ and σ then $\left(\frac{X-\mu}{\sigma}\right) = Z$ follows standard normal distribution with $\mu = 0$ and $\sigma = 1$

3.

Parameter	$(1-\alpha) * 100\%$ confidence interval	Test statistic value
Mean (μ)	Case 1: $\left(\bar{X} \mp Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}\right)$	Case 1: $Z = \frac{\bar{X}-\mu_0}{(\frac{\sigma}{\sqrt{n}})}$
	Case 2: $\left(\bar{X} \mp t_{\frac{\alpha}{2},df} * \frac{S}{\sqrt{n}}\right)$	Case 2: $T = \frac{\bar{X}-\mu_0}{(\frac{S}{\sqrt{n}})}$
	Case 3: $\left(\bar{X} \mp Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}\right)$	Case 3: $Z = \frac{\bar{X}-\mu_0}{(\frac{\sigma}{\sqrt{n}})}$
	Case 4: $\left(\bar{X} \mp Z_{\frac{\alpha}{2}} * \frac{S}{\sqrt{n}}\right)$	Case 4: $Z = \frac{\bar{X}-\mu_0}{(\frac{S}{\sqrt{n}})}$
Variance (σ)	$\left(\frac{(n-1)*S^2}{\chi^2_U}, \frac{(n-1)*S^2}{\chi^2_L}\right)$	

   where Case 1: when population is normal and σ is known, case 2: when population is normal and σ is unknown, case 3: when population is not normal and σ is known, sample size n is large and case 3: when population is not normal and σ is known, sample size n is large.

4. Sample size calculation based on mean confidence interval :

$$n = \left(Z_{\frac{\alpha}{2}} * \frac{\sigma}{E}\right)^2$$

   where E– margin error

5. Sample size calculation based on proportion confidence interval :

$$n = \left(Z^2_{\frac{\alpha}{2}} * \frac{p(1-p)}{E^2}\right)$$

6. Sample mean ($\bar{X}$) and standard deviation (s) can be estimated

$$\bar{X} = \frac{\sum X}{n}, \quad s = \sqrt{\frac{\sum \left(X - \bar{X}\right)^2}{n-1}} \quad = \sqrt{\frac{n\sum X^2 \; - \; (\sum X)^2}{n(n-1)}}$$