Tutorial-03

01) Let X be the IQ scores of engineering job applicants.

$$X \sim N(\mu = 125, \sigma = 10)$$

a) $P(X < 135)$

$$= P\left(\frac{X-\mu}{\sigma} < \frac{135-125}{10}\right)$$

$$= P(Z < 1)$$

$$= 0.841345$$

$$= 0.8413.$$

b) $P(X < x_1) = 0.95$

$$\Rightarrow P\left(Z < \frac{x_1 - 125}{10}\right) = 0.95$$

$$\Rightarrow \frac{x_1 - 125}{10} = 1.645.$$

$$\Rightarrow x_1 = 125 + 16.45$$

$$= 141.45.$$

c) $P(X < 110) = P\left(Z < \frac{110-125}{10}\right)$

$$= P(Z < -1.5)$$

$$= 0.0668.$$

Hence, No. of applicants will not be admitted.

$$800 \times 0.066807 \doteq 53$$

d) $P(x_1 \le X \le x_2) = 0.94$

$P(X \le x_1) = 0.03$

$\Rightarrow P\left(Z \le \frac{x_1 - 125}{10}\right) = 0.03$

$\Rightarrow \frac{x_1 - 125}{10} = -1.88$

$\Rightarrow x_1 = 125 - 1.88 \times 10$

$= 106.20$

$P(X \le x_2) = 0.97$

$\Rightarrow P\left(Z \le \frac{x_2 - 125}{10}\right) = 0.97$

$\Rightarrow \frac{x_2 - 125}{10} = 1.88$

$\Rightarrow x_2 = 125 + 10 \times 1.88$

$= 143.8$.

∴ Between IQ Scores 106.2, 143.8 are symmetrically located about the mean are 94% of the score.

2) Let X be the amount of milk.

$X \sim N(\mu = 200, \sigma^2 = 15^2)$

a) $P(X > 224)$

$= P\left(Z > \frac{224-200}{15}\right)$

$= P(Z > 1.6)$

$= P(Z < -1.6)$

$= 0.0548$.

b) $P(191 < X < 209)$

$= P\left(\frac{191-200}{15} < Z < \frac{209-200}{15}\right)$

$= P(-0.6 < Z < 0.6)$.

$= P(Z < 0.6) - P(Z < -0.6)$

$= 0.7257 - 0.2743$

$= 0.4515$.

c) $P(X > 230)$

$= P\left(\frac{X-\mu}{\sigma} > \frac{230-200}{15}\right)$

$= P(Z > 2)$

$= P(Z < -2)$

$= 0.02275$.

No. of cups that will overflow $= 1000 \times 0.0228$

$\simeq 23$ cups.

c) $P(X < x_1) = 0.25$

$$P\left(Z < \frac{x_1 - 200}{15}\right) = 0.25$$

$$\frac{x_1 - 200}{15} = -0.67$$

$$x_1 = -0.67 \times 15 + 200$$

$$= 189.95 \text{ millimeters.}$$

3) Let x be the inside diameter of a piston ring.

$$x \sim N(\mu=10, \sigma^2=0.03^2)$$

a) $P(x > 10.075)$

$= P\left(\frac{x-\mu}{\sigma} > \frac{10.075-10}{0.03}\right)$

$= P(z > 2.5)$

$= P(z < -2.5)$

$= 0.0062.$

0.62% of the rings have inside diameter exceeding 10.075.

b) $P(9.97 < x < 10.03)$

$= P\left(\frac{9.97-10}{0.03} < z < \frac{10.03-10}{0.03}\right)$

$= P(-1 < z < 1)$

$= P(z < 1) - P(z < -1)$

$= 0.841345 - 0.158655$

$= 0.68269$

$= 0.6827$

c) $P(x < x_1) = 0.15.$

$\Rightarrow P\left(z < \frac{x_1-10}{0.03}\right) = 0.15$

$\frac{x_1-10}{0.03} = -1.04$

$x_1 = -1.04 \times 0.03 + 10$

$= 9.9688 = 9.969 \text{ cm}.$

4) Let x be the length of the pea-pods for its olive-green pea crop.

$X \sim N(11.5, 1.6^2)$

$n = 16$

a) $\sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{1.6}{\sqrt{16}} = 0.4$.

b) $P(\bar{x} > 12.3)$

$= P\left(\frac{\bar{x} - 11.5}{0.4} > \frac{12.3 - 11.5}{0.4}\right)$

$= P(Z > 2)$

$= 1 - P(Z < 2)$

$= 1 - 0.9772$.

$= 0.0228$.

c) $P(\bar{x}_1 < \bar{x} < \bar{x}_2) = 0.68$.

$P(Z < Z_1) = 0.16$

$Z_1 = -1$

$P(Z < Z_2) = 0.84$

$Z_2 = 1$

$\bar{x}_1 = \mu_x + Z\sigma_x$

$= 11.5 + (-1) \times 0.4$

$= 11.1$

$\bar{x}_2 = \mu_x + Z\sigma_x$

$= 11.5 + 1(0.4)$

$= 11.9$

68% of the sample means lie between the value 11.1 and 11.9

5) $np = 100 \times 0.9$ $\quad nq > 5$.

$= 90 > 5$

So we can use normal approximation.

$P(84 \leq x \leq 95) \triangleq P(83.5 < x < 95.5)$

a continuity correction is needed as the response are binary & counts have a binomial distⁿ with $n = 100$, $p = 0.9$.

$\mu = np$
$= 90$

$\sigma = \sqrt{npq}$
$= \sqrt{100 \times 0.9 \times 0.1}$
$= 3$.

a) $P(84 \leq x \leq 95) \triangleq P(83.5 < x < 95.5)$

$= P\left(\frac{83.5 - 90}{3} < z < \frac{95.5 - 90}{3}\right)$

$= P(-2.166 < z < 1.833)$

$= P(z < 1.83) - P(z < -2.17)$

$= 0.9664 - 0.0150$

$= 0.9514$

b) $P(x < 86) \triangleq P(x < 85.5)$

$= P\left(z < \frac{85.5 - 90}{3}\right)$

$= P(z < -1.5)$

$= 0.0668$

6) Let $*$ be the time takes for a student to solve a particular problem.

mean = 15 min

$* \sim \exp(1/15)$

$f(x) = \lambda e^{-\lambda x}$.

$E(x) = 1/\lambda$

a) $P(t < 20) = \int_0^{20} \frac{1}{15} e^{-1/15 t} dt$

$= -e^{-1/15 t} \Big|_0^{20}$

$= -e^{-20/15} + e^0$

$= -e^{-4/3} + 1$

$= 0.7364$

b) $P(T > t) = 0.7$

$\int_t^{\alpha} 1/15 e^{-1/15 t} dt = 0.7$

$-e^{-1/15 t} \Big|_t^{\alpha} = 0.7$

$-e^{-\alpha} + e^{-1/15 t} = 0.7$

$e^{-t/15} = 0.7$

$-t/15 = \ln(0.7)$

$t = -15 \ln 0.7$

$= 5.301$ min.

7) Let T be the time between arrival of successive vehicle at a zebra crossing on a road.

$$T \sim \exp(\lambda = 0.025)$$

a) mean $= 1/\lambda = \frac{1}{0.025} = 40$

$$Var = 1/\lambda^2 = \frac{1}{0.025^2} = 1600$$

b) ~~Probability~~

$$P(T > 30) = \int_{30}^{\infty} \lambda e^{-\lambda x} dx$$

$$= -e^{-\lambda x} \Big|_{30}^{\infty}$$

$$= 0 + e^{-0.025 \times 30}$$

$$= 0.4724$$

8) Let x be the volume of catalyst consumed in each trial.

$$x \sim (\mu = 30, \sigma^2 = 5^2)$$

No. of trails $= 50$

$n \geq 30$

$$\Sigma x_i \sim N(n\mu, n\sigma^2)$$

$$\Sigma x_i \sim N(1500, 1250)$$

$$P(\Sigma x_i < 1600) = P\left(Z < \frac{1600-1500}{\sqrt{1250}}\right)$$

$$= P(Z < 2.8281)$$

$$= 0.9976$$

Prob of unused catalyst left in the sol^n after all 50 trials $= 0.9976$

9) Let x be the GPA of the student.

$$x \sim (\mu = 3.5,\ \sigma = 0.3)$$

$n = 50$,

By C.L.T $n \geq 30$

$$\bar{x} \sim N(\mu, \sigma^2/n)$$

$$\bar{x} \sim N\left(\mu = 3.5,\ \sigma^2/n = \frac{0.3^2}{50}\right)$$

$P(\bar{x} > 3.6)$

$$= P\left(Z > \frac{3.6 - 3.5}{\sqrt{\frac{0.3^2}{50}}}\right)$$

$= P(Z > 2.357)$

$= P(Z < -2.357)$

$= 0.0091$

10) Let x be the no. of individuals cured.

$\mu = np$
$= 100 \times 0.8$
$= 80$

$\sigma = \sqrt{100 \times 0.8 \times 0.2}$
$= 4$.

$np > 5$, $nq > 5$
So we can use normal approximation.

a) Probability of claim rejected when $p = 0.8$

$= P(x < 75)$
$\doteq P(X < 74.5)$
$= P\left(Z < \frac{74.5 - 80}{4}\right)$
$= P(Z < -1.38)$

$= 0.0838$

a continuity correction is needed as ~~Since~~ responses are binary & counts a binomial dist^n with $n = 100$, $p = 0.8$

b) Probability that the claim will accept when $p = 0.7$

$= P(x \geq 75)$
$\doteq P(x > 74.5)$
$= P\left(Z > \frac{74.5 - 70}{4.583}\right)$
$= P(Z > 0.98)$
$= 1 - 0.8365$
$= 0.1635$

(np > 5, nq > 5 we can use normal approximation) continuity correction is needed as responses are binary & counts a binomial dist^n with $n = 100$, $p = 0.7$