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MC 3020 : Probability and statistics

Tutorial-03 **March 2024**

1. In the dynamic city of Ariviyal Nagar, renowned for its scientific and technological advancements, engineering companies constantly seek talented individuals to join their workforce. One such company is TechSolutions, a leading tech firm with a reputation for hiring exceptional engineering professionals. As part of their recruitment process, TechSolutions considers various factors to ensure they hire the best candidates. One crucial aspect is evaluating the IQ scores of engineering job applicants. These IQ scores are known to follow a normal distribution with a mean of 125 and a standard deviation of 10.

   (a) What is the probability that an engineering job applicant has an IQ score less than 135?

   (b) TechSolutions aims to identify top-performing candidates. Among engineering job applicants, what IQ score corresponds to the 95th percentile?

   (c) TechSolutions has a policy of rejecting engineering job applicants with an IQ score below 110. In their recent recruitment drive, there were 800 engineering job applicants. How many applicants were rejected based on this IQ score threshold?

   (d) TechSolutions seeks to hire candidates with above-average intelligence.Between which two IQ scores, symmetrically located around the mean, do 94% of the engineering job applicants' IQ scores fall

2. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

   (a) what fraction of the cups will contain more than 224 milliliters?

   (b) what is the probability that a cup contains between 191 and 209 milliliters?

   (c) how many cups will probably overflow if 230 milliliter cups are used for the next 1000 drinks?

   (d) below what value do we get the smallest 25% of the drinks

3. The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.03 centimeter.

   (a) What proportion of rings will have inside diameters exceeding 10.075 centimeters?

   (b) What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimeters?

   (c) Below what value of inside diameter will 15% of the piston rings fall?

4. An agricultural industry finds that the length of the pea pods for its olive-green pea crop is normally distributed with a mean of 11.5 cm and a standard deviation of 1.6 cm. A sample of 16 pea pods is chosen at random.

   (a) What is the standard deviation of the sampling distribution of the mean of this sampling process?

   (b) What is the probability that the sample mean will exceed 12.3 cm?

   (c) Between which two symmetrically located values will 68% of the sample means lie?

5. The probability that a patient recovers from a delicate heart operation is 0.9. Of the next 100 patients having this operation, what is the probability that

   (a) between 84 and 95 inclusive survive?

   (b) fewer than 86 survive

6. In an engineering faculty, the time it takes for a student to solve a particular problem follows an exponential distribution with a mean of 15 minutes.

   (a) Find the probability that a student completes the problem in less than 20 minutes.

   (b) Find the time t such that there is a 70% chance that a student takes longer than t to solve the problem.

7. The time, T seconds, between the arrival of successive vehicles at a zebra crossing on a road can be modelled by an exponential distribution with parameter $\lambda = 0.025$.

   (a) Write down the mean and the variance of T.

   (b) Find the probability that the arrival of sucessive vehicles after 30 seconds.

8. In a chemical engineering experiment, a 1600 ml solution of catalyst is employed for a series of reactions. Over the course of 50 trials, the volume of catalyst consumed in each trial follows a distribution with a mean of 30 ml and a standard deviation of 5 ml. Assuming the volume of catalyst used in each trial is independent, what is the probability that there is unused catalyst left in the solution after all 50 trials?

9. In an engineering faculty, the average GPA of 200 students in a particular program is 3.5 with a standard deviation of 0.3. If a sample of 50 students is selected randomly, what is the probability that their mean GPA will be greater than 3.6?

10. A drug manufacturer claims that a certain drug cures a blood disease, on the average, 80% of the time.To check the claim, government testers use the drug on a sample of 100 individuals and decide to accept the claim if 75 or more are cured.

    (a) What is the probability that the claim will be rejected when the cure probability is, in fact, 0.8?

    (b) What is the probability that the claim will be accepted by the government when the cure probability is as low as 0.7?