Tutorial - 04

01) 90% C.I for the proportion of engineering faculty students who are proficient in R coding

$1-\alpha = 0.9$

$\alpha = 0.1$

$\alpha/2 = 0.05$.

$$\hat{p} \pm Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= \frac{90}{300} \pm Z_{0.05} \cdot \sqrt{\frac{\frac{90}{300} \times \frac{210}{300}}{300}}$$

$$= 0.3 \pm 1.645 \sqrt{\frac{0.3 \times 0.7}{300}}$$

$$= (0.2565 \,,\ 0.3435)$$

02) $n = \dfrac{(Z_{\alpha/2})^2 \cdot p(1-p)}{E^2}$

E - margin of Error / Half width

$$= \frac{(Z_{0.05})^2 \times 0.5 \times 0.5}{(0.05)^2}$$

$$= \frac{(1.645)^2 \times 0.5 \times 0.5}{(0.05)^2}$$

$$= 270.6025$$

$$\approx 271$$

03) $n = \dfrac{(Z_{\alpha/2})^2 \sigma^2}{E^2}$

$$= \frac{(Z_{0.025})^2 \times (0.1)^2}{(0.03)^2}$$

$$= \frac{1.96^2 \times (0.1)^2}{(0.03)^2}$$

$$= 42.684$$

$$\approx 43.$$

CamScanner

04) $\cdot n = 10$

11, 14, 13, 15, 13, 11, 13, 14, 10, 12

(a) $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\bar{x} = 12.6$

$\Sigma x_i^2 = 1610$

$= \sqrt{\frac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n(n-1)}}$

$\Sigma x_i = 126$

or $\sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n-1}}$

$S = 1.5776$

98% C.I for the mean nitrate content for each 100kg bag in the company

$$\left(\bar{x} - t_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}},\ \bar{x} + t_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$$

$$= \left(12.6 - t_{9,0.01} \times \frac{1.5776}{\sqrt{10}},\ 12.6 + t_{9,0.01} \times \frac{1.5776}{\sqrt{10}}\right)$$

$$= \left(12.6 - 2.821 \times \frac{1.5776}{\sqrt{10}},\ 12.6 + 2.821 \times \frac{1.5776}{\sqrt{10}}\right)$$

$$= (11.1927,\ 14.0073)$$

(b) 95% CI for the variance nitrate content for each 100 kg bag in the company.

$$\left(\frac{(n-1)s^2}{\chi_u^2},\ \frac{(n-1)s^2}{\chi_L^2}\right)$$

$\chi^2_{9,0.025} = 19.02$

$\chi^2_{9,0.975} = 2.70$

$$= \left(\frac{9 \times (1.5776)^2}{19.02};\ \frac{9 \times (1.5776)^2}{2.70}\right)$$

$\chi^2_{n-1,\alpha/2}$

$$= (1.1777,\ 8.2961)$$

$\chi^2_{n-1,1-\alpha/2}$.

95% C.I for the standard deviation nitrate content for each 100kg bag in the company.

$$\left(\sqrt{\frac{(n-1)s^2}{\chi_u^2}}, \sqrt{\frac{(n-1)s^2}{\chi_L^2}}\right)$$

$$= \left(\sqrt{1.1777}, \sqrt{8.2961}\right)$$

$$= (1.0852, 2.8803)$$

05) $\mu = 1.20$

$\sigma = 0.32$

$n = 15$

(a) $\therefore \bar{x} = \frac{\sum_{i=1}^{15} x_i}{n}$

$$= \frac{21.99}{15}$$

$$= 1.466$$

(b) 95% C.I on the mean mercury concentration after the accident

$$\bar{x} \pm Z_{\alpha/2} \cdot S \cdot E(\bar{x})$$

$$= \bar{x} \pm Z_{0.025} \cdot \sigma/\sqrt{n}$$

$$= 1.466 \pm 1.96 \times \frac{0.32}{\sqrt{15}}$$

$$= (1.3041, 1.6280)$$

We are 95% confident the true mean mercury concentration lie between (1.3041, 1.6280) after the accident.

(c) Step 1 : Hypothesis

$H_0 : \mu \leq 1.20$

$H_1 : \mu > 1.20$ (right ~~sided~~ test)

Step 2 : Test statistic value calculation.

$$Z = \frac{\bar{x} - \mu_0}{\left(\sigma/\sqrt{n}\right)}$$

$$= \frac{1.466 - 1.20}{\left(0.32/\sqrt{15}\right)}$$

$$= 3.2194$$

Step 3 : Critical value

$$Z_\alpha = Z_{0.05}$$
$$= 1.645$$

Step 4 : Test value falls in rejection region.

$$1.645 < 3.2194$$

So we reject Ho.

Step 5 :- There is evidence that the mean mercury concentration has increase by the accident at 5% significance level.

$$n = \frac{Z_{\alpha/2}^2 \cdot \sigma^2}{E^2}$$

$$= \frac{Z_{0.015}^2 \cdot 3^2}{(0.5)^2}$$

$$= \frac{2.17^2 \times 3^2}{(0.5)^2}$$

$$= 169.5204$$

$$\simeq 170$$

Range 0 – 12

$\sigma \simeq \frac{R}{4} = \frac{12}{4} = 3$

30 minutes = 0.5 hour

step 1 :

H_0 :- $\mu \leq 50$

H_1 :- $\mu > 50$ (Right tail)

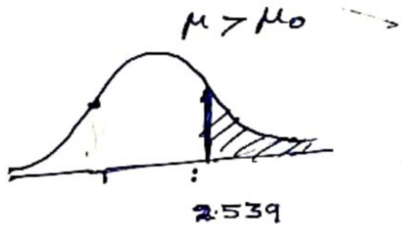
step 2 :

$s = 3$
$n = 20$
$\bar{x} = 52$

Test statistics $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$= \frac{52 - 50}{3/\sqrt{20}}$

≈ 2.98



step 3 : critical value :

$t_{n-1, \alpha}$

$t_{20-1, 0.01} = t_{19, 0.01}$

$= 2.539$

Step 4 :-

$2.98 > 2.539$

test value falls in the critical region.

so reject H_0

Step 5 :-

Mean maximum load capacity is significally higher than 50 tons and the test is carried out with 1% level of significance.

Q8) Step 1 : Hypothesis

5%

Ho :- $\mu = 60$

H_1 :- $\mu \neq 60$

$\sigma = 4$
$n = 25$
$\bar{x} = 58$

Step 2 :- test statistics.

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{58 - 60}{4/\sqrt{25}}$$

$$= -2.5$$

left $\Rightarrow P(Z < Z_0)$
Right $\Rightarrow P(Z > Z_0)$
Two tail $\Rightarrow 2 \times P(Z > |Z_0|)$

Step 3 : P value

P-value $= 2 * P(Z > |-2.5|)$

$$\text{P-value} = 2 * P(Z > |2.5|)$$
$$= 2 * [1 - P(Z < 2.5)]$$
$$= 2 * [1 - 0.9937]$$
$$= 2 * 0.0063$$
$$= 0.0126$$

Step 4 :- If P-value < 0.05 reject Ho.

Here P-value $0.0126 < 0.05$; So reject Ho.

Step 5 :-

So we can conclude that fuel efficiency of its new hybrid model is not 60 miles per gallon (mpg) on average at 5% level of significance.

09)

$$\bar{x} = \frac{\sum_{i=1}^{12} x_i}{12}$$

$$= \frac{4680}{12}$$

$$= 390$$

$n = 12$

$\bar{X} = 390$

$\sum x_i^2 = 1825958$

$\sum x_i = 4680$

$$S = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{n \sum x_i^2 - (\sum x_i)^2}{n(n-1)}}$$

$$= 8.3011$$

95% C.I on the mean number of improperly issued ticket

$$\left(\bar{X} - t_{n-1,\alpha/2} \frac{S}{\sqrt{n}}, \bar{X} + t_{n-1,\alpha/2} \frac{S}{\sqrt{n}}\right)$$

$$= \left(390 - 2.201 \times \frac{8.3011}{\sqrt{12}}, 390 + 2.201 \times \frac{8.3011}{\sqrt{12}}\right)$$

$$= (384.7257, 395.2743)$$

we are 95% confident the true mean number of improperly issued tickets per officer falls between (384.73, 395.27).

(b) Step 1 :- Hypothesis

H_0 :- $\mu \leq 380$

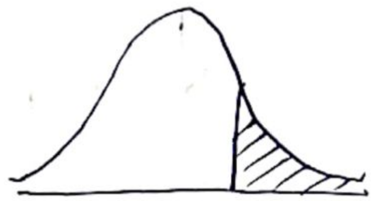
H_1 :- $\mu > 380$ (Right sided test)

Step 2 :- Test statistics.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{390 - 380}{8.3011/\sqrt{12}}$$

$$= 4.1731$$



Step 3 :- critical value

$$t_{n-1,\alpha} = t_{11,\,0.01}$$
$$= 2.718$$

Step 4 :- Test value falls in rejection region.

$$2.718 < 4.1731$$

So we reject H_0.

Step 5 :- There exist sufficient evidence to conclude that the mean number of improperly issued tickets is greater than 380. at ·1% significance level.

(c) Step 1 :- Hypothesis

H_0 :- $\sigma^2 \leq 35$

H_1 :- $\sigma^2 > 35$ (Right sided test)

Step 2 :- Test statistics

$$\chi^2 = \frac{(n-1) \times s^2}{\sigma^2}$$

$$= \frac{11 \times 8.3011^2}{35}$$

$$= 21.6569$$

Step 3 :- Critical value

$$\chi^2_{n-1, \alpha} = \chi^2_{11, 0.05}$$

$$= 19.675$$

Step 4 :- Test value falls in rejection region.

$$19.675 < 21.6569$$

So we reject H_0

Step 5 :- There is sufficient evidence to conclude that the variance of improperly issued tickets is greater than 35 at the 5% significance lev

10)

(a) $H_0 :- p \leq 0.06$

$H_1 :- p > 0.06.$

$\hat{p} \geq \frac{.8}{100} = 0.018.$

Reject H_0 if $\hat{p} \geq 0.08$

(b) Type I error := P(Reject H_0 when H_0 is true)

$= P(\hat{p} \geq 0.08 \mid P_0 = 0.06)$

$= P\left(\frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \geq \frac{0.08 - 0.06}{\sqrt{\frac{0.06(1-0.06)}{100}}} \right)$

$= P(Z > 0.8421)$

$= P(Z > -0.84)$ or $1 - P(Z < 0.84)$

$= 0.2005$

(c) The type I error indicates that the supplier incurs loss after the canning company wrongly rejects the lot because the sample has a higher number of defectives when in fact the lot has fewer than 6% defectives.

The type II error indicates wrongly accepting the lot due to the sample having a lower number of defectives when in fact the lot has higher than 6% defective.

Therefore, the canning company incurs loss.