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MC 3020 : Probability and Statistics

Worksheet **July 2024**

1. The dean of a certain business school is interested in the difference in the proportions of E's (E stands for exceptional) given by Professors Smith and Jones. Suppose the dean has access to 600-grade reports from each professor. It turns out that 80 students received E's from Professor Smith, and 110 students received E's from Professor Jones.

   (a) Form a 90% confidence interval for the true difference in proportions of E's given by the two professors.

   (b) Test whether the rate of E's of Professor Jones is significantly higher than that of Professor Smith. Use a 3% level of significance.

2. In a certain year, the mean interest rate on loans to a random sample of 80 large retailers (that is, those with assets of $\$10,000,000$ or more) was 10.0%, and the standard deviation was 0.6%. Two years later, a random sample of 100 loans to large retailers yielded a mean interest rate of 10.25% and a standard deviation of 0.72%.

   a) Would you conclude that there has been a change in the average level of interest rates for large retailers? Use a 0.01 level of significance.

   b) Find a 96% confidence interval for the difference between the two means.

3. In an effort to link cold environments with hypertension in humans, a preliminary experiment was conducted to investigate the effect of cold on hypertension in rats. Two random samples of 6 rats each were exposed to different environments. One sample of rats was held in a normal environment at 26°C. The other sample was held in a cold 5°C environment. Blood pressure and heart rates were measured for rats for both groups. The blood pressures for the 12 rats are shown in the accompanying table. Assume that the samples are from normal populations.

	26°C		5°C
Rat	Blood Pressure	Rat	Blood Pressure
1	152	7	384
2	157	8	369
3	179	9	354
4	182	10	375
5	176	11	366
6	149	12	423

   (a) Do the data provide sufficient evidence that rats exposed to a 5°C environment have a higher mean blood pressure than rats exposed to a 26°C environment? Use $\alpha = 0.05$.

   (b) Provide a 95% confidence interval on the difference in the two population means

4. An engineering team is developing a new coating material for metal surfaces used in a specific industrial application. They are testing two different formulations to determine if there is a significant difference in the wear resistance between the two coatings. To evaluate this, the team performs wear tests on metal samples coated with each formulation. A total of 24 metal samples are tested, with 12 samples coated using Formulation A (treated group) and 12 samples coated using Formulation B (untreated group). After conducting the wear tests, the wear depth in micrometers for each sample is measured, and the data are summarized in the table below:
Formulation A (Treated Group- Wear Depth in Micrometers):
18,43,28,50,16,32,13,35,38,33,6,7
Formulation B (Untreated Group- Wear Depth in Micrometers):
40,54,26,63,21,37,39,23,48,58,28,39
The engineering team wants to determine if Formulation A provides a significant improvement in wear resistance compared to Formulation B. Assume that the population distributions of the measurements are normal with equal variances.

   (a) Construct a 95% confidence interval for the difference in mean wear depth $\mu_1 - \mu_2$ to assess the size of the difference between the two means.

   (b) Test whether the mean wear depth of metal samples coated with Formulation A is significantly less than the mean wear depth of samples coated with Formulation B using a significance level α of 0.05.

5. Energy conservation is very important. Some scientists think we should give closer scrutiny to the cost (in energy) of producing various forms of food. One recent study compares the mean amount of oil required to produce 1 acre of different types of crops. For example, suppose that we want to compare the mean amount of oil required to produce 1 acre of corn versus 1 acre of cauliflower. The readings in barrels of oil per acre, based on 20-acre plots, seven for each crop, are shown in the table. Assume that the samples are from independent normal populations.

Corn	5.6	7.1	4.5	6.0	7.9	4.8	5.7
Cauliflower	15.9	13.4	17.6	16.8	15.8	16.3	17.1

   a) Find a 90% confidence interval for the difference in the mean amount of oil required to produce these two crops. Assume that the populations have equal variances.

   b) Test whether there is any difference between the means. Assume that the populations have equal variances. Use a 5% significance level.

   c) Find a 90% confidence interval for the difference in the mean amount of oil required to produce these two crops. Assume that the population variances are unequal.

   d) Test whether there is any difference between the means. Assume that the population variances are unequal. Use a 5% significance level.

   e) Test whether the population variances are equal or not. Use a 10% level of significance.