Worksheet

01) a) Jone $P_1 = \frac{110}{600}$

Smith $P_2 = \frac{80}{600}$

$\alpha = 0.1$

$Z_{0.05} = 1.645$

90% C.I for the true difference in proportions of E's given by prof.smith & prof.Jone.

90% C.I for $P_1 - P_2$ is

$$= \left((\hat{P}_1 - \hat{P}_2) - Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}} , \hat{P}_1 - \hat{P}_2 + Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}} \right)$$

$$= \left[\left(\frac{110}{600} - \frac{80}{600}\right) - 1.645\sqrt{\frac{\frac{110}{600} \times \frac{490}{600}}{600} + \frac{\frac{80}{600} \times \frac{520}{600}}{600}} , \left(\frac{110}{600} - \frac{80}{600}\right) + 1.645\sqrt{\frac{\frac{110}{600} \times \frac{490}{600}}{600} + \frac{\frac{80}{600} \times \frac{520}{600}}{600}} \right]$$

$$= (0.05 - 1.645 \times 0.02102 , 0.05 + 1.645 \times 0.02102)$$

$$= (0.0154 , 0.0846)$$

b)

Step 1.

$H_0 : P_1 \leq P_2$

$H_1 : P_1 > P_2$ (Right sided test)

Step 2.

Test statistic

$$Z = \frac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{\bar{P}(1-\bar{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\bar{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{110+80}{600+600} = \frac{190}{1200} = 0.1583$$

$$= \frac{\frac{110}{600} - \frac{80}{600} - 0}{\sqrt{\frac{190}{1200} \times \frac{1010}{1200}\left(\frac{1}{600} + \frac{1}{600}\right)}}$$

$$= 2.3725$$

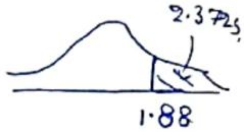
Step 3:

Critical value $Z_{0.03} = 1.88$

Step 4:

Test value > Critical value

Test value falls on the rejection region. So reject H_0



Step 5:

We conclude that the rate of E's professor Jones is significantly higher than that of professor smith & the test is carried out with 3% level of significance.

02)

[Two are independent pop^n dist^n not normal, σ_1^2, σ_2^2 are unknown n_1, n_2 are large.]

a)

Step 1.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ (two sided test)

$n_1 = 80$ $\qquad$ $n_2 = 100$

$\bar{x}_1 = 10$ $\qquad$ $\bar{x}_2 = 10.25$

$s_1 = 0.6$ $\qquad$ $s_2 = 0.72$

Step 2:

Test statistic $Z = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$Var = s_1^2$

$(S.D)^2$

$$= \frac{10.0 - 10.25 - (0)}{\sqrt{\dfrac{0.6^2}{80} + \dfrac{0.72^2}{100}}}$$

$$= -2.5405.$$

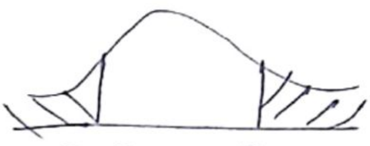
Step 3:

Critical value $= Z_{\frac{0.01}{2}} = 2.575$

Step 4:

Test value falls in acceptance region. $|-2.5405| < 2.575$

So, we donot reject H_0.



Step 5:

There is no significant change in the interest rates for large retailers at 0.01 level of significance.

b)

96% C.I for $\mu_1-\mu_2$ is

$Z_{\frac{0.04}{2}} = 2.05$

$$\left(\bar{x}_1-\bar{x}_2 - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\ ,\ \bar{x}_1-\bar{x}_2 + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\right)$$

$$= \left(10-10.25 - 2.05\sqrt{\frac{0.6^2}{80}+\frac{0.72^2}{100}}\ ,\ 10-10.25 + 2.05\sqrt{\frac{0.60^2}{80}+\frac{0.72^2}{100}}\right)$$

$$= (-0.4517\ ,\ -0.0483)$$

96% C.I for $\mu_2-\mu_1$ is $(0.0483, 0.4517)$.

03)

Samples are from normal popⁿ

σ_1^2, σ_2^2 are unknown. Sample size is small

$F =$

At 26°c	At 5°c
$n_1 = 6$	$n_2 = 6$
$S_1 = 14.7705$	$S_2 = 23.9562$
$\Sigma x_1^2 = 166095$	$\Sigma x_2^2 = 862443$
$\Sigma x_1 = 995$	$\Sigma x_2 = 2271$
$\bar{x}_1 = 165.833$	$\bar{x}_2 = 378.5$

We want to check variances are equal or unequal.

Step 1:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

Step 2:

Test statistic $F = \dfrac{S_1^2}{S_2^2} = \dfrac{14.7705^2}{23.9562^2} = 0.3801$

Step 3:

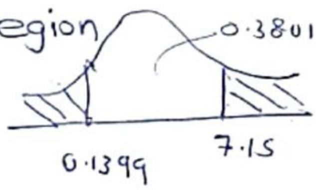
Critical value $F_{0.025,5,5} = 7.15$

$F_{0.975,5,5} = \dfrac{1}{7.15} = 0.1399$

Step 4: $0.1399 < F = 0.3801 < 7.15$

Test value falls in acceptance region

Donot reject H_0.



Step 5:

We can conclude that the variances are not significantly different at 5% level of significance.

Step 1:

$H_0 : \mu_{26} - \mu_5 \geq 0$ OR $\mu_{26} \geq \mu_5$

$H_1 : \mu_{26} - \mu_5 < 0$ $\quad$ $\mu_{26} < \mu_5$ (left sided test)

Step 2:

Test statistic

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_{26} - \mu_5)}{S_p\sqrt{1/n_1 + 1/n_2}}$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$= \frac{165.833 - 378.5 - 0}{19.9006\sqrt{\frac{1}{6}+\frac{1}{6}}}$$

$$= \sqrt{\frac{5\times 14.7705^2 + 5\times 23.9562^2}{10}}$$

$$= 19.9006$$

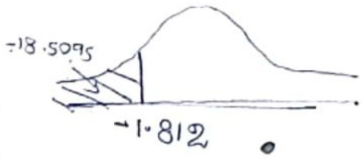
$$= -18.5095$$

Step 3:

Critical value $= t_{0.05, 10} = -1.812$

Step 4:

Test value $= -18.5096 <$ Critical value $= -1.812$



Test value falls in rejection region.
So reject H_0.

Step 5:

We can conclude that μ_{26} is less than μ_5. at 5% level of significant.

b) $t_{n_1+n_2-2,\alpha/2} = t_{10,0.025} = 2.228$

95% C.I on the difference in two pop^n mean $\mu_{26}-\mu_5$ is

$$\left(\bar{x}_1-\bar{x}_2 - t_{n_1+n_2-2,\alpha/2}\, S_p\sqrt{1/n_1+1/n_2}\ ,\ \bar{x}_1-\bar{x}_2 + t_{n_1+n_2-2,\alpha/2}\, S_p\sqrt{1/n_1+1/n_2}\right)$$

$$= \left(165.833 - 378.5 - 2.228 \times 19.9006\sqrt{1/6+1/6}\ ,\ 165.833-378.5+2.228\times 19.9006\sqrt{1/6+1/6}\right)$$

$$= (-238.2659,\ -187.0681)$$

It indicates that the average warm temperature rat blood pressure is between 187 and 238 units lower than the average 5°c rat blood pressure.

04)

Formulation A (Treated Gp)

$\bar{x}_A = \frac{319}{12} = 26.5833$

$S_1^2 = \frac{n_A \sum x_A^2 - (\sum x_A)^2}{n_A(n_A - 1)}$

$= \frac{12 \times 10749 - (319)^2}{12 \times (12-1)}$

$= 206.2651$

$n_A = 12$

Formulation B (Untreated Gp)

$\bar{x}_B = \frac{476}{12} = 39.6667$

$S_2^2 = \frac{n_B \sum x_B^2 - (\sum x_B)^2}{n_B(n_B - 1)}$

$= \frac{12 \times 20994 - (476)^2}{12 \times (12-1)}$

$= 192.0606.$

$n_B = 12.$

95% C.I for the difference in mean wear depth $\mu_A - \mu_B$ is.

$$\bar{x}_A - \bar{x}_B \pm t_{22, 0.025} S_p \sqrt{1/n_A + 1/n_B}.$$

$$S_p = \sqrt{\frac{(n_A - 1) S_A^2 + (n_B - 1) S_B^2}{n_A + n_B - 2}}$$

$$= \sqrt{\frac{11 \times 206.2651 + 11 \times 192.0606}{12 + 12 - 2}}$$

$$= 14.1125.$$

$$t_{n_A + n_B - 2, \alpha/2} = t_{22, 0.025} = 2.2074.$$

$$= (26.5833 - 39.6667) \pm 2.2074 \times 14.1125 \sqrt{1/12 + 1/12}$$

$$\Rightarrow (-13.0834 \pm 2.074 \times 5.7614)$$

$$= (-25.0362, -1.1342)$$

Step 1:

$H_0 : \mu_1 \geq \mu_2$

$H_1 : \mu_1 < \mu_2$ (left tail test).

Step 2:

Test statistic

$$T = \frac{\bar{X}_A - \bar{X}_B - 0}{S_p \sqrt{1/n_A + 1/n_B}}$$

$$= \frac{26.5833 - 39.6667}{14.1125\sqrt{1/12 + 1/12}}$$

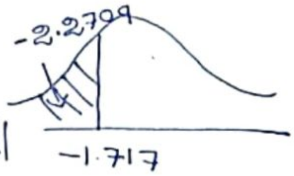
$$= -2.2709$$

Step 3:

Critical value $= t_{n_A+n_B-2,\alpha} = t_{22,0.05} = -1.717$.

Step 4:

Test value $= -2.2704 < -1.717$



Since test value falls in rejection region. So we can reject the null hypothesis.

Step 5:

Therefore we can conclude that the mean wear depth of metal samples coated with formulation A is significantly less than the mean wear depth of samples coated with formulation B at the $\alpha = 0.05$ significance level.

05)

Corn	5.6	7.1	4.5	6.0	7.9	4.8	5.7
Cauliflower	15.9	13.4	17.6	16.8	15.8	16.3	17.1

Samples from independent normal popⁿ given that assume that the popⁿ variances are unknown and equal

a)

$$\bar{x}_1 = \frac{\sum x_1}{n} = \frac{41.6}{7} = 5.9429.$$

$$s_1^2 = \frac{n\sum x_1^2 - (\sum x_1)^2}{n(n-1)} = \frac{7\times(255.96) - (41.6)^2}{7\times(7-1)} = 1.4562$$

$$\bar{x}_2 = \frac{\sum x_2}{n} = \frac{112.9}{7} = 16.1286$$

$$s_2^2 = \frac{n\sum x_2^2 - (\sum x_2)^2}{n(n-1)} = \frac{7\times 1832.11 - (112.9)^2}{7\times 6} = 1.8657$$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)\times 1.4561 + (7-1)1.8657}{7+7-2}}$$

$$= 1.2888 \qquad t_{12,0.05} = 1.782$$

90% C.I for $\mu_1 - \mu_2$ is

$$\left(\bar{x}_1 - \bar{x}_2 - t_{n_1+n_2-2,\alpha/2} S_p\sqrt{1/n_1 + 1/n_2},\ \bar{x}_1 - \bar{x}_2 + t_{n_1+n_2-2,\alpha/2} S_p\sqrt{1/n_1+1/n_2}\right)$$

$$= \Big((5.9429 - 16.1286) - 1.782\times 1.2888\sqrt{1/7 + 1/7},$$
$$5.9429 - 16.1286 + 1.782\times 1.2888\sqrt{1/7+1/7}\Big)$$

$$= (-11.4133, -8.9579)$$

90% C.I for $\mu_2 - \mu_1$ is $(8.9579, 11.4133)$

b) Given that populations have equal variances.

Step 1:

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ (two sided test

Step 2.

Test statistic $T = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$

$= \dfrac{5.9429 - 16.1286 - 0}{1.2888 \sqrt{1/7 + 1/7}}$

$= -14.7856$

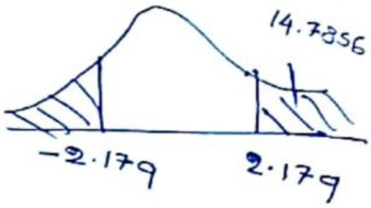
Step 3:

Critical value $t_{n_1+n_2-2, \alpha/2} = t_{12, 0.025} = 2.179$

Step 4:

$|-14.7856| > 2.179$

Test value falls in rejection region. So, we can reject H_0.



Step 5:

We can conclude that there is a significant different between the means at 5% level of significance.

(population variances are unknown and notequal)

90% C.I for $\mu_1 - \mu_2$ is

$$\left(\bar{x}_1 - \bar{x}_2 - t_{v,\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\ ,\ \bar{x}_1 - \bar{x}_2 + t_{v,\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \right)$$

$$A = \frac{s_1^2}{n_1} = \frac{1.4562}{7} = 0.2080 \qquad B = \frac{s_2^2}{n_2} = \frac{1.8657}{7} = 0.2665$$

$$v = \frac{(A+B)^2}{A^2/n_1-1 + B^2/n_2-1} = \frac{(0.2080+0.2665)^2}{\frac{0.2080^2}{6}+\frac{0.2665^2}{6}} = 11.8203$$

$$\simeq 12$$

$$t_{12,0.10/2} = 1.782$$

$$= \left(5.9429 - 16.1286 - 1.782 \times \sqrt{\frac{1.4562}{7}+\frac{1.8657}{7}}\ ,\ 5.9429 - 16.1286 + 1.782\sqrt{\frac{1.4562}{7}+\frac{1.8657}{7}} \right)$$

$$= (-11.4133,\ -8.9581)$$

90% C.I for $\mu_2 - \mu_1$ is

$$(8.9581,\ 11.4133)$$

d) Population variances are unknown & unequ

Step 1.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$. (two sided test)

Step 2:

Test statistic

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{5.9429 - 16.1286 - 0}{\sqrt{\frac{1.4562}{7} + \frac{1.8657}{7}}}$$

$$= -14.7859$$

Step 3:

Critical value $t_{v, \alpha/2} = t_{12, 0.025} = 2.179$

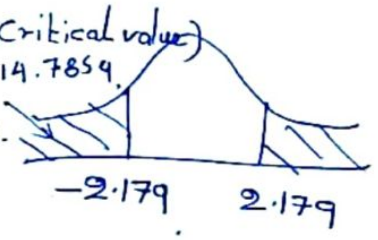
$d.f = 12.8203 \simeq 12$ (from part c)

Step 4.

Test value $|t| = |-14.7859| > 2.179$ (Critical value)

Test value falls rejection region.

So reject H_0.



Step 5:

We can conclude that there is a significant different between the means at 5% level of significance.

F_{α, v_1, v_2} or $F^{v_1}_{v_2, \alpha}$

$\alpha = 0.05$

$\alpha = 10\%$

$v_2 \diagdown v_1$	Numerator d.f
denominator d.f	

Step 1.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2:

Test statistic $F = \dfrac{s_1^2}{s_2^2} = \dfrac{1.4562}{1.8657}$

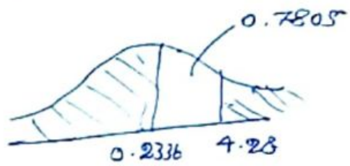
$= 0.7805$

$F_{\alpha, v_1, v_2} = \dfrac{1}{F_{1-\alpha, v_2, v_1}}$

Step 3:

Critical value $= F_{0.05, 6, 6} = 4.28$

$F_{0.95, 6, 6} = \dfrac{1}{F_{0.05, 6, 6}} = \dfrac{1}{4.28} = 0.2336$



Step 4: $0.2336 < F < 4.28$

Test value falls in acceptance region

So, Donot reject H_0.

Step 5:

We conclude that the variances are not significantly different & the test is carried out 10% level of significance.