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MC 3020 : Probability and statistics

Tutorial-01 **February 2024**

1. In a certain factory, machines I, II, and III are all producing springs of the same length. Machines I, II, and III produce 1%, 4%, and 2% defective springs, respectively. Of the total production of springs in the factory, Machine I produces 30%, Machine II produces 25%, and Machine III produces 45%.

   (a) If one spring is selected at random from the total springs produced in a given day, determine the probability that it is defective.

   (b) Given that the selected spring is defective, find the conditional probability that it was produced by Machine II

2. A certain material is fed to a two-step process. For this process, the probabilities of a malfunction are $P(B_1) = 0.03$ and $P(B_2) = 0.05$, where the factors B_1 and B_2 represent a malfunction in Steps 1 and 2, respectively. A sample of the final product is taken and found to be unacceptable. Our experience over the previous two months indicates that a defective product will be obtained 20% of the time if Section 1 of the process malfunctions and 36% of the time if Section 2 malfunctions. That means $P(E|B_1) = 0.2$ and $P(E|B_2) = 0.36$. In which part of the process does the fault probably lie?

3. Consider three boxes of computer parts. In Box X, there are 15 RAM sticks, with 3 being defective; Box Y contains 8 hard drives, and 2 are defective; and Box Z holds 12 CPUs, with 4 being defective. A box is chosen at random, and then a part is drawn from that box randomly. Determine the probability that the drawn part is defective.

4. Three bags, A, B and C, each contains coloured balls. Bag A contains 4 red balls and 2 yellow balls only. Bag B contains 4 red balls and 1 yellow ball only. Bag C contains 6 red balls only. In a game, Mike randomly takes a ball from bag A, records the colour, and places it in bag C. He then takes a ball at random from bag B, records the colour and places it in bag C. Finally, Mike takes a ball at random from bag C and records the colour.

   (a) Draw the tree diagram to illustrate the game with all probabilities.

   (b) Show that the probability that Mike records a yellow ball exactly twice is 1/10.

   (c) Given that Mike records exactly 2 yellow balls, find the probability that the ball is drawn from bag A is red.

5. A college finds that 10% of students have taken a distance learning class and that 40% of students are part time students. Of the part time students, 20% have taken a distance learning class. Let D = event that a student takes a distance learning class and E = event that a student is a part time student

   (a) Find P(D AND E).

   (b) Find $P(E|D)$.

(c) Find P(D OR E).

(d) Using an appropriate test, show whether D and E are independent.

(e) Using an appropriate test, show whether D and E are mutually exclusive.

6. Suppose we have a group of 200 employees in a company categorized by their departments and job titles as follows:

	Sales	Marketing	HR
Male	50	30	20
Female	25	40	35

(a) If we randomly select one employee, what is the probability of getting a Sales employee, given that a male was selected?

(b) If we randomly select one employee, what is the probability of getting a male, given that a Sales employee was selected? Is this the same result found in (a)?

(c) If we randomly select one employee, what is the probability of getting a female, given that an HR employee was selected?

(d) If we randomly select one employee, let A be the event that the selected employee is a female and B be the event that the selected employee is from the Marketing department. Are events A and B mutually exclusive?

7. The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and the filter need changing is 0.14.

(a) If the oil has to be changed, what is the probability that a new oil filter is needed?

(b) If a new oil filter is needed, what is the probability that the oil has to be changed?

8. A truth serum has the property that 90% of the guilty suspects are properly judged while, of course, 10% of the guilty suspects are improperly found innocent. On the other hand, innocent suspects are misjudged 1% of the time. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime, and the serum indicates that he is guilty, what is the probability that he is innocent?

9. A computer system is built so that if component K_1 fails, it is by passed and K_2 is used. If K_2 fails, then K_3 is used. Suppose that the probability that K_1 fails is 0.01, that K_2 fails is 0.03, and that K_3 fails is 0.08. Moreover, we can assume that the failures are mutually independent events. Then what is the probability that the system does not failure?

10. Suppose there are 10 employees in a company, consisting of 6 engineers and 4 managers. Three employees are selected randomly to attend a training workshop. Find the probabilities for the following scenarios:

(a) All 3 selected will be engineers.

(b) All 3 selected will be managers.

(c) 2 engineers and 1 manager will be selected.

(d) 1 engineer and 2 managers will be selected.