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MC 3020 : Probability and statistics

Tutorial-05 **May 2024**

1. An educational researcher designs a study to compare the effectiveness of teaching English to non-English speaking people by a computer software program and by the traditional classroom system. The researcher randomly assigns 125 students from a class of 300 to instruction using the computer. The remaining 175 students are instructed using the traditional method. At the end of a 6-month instructional period, all 300 students are given an examination with the results reported in given table

Exam Results	**Computer Instruction**	**Traditional Instruction**
Pass	94	113
Fail	31	62

   (a) Form 95% confidence interval for the difference in proportions of students passing the examination between instruction using the computer software program and the traditional method

   (b) Does instruction using the computer software program appear to increase the proportion of students passing the examination in comparison to the pass rate using the traditional method of instruction? Use a $\alpha = 0.01$.

2. An experiment was conducted to evaluate the effectiveness of a new construction material in enhancing the structural integrity of concrete beams. A random sample of 24 beams, all with similar dimensions and initial structural conditions, was selected. Twelve of the beams were reinforced with the new material, while the remaining twelve were left un-reinforced. After a 6-month period of exposure to varying loads and environmental conditions, the beams were tested for their load-bearing capacity, and the results are summarized in the table below:

Beam Number	1	2	3	4	5	6	7	8	9	10	11	12
Reinforced Beams	18	43	28	50	16	32	13	35	38	33	6	7
Un-reinforced Beams	40	54	26	63	21	37	39	23	48	58	28	39

   Assume that the load-bearing capacity measurements follow a normal distribution with equal variances.

   (a) Test whether the mean load-bearing capacity of the reinforced beams is greater than the mean for the un-reinforced beams. Use a significance level of $\alpha = 0.05$.

   (b) Place a 95% confidence interval on the difference $\mu_1 - \mu_2$ to assess the magnitude of the difference in load-bearing capacity between the two types of beams.

3. A group of engineering students is investigating the effectiveness of a new ergonomic chair design in reducing discomfort during long hours of computer usage. Seven participants were recruited for the study, and their discomfort levels were assessed before and after using the new chair prototype for a 6-week period. Discomfort levels were rated on a scale from 1 to 10, with higher scores indicating greater discomfort.

Participant	1	2	3	4	5	6	7
Before	6	7	5	4	3	8	7
After	4	3	6	4	3	7	7

Assuming that discomfort level changes follow an approximately normal distribution, the engineering students are interested in evaluating the impact of their chair design on reducing discomfort.

(a) Construct a 95% confidence interval for the mean difference in discomfort level changes experienced by the participants.

(b) Can it be concluded that the discomfort level has changed significantly at the 5% level of significance?

4. A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Seven subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table.(Cholesterol level is measured in milligrams per deciliter.)

Subject	1	2	3	4	5	6	7
Before	210	235	208	190	172	244	232
After	190	170	210	188	173	228	232

Assume the variable (person's cholesterol level changes) is approximately normally distributed.

(a) Construct 99% confidence interval for the mean difference of person's cholesterol level changes.

(b) Can it be concluded that the cholesterol level has been changed at the 10% level of significance?

5. A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are

$$\text{Brand A: } \bar{x_1} = 36{,}300 \text{ kilometers, } s_1 = 5000 \text{ kilometers.}$$

$$\text{Brand B: } \bar{x_2} = 38{,}100 \text{ kilometers, } s_2 = 6100 \text{ kilometers.}$$

Compute a 95% confidence interval for $\mu_A - \mu_B$ assuming the populations to be approximately normally distributed. You may not assume that the variances are equal.

6. In a study conducted as part of the MC3020 course, students aimed to analyze the performance of two different algorithms, Algorithm A and Algorithm B, in optimizing computational efficiency for a specific engineering problem. A sample of 20 engineering students participated in the study, where ten students were randomly assigned to Algorithm A and the other ten to Algorithm B. After implementing the algorithms, each student measured the time taken to solve a set of computational problems, and their results were recorded.

The time taken by students using Algorithm A was as follows:

$$5.6, 6.2, 5.8, 5.4, 5.9, 6.3, 5.7, 6.1, 5.5, 6.0.$$

The time taken by students using Algorithm B was as follows:

$$5.1, 5.8, 6.0, 5.3, 5.7, 5.4, 5.9, 6.2, 5.5, 5.9.$$

Assume that the time measurements are normally distributed.

(a) Find a 95% confidence interval for the difference in the mean time taken between Algorithm A and Algorithm B. Assume that the populations have equal variances.

(b) Test whether the populations of time taken by students using Algorithm A and Algorithm B have equal variances. Use a 5% level of significance.

(c) Test whether there is any difference between the mean time taken by Algorithm A and Algorithm B. Use a 5% significance level.

(d) Compare the conclusions drawn from the confidence intervals and hypothesis tests. Do both methods lead to the same conclusion? If not, discuss the implications of the differing conclusions.

7. A research study aims to compare the effectiveness of two teaching methods, Method A and Method B, in improving students' understanding of thermodynamics concepts. A sample of 20 students from the engineering faculty participated in the study. Ten students were randomly assigned to Method A, and the other ten were assigned to Method B. After completing the course, each student took a standardized test on thermodynamics, and their scores were recorded.
The scores obtained by students in Method A were as follows:

$$78, 82, 85, 79, 81, 77, 83, 86, 80, 84$$

The scores obtained by students in Method B were as follows:

$$75, 79, 82, 76, 80, 77, 81, 83, 78, 82$$

Assume that the scores are normally distributed

(a) Find a 90% confidence interval for the difference in the mean test scores between Method A and Method B. Assume that the populations have equal variances.

(b) Test whether there is any difference between the mean test scores of Method A and Method B. Assume that the populations have equal variances. Use a 5% significance level.

(c) Find a 90% confidence interval for the difference in the mean test scores between Method A and Method B. Assume that the population variances are unequal.

(d) Test whether there is any difference between the mean test scores of Method A and Method B. Assume that the population variances are unequal. Use a 5% significance level.

(e) Test whether the population variances of the test scores obtained by students in Method A and Method B are equal or not. Use a 10% level of significance.