MC3020 : Probability & Statistics.

Tutorial-01 Solutions.

1) M_1 be the spring produced by machine 1.
M_2 be the spring produced by machine 2.
M_3 be the spring produced by machine 3.

D be the Defective spring

$P(D/M_1) = 0.01$

$P(D/M_2) = 0.04$

$P(D/M_3) = 0.02$.

$P(M_1) = 0.3 \quad P(M_2) = 0.25 \quad P(M_3) = 0.45$

a) $P(D) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3)$

$= P(D/M_1) \cdot P(M_1) + P(D/M_2) \cdot P(M_2) + P(D/M_3$

$= 0.01 \times 0.3 + 0.04 \times 0.25 + 0.02 \times 0.45$

$= 0.022$.

b) $P(M_2|D) = \dfrac{P(D|M_2) \cdot P(M_2)}{P(D)}$

$= \dfrac{0.04 \times 0.25}{0.022}$.

$= \dfrac{10}{22}$

$= 0.4545$.

2) B_1, B_2 represent a malfunction in steps 1 & 2

$P(B_1) = 0.03$

$P(B_2) = 0.05$

E be the defective product obtained.

$P(E|B_1) = 0.2$

$P(E|B_2) = 0.36$

By using bay's law.

$$P(B_1|E) = \frac{P(E|B_1) \times P(B_1)}{P(E|B_1) \times P(B_1) + P(E|B_2) \times P(B_2)}$$

$$= \frac{0.2 \times 0.03}{0.2 \times 0.03 + 0.36 \times 0.05}$$

$$= 0.25$$

$$P(B_2|E) = \frac{P(E|B_2) \times P(B_2)}{P(E|B_1) \cdot P(B_1) + P(E|B_2) \cdot P(B_2)}$$

$$= \frac{0.36 \times 0.05}{0.2 \times 0.03 + 0.36 \times 0.05}$$

$$= 0.75$$

From these results, Section 2 of the process is the more likely to need Corrective maintenance.

(iv)

X	Y	Z
15	8	12
3D	2D	4D

X: Event that box X is chosen

Y: Event that box Y is chosen.

Z: Event that box Z is chosen.

D: Event that drawn part is defective.

$$P(X) = P(Y) = P(Z) = \frac{1}{3}$$

$$P(D) = P(D|X) \cdot P(X) + P(D|Y) \cdot P(Y) + P(D|Z) \cdot P(Z)$$

$$= \frac{3}{15} \times \frac{1}{3} + \frac{2}{8} \times \frac{1}{3} + \frac{4}{12} \times \frac{1}{3}$$

$$= \frac{1}{15} + \frac{1}{12} + \frac{1}{9}$$

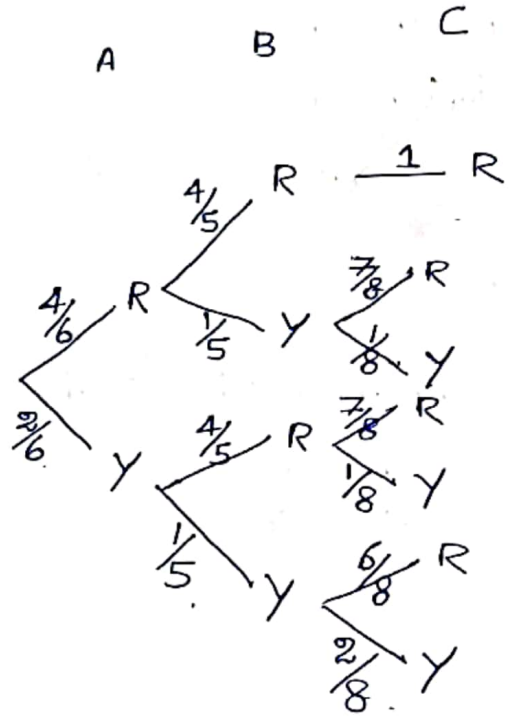
$$= 0.2611$$

mass

4)

A	B	C
4R	4R	6R
2Y	1Y	

a)



b) P(Record yellow exactly twice)

$= P(RYY) + P(YRY) + P(YYR)$

$= \frac{4}{6} \times \frac{1}{5} \times \frac{1}{8} + \frac{2}{6} \times \frac{4}{5} \times \frac{1}{8} + \frac{2}{6} \times \frac{1}{5} \times \frac{6}{8}$

$= \frac{1}{60} + \frac{1}{30} + \frac{1}{20}$

$= \frac{1}{10}$

c) Let x be the event A is red.

z be the event exactly 2 Yellow ball

$x = \{\{RRR\}, \{RYR\}, \{RYY\}\}$

$z = \{\{RYY\}, \{YRY\}, \{YYR\}\}$

$x \cap z = \{\{RYY\}$

$$P(x/z) = \frac{P(x \cap z)}{P(z)}$$

$$= \frac{P(RYY)}{P(z)}$$

$$= \frac{\frac{4}{6} \times \frac{1}{5} * \frac{1}{8}}{\frac{1}{10}}$$

$$= \frac{1}{6}$$

$$= 0.1667$$

5)

D be the event that a student takes a distance learning class

E be the event that a student is a part time student.

$P(D) = 0.1$

$P(E) = 0.4$

$P(D/E) = 0.2$

a) $P(D \text{ AND } E) = P(D \cap E)$

$= P(D|E) \cdot P(E)$

$= 0.2 \times 0.4$

$= 0.08$

b) $P(E|D) = \dfrac{P(D|E) \cdot P(E)}{P(D)}$

$= \dfrac{0.08}{0.1}$

$= \dfrac{4}{5} = 0.8$

c) $P(D \text{ or } E) = P(D \cup E)$

$= P(D) + P(E) - P(D \cap E)$

$= 0.1 + 0.4 - 0.08$

$= 0.42$

d) $P(D \cap E) = 0.08$ OR $P(D|E) \neq P(D)$

$\neq 0.1 \times 0.4$

$= P(D) \times P(E)$

Since $P(D \cap E) \neq P(D) \cdot P(E)$, D and E are not independent.

e) $P(D \cap E) = 0.08 \neq 0$

Since $P(D \cap E) \neq 0$, D & E are not mutually exclusive

6)

	Sales	Marketing	HR	
Male	50	30	20	100
Femde	25	40	35	100
	75	70	55	200

Let M be Male
F be Female
S be sales
M_1 be marketing
H be HR

a) $P(S|M) = \frac{P(S \cap M)}{P(M)}$ OR $P(S/M) = \frac{50}{100}$

$= \frac{50/200}{100/200}$ $= \frac{1}{2}$

$= \frac{50}{100}$

$= \frac{1}{2} = 0.5$

b) $P(M|S) = \frac{P(M \cap S)}{P(S)}$

$= \frac{50/200}{75/200}$

$= \frac{2}{3} = 0.6667$

No.

c) $P(F|H) = \frac{P(F \cap H)}{P(H)}$

$= \frac{\frac{35}{200}}{\frac{55}{200}}$

$= \frac{7}{11} = 0.6363$

d) A – Selected employee is a female

B – Selected employee is from the marketing department.

$P(A \cap B) = \frac{40}{200} = \frac{1}{5} \neq 0$

Events A and B are not mutually exclusive

7)

C be the event an oil change is needed.

F be the event an oil filter is needed.

$P(C) = 0.25 \qquad P(F) = 0.40$

$P(C \cap F) = 0.14$

a) $P(F|C) = \frac{P(F \cap C)}{P(C)}$

$= \frac{0.14}{0.25}$

$= 0.56$

b) $P(C|F) = \frac{P(C \cap F)}{P(F)}$

$= \frac{0.14}{0.40}$

$= 0.35$

8)

G_1 – Guilty of committing a crime.
I – Innocent of the crime.
G_1 – Judged Guilty of the crime.
I_1 – Judged Innocent of the crime.

$P(G_1|G) = 0.9$ $\quad P(I_1|G) = 0.1$

$P(G_1|I) = 0.01$ $\quad P(G) = 0.05$ $\quad P(I) = 0.95$

$$P(I|G_1) = \frac{P(G_1|I) \cdot P(I)}{P(G_1)}$$

$$= \frac{P(G_1|I) \cdot P(I)}{P(G_1|I) \cdot P(I) + P(G_1|G) \cdot P(G)}$$

$$= \frac{0.01 \times 0.95}{0.9 \times 0.05 + 0.01 \times 0.95}$$

$$= 0.1743119$$

$$= 0.1743$$

9) Let A be the event that the system fails.

$P(K_1 \text{ fails}) = 0.01$

$P(K_2 \text{ fails}) = 0.03$

$P(K_3 \text{ fails}) = 0.08$.

$P(A) = P(K_1 \text{ fails}) \cdot P(K_2 \text{ fails}) \cdot P(K_3 \text{ fails})$.

$= 0.01 \times 0.03 \times 0.08$

$= 0.000024$.

$P(\text{System doesnot fail}) = 1 - P(A)$

$= 1 - 0.000024$

$= 0.999976$.

10)

6 Engineers
4 Managers } ⇒ 3 are selected.

a) P(All 3 selected will be engineers)

$$= \frac{{}^6C_3 \times {}^4C_0}{{}^{10}C_3}$$

$$= \frac{\frac{6!}{3!\,3!} \times \frac{4!}{4!\,0!}}{\frac{10!}{7!\,3!}}$$

$$= \frac{4\times5\times6}{3\times2} \div \frac{8\times9\times10}{3\times2}$$

$$= \frac{4\times5\times6}{8\times9\times10}$$

$$= \frac{1}{6} = 0.1667$$

b) P(All 3 selected will be managers)

$$= \frac{{}^4C_3 \times {}^6C_0}{{}^{10}C_3}$$

$$= \frac{\frac{4!}{3!\,1!} \times \frac{6!}{6!\,0!}}{\frac{10!}{7!\,3!}}$$

$$= \frac{1}{30} = 0.0333$$

c) P(2 engineers & 1 manager)

$$= \frac{{}^6C_2 \times {}^4C_1}{{}^{10}C_3}$$

$$= \frac{\frac{6!}{4!\,2!} \times \frac{4!}{3!\,1!}}{\frac{10!}{7!\,3!}}$$

$$= \frac{\frac{5 \times 6 \times 4}{2}}{\frac{8 \times 9 \times 10}{3 \times 2}}$$

$$= \frac{1}{2} = 0.5$$

d) P(1 engineer & 2 managers)

$$= \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$$

$$= \frac{\frac{6!}{5!\,1!} \times \frac{4!}{2!\,2!}}{\frac{10!}{7!\,3!}}$$

$$= \frac{\frac{6 \times 3 \times 4}{2}}{\frac{8 \times 9 \times 10}{3 \times 2}}$$

$$= 0.3$$