MC3020 - Tutorial 02 Answers

01)

(a) By the property of the discrete probability mass function

$$\sum_{\forall x} P(X=x) = 1$$

This implies, $0.20 + k + 0.25 + 0.10 + 0.10 + 0.05 = 1$

$$k = 1 - 0.70$$
$$k = 0.30$$

(b)

$$\mu = E(X)$$
$$= \sum_{\forall x} x \cdot P(X=x)$$
$$= 1 \times 0.20 + 2 \times 0.30 + 3 \times 0.25 + 4 \times 0.10 + 5 \times 0.10 + 6 \times$$
$$= 2.75$$

(c)

$$\sigma^2 = Var(X) = \sum_{\forall x} (x-\mu)^2 \cdot P(X=x)$$

or

$$= E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{\forall x} x^2 \cdot P(X=x)$$
$$= 1^2 \times 0.20 + 2^2 \times 0.30 + 3^2 \times 0.25 + 4^2 \times 0.10 + 5^2 \times 0.1$$
$$+ 6^2 \times 0.05$$
$$= 9.55$$

$$E(X) = 2.75$$

$$\sigma^2 = Var(X) = E(X^2) - (E(X))^2$$
$$= 9.55 - 2.75^2$$
$$= 1.9875$$

$$S.D(X) = \sqrt{Var(X)} = \sqrt{1.9875} \approx 1.4098.$$

(d) $p(x<3) = p(x=2) + p(x=1)$

$= 0.20 + 0.30$

$= 0.50$

(e) $p(x \geq 4) = 1 - p(x<4)$

$= 1 - (p(x=3) + p(x=2) + p(x=1))$

$= 1 - (0.20 + 0.30 + 0.25)$

$= 0.25$

02) a) Let x be the no. of proposals which is approved by Finance committee.

$$X \sim Bin(n=5,\ p=0.6)$$

$$P(X=x) = \binom{5}{x}(0.6)^x(0.4)^{5-x} \qquad x = 0,1,2,3,4,5.$$

a) $P(X=1) = \binom{5}{1}(0.6)^1(0.4)^{5-1}$

$= 0.0768$

b) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \binom{5}{0}(0.6)^0(0.4)^{5-0}$

$= 1 - 0.0102$

$= 0.9898$

c) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \binom{5}{0}(0.6)^0(0.4)^{5-0} + \binom{5}{1}(0.6)^1(0.4)^{5-1} +$

$\binom{5}{2}(0.6)^2(0.4)^{5-2} + \binom{5}{3}(0.6)^3(0.4)^{5-3}$

$= 0.6630$

3)

Let x be the no. of traffic crashes occured on Jaffna roadways on any given day.

$$x \sim poi(\lambda = \frac{842}{365} = 2.3068)$$

p.m.f $P(x=x) = \frac{e^{-2.3068} \cdot 2.3068^{x}}{x!}$ $x = 0, 1, 2 \ldots$

a) Expected no. of traffic crashes $= \lambda = 2.3068$.

b) Let y be the no. of traffic crashes occureed on Jaffna roadways on any given day.

$$\lambda = \frac{842}{365} \times 30 = 69.2055$$

$$x \sim poi(69.2055)$$

$$\sqrt{\lambda} = \sqrt{69.2055} = 8.319$$

c) $P(x=20) = \frac{e^{-2.3068} \times 2.3068^{0}}{0!}$

$= 7.6365 \times 10^{-13}$

$\simeq 0$.

d) $P(x \geq 5) = 1 - P(x < 5)$

$= 1 - (P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4))$

$= 1 - 0.9154$

$= 0.0846$.

4)

1)

a) Binomial.

Fixed no. of trials, each one can be correctly placed with an individual probability of 0.537 which is constant for all bulbs, the bulbs are independent to one another. Therefore, Binomial distribution would best be used to model the no. of bulbs that will be stuck in the socket.

b) Let x be the no. of bulbs that will be stuck in the socket.

p.m.f $\quad P(X=x) = \binom{8}{x} 0.537^{x}(1-0.537)^{8-x}.$

$x = 0, 1 \ldots 8.$

$P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + P(X=8)$

$= 0.4467472$

$= 0.4467.$

c) Mean of the distribution of the no. of stuck bulbs.

$= n * p \quad = 8 \times 0.537 = 4.296.$

Variance of the dist^{n} of the no. of stuck bulbs.

$= n * p.(1-p) = 1.989048 = 1.9890$

CamScanner

5)

a) The poisson distⁿ counts events in a time period.
A rate of $12/h = 2/10min$.

b) Let x be the no. of photos takes in 10 min period.
Then p.m.f is given by

$$P(X=x) = \frac{e^{-2} \cdot 2^x}{x!} \qquad x = 0, 1, \ldots$$

Therefore no. speeding motorists is given by

$$P(X=0) = 0.13534.$$

c) The probability that the police officer catches at-least three motorists in 15 min period is given by $P(X \geq 3)$,

$$\lambda = \frac{12}{60} \times 15 = 3.$$

$$\begin{aligned} P(X \geq 3) &= 1 - P(X < 3) \\ &= 1 - P(X \leq 2) \\ &= 1 - 0.42319 \\ &= 0.57681 \\ &= 0.5768. \end{aligned}$$

06) let x be the compressive strength of concrete - sample

(a)

$$P(25 < x < 35) = P(X=30)$$

$$= 0.30$$

$$= 0.3$$

(b) To check this is legitimate probability distribution we need to verify the $\sum_{\forall x} P(X=x) = 1$

$$0.10 + 0.20 + 0.30 + 0.25 + 0.15 = 1$$

$\therefore$ This is legitimate probability distribution.

(c) $$P(X \geq 30) = P(X=30) + P(X=35) + P(X=40)$$

$$= 0.30 + 0.25 + 0.15$$

$$= 0.70.$$

(d) $$P(X \geq 25) = 1 - P(X < 25)$$

$$= 1 - P(X=20)$$

$$= 1 - 0.10$$

$$= 0.90$$

(e) Mean $\mu = E(X) = \sum_{\forall x} x \cdot P(X=x)$

$$= 20 \times 0.10 + 25 \times 0.20 + 30 \times 0.30 + 35 \times 0.25 + 40 \times 0.15$$

$$= 30.75$$

$$S.D(X) = \sqrt{Var(X)}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{\forall x} x^2 \cdot P(X=x)$$

$$= 20^2 \times 0.10 + 25^2 \times 0.20 + 30^2 \times 0.30 + 35^2 \times 0.25 + 40^2 \times 0.15$$

$$= 981.25$$

$$Var(X) = 981.25 - (30.75)^2$$

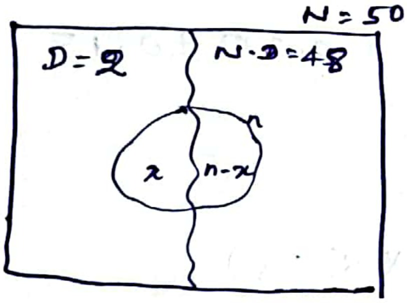
$$= 981.25 - 945.5625$$

$$= 35.6875$$

$$S.D(X) = \sqrt{35.6875}$$

$$= 5.9739$$

07)



$X \sim$ ~~[illegible]~~ ,

(a) If $n=10$

Let x be the no. of defective electronic componen[ts] in a sample.

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\binom{5}{0}\binom{45}{10}}{\binom{50}{10}}$$

$$= 1 - 0.31056$$

$$= 0.6894$$

(b) If $n=15$

$$P(X\geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\binom{5}{0}\binom{45}{15}}{\binom{50}{15}}$$

$$= 1 - 0.1532$$

$$= 0.8468$$

(c) If $n=20$

$$P(X\geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\binom{5}{0}\binom{45}{20}}{\binom{50}{20}}$$

$$= 1 - 0.0673$$

$$= 0.9327$$

(d) When there is n, $P(n) = 0.95$

$$0.95 = 1 - \frac{\binom{5}{0}\binom{45}{n}}{\binom{50}{n}}$$

$$0.05 = \frac{\binom{45}{n}}{\binom{50}{n}}$$

$$= \frac{45!/(45-n)!\cdot n!}{50!/(50-n)!\cdot n!}$$

$$\Rightarrow \frac{(50-n)(49-n)(48-n)(47-n)(46-n)}{50\times 49\times 48\times 47\times 46}$$

for $n = 20 \Rightarrow 0.0673$

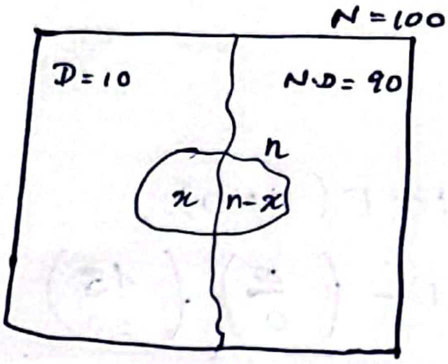
$21 \Rightarrow 0.0560$

$22 \Rightarrow 0.0464$

$23 \Rightarrow 0.0381$

$\therefore n = 22$

08)



$$P(X=x) = \frac{\binom{10}{x}\binom{90}{n-x}}{\binom{100}{n}}$$

Let X be the no. of defective identical components in a sample

$X \sim$ Hyper·Geometric( )

If n=20

(a) $P(X=3) = \dfrac{\binom{10}{3}\binom{90}{17}}{\binom{100}{20}}$

$= 0.2092$

If n=30

(b) $P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - \big(P(X=4) + P(X=3) + P(X=2) + P(X=1) + P(X=0) \big)$

$$= 1 - \left[\frac{\binom{10}{4}\binom{90}{26}}{\binom{100}{30}} + \frac{\binom{10}{3}\binom{90}{27}}{\binom{100}{30}} + \frac{\binom{10}{2}\binom{90}{28}}{\binom{100}{30}} + \frac{\binom{10}{1}\binom{90}{29}}{\binom{100}{30}} + \frac{\binom{10}{0}\binom{90}{30}}{\binom{100}{30}} \right]$$

$= 1 - [0.861597]$

$= 0.1384$

If $n = 40$

(c) $P(X=0) = \dfrac{\binom{10}{0}\binom{90}{40}}{\binom{100}{40}}$

$= 4.3554 \times 10^{-3}$