Tutorial-02   **March 2024**

1. The number of patients waiting for acupuncture treatments at a clinic has the following probability distribution:

x	1	2	3	4	5	6
P(X = x)	0.20	k	0.25	0.10	0.10	0.05

   (a) Determine the value of k?

   (b) What is the expected number of patients waiting for treatment?

   (c) What is the standard deviation of the number of patients?

   (d) What is the probability that fewer than three patients are waiting for treatment?

   (e) What is the probability that at least four patients are waiting for treatment?

2. You have submitted five proposals for upgrading the manufacturing facilities in your process area. From past experience you feel that the chance for any one project to be approved by the Finance Committee is 0.6. Accepting that p = 0.6 and that selection is a random event, what are the chances

   (a) that one project will be approved?

   (b) that at least one project will be approved?

   (c) that at most three projects will be approved?

3. On an average in 2010, there were 842 traffic crashes occurred on Jaffna roadways (Road Accident Data by Vehicle type – 2010, http://www.data.gov.lk). (there are 365 days in 2010)

   (a) What is the expected number of traffic crashes on any given day of 2010?

   (b) What is the standard deviation of the number of the traffic crashes occurred on Jaffna roadways on any given month of 2010?

   (c) What is the probability that there were exactly 20 traffic crashes on Jaffna roadways in any given day of 2010?

   (d) What is the probability that there were at least 5 traffic crashes on Jaffna roadways in any given day of 2010?

4. If you put an ordinary incandescent bulb into a light fixture designed for a high efficiency (low heat) bulb, it is possible–but not certain–that the contact point will melt and the light bulb will become stuck in the socket. Extra labor is then required to remove it and clean the fixture's contact point. An electrician estimates that these extra tasks must be performed 0.5370 of the time when replacing an incorrect bulb. The electrician has been called to a house on another matter, and notices they have used the wrong light bulb type in five of their downlights.

(a) If the electrician points out the hazard and is employed to replace the bulbs, what distribution would best be used to model the number of bulbs that will be stuck in the socket?

(b) If the electrician is working his way through a large apartment block, and each apartment has eight incorrect bulbs that must be replaced, what is the long run proportion of times at least five of the bulbs will be stuck and require the extra steps?

(c) What is the mean and variance of the distribution of the number of stuck bulbs?

5. The traffic Police officer's radar speed gun causes grief for many drivers who use the Jaffna-Kandy (A9) road. A particular location between Ariviyal Nagar and Iranaimadu junction in Kilinochchi catches 12 speeding motorists per hour. This figure is an average of all hours between 9am and 5pm.

(a) Which distribution would best be used to model the number of speeding motorists in a 10 minutes period? and what proportion of 10 minutes period would you expect the Police officer's radar speed gun to remain unused to deliver a fine?

(b) What is the probability that the Police officer catches at-least three motorists in 15 minutes period?

6. In a civil engineering lab, the compressive strength of concrete samples is tested. The strength (in megapascals, MPa) of each sample follows a certain probability distribution, as shown below:

Strength (MPa)	20	25	30	35	40
Probability	0.10	0.20	0.30	0.25	0.15

(a) What is the probability that the compressive strength of the next concrete sample will be between 25 MPa and 35 MPa?

(b) Verify if the given probability distribution is legitimate.

(c) Determine the probability that the next concrete sample will have a compressive strength of at least 30 MPa.

(d) Find the probability that the compressive strength of the next concrete sample will be 25 MPa or higher.

(e) Calculate the mean and standard deviation of the compressive strength distribution.

7. In a manufacturing process, a batch of 50 electronic components is produced, of which 5 are defective and 45 are non-defective. An engineer is tasked with randomly selecting components for testing, and if any defective component is found, the entire batch is rejected.

(a) If a sample of 10 components is randomly selected for testing, what is the probability that the batch will be rejected?

(b) If a sample of 15 components is randomly selected for testing, what is the probability that the batch will be rejected?

(c) If a sample of 20 components is randomly selected for testing, what is the probability that the batch will be rejected?

(d) If management wants a 0.95 probability of rejecting a batch with 5 defective and 45 non-defective components, how large a sample size would you recommend for testing?

8. An engineering student at the University of Jaffna is conducting a study on the failure rates of electrical components in a certain system. In a large inventory, there are 100 identical components, of which 10 are known to be defective and 90 are non-defective. The student plans to randomly select a sample of components for testing.

   (a) If the student selects a sample of 20 components, what is the probability that exactly 3 defective components are found in the sample?

   (b) If the student selects a sample of 30 components, what is the probability of finding at least 5 defective components in the sample?

   (c) If the student selects a sample of 40 components, what is the probability of finding no defective components in the sample?

   (d) If the student wants to ensure a 95% confidence level of finding at least 10 defective components in the sample, how large of a sample size should they choose?

"Probability guides our every step. In engineering, understanding uncertainty is key. By mastering probability and statistics, students gain the tools to navigate complex challenges with confidence."
- Joseph Butler