Tutorial-05

01)

$P_c = \frac{94}{125} = 0.7520$ $\qquad n_c = 125$

$n_T = 175$

$P_T = \frac{113}{175} = 0.6457$ $\qquad z_{0.025} = 1.96$

(a) 95% C.I for the difference in proportions of students passing the examination between instruction using the computer software program & the traditional method.

95% C.I for $P_c - P_T$

$$= \left((\hat{P}_c - \hat{P}_T) - Z_{\alpha/2}\sqrt{\frac{\hat{P}_c(1-\hat{P}_c)}{n_c} + \frac{\hat{P}_T(1-\hat{P}_T)}{n_T}},\ (\hat{P}_c - \hat{P}_T) + Z_{\alpha/2}\sqrt{\frac{\hat{P}_c(1-\hat{P}_c)}{n_c} + \frac{\hat{P}_T(1-\hat{P}_T)}{n_T}} \right)$$

$$= \left((0.7520 - 0.6457) - 1.96\sqrt{\left(\frac{0.752 \times 0.248}{125} + \frac{0.6457 \times 0.3543}{175}\right)},\ (0.7520 - 0.6457) + 1.96\sqrt{\frac{0.752 \times 0.248}{125} + \frac{0.6457 \times 0.3543}{175}} \right)$$

$$= (0.1063 - 1.96 \times 0.0529,\ 0.1063 + 1.96 \times 0.0529)$$

$$= (0.0026,\ 0.2100)$$

(b)

Step 1

$H_0 : P_C \leq P_T$

$H_1 : P_C > P_T$

$$\bar{P} = \frac{94+113}{125+175} = \frac{207}{300} \approx 0.69$$

Step 2.

$$\text{Test statistic} = \frac{\hat{P}_C - \hat{P}_T - (P_C - P_T)}{\sqrt{\bar{P}(1-\bar{P})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$= \frac{(0.7520 - 0.6457) - 0}{\sqrt{0.69(1-0.69)\left(\frac{1}{125}+\frac{1}{175}\right)}}$$

$$= \frac{0.1063}{0.0542}$$

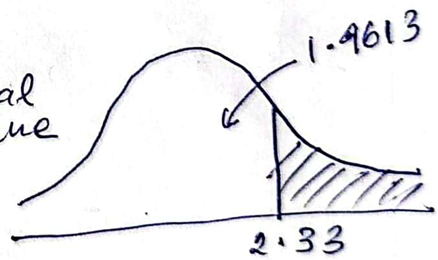
$$= 1.9613$$

Step. 3

Critical Value $= Z_{0.01} = +2.33.$

Step 4

Test value $1.9613 < 2.33$ critical Value



Test value falls on acceptance region, so we donot reject Ho

Step 5

we can conclude that the instruction using the computer ~~.~~software is not appear to increase the proportion of students passing the examinaton in comparision to the pass rate using the traditional method of instruction.

Q2) Pop^n $dist^n$ normal, pop^n variance unknown with equal variances.

R- Reinforced beams.

$\overline{x}_R = 26.5833$

$S_R = 14.3619$

$n_R = 12$

U- Un-reinforced beams.

$\overline{x}_U = 39.6667$

$S_U = 13.8586$

$n_U = 12$

$$S = \sqrt{\frac{n\sum x_i^2 - (\sum x_i)^2}{n(n-1)}}$$

Step 1:

$H_0 : \mu_R \leq \mu_U$ OR/ $\mu_R - \mu_U \leq 0$

$H_1 : \mu_R > \mu_U$ $\quad$ $\mu_R - \mu_U > 0$ (Right sided test)

Step 2:

Test statistic

$$T = \frac{\overline{x}_R - \overline{x}_U - (\mu_R - \mu_U)}{S_p\sqrt{1/n_R + 1/n_U}}$$

$$= \frac{26.5833 - 39.6667 - 0}{14.1125\sqrt{1/12 + 1/12}}$$

$$= \frac{-13.0834}{5.7614}$$

$$= -2.2709$$

$$S_p = \sqrt{\frac{(n_R-1)S_R^2 + (n_U-1)S_U^2}{n_R+n_U-2}}$$

$$= \sqrt{\frac{11\times(14.3619)^2 + 11\times(13.8586)^2}{22}}$$

$$= \sqrt{\frac{4381.5746}{22}}$$

$$= \sqrt{199.1625}$$

$$= 14.1125$$

Step 3:

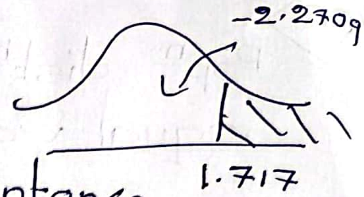
Critical value

$$t_{n_R+n_U-2,\alpha} = t_{22,0.05}$$

$$= 1.717$$

Step 4:

Test value $= -2.2709 < 1.717$



Since test value falls in the acceptance region. So we can't reject H_0.

Step 5:

We can conclude that the mean loading capacity of the reinforced beams is not greater than the loading capacity of the un-reinforced beams at 5% level of significance.

b) 95% C.I for $\mu_R - \mu_U$ is

$$= \bar{x}_R - \bar{x}_U \pm t_{n_R+n_U-2, \alpha/2} \, S_p \sqrt{1/n_R + 1/n_U}.$$

$$= 26.5833 - 39.6667 \pm t_{22, 0.025} \times 14.1125 \sqrt{1/12 + 1/12}$$

$$= -13.0834 \pm 2.074 \times 14.1125 \sqrt{1/6}$$

$$= (-25.0362, -1.1342)$$

σ unknown , Normal $dist^n$, dependent sample

03)

Participant	1	2	3	4	5	6	7
Before	6	7	5	4	3	8	7
After	4	3	6	4	3	7	7
D	2	4	-1	0	0	1	0

$\bar{D} = 0.8571$

$S_D = 1.6762$

(a) 95% C.I for the mean difference

$$\bar{D} \pm t_{n-1,\alpha/2} \cdot \frac{S_D}{\sqrt{n}}$$

$$= 0.8571 \pm t_{6,0.025} \cdot \frac{1.6762}{\sqrt{7}}$$

$$= 0.8571 \pm 2.447 \times \frac{1.6762}{\sqrt{7}}$$

$$= (-0.6932 , 2.4074)$$

σ unknown

(b) Step 1

$H_0 :- \mu_D = 0$

$H_1 :- \mu_D \neq 0$

Step 2

Test Statistic $T = \dfrac{\bar{D} - \mu_D}{S_D/\sqrt{n}}$

$=$ [redacted] $\dfrac{0.8571 - 0}{1.6762/\sqrt{7}}$

$= 1.3529$

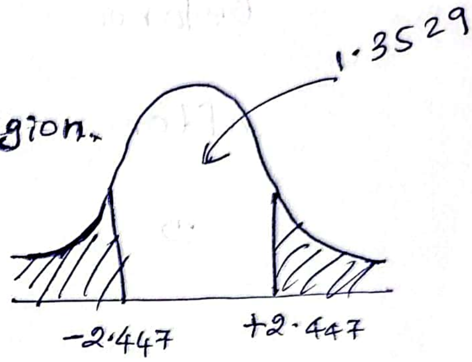
Step 3

Critical value

$$t_{n-1,\alpha/2} = \pm t_{6,0.025}$$
$$= \pm 2.447$$

Step 4

Test value falls in acceptance region.
So we donot reject H_0



Step 5

We can conclude that the discomfort level has not been changed and the test is carried out with 5% level of significance.

04)

Subject	1	2	3	4	5	6	7
Before	210	235	208	190	172	244	232
After	190	170	210	188	173	228	232
D	20	65	-2	2	-1	16	0

$\bar{D} = 14.2857$

$S_D = 24.0188$

(a) 99% C.I for the mean difference

$$\bar{D} \pm t_{n-1,\alpha/2} \cdot S_D/\sqrt{n}$$

$$= 14.2857 \pm t_{6,0.005} \cdot \frac{24.0188}{\sqrt{7}}$$

$$= 14.2857 \pm 3.707 \times \frac{24.0188}{\sqrt{7}}$$

$$= (-19.3674, 47.9388)$$

(b) Step 1

$H_0 :- \mu_D = 0$

$H_1 :- \mu_D \neq 0$

Step 2

Test statistic $T = \dfrac{\bar{D} - \mu_D}{S_D/\sqrt{n}}$

$= \dfrac{14.2857 - 0}{24.0188/\sqrt{7}}$

$= 1.5736$

Step 3

Critical value

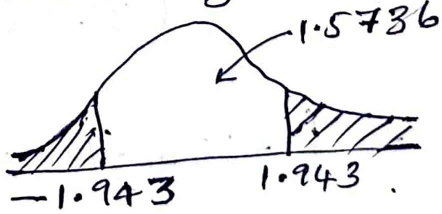
$\pm t_{n-1, \alpha/2} = \pm t_{6, 0.05}$

$= \pm 1.943$.

Step 4

Test value falls in acceptance region.

So we donot reject H_0



Step 5

We can conclude that the cholesterol has not been changed and the test is carried out with $\cdot 10\%$ level of significance.

normal dist^n, unequal variance

05)

95% C.I for $\mu_A - \mu_B$

$$= \left((\bar{x}_1 - \bar{x}_2) - t_{df, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \,,\ (\bar{x}_1 - \bar{x}_2) + t_{v, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

$$d.f = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

$t_{21, 0.025} = 2.080$

A : $\bar{x}_1 = 36300$ $s_1 = 5000$

B : $\bar{x}_2 = 38100$ $s_2 = 6100$

$$= \frac{\left(\frac{5000^2}{12} + \frac{6100^2}{12} \right)^2}{\frac{\left(\frac{5000^2}{12}\right)^2}{11} + \frac{\left(\frac{6100^2}{12}\right)^2}{11}}$$

$= 21.1839$

$\simeq 21$

$$= \left((36300 - 38100) - 2.080 \cdot \sqrt{\frac{5000^2}{12} + \frac{6100^2}{12}} \,, \right.$$

$$\left. (36300 - 38100) + 2.080 \sqrt{\frac{5000^2}{12} + \frac{6100^2}{12}} \right)$$

$= (-6535.903, 2935.903)$

6)

Algorithm A

$\bar{x}_A = 5.85$

$S_A = 0.3028$

$n_A = 10$

Algorithm B.

$\bar{x}_B = 5.68$

$S_B = 0.3458$

$n_B = 10$

Assume pop^n have equal variances.

a) 95% C.I for the ~~the~~ mean time taken between Algorithm A & B.

$$(\bar{x}_A - \bar{x}_B) \pm t_{n_A+n_B-2,\alpha/2}\, S_p \sqrt{1/n_A + 1/n_B}$$

$$S_p = \sqrt{\frac{(n_A-1)S_A^2 + (n_B-1)S_B^2}{n_A+n_B-2}}$$

$$= \sqrt{\frac{(10-1)\times 0.3028^2 + (10-1)\times 0.3458^2}{10+10-2}}$$

$$= 0.3250$$

$$=(5.85-5.68) \pm t_{10+10-2,0.025} \times 0.325 \sqrt{1/10 + 1/10}$$

$$= 0.17 \pm t_{18,0.025} \times 0.325 \times 0.4472$$

$$= 0.17 \pm 2.101 \times 0.325 \times 0.4472$$

$$= (-0.1354, 0.4754)$$

95% C.I for $\mu_A - \mu_B$ is $(-0.1354, 0.4754)$

CamScanner

b) Step 1.

$H_0 : \sigma_A^2 = \sigma_B^2$

$H_1 : \sigma_A^2 \neq \sigma_B^2$.

$$F_{\alpha, v_1, v_2} = \frac{1}{F_{1-\alpha, v_2, v_1}}$$

Step 2:

Test statistic

$$F = \frac{S_A^2}{S_B^2} = \frac{0.0917}{0.1196} = 0.7667$$

Step 3:

Critical value $= F_{0.025,9,9} = 4.03$

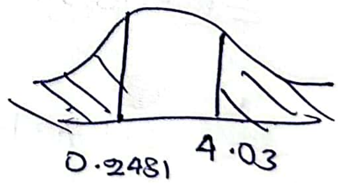
$$F_{0.975,9,9} = \frac{1}{F_{0.025,9,9}} = \frac{1}{4.03} = 0.2481$$

Step 4:

$0.2481 < F < 4.03$

Test value falls in acceptance region.

So donot reject H_0.



Step 5:

We conclude that the variances are not significantly different & the test is carried out 5% level of significance.

pop^n variances are unknown & equal (from part b)

c) Step 1:

$H_0: \mu_A - \mu_B = 0$ or $\mu_A = \mu_B$

$H_1: \mu_A - \mu_B \neq 0$ $\quad$ $\mu_A \neq \mu_B$ (two sided test)

Step 2:

Test statistic:

$$T = \frac{\bar{x}_A - \bar{x}_B - (\mu_A - \mu_B)}{S_p\sqrt{1/n_A + 1/n_B}}$$

$$= \frac{5.85 - 5.68 - 0}{0.3250\sqrt{1/10 + 1/10}}$$

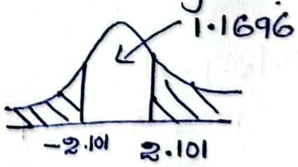
$$= 1.1696$$

Step 3:

Critical value $t_{n_A+n_B-2,\alpha/2} = t_{18,0.025} = 2.101$

Step 4:

Test statistic value falls in acceptance region.
So donot reject H_0.



Step 5:

We can conclude that there is no significant different between the mean time taken by Algorithm A & Algorithm B at 5% level of significance.

d) Yes.

From both the C.I & the hypothesis test, it can be observed that the interval contains zero & hypothesis test lead to the conclusion that there is no difference in the mean time taken by Algorithm A & B at 5% level of significance.

$S_A = 85, 90, 88, 92, 87, 89, 91, 86, 90, 88$

$S_B = 75, 88, 85, 82, 89, 86, 83, 85, 87, 80$

7) **Method A** | **Method B**

a) $\bar{x}_A = 88.6$ | $\bar{x}_B = 84$

$S_A^2 = 4.933$ | $S_B^2 = 17.556$

$n_A = 10$ | $n_B = 10$

Step 1:

$H_0 : \sigma_A^2 = \sigma_B^2$

$H_1 : \sigma_A^2 \neq \sigma_B^2$

Step 2:

Test statistic

$$F = \frac{S_A^2}{S_B^2} = \frac{4.933}{17.556} = 0.281$$

Step 3:

Critical value $= F_{0.05,9,9} = 3.18$

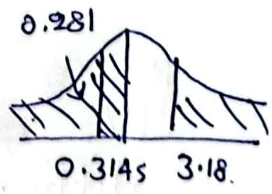
$$F_{0.95,9,9} = \frac{1}{F_{0.05,9,9}} = \frac{1}{3.18} = 0.3145$$

Step 4:

$F = 0.281 < 0.3145$.

Test value falls in rejection region

$\therefore$ Reject H_0.



Step 5:

We conclude that the popⁿ variances of the test scores obtain by students in Method A & Method B are not equal. the test is carried out at 10% level of significance.

b)

Two popⁿ are independent & normally distributed
σ_A^2, σ_B^2 are unknown & unequal (from part A at 10% level of significance).

Step 1:

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$.

Step 2:

Test statistic

$$T = \frac{\bar{X}_A - \bar{X}_B - (\mu_A - \mu_B)}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$$

$$= \frac{(88.6 - 84) - 0}{\sqrt{\frac{4.933}{10} + \frac{17.556}{10}}}$$

$$= 3.0674.$$

Step 3:

Critical value $= t_{v, 0.05} = \pm t_{14, 0.05} = \pm 1.761$

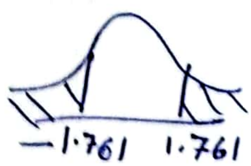
$$v = \frac{\left(\frac{4.933}{10} + \frac{17.556}{10}\right)^2}{\frac{\left(\frac{4.933}{10}\right)^2}{9} + \frac{\left(\frac{17.556}{10}\right)^2}{9}} = 13.6876 \approx 14.$$

Step 4:

Test value $= 3.0674 > 1.761$

Test value falls in rejection region

$\therefore$ Reject H_0.



Step 5:

We can conclude that there is significant difference between method A & B. The test is carried out at 10% level of significance.

c) Step 1.

$H_0: \sigma_A^2 = \sigma_B^2$

$H_1: \sigma_A^2 \neq \sigma_B^2$.

Step 2:

Test statistic:

$$F = \frac{S_A^2}{S_B^2} = \frac{4.933}{17.556} = 0.281.$$

Step 3:

$$\text{Critical value} = F_{0.01,9,9} = 5.35.$$

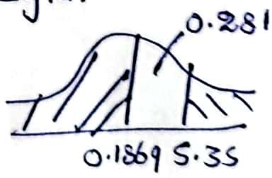
$$F_{0.99,9,9} = \frac{1}{F_{0.01,9,9}} = 0.1869.$$

Step 4: Test value

$0.1869 < 0.281 < 5.35$.

Test value falls in acceptance region (non-critical region).

$\therefore$ Donot reject H_0.



Step 5:

We can conclude that the pop^n variances of the test scores obtained by students in Method A & Method B are equal (There is no difference) & the test is carried out at 2% level of significance.

Two popⁿ are independent & normally distributed σ_1^2, σ_2^2 are unknown but equal at 2% level of significance.

Step 1:

$H_0: \mu_A = \mu_B$

$H_1: \mu_A \neq \mu_B$ (two sided test)

Step 2:

Test statistic

$$T = \frac{\bar{x}_A - \bar{x}_B - (\mu_A - \mu_B)}{S_p\sqrt{1/n_A + 1/n_B}}$$

$$= \frac{(88.6 - 84) - 0}{3.3533\sqrt{1/10 + 1/10}}$$

$$= 3.0674$$

$$S_p = \sqrt{\frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2}{n_A + n_B - 2}}$$

$$= \sqrt{\frac{9 \times 4.933 + 9 \times 17.556}{18}}$$

$$= 3.3533$$

Step 3:

Critical value $= t_{n_A+n_B-2, \alpha/2} = \pm t_{18, 0.01} = \pm 2.552$

Step 4:

Test value $= 3.0674 > 2.552$

Test value falls in rejection region.

$\therefore$ Reject H_0.

Step 5:

We can conclude that there is significant different between method A & B. The test is carried out at 2% level of significance.