Solutions:

(1) We assume that m and n are both perfect numbers,

Then, By definition, we can write,

$m = p^2$ and $n = q^2$ where p and q are integers

we have to proof m.n is a perfect number.

Now Consider, $m.n = p^2 . q^2$

$= (p.q)^2$

$\Rightarrow m.n = (pq)^2$; $p.q$ is an integer

Hence, m.n is a perfect number.

(2) We assume that r and s are rational numbers

By the definition, we can write

$r = a/b$ where a and b are integers, $b \neq 0$

$s = c/d$ where c and d are integers, $d \neq 0$

Now consider,

$r + s = a/b + c/d$

$= ad + bc/bd$

$r+s = \dfrac{(ad+bc)}{bd}$ —— (1)

Since $b \neq 0$ and $d \neq 0 \Rightarrow b.d \neq 0$

$\Rightarrow \quad r+s = \dfrac{(ad+bc)}{bd}$ where $bd \neq 0$ and $(ad+bc)$, & bd - are intergers

Therefore, $r+s$ is rational number.

~~Solutions:~~

~~Suppose that~~

Solutions:

We assume that $\sqrt{2}$ is not irrational number

$\Rightarrow$ $\sqrt{2}$ is rational number

By the definition of rational number, we can write $\sqrt{2}$ as ratio of two integers.

$\Rightarrow \sqrt{2} = p/q$; where $q \neq 0$

$\Rightarrow 2 = p^2/q^2 \Rightarrow p^2 = 2q^2$

As we know 2 divides $2q^2$, so 2 divides p^2 as well

Hence 2 is prime, so 2 divides p — ①

Since, 2 divides p

$\Rightarrow$ we can write $p = 2k$; k is an integer

$p^2 = 4k^2$

$2q^2 = 4k^2$

$q^2 = 2k^2$

$\Rightarrow$ As we know 2 divides $2k^2$, so divides q^2

But 2 is prime, so 2 divides q

Thus p and q have a common factor 2

This statement contradicts that p and q have no common factors.

Hence, $\sqrt{2}$ is not a rational number,

$\Rightarrow \sqrt{2}$ is irrational

Suppose we assume that if $3n+2$ is odd then n is even number.

Since n is even number, we can write

$$n = 2k \; ; \; k \text{ is an integer}$$

consider,

$$3n+2 = 3(2k)+2$$

$$= 6k+2$$

$$= 2\underbrace{(3k+1)}_{\text{integer}}$$

$\Rightarrow$ $3n+2$ is an even number.

This is contradict to our assumption.

Therefore, "if $3n+2$ is odd then n is odd" ∎