UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING
END SEMESTER EXAMINATION — October 2017
MC 4010 : DISCRETE MATHEMATICS
(Duration: 2 hours)

(This question paper contains SIX questions. Answer **ALL** questions.)

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1. Let p , q and r be three statements.

   (a) Find the truth table for the proposition: $> [q \leftrightarrow (r \to > p)] \wedge [(q \vee p) \leftrightarrow r]$

   (b) Simplify the proposition: $(p \to > q) \vee [(p \to r) \wedge (> q \to r)]$

   (c) Prove the following without using truth table

      i. $[(p \to q) \wedge > q] \to > p$

      ii. $[(p \to (q \to r)] \to [(p \to q) \to (p \to r)]$

   (d) Using only the laws of propositions show that

   $$[(p \to q) \wedge (r \to s)] \to [(p \vee r) \to (q \vee s)]$$

   is a Tautology.

   (e) Test the validity of the following arguments :$p \to (q \wedge > r)$, $s \to r$, $p| => s$.

[15 marks]

2. Write R program codes:

   (a) To generate a random variable using the inverse transformation method for the following distribution function

   $$F(x) = \frac{x^2 + x}{2}, \ 0 < x < 1.$$

   (b) To generate 5000 pairs of normal random variables using Box-Muller algorithm. Plot the histogram. Check the normality assumption.

   (c) To generate 100 random numbers for the following distribution, using the rejection method

   $$f(x) = \begin{cases} \frac{4}{\pi}\frac{1}{1+x^2}, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

   (d) To find the mean using 1000 samples using an expression for X, where X have a standard Cauchy distribution

   $$F_x = \frac{1}{\pi}\tan^{-1}(x) + \frac{1}{2}.$$

   and $U \sim Uniform(0, 1)$.

[15 marks]

[CONTINUED]

3. (a) Write R program codes to find the roots of the following equation

$$2x^2 + 8x + 7 = 0.$$

(b) Write R program codes to simulate 100 observations the following integral, using Monte Carlo method.

$$\int_0^{\frac{\pi}{2}} 4\sin(3x)\exp\left(-x^2\right)dx$$

(c) Write R program codes to simulate 1000 observations from normal distribution with $\mu = 25$ and $\sigma = 3$ [Use set.seed (1)]. Plot the empirical cumulative distribution function $F_n(x)$ for this sample and overlay the true cumulative distribution function $F_n(x)$.

(d) Monthly mortgage payment P, is calculated using following formula.

$$P = A\left(\frac{\frac{r}{1200}}{1-\left(1+\frac{r}{1200}\right)^{(-12y)}}\right)$$

where A is the loan amount, r is the nominal interest rate (assumed convertible monthly), and y is the number of years. Write the R programming function that computes the monthly payment. Monthly payment should be display in two decimal points.

[20 marks]

4. (a) Consider the third-order homogenous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$

i. Find the general solution.

ii. Find the solution with initial conditions $a_0 = 3$, $a_1 = 4$ and $a_2 = 12$

(b) Let $f(n) = 5f(n/2) + 3$ and $f(1) = 7$. Find $f(2^k)$, where k is a positive integer. Also estimate $f(n)$ if f is an increasing function.

(c) If $f(x) = x\sqrt{x+1}$, what can you say about the Big-Θ behaviour of solutions to

$$T(n) = \begin{cases} 2T([n/3]) + f(n) & n > 1 \\ d & n = 1 \end{cases}$$

(d) What is the form of general solution if r is a root of multiplicity m of the characteristic polynomial.? What can you say about its linear combination.

[15 marks]

[CONTINUED]

5. (a) Prove that for any finite sets A, B and C,

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$$

(b) Show that $A = B$ if $A = \{x \in \mathbb{Z} | x = 3 + 5p, \text{for some } p \in \mathbb{Z}\}$ and $B = \{x \in \mathbb{Z} | x = -7 + 5p, \text{for some} p \in \mathbb{Z}\}$.

(c) Prove that for every $n \geq 1$, $n^3 + 2n$ is divisible by 3.

(d) Discover and prove a theorem about the sizes of 3^n and $n!$.

(e) Show that $\sum_{k=0}^{n} (-1)^k C(n,k) = 0$

(f) Prove that for all integers $n \geq 2$, $P(n,r) + P(n,1) = n^2$

[20 marks]

6. (a) Let $F = \{(1,1), (-1,1), (2,4), (-2,4)\}$.

i. Is F a function from $A = \{1,2,3\}$ to $B = \{1,4\}$?

ii. Is F a function from $A = \{\pm 1, \pm 2\}$ to $B = \{1,2,3,4\}$?

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^3 + 1$$

i. Show that f is a bijection.

ii. What is $f^{-1}(x)$

(c) Use the notation $\{x : P(x)\}$, where $P(x)$ is a propositional function, to describe each of the following sets.

i. $\{3, 6, 9, 12, 15, \ldots, 27, 30\}$

ii. $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\}$

iii. The set of integers which can be written as the sum of the squares.

(d) Prove the following De Morgan's law using induction.

$$\overline{A_1 \cup \ldots \cup A_n} = \bar{A}_1 \cap \ldots \cap \bar{A}_n.$$

[15 marks]

[TOTAL=100 marks]

[END]