UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

END SEMESTER EXAMINATION - MARCH 2021

MC4010: Discrete Mathematics

Permitted Materials: Calculator; Statistical table

Duration: Two hours *Registration Number:*

Instructions

1. This is a **close book** exam.
2. This paper contains **SIX (06)** questions
3. Answer **all** questions in the **SPACE** provided.
4. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
5. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
6. This examination accounts for **60%** of module assessment. Total maximum mark attainable is **100**.
7. Write your **registration number** in the question-answer book. Also write your registration number on each sheet.

Question	Marks
1	
2	
3	
4	
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6	
TOTAL	

1. (a) Test the validity of the following argument: 'If you insulted Bob then I'll never speak to you again. You insulted Bob so I'll never speak to you again.'

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(b) Let p be the proposition 'Today is Monday' and q be the proposition' I'll go to London'. Write the following propositions symbolically.

i. I'll go to London and today is not Monday.

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ii. If and only if today is not Monday then I'll go to London.

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(c) Construct truth tables for the following compound propositions.

i. $\overline{q} \rightarrow p$.

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ii. $\overline{p} \leftrightarrow \overline{q}$.

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(d) Show that $(p \wedge \overline{q}) \wedge (\overline{p} \vee q)$ is a contradiction.

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(e) Symbolize the proposition 'Every day I go jogging'.

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[15 marks]

2. (a) True or False: If $X \subseteq Y$, then $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$. Justify your answer.

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(b) Describe the set $\mathbb{Z} = \{x \in \mathbb{Z} | x = 2n, n \in \mathbb{Z}\} \cup \{x \in \mathbb{Z} | x = 2n+1, n \in \mathbb{Z}\}$

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(c) Determine the cardinality of $\{\Phi, \{\Phi\}, \{\{\Phi\}\}\}$.

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(d) Show that $\{x : 2x^2 + 5x - 3 = 0\} \subseteq \{x : 2x^2 + 7x + 2 = 3/x\}$.

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(e) Show that the set $\{x \in \mathbb{Z} | x = 2n \text{ and} x = 2m + 1 \text{ for some } m, n \in \mathbb{Z}\}$ is empty.

(f) Prove that $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

[13 marks]

3. (a) Let $S = \{1, 2, 3, 4\}$. The relation

$$R = \{(1,1), (1,2), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,1)\}$$

is defined on the set S. Which of the properties reflexive, symmetric and transitive does R possess?

(b) A relation R is defined on $\mathbb{Z}$ by aRb if $3a - 7b$ is even. Prove that R is an equivalence relation.

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(c) The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 2x + 5$. Determine whether the function f is not onto.

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(d) Determine whether the function $\phi : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $\phi(x) = \sqrt[3]{x}$ for all $x \in \mathbb{R}$ is bijective.

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[12 marks]

4. (a) How many bit strings of length four do not have two consecutive 1s?

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(b) A local bank requires customers to choose a four-digit code to use with an ATM card. The code must consist of two letters in the first two positions and two digits in the other two positions. The bank has 75,000 customers.

i. Use the Multiplication Principle to calculate the number of possible distinct codes.

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ii. Show that at least two customers choose the same four-digit code.
Hint:Using the generalized pigeonhole principle, i.e.,one box containing at least $\lceil \frac{N}{k} \rceil$ objects.

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(c) i. Suppose that the roots of the characteristic equation of linear homogeneous recurrence relation are $3, 3, 3, 3, 4, 4$, and -1. What is the form of general solution?

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ii. Find the solution of the Fibonacci recurrence relation $a_n = a_{n-1} + a_{n-2}$ with initial conditions $a_0 = 0$ and $a_1 = 1$.

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(d) The following function $M : \mathbb{Z}^+ \longrightarrow \mathbb{Z}$ was defined by John McCarthy, a pioneer in the theory of computation and in the study of artificial intelligence:

$$M(n) = \begin{cases} n - 10 & if\ n > 100 \\ M(M(n+11)) & if\ n \leq 100 \end{cases} \tag{1}$$

For all positive integers n. Find $M(99)$.

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[20 marks]

5. (a) i. Prove that, $T(n) = a_0 + a_1 n + a_2 n^2 + a_3 n^3$ is $O(n^3)$

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ii. Use the binomial theorem to find the coefficient of x^8y^5 in $(x+y)^{13}$.

iii. Prove by contra-positive: Let $x \in \mathbb{Z}$. If $x^2 - 6x + 5$ is even, then x is odd.

(b) Algorithm A and B spend exactly $T_A(n) = 0.1n^2 \log_{10} n$ and $T_B(n) = 2.5n^2$ microseconds respectively, for a problem of size n. Choose the algorithm, which of better in the big- Oh sense, and find out the problem size n_0 such that for any larger size $n > n_0$ the chosen algorithm outperforms the other. If your problems are of the size $n < 10^9$, which algorithm will you recommend to use.

(c) An algorithm with time complexity $O(f(n))$ and processing time $T(n) = cf(n)$, where $f(n)$ is a known function of n, spends 10 seconds to process 1000 data items. How much time will be spent to process 100,000 data items if $f(n) = n$ and $f(n) = n^2$?

[20 marks]

6. (a) A bit string of length four is generated at random so that each of the 16 bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0? (We assume that 0 bits and 1 bits are equally likely.)

(b) The number of messages sent per hour over a computer network has the following distribution.

X(Number of messages)	10	11	12	13	14	15
P(X=x)	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages sent per hour.

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(c) During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

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(d) A website receives hits at a rate of 300 per hour.

i. State a distribution that is suitable to model the number of hits obtained in one minute interval.

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ii. Find the probability of

A. 10 hits in a given minute.

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B. at least 15 hits in 2 minutes.

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[20 marks]

—— End of Examination ——