**Department of Inter Disciplinary Studies,**
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**University of Jaffna, Sri Lanka**
**MC 4010 : Discrete Mathematics**

**Tutorial-01** **February 2023**

1. Find the cardinality in the following sets,
   (a) $A = \{x \in \mathbb{R} : x^2 + 2x + 2 = 0\}$
   (b) $B = \{a, b, c, \{a, b, c\}\}$.
   (c) $C = \{x \in \mathbb{Q} : x < 1\}$

2. Let $X = [0, 5)$, $Y = [2, 4]$, $Z = (1, 3]$ and $W = (3, 5)$ be intervals in $\mathbb{R}$. Find in each of the following sets:
   (a) $Y \cup Z$
   (b) $Z \cap W$
   (c) $X - (Z \cup W)$
   (d) $\overline{Z}$

3. Let $A = \{x \in \mathbb{R} : x^2 - 4 > 0\}$ and $B = \{x \in \mathbb{R} : x^2 - 9 > 0\}$.
   (a) Prove that, $A \subset B$.
   (b) Using part(a), complete the prove that $A \neq B$.

4. Let $A = \{x \in \mathbb{R} \mid -3 < x < -2\}$ and $B = \{x \in \mathbb{R} \mid x^2 + 6x - 5 < 0\}$. Prove that $A = B$.

5. Show that the following two sets are equal:

   A= $\{x \in \mathbb{Z} | x = 1 + 3q, \text{for some } q \in \mathbb{Z}\}$.
   B= $\{x \in \mathbb{Z} | x = -2 + 3q, \text{for some } q \in \mathbb{Z}\}$.

6. Simplify the set, $(A \cup B) \cap (\overline{\overline{A} \cap B})$.

7. Prove that $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

8. Show that the function $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R} \setminus \{-1\}$ given that;
$$f(x) = \frac{x-3}{x+1}$$
   is a bijective function.

9. Show that the function $f : \mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$ defined by;
$$f(x) = \frac{x}{1+|x|}, \quad \forall x \in \mathbb{R}$$
   is a bijective function.

10. Since $f : \mathbb{N} \to \mathbb{N}$ is given by;

$$f(x) = \begin{cases} x+1 \text{ if } x \text{ is odd} \\ x-1 \text{ if } x \text{ is even} \end{cases}$$

Show that $f$ is both 1-1 and onto.

11. Prove the following:

    (a) If $f : A \to B$ is bijective, then $f^{-1} : B \to A$ is unique.

    (b) If $f : A \to B$ is bijective, then $f^{-1} : B \to A$ is also bijective.

12. Define the relation $\sim$ on $\mathbb{Q}$ by;

$$x \sim y \text{ if and only if } \frac{x-y}{2} \in \mathbb{Z}$$

Show that $\sim$ is an equivalence relation. Describe the equivalence classes $[0], [1], \left[\frac{1}{2}\right]$.

13. Let $S$ be a relation on the set $R$ of all real numbers defined by;

$$S = \left\{(a,b) \in \mathbb{R}^2 |\ a^2 + b^2 = 1\right\}.$$

Prove that $S$ is not an equivalence relation on $\mathbb{R}$.

14. Let $R$ be relation defined on the set of natural numbers $\mathbb{N}$ as follows;

$$R = \{(x,y); x \in \mathbb{N}, y \in \mathbb{N},\ 2x + y = 41\}$$

Find the domain and range of the relation $R$. Also verify whether $R$ is reflexive, symmetric and transitive.

15. Let $R$ and $S$ be partial order on a set $A$. Determine whether the union relation $R \cup S$ is also partial order on $A$.

16. Let $f$ be a function from $A$ to $B$. Let $S$ and $T$ be subsets of $B$. Show that

    (a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$.

    (b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.

17. Let $f$ be a function from the set $A$ to the set $B$. Let $S$ and $T$ be subsets of $A$. Show that

    (a) $f(S \cup T) = f(S) \cup f(T)$.

    (b) $f(S \cap T) \subseteq f(S) \cap f(T)$.

18. Show that $A \oplus B = (A - B) \cup (B - A)$.

19. Draw a Venn diagram for the symmetric difference of the sets $A$ and $B$.

20. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbb{R}$ to $\mathbb{R}$.

**Tutors will conduct this tutorial discussion on the 20th and 22nd of Feb, 2023.*