UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

End Examination Test – February 2022

MC 4010 - Discrete Mathematics

1. (a) List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \epsilon$ R and fill the table whether each of these relations on the set A is Reflexive, Symmetric, Anti-Symmetric, and Transitive
(Hint: zero is divisible by any number except by itself)

   i. $a = b$

   ii. $a + b = 4$

   iii. $a|b$ (a divides b)

   iv. $\gcd(a, b) = 1$(Great Common Divisor).

   v. $\text{lcm}(a, b) = 2$(Least Common Multiple)

No	Reflexive	Symmetric	Anti-Symmetric	Transitive
i				
ii				
iii				
iv				
v				

Mark a "✓" whether each of this Relation have that property

(b) Let $X = \{1, 2, 3, 4\}$ and $Y = \{5, 6, 7, 8, 9\}$. Let F $=\{(1, 5), (2, 7), (4, 9), (3, 8)\}$.

   i. Show that F is a function from X to Y and check whether F is bijective

   ii. Find F^{-1}, and list its elements.

   iii. Is F^{-1} a function? Why, or why not?

2. (a) How many bit strings of length 10 contain

   i. exactly four 1s?

   ii. at most four 1s?

   iii. an equal number of 0s and 1s?

(b) How many permutations of the letters ABCDEFG contain

   i. the string CFGA?

   ii. the strings ABC and DE?

   iii. the strings ABC and CDE?

   iv. the strings CBA and BED?

(c) i. show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9

   ii. How many numbers must be selected from the set 1, 2, 3, 4, 5, 6 to guarantee that at least one pair of these numbers add up to 7?

3. (a) i. Find the expansion of $(x+y)^6$
   ii. Find the coefficient of x^5y^8 in $(x+y)^{13}$.
   iii. Give a formula for the coefficient of x^k in the expansion of $(x^2 - \frac{1}{x})^{100}$, where k is an integer

   (b) Give as good a big-O estimate as possible for each of these functions.
   i. $(n^2+8)(n+1)$
   ii. $(n\log n + n^2)(n^3+2)$
   iii. $(n! + 2^n)(n^3 + log(n^2+1))$

   (c) Suppose that you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly $n(\log n)$ operations and the second algorithm uses exactly $n^{\frac{3}{2}}$ operations. As n grows, which algorithm uses fewer operations?

4. (a) Consider the following darts game: The target consists of a bull's-eye, which is a circle of radius 1, surrounded by a middle ring of outer radius 3 and inner radius 1; this region is in turn surrounded by another ring of outer radius 4 and inner radius 3. If you hit the bull's-eye, you get \$10 . If you hit the middle ring,you get \$5, and if you hit the outer ring, you get \$2. Suppose the probability that you hit a region is proportional to its area and the probability to miss the entire circle is $\frac{1}{5}$
   i. Set up an underlying sample space Ω and its probability density p.
   ii. Define a random variable X on Ω.
   iii. Define a sample space Ω_x and a probability distribution p_x on Ω_x
   iv. what is the Expected amount per throw you can get it from this game
   v. The game score decides by hitting the bulls eye. If you have 10 chances to throw then what is the distribution related to finding the score of this game and find the probability to get exactly four points.
   vi. If you have a four chances to throw then what is the probability to get more than or equal to \$30 in this game

   (b) The average rate of job submissions in a busy computer centre is 4 per minute If it can be assumed that the number of submissions per minute interval is Poisson distribution, calculate the probability that
   i. 0 job submissions;
   ii. exactly 2 job submissions;
   iii. at most 3 job submissions.
   iv. what is the average number of submissions in 2 minutes
   v. Whats is the average time taken between two job submissions