

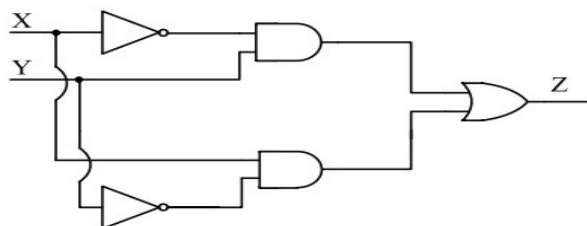


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MC 4010 : Discrete Mathematics

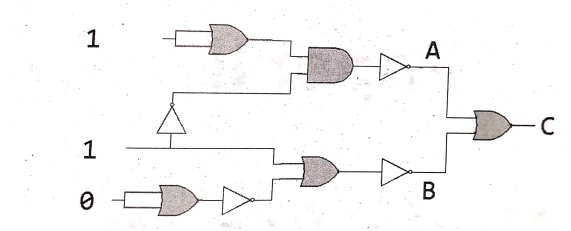
Tutorial-02

March 2023

1. Express these system specifications using the propositions  $p$  “The user enters a valid password,”  $q$  “Access is granted,” and  $r$  “The user has paid the subscription fee” and logical connectives (including negations).
  - a) “The user has paid the subscription fee, but does not enter a valid password.”
  - b) “Access is granted whenever the user has paid the subscription fee and enters a valid password.”
  - c) “Access is denied if the user has not paid the subscription fee.”
  - d) “If the user has not entered a valid password but has paid the subscription fee, then access is granted.”
2. Show that the following are tautologies
  - a)  $(p \wedge (q \wedge r)) \rightarrow [((r \wedge p) \wedge q) \vee q]$
  - b)  $[p \rightarrow (r \rightarrow q)] \leftrightarrow [(p \wedge r) \rightarrow q]$
  - c)  $[p \rightarrow q] \rightarrow [(r \vee p) \rightarrow (r \vee q)]$
  - d)  $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
3. Construct a truth table for each of these compound propositions.
  - a)  $p \leftrightarrow \neg p$
  - b)  $p \oplus (p \vee q)$
  - c)  $(p \wedge q) \rightarrow (p \vee q)$
  - d)  $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
  - e)  $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
4. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the following outputs from input bits  $p$ ,  $q$ , and  $r$ .
  - a)  $(p \wedge \neg r) \vee (\neg q \wedge r)$
  - b)  $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$
5. In the circuit below,  $X$  and  $Y$  are digital inputs and  $Z$  is a digital output. Find the output  $Z$  and verify that, output is equivalent to XOR gate.



6. Find the values of A, B, C



7. which of the following pairs of propositional forms are logically equivalent,

- a)  $p \rightarrow (q \wedge r), (p \rightarrow q) \wedge (p \rightarrow r);$
- b)  $p \leftrightarrow q, (\neg p \vee q) \wedge (p \vee \neg q);$
- c)  $p \leftrightarrow q, (p \wedge q) \vee (\neg p \wedge \neg q);$
- d)  $(p \rightarrow q) \rightarrow r, (p \wedge \neg r) \rightarrow \neg q;$
- e)  $p \rightarrow (q \vee r), [p \rightarrow (q \vee p)] \rightarrow r;$

8. Let p, q, r be three statements. Simplify the following expressions,

- a)  $\neg(p \vee q) \vee (\neg p \wedge q);$
- b)  $[(p \rightarrow q) \rightarrow p] \rightarrow p;$
- c)  $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p;$

9. Determine the validity of the following argument

- a) Robbery was the motive for the crime only if the victim had money in his pockets. But robbery or vengeance was the motive for the crime. Therefore, vengeance must have been the motive for the crime.
- b) If students play cricket they study well. Either students do not study well or they pass annual examinations. But students are failing the annual examination. Therefore, students do not play cricket.
- c) Either John is exhausted or he is sick. If he is exhausted then he is contrary. He is not contrary. Therefore he is sick.

10. Let  $C(x)$  be the statement “x has a cat,” let  $D(x)$  be the statement “x has a dog,” and let  $F(x)$  be the statement “x has a parrot.” Express each of these statements in terms of  $C(x)$ ,  $D(x)$ ,  $F(x)$ , quantifiers, and logical connectives. Let the domain consist of all students in your class.

- a) A student in your class has a cat, a dog, and a parrot.
- b) All students in your class have a cat, a dog, or a parrot.
- c) Some student in your class has a cat and a ferret, but not a dog.
- d) No student in your class has a cat, a dog, and a parrot.
- e) For each of the three animals, cats, dogs, and parrots, there is a student in your class who has this animal as a pet.

11. Find the logic expression for given truth table

a)

p	q	r	A
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

b)

p	q	r	A
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F
F	F	F	T