MC4010- Discrete Mathematics

Set Theory

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1 Sets and Membership

A set is often described by a list of its elements between braces. For example,

$$S = \{1,\ 2,\ 3,\ 4,\ 5\}$$

is the set whose elements are the first 5 positive integers. The order in which the elements are listed is not important. For example, $\{1,\ 2,\ 3\}$ and $\{2,\ 1,\ 3\}$ represent the same set.

Sometimes it may be impossible to list the elements of a set. For example, one cannot list the set $\mathbb{R}$ of all real numbers. Sometimes a partial list is given, when there are infinitely many elements in a set. For example,

$$\mathbb{N} = \{1,\ 2,\ 3,\ \ldots\}$$

represents the set of natural numbers (positive integers). The notation $\mathbb{Z}^+$ is also used to denote this set in some literature. The set of integers is denoted by

$$\mathbb{Z} = \{0,\ \pm 1,\ \pm 2,\ \pm 3,\ \ldots\}.$$

Notation

We shall generally use upper-case letters to denote sets and lower-case letters to denote elements. (This convention will sometimes be violated, for example when the elements of a particular set are themselves sets.) The symbol $\in$ denotes 'belongs to' or 'is an element of'. Thus

$$a \in A \text{ means (the element) } a \text{ belongs to (the set) } A$$

and

$$a \notin A \text{ means } \neg(a \in A) \text{ or } a \text{ does not belong to } A.$$

$D = \{\ \}$, the **empty set** (or **null set**), which contains no elements. This set is usually denoted $\varnothing$.

$$\mathbb{N} = \{1,\ 2,\ 3,\ \ldots\}$$

$$\mathbb{Z} = \{0,\ \pm 1,\ \pm 2,\ \pm 3,\ \ldots\}.$$

The set $\mathbb{Q} = \left\{ \frac{n}{m} \,\middle|\, m, n \in \mathbb{Z},\ m \neq 0 \right\}$ is the set of all **rational numbers**. The set $\mathbb{C} = \{x + yi \mid x, y \in \mathbb{R}\}$ is the set of all **complex numbers** ($i^2 = -1$).

Examples

1. The set B above could be defined as $B = \{n : n$ is an even, positive integer$\}$, or $B = \{n : n = 2m$, where $m > 0$ and m is an integer$\}$, or, with a slight change of notation, $B = \{2m : m > 0$ and m is an integer$\}$.

   Note that although the propositional functions used are different, the same elements are generated in each case.

2. The set C_n above could be defined as $C_n = \{p : p$ is an integer and $1 \leqslant p \leqslant n\}$.

3. The set $\{1, 2\}$ could alternatively be defined as $\{x : x^2 - 3x + 2 = 0\}$. We say that $\{1, 2\}$ is the **solution set** of the equation $x^2 - 3x + 2 = 0$.

4. The empty set $\varnothing$ can be defined in this way using any propositional function $P(x)$ which is true for no objects x. For instance, rather frivolously,

$$\varnothing = \{x : x \text{ is a green rabbit with long purple ears}\}.$$

Equality of Sets

Two sets are defined to be **equal** if and only if they contain the same elements; that is, $A = B$ if $\forall x[x \in A \leftrightarrow x \in B]$ is a true proposition, and conversely. The order in which elements are listed is immaterial. Also, it is the standard convention to disregard repeats of elements in a listing. Thus the following all define the same set:

$$\{1, -\tfrac{1}{2}, 1066, \pi\}$$
$$\{-\tfrac{1}{2}, \pi, 1066, 1\}$$
$$\{1, -\tfrac{1}{2}, -\tfrac{1}{2}, \pi, 1066, -\tfrac{1}{2}, 1\}.$$

Exercise

1. *True or false:*

   (a) $-1 \in \mathbb{Q}$.

   (b) $0 \notin \mathbb{Q}$.

   (c) $1 \in \mathbb{C}$.

   (d) $\emptyset \in \emptyset$.

   (e) $\{0, 1\} = \{1, 0\}$.

(a) True,

(b) False,

(c) True,

(d) False,

(e) True.

2. *Describe the set* $A = \{x \in \mathbb{Z} \mid |x| \leq 2\}$ *by listing its elements.*

$A = \{ -2, -1, 0, 1, 2 \}$

3. *Let* $\mathbb{Z}_3 = \{0, 1, 2\}$. *Describe the set*

$$B = \{x \in \mathbb{Q} \mid 2x \in \mathbb{Z}_3\}$$

by listing its elements.

$x \in B$ if and only if $2x = 0, 1,$ or 2.

Hence $x = 0, \frac{1}{2},$ or 1.

Thus $B = \{0, \frac{1}{2}, 1\}$.

4. *Let* $\mathbb{Z}_2 = \{0, 1\}$. *Describe the set*

$$X = \{z \in \mathbb{C} \mid z = x + yi \text{ and } x, y \in \mathbb{Z}_2\}$$

by listing its elements.

$$X = \{0, i, 1, 1 + i\}$$

5. *Describe the set*

$$Y = \{x \in \mathbb{C} \mid x^2 + 1 = 0\}$$

by listing its elements if it is not empty.

$$Y = \{i, -i\}$$

6. *Describe the set*

$$Z = \{z \in \mathbb{C} \mid z^3 = -1\}$$

by listing its elements.

$z^3 = -1$ if and only if

$$z^3 + 1 = 0$$

Now

$$z^3 + 1 = (z + 1)(z^2 - z + 1).$$

Hence either $z = -1$ or

$$z = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm 3i}{2}$$

Definition

If A is a finite set its **cardinality**, $|A|$, is the number of (distinct) elements which it contains.

If A has an infinite number of elements, we say it has **infinite cardinality**†, and write $|A| = \infty$.

Other notations commonly used for the cardinality of A are $n(A)$, $\#(A)$ and $\bar{\bar{A}}$.

Examples

1. $|\varnothing| = 0$ since $\varnothing$ contains no elements.
2. $|\{\pi, 2, \text{Attila the Hun}\}| = 3$.
3. If $X = \{0, 1, \ldots, n\}$ then $|X| = n + 1$.
4. $|\{2, 4, 6, 8, \ldots\}| = \infty$.

For example, let $X = \{\{1,2\}\}$. Then X contains only a single element, namely the set $\{1,2\}$, so $|X| = 1$. It is clearly important to distinguish between the set $\{1,2\}$ (which has cardinality 2) and X, the set which has $\{1,2\}$ as its only element. Similarly, the sets $\varnothing$ and $\{\varnothing\}$ are different. The latter is non-empty since it contains a single element—namely $\varnothing$. Thus $|\{\varnothing\}| = 1$.

Examples

1. Let $A = \{1, \{1,2\}\}$. Note that A has two elements, the number 1 and the set $\{1,2\}$. Therefore, $|A| = 2$.

2. Similarly,
$$
\begin{aligned}
|\{1, 2, \{1,2\}\}| &= 3,\\
|\{\varnothing, \{1,2\}\}| &= 2,\\
|\{\varnothing, \{\varnothing\}\}| &= 2,\\
|\{\varnothing, \{\varnothing\}, \{1,2\}\}| &= 3,\\
|\{\varnothing, \{\varnothing, \{\varnothing\}\}\}| &= 2, \text{ etc.}
\end{aligned}
$$

Exercises

1. List the elements of each of the following sets, using the '...' notation where necessary:

(i) $\{x : x \text{ is a positive (integer) multiple of three}\}$
(ii) $\{x : (3x-1)(x+2)=0\}$
(iii) $\{x : x \text{ is an integer and } (3x-1)(x+2)=0\}$
(iv) $\{x : 2x \text{ is a positive integer}\}$.

(i) $\{3, 6, 9, 12, \ldots\}$
(ii) $\{1/3, -2\}$
(iii) $\{-2\}$
(iv) $\{1/2, 1, 3/2, 2, 5/2, 3, \ldots\}$.

2. Let $X = \{0, 1, 2\}$. List the elements of each of the following sets:

(i) $\{z : z = x + y \text{ where } x \in X \text{ and } y \in X\}$

(ii) $\{z : z \in X \text{ or } -z \in X\}$

(iii) $\{z : z \text{ is an integer and } z^2 \in X\}$.

Solutions

(i) $\{0, 1, 2, 3, 4\}$

(ii) $\{-2, -1, 0, 1, 2\}$

(iii) $\{-1, 0, 1\}$.

3. Determine the cardinality of each of the following sets:

(i) $\{x : \sqrt{x} \text{ is an integer}\}$

(ii) $\{a, b, c, \{a, b, c\}\}$

(iii) $\{\{a, b, c\}, \{a, b, c\}\}$

(iv) $\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}\}$.

Solutions

(i) ∞

(ii) 4

(iii) 1

(iv) 3.

4. Use the notation $\{x : P(x)\}$, where $P(x)$ is a propositional function, to describe each of the following sets.

(i) $\{3, 6, 9, 12, 15, \ldots, 27, 30\}$.

(ii) $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\}$.

(iii) The set of integers which can be written as the sum of the squares

Solutions

(i) $\{x : x \text{ is an integer multiple of } 3 \text{ and } 3 \leqslant x \leqslant 30\}$

(ii) $\{x : x \text{ is a prime number}\}$

(iii) $\{x : x = n^2 + m^2 \text{ for some integers } n \text{ and } m\}$

Subsets

Definition

The set A is a **subset** of the set B, denoted $A \subseteq B$, if every element of A is also an element of B. Symbolically, $A \subseteq B$ if $\forall x[x \in A \rightarrow x \in B]$ is true, and conversely.

If A is a subset of B, we say that B is a **superset** of A, and write $B \supseteq A$.

Clearly every set B is a subset of itself, $B \subseteq B$. (This is because, for any given x, $x \in B \to x \in B$ is 'automatically' true.) Any other subset of B is called a **proper subset** of B. The notation $A \subset B$ is used to denote 'A is a proper subset of B'. Thus $A \subset B$ if and only if $A \subseteq B$ and $A \neq B$.

It should also be noted that $\varnothing \subseteq A$ for every set A.

Examples

1. $\{2,4,6,\ldots\} \subseteq \{1,2,3,\ldots\} \subseteq \{0,1,2,\ldots\}$. Of course, we could have used the proper subset symbol $\subset$ to link these three sets instead.

2. Similarly: {women} $\subseteq$ {people} $\subseteq$ {mammals} $\subseteq$ {creatures};
{*War and Peace*} $\subseteq$ {novels} $\subseteq$ {works of fiction};
{*Mona Lisa*} $\subseteq$ {paintings} $\subseteq$ {works of art}; etc.

Again, in each of these we could have used $\subset$ instead.

Theorem

Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

Examples

1. Show that $\{x : 2x^2 + 5x - 3 = 0\} \subseteq \{x : 2x^2 + 7x + 2 = 3/x\}$.

2. Let $A = \{\{1\}, \{2\}, \{1, 2\}\}$ and let B be the set of all non-empty subsets of $\{1, 2\}$. Show that $A = B$.

3. Prove that if $A \subseteq B$ and $C = \{x : x \in A \vee x \in B\}$, then $C = B$.

Some special sets of numbers which are frequently used as universal sets are the following.

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ the set of **natural numbers**.

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ the set of **integers**†.

$\mathbb{Q} = \{p/q : p, q \in \mathbb{Z} \text{ and } q \neq 0\}$ the set of fractions or **rational numbers**.

$\mathbb{R} =$ the set of **real numbers**; real numbers can be thought of as corresponding to points on a number line or as numbers written as (possibly infinite) decimals.

$\mathbb{C} = \{x + iy : x, y \in \mathbb{R} \text{ and } i^2 = -1\}$ the set of **complex numbers**.

Clearly the following subset relations hold amongst these sets:

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}.$$

Exercise

1. *True or false (Justify your answers):*
   (a) $\mathbb{Z} \subseteq \mathbb{Q}$.
   (b) $\mathbb{R} \subseteq \mathbb{C}$.
   (c) $\emptyset \subseteq \emptyset$.
   (d) $\{\emptyset\} \subseteq \mathbb{Z}$.

2. *Find all subsets of* $A = \{z \in \mathbb{C} \mid z^2 = -1\}$.

3. *Find all subsets of* $\{\emptyset\}$.

Equality

If X and Y are sets such that $X \subseteq Y$ and $Y \subseteq X$, then we say that sets X and Y are **equal** (to each other). Otherwise X and Y are said to be **distinct** (from each other). Thus sets X and Y are equal if and only if they contain the same elements. In this case we write $X = Y$; otherwise we write $X \neq Y$. The proposition $X = Y$ is called an **equation**. Note that $X = X$, and that if $X = Y$ then $Y = X$.

Note that since $\emptyset$ is a subset of any set, $\emptyset$ is unique.

$$\boxed{\begin{array}{ll} & X = Y \text{ if} \\ (1) & (x \in X) \Rightarrow (x \in Y), \\ (2) & (x \in Y) \Rightarrow (x \in X) \end{array}}$$

Example

Let

$$\begin{aligned} A &= \{x \in \mathbb{Z} \mid x = 3 + 5p, \text{ for some } p \in \mathbb{Z}\}, \\ B &= \{x \in \mathbb{Z} \mid x = -7 + 5p, \text{ for some } p \in \mathbb{Z}\}. \end{aligned}$$

We show that $A = B$.

Theorem *Let X, Y, Z be sets.*

(a) $X = X$.

(b) If $X = Y$ then $Y = X$.

(c) If $X = Y$ and $Y = Z$ then $X = Z$.

Exercise

1. *Show that the following two sets are equal:*

$$\begin{aligned} A &= \{x \in \mathbb{Z} \mid x = 1 + 3q, \ \textit{for some } q \in \mathbb{Z}\}, \\ B &= \{x \in \mathbb{Z} \mid x = -2 + 3q, \ \textit{for some } q \in \mathbb{Z}\}. \end{aligned}$$

2. *Let p be any odd number. Show that*

$$\{x \mid x = p + 2m, \text{ for some } m \in \mathbb{Z}\}$$

is equal to the set of all odd numbers.

3. *Show that the set*

$$\{x \in \mathbb{Z} \mid x = 2n, \;\; and \; x = 2m + 1 \; for \; some \; m, n \in \mathbb{Z}\}$$

is empty.

Proper Subsets

Let X and Y be sets. If $X \subseteq Y$ and $X \neq Y$, then we say that X is a **proper** subset of Y. In this case we write $X \subsetneqq Y$, or sometimes just $X \subset Y$. The proposition $X \subsetneqq Y$ is called a **proper** inclusion. Note that $X \subseteq Y$ if and only if $X \subsetneqq Y$ or $X = Y$.

Theorem *Let X, Y, Z be sets. If $X \subsetneqq Y$ and $Y \subsetneqq Z$, then $X \subsetneqq Z$.*

The Power Set of a Set

If X is a set, the **power set** $\mathcal{P}(X)$ of X is the set of all subsets of X.

if x and y are distinct objects then

$$\mathcal{P}(\{x, y\}) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}.$$

The number of elements in a finite set X is the **cardinality** of X, and is denoted by $|X$

Let X be a set with $n \geq 1$ elements. Then for each $x \in X$, there are two possibilities: either x is an element of a given subset or it is not. Hence there are 2^n ways of constructing subsets of X. Thus

$$|\mathcal{P}(X)| = 2^{|X|}.$$

Exercise

1. *Find the power set of each of the following sets.*

   (a) $X = \{0\}$.

   (b) $\mathbb{Z}_2 = \{0, 1\}$.

2. *Find the power set of* $A = \{\pm 1, \pm i\}$. *Arrange the subsets of* A *in chains of inclusions.*

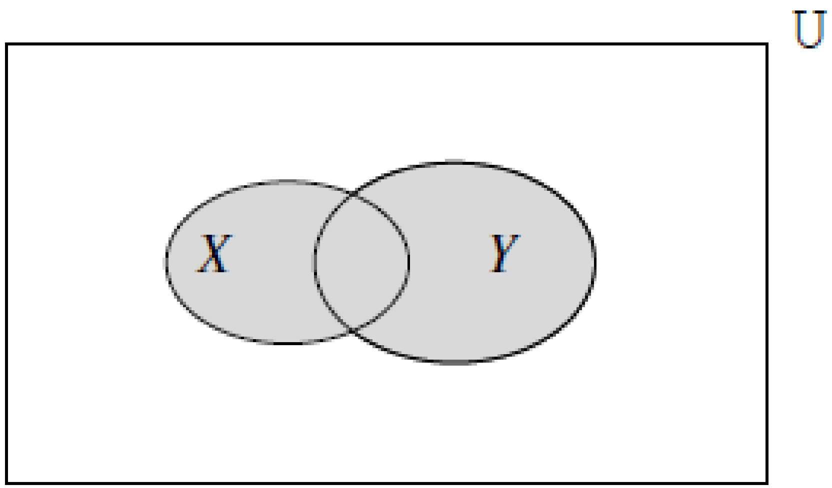
3. *Prove that if* $|A| = |B|$, *then* $\mathcal{P}(A)$ *and* $\mathcal{P}(B)$ *have the same number of elements.*

4. *Let A be a non-empty set, and $a \in A$. Prove that A has twice as many subsets as* $A - \{a\} = \{x \in A \mid x \neq a\}$.

Unions of Sets

The **union**, $X \cup Y$, of sets X and Y is defined as the set of all objects that are in either X or Y. Of course some such objects might be in both. As an example, $\{x, y\} \cup \{x, z\} = \{x, y, z\}$. In a Venn diagram, $X \cup Y$ is represented by the shaded region in Figure

$$\boxed{\begin{array}{c} x \in X \cup Y \text{ if and only if} \\ x \in X \text{ or } x \in Y \end{array}}$$



$X \cup Y$ shaded

Example

The set of integers is the union of the set of even numbers and the set of odd numbers. That is

$$\mathbb{Z} = \{x \in \mathbb{Z} | x = 2n,\ n \in \mathbb{Z}\} \cup \{x \in \mathbb{Z} | x = 2n + 1,\ n \in \mathbb{Z}\}.$$

If a and b are integers with $a \leq b$ and $X_a, X_{a+1}, \ldots, X_b$ are sets then we write

$$\bigcup_{i=a}^{b} X_i = X_a \cup X_{a+1} \cup \ldots \cup X_b.$$

If X and Y are sets then clearly

$$X \subseteq X \cup Y$$

and

$$Y \subseteq X \cup Y.$$

The commutativity of the disjunction of propositions immediately implies the commutative law for unions of sets:

$$X \cup Y = Y \cup X.$$

Similar considerations show that

$$X \cup \emptyset = X$$

and

$$X \cup X = X,$$

and that

$$X \cup Y = Y$$

if $X \subseteq Y$. In particular,

$$X \cup \mathtt{U} = \mathtt{U},$$

where $\mathtt{U}$ is the universal set.

Exercise

1. *Show that* $X \cup Y = X$ *if and only if* $Y \subseteq X$.

2. *True or false: For any sets* X, Y, $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$.

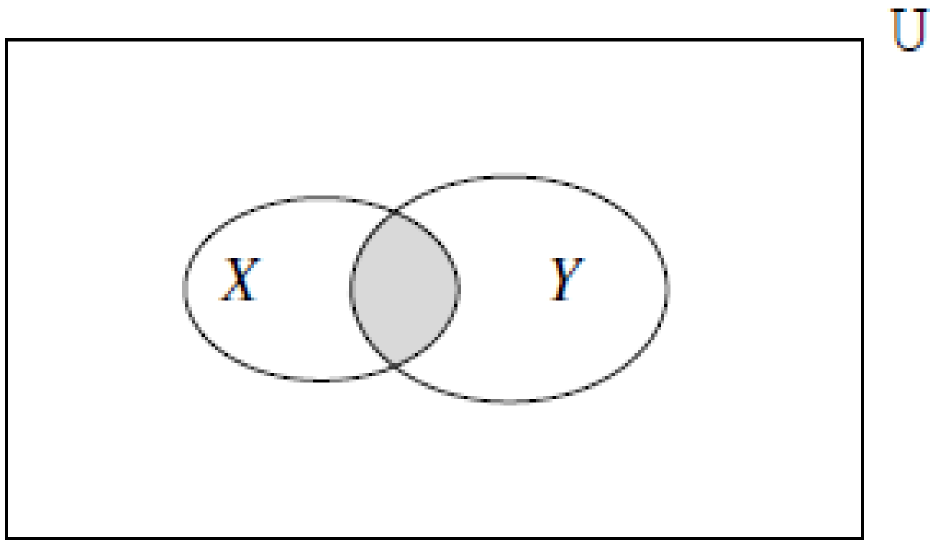
3. *True or false: If* $X \subseteq Y$, *then* $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$.

Intersections of Sets

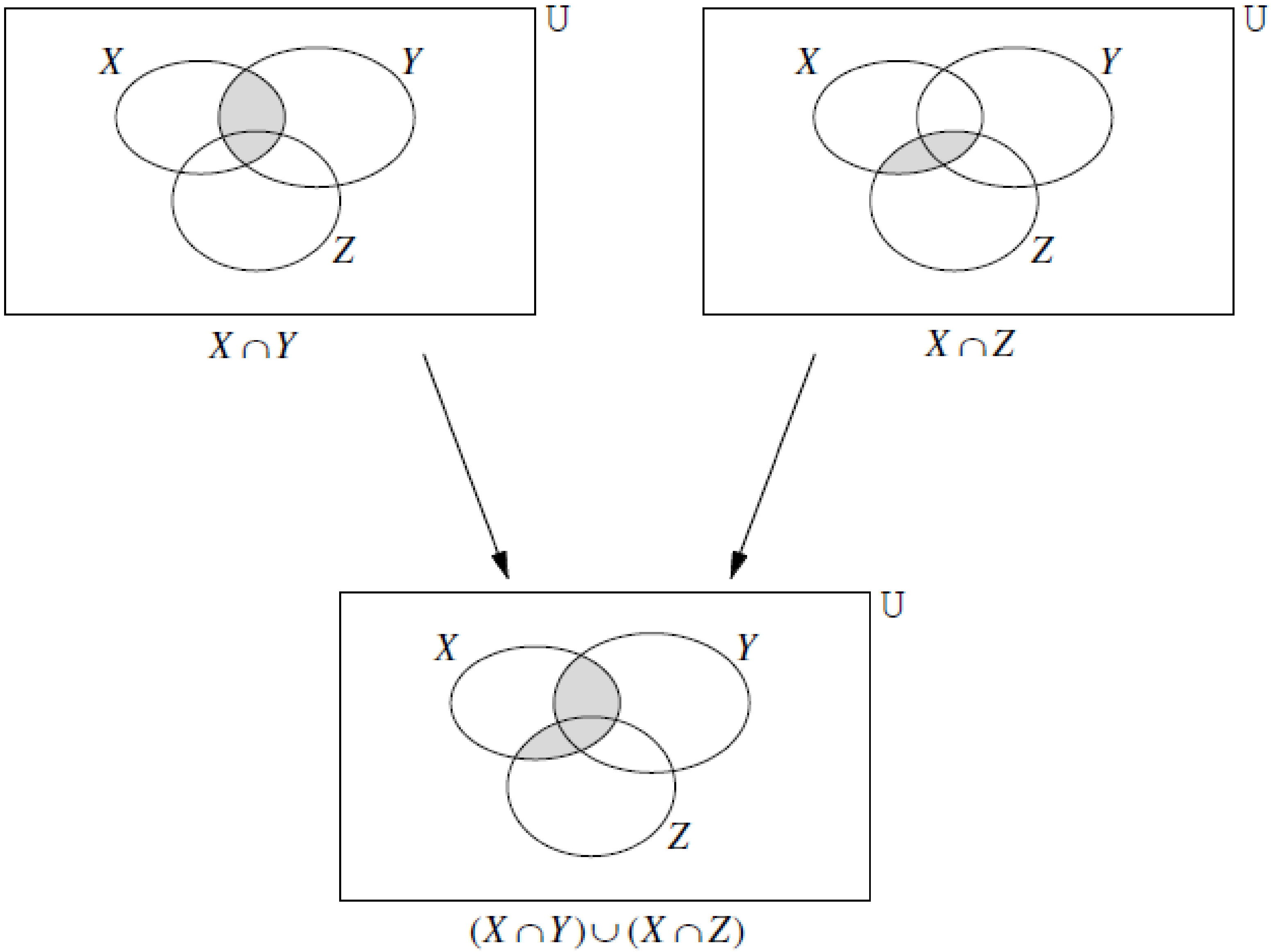
Let X and Y be sets. Then the set $\{x \in X | x \in Y\}$ is called the **intersection** of X and Y, and is denoted by $X \cap Y$. Its elements are the objects contained in both X and Y.

$$\boxed{\begin{array}{c} x \in A \cap B \text{ if and only if} \\ x \in A \text{ and } x \in B \end{array}}$$

The region representing $X \cap Y$ in a Venn diagram is the overlap between the region representing X and that representing Y. It is indicated by the shaded area in Figure



$X \cap Y$ shaded


U
X
Y
Z
$X \cap Y$
U
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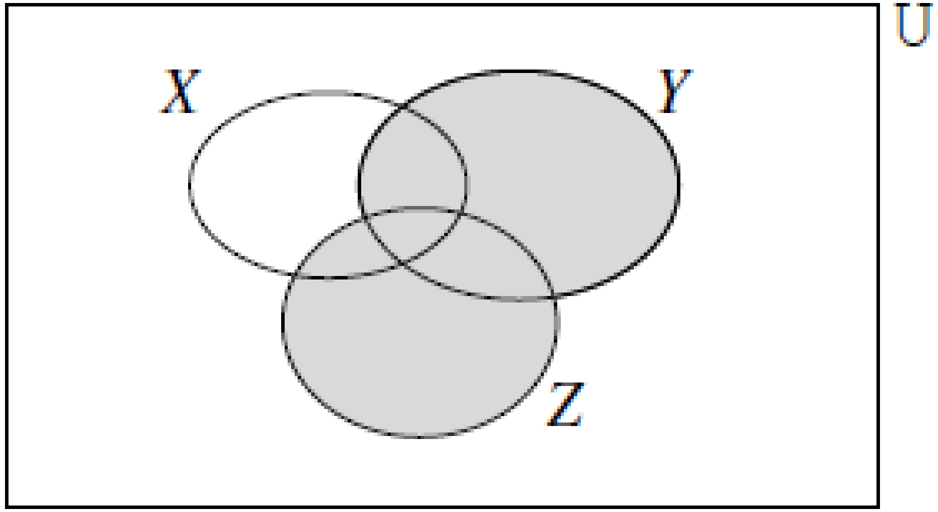
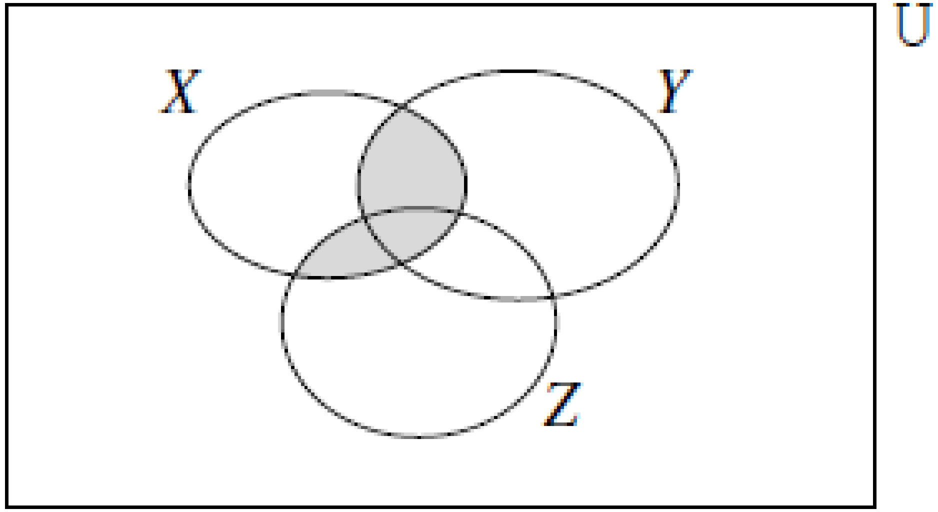
$X \cap Z$
U
X
Y
Z
$(X \cap Y) \cup (X \cap Z)$



$Y \cup Z$



$X \cap (Y \cup Z)$

Exercise

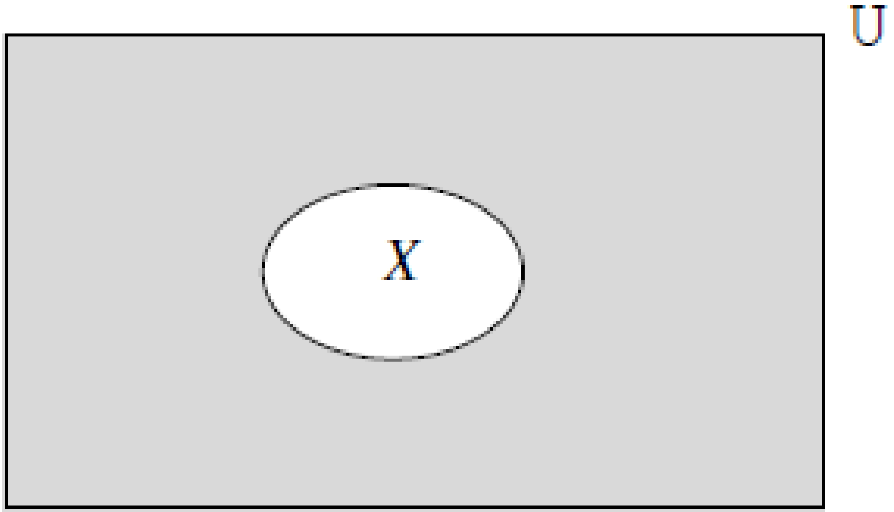
1. *Show that* $X \cap Y = X$ *if and only if* $X \subseteq Y$.

2. *Show by using Venn diagrams that* $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$.

The Complement of a Set with respect to Another

Let X be a set. Then the set of all elements (in a universal set $\mathtt{U}$) that are not in X is called the **complement** of X, denoted by $\overline{X}$

$$\overline{X} = \{x \in \mathtt{U} \mid x \notin X\}.$$



$\overline{X}$ shaded

Example

Let $\mathtt{U} = \mathbb{Z}^+$ and $X = \{2n \mid n \in \mathbb{Z}^+\}$, the set of even positive integers. Then

$$\overline{X} = \{2n - 1 \mid n \in \mathbb{Z}^+\},$$

the set of odd positive integers.

Theorem *Let X be any set. Then*

(1) $X \cap \overline{X} = \emptyset$.

(2) $X \cup \overline{X} = U$.

(3) $\overline{\overline{X}} = X$.

(4) $\overline{\emptyset} = U$.

(5) $\overline{U} = \emptyset$.

Theorem *For any sets X and Y,*

(1) $X - Y = X \cap \overline{Y}$.

(2) $X - Y = X - (X \cap Y)$.

Example
Let

$$X = \{1,\ 2,\ 3,\ 4,\ 5\},$$
$$Y = \{-1,\ 2,\ 4,\ 6\}.$$

Then

$$X - Y = \{1,\ 3,\ 5\},$$

and

$$Y - X = \{-1,\ 6\}.$$

The following theorem is the analogue of de Morgan's laws, and is an immediate consequence of them. Try drawing a Venn diagram.

Theorem *Let X and Y be sets. Then*

(a) $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$ *and*

(b) $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$.

1. *Prove that for any sets* X *and* Y, $X - Y = X \cap \overline{Y}$.

3. *True or false*

$$A - (B - C) = (A - B) - C.$$

4. *Generalise de Morgan's law to*

$$(\overline{A_1 \cup \ldots \cup A_n}) = \overline{A}_1 \cap \ldots \cap \overline{A}_n.$$

Prove it by induction.

5. *True or false:*

$$X - (Y \cup Z) = (X - Y) \cap (X - Z)$$

for any sets X, Y *and* Z.

Let X, Y be sets. We define the **symmetric difference** *of X and Y to be*

$$X \oplus Y = (X - Y) \cup (Y - X).$$

Prove by induction that the elements of
$A_1 \oplus \ldots \oplus A_n$ are exactly those that are members of an odd number of $A_1, A_2, \ldots, A_n$.

Fundamental Properties of Sets

Theorem *Let X, Y, Z be subsets of U. Then*

(a) $X \cup Y = Y \cup X$ *and* $X \cap Y = Y \cap X$; *and*

(b) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ *and* $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$.

(c) *Moreover* $X \cup \emptyset = X$, *and* $X \cap U = X$.

(d) *Finally* $X \cap \overline{X} = \emptyset$, *and* $X \cup \overline{X} = U$.

Theorem *Let X, Y, Z be sets. Then*

(a) $X \cup (X \cap Y) = X \cap (X \cup Y) = X$; *and*

(b) $X \cup (Y \cup Z) = (X \cup Y) \cup Z$ *and* $X \cap (Y \cap Z) = (X \cap Y) \cap Z$.

Exercises

1. List the elements of each of the following sets, using the '...' notation where necessary:

(i) $\{x : x \text{ is an integer and } -3 < x < 4\}$
(ii) $\{x : x \text{ is a positive (integer) multiple of three}\}$
(iii) $\{x : x = y^2 \text{ and } y \text{ is an integer}\}$
(iv) $\{x : (3x-1)(x+2) = 0\}$
(v) $\{x : x \geqslant 0 \text{ and } (3x-1)(x+2) = 0\}$
(vi $\{x : x \text{ is an integer and } (3x-1)(x+2) = 0\}$
(vii) $\{x : x \text{ is a positive integer and } (3x-1)(x+2) = 0\}$
(viii) $\{x : 2x \text{ is a positive integer}\}$.

4. Use the notation $\{x : P(x)\}$, where $P(x)$ is a propositional function, to describe each of the following sets.

(i) $\{1, 2, 3, 4, 5\}$.
(ii) $\{3, 6, 9, 12, 15, \ldots, 27, 30\}$.
(iii) $\{1, 3, 5, 7, 9, 11, \ldots\}$.
(iv) $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\}$.
(v) $\{\mathrm{a}, \mathrm{e}, \mathrm{i}, \mathrm{o}, \mathrm{u}\}$.
(vi) The set of integers which can be written as the sum of the squares of two integers.
(vii) The set of all integers less than 1000 which are perfect squares.
(viii) The set of all numbers that are an integer multiple of 13.
(ix) {Afghanistan, Albania, Algeria, . . . , Zambia, Zimbabwe}.
(x) {*Love's Labour's Lost*, *The Comedy of Errors*, *The Two Gentlemen of Verona*, . . . , *The Tempest*, *The Winter's Tale*, *The Famous History of the Life of King Henry VIII*}.

Show that $\{x : 2x^2 + 5x - 3 = 0\} \subseteq \{x : 2x^2 + 7x + 2 = 3/x\}$.

Let $A = \{\{1\}, \{2\}, \{1, 2\}\}$ and let B be the set of all non-empty subsets of $\{1, 2\}$. Show that $A = B$.

Prove that if $A \subseteq B$ and $C = \{x : x \in A \vee x \in B\}$, then $C = B$.

For each of the following, draw a Venn-Euler diagram and shade the region corresponding to the indicated set.

(a) $A - (B \cap C)$ (b) $(A - B) \cup (A - C)$.

Show that $A - (B \cap C) = (A - B) \cup (A - C)$ for all sets A, B and C.