UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

END SEMESTER EXAMINATION– SEPTEMBER 2019

DISCRETE MATHEMATICS

MC 4010

Writing Time: TWO Hours, *Registration Number:*

Instructions

1. This is a **close book** exam.
2. This paper contains **SIX (06)** questions
3. Answer **all** questions in the **SPACE** provided.
4. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
5. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
6. This examination accounts for **60%** of module assessment. Total maximum mark attainable is **100**.
7. Write your **registration number** in the question-answer book. Also write your registration number on each sheet.

Question	Marks
1	
2	
3	
4	
5	
6	
TOTAL	

1. (a) Consider the following propositions:

p: Mathematicians are generous.
q: Spiders hate algebra.

Write the compound propositions symbolized by: $\overline{p} \leftrightarrow \overline{q}$. ....
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....

(b) Test the validity of the following argument: 'If you are a mathematician then you are clever. You are clever and rich. Therefore if you are rich then you are a mathematician ....
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(c) Prove that $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$. ....
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(d) Show that $(\overline{p} \vee \overline{q})$ and $(\overline{p \wedge q})$ are logically equivalent, i.e. that $(\overline{p} \vee \overline{q}) \equiv (\overline{p \wedge q})$.
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(e) Show that $(p \wedge q) \vee (\overline{p \wedge q})$ is a tautology. ....

(f) Show that the following two propositions are logically equivalent.

i. If it rains tomorrow then, if I get paid, Ill go to Paris.
ii. If it rains tomorrow and I get paid then Ill go to Paris.

[20 marks]

2. (a) Let X, Y be sets. We define the symmetric difference of X and Y to be

$$X \oplus Y = (X - Y) \cup (Y - X)$$

i. Draw a diagram to represent $X \oplus Y$ ....

ii. Prove by a sequence of Venn diagrams that

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

....

iii. Prove by a sequence of Venn diagrams that

$$A \oplus B = (A \cup B) - (A \cap B)$$

....

(b) Determine if each of the following relations is (1) reflexive, (2) symmetric, (3) anti- symmetric, or (4) transitive.

i. on $\mathcal{Z}$, $m \sim n$ if and only if $|m - n| = 2$; ....

ii. on $\mathcal{Z}$, $m \sim n$ if and only if $|m| = |n|$; ....

iii. on $\mathcal{R}^2$, $(a, b) \sim (c, d)$ if and only if $(a, b) = \lambda(c, d)$ for some $\lambda \neq 0$ ....

iv. on $\mathcal{Z}$, $m \sim n$ if and only if $mn \geq 0$.; ....

(c) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^3 + 1$$

i. Show that f is a bijection. ....

ii. What is $f^{-1}(x)$ ....

[20 marks]

3. Let a_n be the number of bit strings of length n that do not have two consecutive 0 s.

(a) Find a recurrence relation and give initial conditions for a_n. ....

(b) How many such bit strings are there of length 5 ? ....

(c) Find the general solution for a_n. ....

(d) Suppose that the roots of the characteristic equation of linear homogeneous recurrence relation are $2, 2, 5, 5, 5, 5$, and -3. What is the form of general solution? ....

(e) Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with initial conditions $a_0 = 1, a_1 = -2$ and $a_2 = -1$. ....

[15 marks]

4. (a) How many 10-digit binary strings begin in 1 or end in 1? ....

(b) How many different license plates are available if each plate contains a sequence of 3 letters followed by 3 digits? ....

(c) This problem concerns lists of length 6 made from the letters A, B, C, D, E, F, G, H. How many such lists are possible if repetition is not allowed and the list contains two consecutive vowels? ....

(d) i. Construct the first 6 rows of Pascal's triangle. ....

ii. Use the binomial theorem to find the coefficient of x^8 in $\left(x+2\right)^{13}$. ......

(e) How many bit strings of length four do not have two consecutive 1s ? ....

[15 marks]

5. (a) A rectangular floor measures $299cm \times 221cm$. Use the **Euclidean algorithm** to find the largest square tiles that can be used to cover the floor exactly?

(b) Decrypt the message $RTOLK\ TOLK$ which is encrypted using the affine transformation $C \equiv 3P + 24 \bmod 26$.

(c) Decrypt the ciphertext $LEWLYPLUJL\ \ PZ\ \ H\ \ NYLHA\ \ ALHJOLY$,that was encrypted with the shift cipher with key (shift) 7.

(d) Encrypt the plain text $MILLENNIUM$ using the key $MATH$ for a Vigenere cipher.

(e) Prove from the definition that running time $T(n) = 5n\log_2 n + 8n - 200$ is $O(n\log_2 n)$ for all $n \geq 2$.

[15 marks]

6. (a) One of the two software packages, A or B, should be chosen to process very big databases, containing each up to 10^{12} records. Average processing time of the package A is $T_A(n) = 0.1n\log_{10} n$ microseconds, and the average processing time of the package B is $T_B(n) = 2n$ microseconds.

   i. Which algorithm has better performance in a "Big-Oh" sense?

   ii. Work out exact conditions when these packages outperform each other.

(b) A sorting method with "Big-Oh" complexity $O(n \log n)$ spends exactly 1 millisecond to sort $1,000$ data items. Assuming that time $T(n)$ of sorting n items is directly proportional to $n \log n$, that is, $T(n) = cn \log n$.

i. Derive a formula for $T(n)$, given the time $T(N)$ for sorting N items.

ii. Estimate how long this method will sort $1,000,000$ items.

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[15 marks]

——— *End of Examination* ———