UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

END SEMESTER EXAMINATION– AUGUST 2018

DISCRETE MATHEMATICS

MC 4010

Reading Time: 10 Minutes

Writing Time: TWO Hours

Permitted Materials: Calculators; Mathematical tables

Instructions

1. This is a **Closed-book exam** exam.
2. This paper contains **TWO (2)** parts:
   (a) **Part 1** contains **40** questions: Each question in this part is a **multiple choice** with four answer choices. Read each question and answer carefully and choose the ONE best answer. There will be **NEGATIVE MARKING** **(each correct answer, you will get +1 point, and for 2 wrong answers, you will loose 1 point)** for the wrong answers.
   (b) **Part 2** contains **04** questions.
3. Answer **all** questions in the space provided.
4. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
5. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
6. This examination accounts for **60%** of module assessment. Total maximum mark attainable is **100**.
7. Write your **registration number**, **the module code** and the **title of the paper** in the answer book. Also write your registration number on each additional sheet attached.

Part 1[40 marks]

*Answer **all** questions in this part and you are advised to spend 50 minutes answering the questions in Part 1. You must mark your answers on the separate answer sheet.*

1. A function in which second elements of order pair are distinct is called

   (a) onto function
   (b) one -one function
   (c) identity function
   (d) inverse function

2. A finite non-empty set of symbol is called

   (a) alphabet
   (b) letter
   (c) string
   (d) language

3. The number of vertices in a full binary tree is

   (a) odd (b) even (c) equal (d) 0

4. What is the secret message produced from the message 'MEET YOU' using the Caesar cipher?

   (a) JBBQ VLR
   (b) LHHW WKQ
   (c) PHHW BRX
   (d) LQQW BRX

5. Which of the following statements is the contra-positive of the statement, 'you win the game if you know the rules but are not overconfident'?

   (a) If you lose the game then you don't know the rules or you are overconfident.
   (b) A sufficient condition that you win the game is that you know the rules or you are not overconfident.
   (c) If you know the rules or are overconfident then you lose the game.
   (d) If you know the rules and are overconfident then you win the game.

6. Floor(-3.6782)+ Ceil(2.965)

   (a) -2 (b) 2 (c) 0 (d) -1

7. Geometric distribution has ... parameters

   (a) 3
   (b) 2
   (c) 1
   (d) 4
   (e) None

8. A random variable X has a binomial distribution with $n = 10$ and $p = 0.3$. The variance of X is

   (a) 10 (b) 12 (c) 2.1 (d) 21

9. Which of the following is not the property of binomial distribution?

   (a) n is fixed
   (b) has two outcomes
   (c) Trials are independent
   (d) Probability of success varies from trial to trial

10. Consider the statement,'If n is divisible by 30 then n is divisible by 2, 3 and by 5'. Which of the following statements is equivalent to this statement?

    (a) If n is not divisible by 30 then n is divisible by 2 or divisible by 3 or divisible by 5
    (b) If n is not divisible by 30 then n is not divisible by 2 or not divisible by 3 or not divisible by 5
    (c) If n is divisible by 2 and divisible by 3 and divisible by 5 then n is divisible by 30
    (d) If n is not divisible by 2 or not divisible by 3 or not divisible by 5 then n is not divisible by 30

11. In cryptography, what is cipher?

    (a) algorithm for performing encryption and decryption
    (b) encrypted message
    (c) decrypted message
    (d) none of the mentionedabove

12. What is the value of 11 mod 7 and -11 mod 7?

    (a) 4 and 5 (b) 4 and -4 (c) 5 and 3 (d) 4 and 4

13. Which of the following problems can be solved using recursion?

    (a) Factorial of a number
    (b) n^{th} Fibonacci number
    (c) Length of a string
    (d) All of the mentioned

14. Which of the following pairs is not congruent modulo 11?

    (a) $25, 13$ (b) $-31, 12$ (c) $33, 10$ (d) $-64, 8$

15. If there are n distinct components in a statement then there are ..... combinations of values in the truth table.

(a) 2^n (b) $n+1$ (c) n (d) $n+2$

16. If truth table answer column has the truth values of both **true** and **false**, then it is said to be

    (a) tautology
    (b) contradiction
    (c) contingency
    (d) equivalence relation

17. To prove the statement P is tautologically equivalent to the statement Q, it is enough to prove that

    (a) P conditional Q is a contradiction
    (b) P conditional Q is a tautology
    (c) P bi-conditional Q is a contradiction
    (d) P bi-conditional Q is a tautology

18. Consider the statement,'either $-2 \leq x \leq -1$ or $1 \leq x \leq 2$'. The negation of this statement is

    (a) $x < -2$ or $2 < x$ or $-1 < x < 1$
    (b) $x < -2$ or $2 < x$
    (c) $-1 < x < 1$
    (d) $x < -2$ or $2 < x$ or $-1 < x < 1$

19. A relation R in $\{1, 2, 3, 4, 5, 6\}$ is given by $\{(1, 2), (2, 3), (3, 4), (4, 4), (4, 5)\}$. This relation is

    (a) reflexive
    (b) symmetric
    (c) transitive
    (d) non of the above

20. How many functions are there from a set with m elements to a set with n elements?

    (a) $n!m!$
    (b) n^m
    (c) m^n
    (d) $n \times (n-1) \times (n-2) \times ..... \times (n-m+1)$

21. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

    (a) $5,300,200$
    (b) $5,200,300$
    (c) $4,300,200$
    (d) $4,200,300$

22. How many different license plates are available, if each plate contains a sequence of 3 letters followed by 3 digits?

(a) $26C_3 \times 10C_3$
(b) $26^3 \times 10^3$
(c) $26P_3 \times 10P_3$
(d) None of them.

23. Each user on a computer system has a password which is 6 to 8 characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

(a) $26^6 - 26^7 - 26^8$
(b) $10^6 + 10^7 + 10^8 + 26^6 + 26^7 + 26^8$
(c) $36^6 + 36^7 + 36^8$
(d) $36^6 + 36^7 + 36^8 - 26^6 - 26^7 - 26^8$

24. How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

(a) 102 (b) 100 (c) 101 (d) None of them

25. How many bit strings of length eight either start with a bit value 1 or end with bits values 00?

(a) 128 (b) 64 (c) 32 (d) 160

26. A computer company receives 350 applications from computer graduates for a job. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

(a) 220 (b) 367 (c) 124 (d) 316

27. Consider the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 2$. Which of the following is an explicit solution to this recurrence relation, provided the constants A and B?

(a) $a_n = A3^n + B$
(b) $a_n = A(-3)^n + B(-1)^n$
(c) $a_n = A3^n + B(-1)^n$
(d) None of them

28. How many seven digit telephone number can be made up using the digits $0 - 9$, without repetition?

(a) $604,800$
(b) $3,628,800$
(c) $608,400$
(d) None of them

29. Which of the following is not a condition of the binomial distribution?

(a) only 2 possible outcomes
(b) have constant probability of success
(c) must have at least 3 trials
(d) trials must be independent

30. The linear combination of $gcd(24, 138) = 6$ is

(a) $138 \times (-1) + 24 \times 6$
(b) $138 \times (6) + 24 \times (-1)$
(c) $138 \times (3) + 24 \times 2$
(d) $138 \times (6) + 24 \times 6$

31. In a Poisson probability distribution

(a) the mean and variance of the distribution are same
(b) the probability of success is always grater than 5
(c) the number of trials is always less than 5
(d) it always contains a contingency table

32. Each trial in Binomial distribution has

(a) one outcome
(b) two outcome
(c) three outcome
(d) four outcome

33. Which of the following statements is **False**?

(a) $C - (B \cup A) = (C - B) - A$
(b) $A - (C \cup B) = (A - B) - C$
(c) $B - (A \cup C) = (B - C) - A$
(d) $A - (B \cup C) = (B - C) - A$

34. A discrete probability distribution may be represented by

(a) a table
(b) a graph
(c) a mathematical equation
(d) all of these

35. Let $P(A)$ denote the power set of A. If $P(A) \subseteq B$ then

(a) $2^{|A|} \leq |B|$
(b) $2^{|A|} \geq |B|$
(c) $2^{|A|} < |B|$
(d) $2^{|A|} \geq 2^{|B|}$

36. In proving $\sqrt{5}$ as irrational, we begin with assumption $\sqrt{5}$ is rational in which type of proof?

(a) Direct proof
(b) Proof by contradiction
(c) Mathematical induction
(d) Constructive proof

37. Logic gate in which output is zero for inputs in which one input is one and other inputs are zero is classified as

(a) AND gate
(b) NOT gate
(c) OR gate
(d) OUT gate

38. Let $\sum = \{x, y\}$ be an alphabet. The strings of length seven over $\sum$ are listed in dictionary order. What is the first string after $xxxxyxx$ that is a palindrome?

(a) $xxxxyxy$
(b) $xxyxyxx$
(c) $xyxxxyx$
(d) $xxxyxxx$

39. The truth table for$(p \vee q) \vee (p \wedge r)$ is the same as the truth table for

(a) $(p \vee q) \wedge (p \vee r)$
(b) $p \vee q$
(c) $(p \vee q) \wedge r$
(d) $(p \wedge q) \vee p$

40. Let $f : X \to Y$, $g : Y \to Z$ and $h = g \circ f : X \to Z$. Suppose g is one to one and onto. Which of the following is **false**?

(a) If f is one to one then h is one to one and onto.
(b) If f is not onto then h is not onto
(c) If f is onto then h is onto.
(d) If f is one to one then h is one to one.

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Registration Number: _____

Part II[60 marks]

*Answer **all** questions in this part and you are advised to spend 70 minutes answering the questions in Part II. You must answer in the space provided.*

Question 1

1. (a) A rectangular floor measures $299cm \times 221cm$. Use the **Euclidean algorithm** to find the largest square tiles that can be used to cover the floor exactly?

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(b) If two factories c and b manufactures a mowing machine every 299 and 221 days respectively, calculate the number of days it will take for them to manufacture at the same time.

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2. (a) Find the solution of the recurrence relation $a_{n+2} + a_{n+1} - 12a_n = 0, n \geq 0$ satisfying the initial conditions $a_o = 1$ and $a_1 = 1$

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(b) Find the general solution of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

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3. Let $f : \mathbb{N} \to \mathbb{R}$ be defined by

$$f(n) = \frac{n^2 + log_2 n}{n+1}; n \geq 1$$

Prove from the definition that $f(n) = O(n)$.

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4. The following function $M : \mathbb{Z}^+ \to \mathbb{Z}$ was defined by John McCarthy, a pioneer in the theory of computation and in the study of artificial intelligence:

$$G(n) = \begin{cases} n - 10; & \text{if } n > 100 \\ M(M(n+11)); & \text{if } n \leq 100 \end{cases}$$

For all positive integers n, find $M(99)$.

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Question 2

1. (a) Use the Vigenere cipher with encrypting key $SECRET$ to encrypt the message $DO\ NOT\ OPEN\ THE\ ENVELOPE$.

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(b) Decrypt the message $RTOLK\ TOLK$ which is encrypted using the affine transformation $C \cong 3P + 24 \bmod 26$.

2. If two of the most common letters in a long cipher text, encrypted by an Affine transformation $C \cong aP + b$ (mod26) are X and Q respectively then by finding the most likely values for a and b, encrypt the plaintext $ATTACK\ AT\ DAWN$.[Hint: In English the two most frequently occurring alphabets are E and T receptively.]

3. (a) Suppose you have a computer with eight empty slots for interface cards, two parallel ports for printers, and four serial ports for modems, scanners or mice. Suppose you have three interface cards, one printer, one mouse, and one modem. In how many ways you can connect them to your computer?

(b) There are 40 computer programmers for a job. 25 know Java, 28 know oracle and 7 know neither language. Using principle of inclusion-exclusion find how many programmers know both languages.

4. A recruiting firm finds that 30 of the applicants for a certain industrial job have advanced training in computer programming. Applicants are selected at random from the pool and are interviewed sequentially. Find the probability that the first applicant having advanced training is found on the fifth interview.

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Question 3

1. Let $F = \{(1,1),(-1,1),(2,4),(-2,4)\}$.

   (a) Is F a function from $A = \{1,2,3\}$ to $B = \{1,4\}$?

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   (b) Is F a function from $A = \{\pm 1, \pm 2\}$ to $B = \{1,2,3,4\}$?

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2. Show that $\{x : 2x^2 + 5x - 3 = 0\} \subseteq \{x : 2x^2 + 7x + 2 = 3/x\}$.

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3. Prove that $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$.

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4. Prove that for any finite sets A, B and C,

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$$

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5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^3 + 1$$

(a) Show that f is a bijection.

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(b) What is $f^{-1}(x)$.

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Question 4

1. Consider the following propositions:

p: Mathematicians are generous.
q: Spiders hate algebra.
Write the compound propositions symbolized by:

(i) $p \vee \overline{q}$

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(ii) $(\overline{q \wedge p})$

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(iii) $\overline{p} \to q$

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(iv) $\overline{p} \leftrightarrow \overline{q}$.

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2. Construct truth tables for the following compound propositions.

   (i) $\overline{p} \vee q$

   (ii) $\overline{p} \wedge \overline{q}$

   (iii) $\overline{q} \rightarrow p$

   (iv) $\overline{p} \leftrightarrow \overline{q}$.

3. Prove that $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$.

——— *End of Examination* ———